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Acceleration as a circular motion along an imaginary circle

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Abstract

We describe a quantum fluid undergoing constant acceleration a in the grand canonical ensemble, in thermal equilibrium at finite inverse temperature β_T. Writing the action of the density operator $\hat{\rho}$ as a Poincare transformation with imaginary parameters, we derive the Kubo-Martin-Schwinger (KMS) relation characterizing the two-point functions. The KMS relation sets boundary conditions for the Euclidean propagator, identifying points in the $\tau - z$ plane on a circle separated by an angle equal to the thermal acceleration $\alpha = a\beta_T$. When $\alpha/2\pi = p/q$ is a rational number, we find a fractalization of thermodynamics, similar to the case of states under imaginary rotation. **(Based on Ref. [1])**

1 Global thermodynamic equilibrium

- In kinetic theory, a fluid is in thermal equilibrium if $f_{\mathbf{k}} = [e^{\beta_\mu k^\mu - \alpha} + \varepsilon]^{-1}$, with $\varepsilon = \pm 1$ for Fermi-Dirac/Bose-Einstein statistics.
- Imposing the Boltzmann equation, $k^\mu \partial_\mu f_{\mathbf{k}} = C[f] = 0$, the temperature four-vector $\beta^\mu = u^\mu / T(x)$ satisfies the Killing equation, $\partial_\mu \beta_\nu + \partial_\nu \beta_\mu = 0$, while $\alpha = \mu(x)/T(x) = \text{const}$.
- β^μ can be written in terms of the 10 Killing vectors of Minkowski space [2]:

$$\beta^\mu = b^\mu + \varpi^\mu{}_\nu x^\nu, \tag{1}$$

with constant b^μ and $\varpi^{\mu\nu} = -\varpi^{\nu\mu}$, such that $\beta_\mu \beta^\mu = 1/T^2(x) > 0$.
- For **constant acceleration:** $\beta^\mu \partial_\mu = \beta_T[(1 + az)\partial_t + at\partial_z]$, $[\beta_T, a \equiv \text{const}]$
- The density operator corresponding to Eq. (1) reads [3, 4]

$$\hat{\rho} = \exp\left(-b_\mu \widehat{P}^\mu + \frac{1}{2}\varpi_{\mu\nu}\widehat{J}^{\mu\nu}\right) = e^{-\tilde{b}(\varpi)\cdot\widehat{P}} e^{\varpi:\widehat{J}/2}, \tag{2}$$

where for simplicity, we set $\alpha = \mu = 0$ and $\hat{\rho}$ was factorized using

$$\tilde{b}^\mu = \sum_{k=0}^{\infty} \frac{i^k}{(k+1)!}(\varpi^\mu{}_{\nu_1}\varpi^{\nu_1}{}_{\nu_2}\cdots\varpi^{\nu_{k-1}}{}_{\nu_k})b^{\nu_k}. \tag{3}$$

- For **constant acceleration:** $\tilde{b}^\mu = B\delta^\mu_0 + A\delta^\mu_3$, $\varpi^{\mu\nu} = \alpha(-g^{\mu t}g^{\nu z} - g^{\mu z}g^{\nu t})$ and

$$\hat{\rho} = e^{-B\widehat{H} + A\widehat{P}^z} e^{\alpha\widehat{K}^z}, \quad A = \frac{i}{a}(1 - \cos\alpha), \quad B = \frac{\sin\alpha}{a}, \tag{4}$$

with $\alpha = \sqrt{-\alpha^\mu\alpha_\mu} = a\beta_T = \text{const}$ and $\alpha^\mu = a^\mu/T(x)$ the thermal acceleration, while

$$a^\mu\partial_\mu = (u^\nu\partial_\nu u^\mu)\partial_\mu = aT^2(x)\beta_T^2[at\partial_t + (1 + az)\partial_z]. \tag{5}$$

2 Quantum thermal KMS relation

- The quantum field operator $\widehat{\Phi}(x)$ is covariant under Poincaré transformations:

$$e^{i\tilde{b}\cdot\widehat{P}}\widehat{\Phi}(x)e^{-i\tilde{b}\cdot\widehat{P}} = \widehat{\Phi}(x + \tilde{b}), \qquad \widehat{\Lambda}\widehat{\Phi}(x)\widehat{\Lambda}^{-1} = D[\Lambda^{-1}]\widehat{\Phi}(\Lambda x), \tag{6}$$

where $\Lambda = e^{-\frac{i}{2}\varpi:\mathcal{J}}$, $(\mathcal{J}^{\mu\nu})_{\alpha\beta} = i(\delta^\mu_\alpha\delta^\nu_\beta - \delta^\mu_\beta\delta^\nu_\alpha)$ and $D[\Lambda]^{-1} = e^{\frac{i}{2}\varpi:S}$ is the spin part of the inverse Lorentz transformation. [$\mathbf{S} = 0$ for the scalar field; and $\mathbf{S} = \frac{i}{2}\gamma^5\gamma^0\boldsymbol{\gamma}$ for the Dirac field]
- The density operator acts like a Poincaré transformation with imaginary parameters:
- For **constant acceleration:** $\hat{\rho}\widehat{\Phi}(t,\varphi)\hat{\rho}^{-1} = e^{-\alpha S^{0z}}\widehat{\Phi}(\tilde{t},\tilde{z})$, where $[S^{0z} = \frac{i}{2}\gamma^0\gamma^z]$

$$\tilde{t} = \cos(\alpha)t + i\sin(\alpha)z + \frac{i}{a}\sin(\alpha), \qquad \tilde{z} = i\sin(\alpha)t + \cos(\alpha)z - \frac{1}{a}[1 - \cos(\alpha)]. \tag{7}$$

- We consider now the scalar/Dirac Wightman functions:

$$G^+(x,x') = \langle\hat{\Phi}(x)\hat{\Phi}(x')\rangle, \qquad S^+(x,x') = \langle\hat{\Psi}(x)\hat{\overline{\Psi}}(x')\rangle,$$
$$G^-(x,x') = \langle\hat{\Phi}(x')\hat{\Phi}(x)\rangle, \qquad S^-(x,x') = -\langle\hat{\overline{\Psi}}(x')\hat{\Psi}(x)\rangle, \tag{8}$$

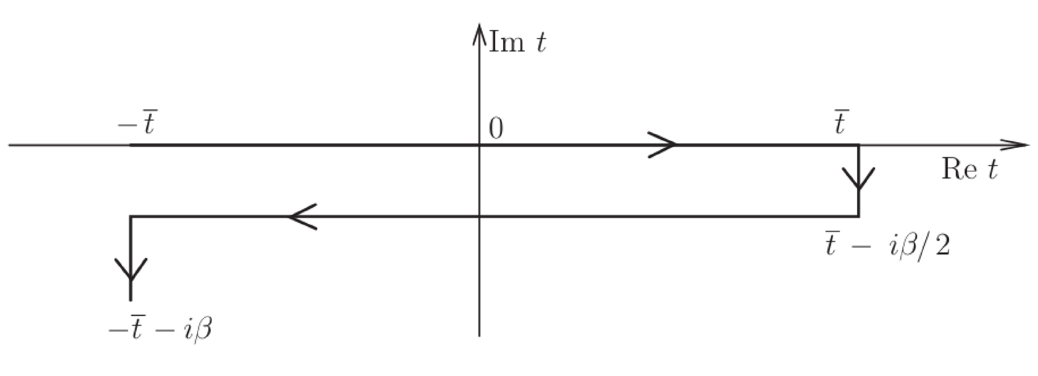
with $\langle\widehat{A}\rangle = Z^{-1}\text{Tr}(\hat{\rho}\widehat{A})$ and $Z = \text{Tr}(\hat{\rho})$. Using $\langle\widehat{A}\rangle = \text{Tr}(\hat{\rho}\widehat{B}\hat{\rho}\widehat{A}\hat{\rho}^{-1})$ leads to the KMS relations:
- For **constant acceleration [1]:**

$$G^+(t,z;x') = G^-(\tilde{t},\tilde{z};x'), \quad S^+(t,\varphi;x') = -e^{-\alpha S^{0z}}S^-(t + i\beta_T, \varphi + i\beta_T\Omega; x'). \tag{9}$$

3 Feynman propagator under constant acceleration

- We now consider the Feynman two-point function,

$$G_F(x,x') = \Theta_C(x,x')G^+(x,x') + \Theta_C(x',x)G^-(x,x'),$$



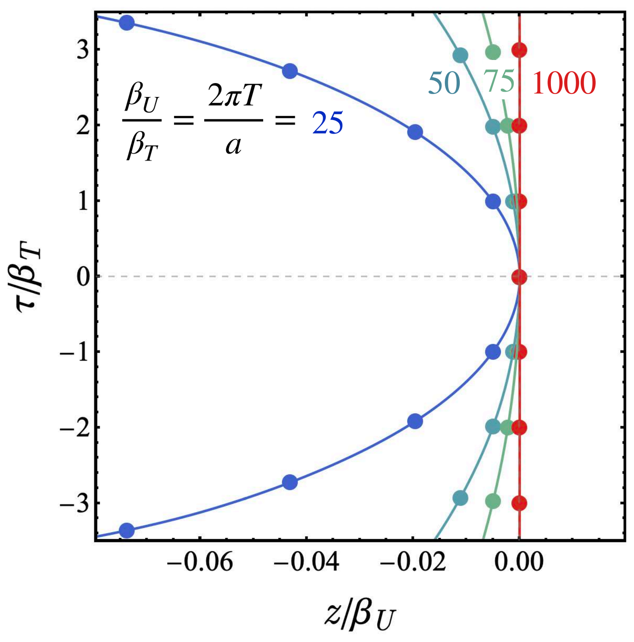
where $\Theta_C(x,x')$ is the causal step function along the thermal contour.

- The KMS relations imply:

$$G_F(\tilde{t},\tilde{z};x') = G_F(t,z;x'), \quad S_F(\tilde{t},\tilde{z};x') = -e^{-\alpha S^{0z}}S_F(t,z;x'). \tag{10}$$

- Keeping these relations with respect to the imaginary time $\tau = it$, a formal solution can be written in terms of the Euclidean vacuum propagators,

$$G_E^{(\alpha)}(\tau,z;X') = \sum_{j=-\infty}^{\infty} G_E^{\text{vac}}(\tau_{(j)}, z_{(j)}; X'),$$
$$S_E^{(\alpha)}(\tau,z;X') = \sum_{j=-\infty}^{\infty} (-1)^j e^{-j\alpha S^{0z}} S_E^{\text{vac}}(\tau_{(j)}, z_{(j)}; X').$$



with $j \in \mathbb{Z}$ and

$$\tau_{(j)} = \tau\cos(j\alpha) - \frac{1}{a}(1 + az)\sin(j\alpha),$$
$$z_{(j)} = \tau\sin(j\alpha) + \frac{1}{a}(1 + az)\cos(j\alpha) - \frac{1}{a}.$$

- Using $G_E^{\text{vac}}(X,X') = 1/(4\pi^2\Delta X^2)$ and $S_E^{\text{vac}}(X,X') = \gamma^E_\mu\partial_\mu G_E^{\text{vac}}$, we find the energy density $\mathcal{E} = u_\mu T^{\mu\nu}u_\nu$ to be (in agreement with Refs. [3, 4])

$$\mathcal{E}^\xi_{\text{KG}} = \frac{3[\alpha T(x)]^4}{16\pi^2}[G_4(\alpha) + 4\xi G_2(\alpha)], \qquad \mathcal{E}_D = \frac{3[\alpha T(x)]^4}{4\pi^2}S_4(\alpha), \tag{11}$$

where $G_n(\alpha) = \sum_{j=1}^\infty [\sin(j\alpha/2)]^{-n}$ and $S_n(\alpha) = -\sum_{j=1}^\infty (-1)^j\cos(j\alpha/2)/[\sin(j\alpha/2)]^n$.

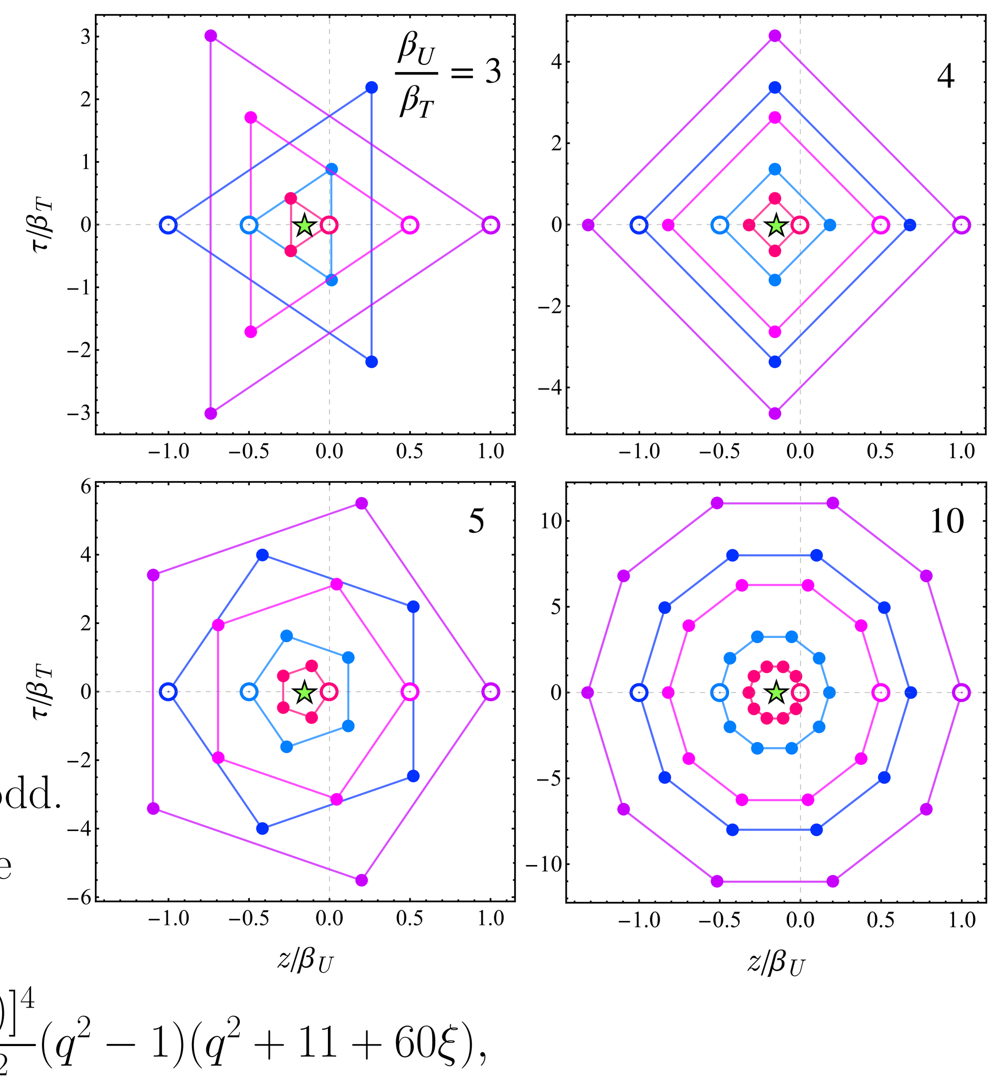
4 Rational acceleration

- In the case when $\alpha/2\pi = p/q$ is a rational number, the contour closes on itself after q iterations:

$$\tau_{(q)} = \tau, \qquad z_{(q)} = z,$$

such that

$$G_E^{(p,q)}(\tau,z) = \sum_{j=0}^{q-1} G_E^{\text{vac}}(\tau_{(j)}, z_{(j)}),$$
$$S_E^{(p,q)}(\tau,z) = \sum_{j=0}^{q-1}(-1)^j e^{-j\alpha S^{0z}} \times S_E^{\text{vac}}(\tau_{(j)}, z_{(j)}).$$



- Since $e^{-q\alpha S^{0z}} = (-1)^p$, $S_E^{(p,q)}(\tau_{(q)}, z_{(q)}) = (-1)^{p+q}S_E^{p,q}(\tau,z)$, such that $S_E^{(p,q)} = 0$ when $p + q$ is odd.
- In this case, both G_n and S_n can be computed in closed form. We find:

$$\mathcal{E}_\xi^{(p,q)} = \frac{[\alpha T(x)]^4}{480\pi^2}(q^2 - 1)(q^2 + 11 + 60\xi),$$
$$\mathcal{E}_D^{(p,q)} = \frac{[\alpha T(x)]^4}{960\pi^2}(q^2 - 1)(7q^2 + 17)\frac{1 + (-1)^{p+q}}{2}. \tag{12}$$

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