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Dirac fermions under imaginary rotation

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Abstract

Recent years have seen an increase in the interest to investigate the thermodynamic properties of strongly-interacting systems under rotation. Such studies are usually performed using lattice gauge techniques on the Euclidean manifold and with an imaginary angular velocity, $\Omega = i\Omega_I$. When $\nu = \beta\Omega_I/2\pi$ is a rational number, the thermodynamics of free scalar fields "fractalizes" in the large volume limit, that is, it depends only on the denominator q of the irreducible fraction $\nu = p/q$ [1].

The present study considers the same problem for free, massless, fermions at finite temperature $T = \beta^{-1}$ and chemical potential μ and confirms that the thermodynamics fractalizes when $\mu = 0$. Curiously, fractalization has no effect on the chemical potential μ, which dominates the thermodynamics when q is large. The fractal behavior is shown analytically for the fermionic condensate (this poster), the charge currents and the energy-momentum tensor [3]. For these observables, the limits on the rotation axis are validated by comparison to the results obtained in [2] for the case of real rotation. Enclosing the system in a fictitious cylinder of radius R and length L_z allows constructing averaged thermodynamic quantities that satisfy the Euler relation and fractalize (see [3]).

Thermal states in the macrocanonical ensemble

Rotating massless fermions with vector chemical potential

- Rotation frequency $\Omega \mapsto i\Omega_I$ and chemical potential μ
- Free, massless fermions, with charge $\widehat{Q} = \int d^3x\, \widehat{\Psi}^\dagger \widehat{\Psi}$ (conserved)
- Quantized field $\widehat{\Psi}(x) = \sum_j \left[U_j(x)\hat{b}_j + V_j(x)\hat{d}_j^\dagger\right]$
- The modes U_j and V_j are chosen to be eigenfunctions of the Hamiltonian $H = i\partial_t = -i\gamma^0\boldsymbol{\gamma}\cdot\boldsymbol{\nabla} + M\gamma^0$, vertical momentum $P^z = -i\partial_z$, angular momentum along the z axis $J^z = -i\partial_\varphi + S^z$, and helicity operator $h = \mathbf{S}\cdot\mathbf{P}/P$, where the spin matrix $\boldsymbol{S}$ is given by $\boldsymbol{S} = \frac{1}{2}\gamma^5\gamma^0\boldsymbol{\gamma} = \frac{1}{2}\begin{pmatrix}\boldsymbol{\sigma} & 0 \\ 0 & \boldsymbol{\sigma}\end{pmatrix}$, as to diagonalize the density operator.
- Density operator describind rigidly-rotating thermal states
$$\hat{\rho} = \exp\left\{-\beta\left(:\widehat{H}: -\mu :\widehat{Q}: -\Omega :\widehat{J}^z:\right)\right\},$$
- Partition function $\mathcal{Z} = \mathrm{Tr}(\hat{\rho})$.
- Thermal expectation values $\langle\widehat{A}\rangle = \mathcal{Z}^{-1}\,\mathrm{Tr}(\hat{\rho}\widehat{A})$

Definitions

- Thermal expectation value energy
$$\mathcal{E}_j^\sigma \equiv E_j - \sigma\mu, \quad \sigma = \pm 1.$$
- Rotating frame effective energy
$$\tilde{\mathcal{E}}_j^\sigma \equiv \mathcal{E}_j^\sigma - \Omega m_j = \tilde{E}_j - \sigma\mu.$$
- Rotating frame (comoving) energy $\tilde{E}_j \equiv E_j - \Omega m_j$

Imaginary thermal expectation values

- Thermal expectation values become for operator $\widehat{A}$
$$A_\beta^\Omega \equiv \langle :\widehat{A}: \rangle_\beta^\Omega = \sum_{j,\sigma} \mathcal{C}_\sigma(\mathcal{A}) \frac{\mathcal{A}(U_j, U_j)}{e^{\beta\tilde{\mathcal{E}}_j^\sigma} + 1},$$
- Thermal factor becomes (expansion holds for any $\mathcal{E}_j^\sigma$)
$$\frac{1}{\exp(\tilde{\mathcal{E}}_j^\sigma) + 1} \equiv \frac{1}{\exp[\beta\left(\mathcal{E}_j^\sigma - i\Omega_I m_j\right)] + 1}$$
$$= \theta(\mathcal{E}_j^\sigma)\sum_{v=1}^\infty (-1)^{v+1} e^{-v\beta\mathcal{E}_j^\sigma} e^{iv\beta\Omega_I m_j} + \theta(-\mathcal{E}_j^\sigma)\sum_{v=0}^\infty (-1)^v e^{v\beta\mathcal{E}_j^\sigma} e^{-iv\beta\Omega_I m_j}$$
- Thermal expectation values split as
$$A_\beta^{i\Omega_I} \equiv \langle :\widehat{A}: \rangle_\beta^{i\Omega_I} \equiv A_{v=0}^{i\Omega_I} + \Delta A^{i\Omega_I}$$
- Here we will focus on the fermion condensate $\bar{\Psi}\Psi$. A more extensive analysis, where other observables (like the currents and the energy momentum tensor) are considered can be found in [3].

References

[1] V. E. Ambruş, M. N. Chernodub, Phys. Rev. D **108** (2023), 085016.

[2] V. E. Ambruș, JHEP **08** (2020), 016.

[3] T. Pătuleanu, D. Fodor, V. E. Ambruș, C. Crucean, Work in progress

[4] M. N. Chernodub, arXiv:2210.05651 [quant-ph].

Fermion condensate fractalization

- The imaginary frequency thermal expectation value for the fermion condensate:
$$\frac{FC^{i\Omega_I}}{M} = \frac{\mu^2}{2\pi^2} + \frac{2}{\pi^2}\sum_{v=1}^\infty \frac{1}{(v\beta)^2}(-1)^{v+1}\frac{c_v}{1+\alpha_v^2}\cos\left(v\beta\mu\alpha_v\right),$$
where $s_v = \sin\left(\frac{v\beta\Omega_I}{2}\right)$, $c_v = \cos\left(\frac{v\beta\Omega_I}{2}\right)$ and $\alpha_v = \frac{2\rho}{v\beta}s_v$,
- On the rotation axis
$$\left.\frac{FC^{i\Omega_I}}{M}\right|_{\rho\to 0} = \frac{\mu^2}{2\pi^2} + \frac{1}{6\beta^2} - \frac{\Omega_I^2}{8\pi^2} = \left[\frac{\mu^2}{2\pi^2} + \frac{1}{6\beta^2} + \frac{\Omega^2}{8\pi^2}\right]_{\Omega\to i\Omega_I},$$
which agrees with the results obtained for real rotation [2].
- Consider $\nu = \beta\Omega_I/2\pi = p/q$ rational frequencies (as irreducible fractions)
- Take $v = r + qQ$, denote $l = 2\pi\rho/\beta$ and $x_r = l s_r/\pi q$
- Split $\sum_{v=1}^\infty \equiv \sum_{r+qQ=1}^\infty \equiv \sum_{Q=0}^\infty \sum_{r=1}^q$
- Fractalized value of the fermion condensate:
$$\frac{FC^{i\Omega_I}}{M} = \frac{\mu^2}{2\pi^2} + \frac{2}{\pi^2\beta^2q^2}\sum_{r=1}^q (-1)^{r+1}c_r\cos\left(\beta\mu q x_r\right)\sum_{Q=0}^\infty \frac{(-1)^{kQ}}{\left(Q+\frac{r}{q}\right)^2 + x_r^2}$$
- When $r = q$, $c_r = (-1)^{pQ}$ and $s_r = x_r = 0 \Rightarrow$ position-independent term.
- The terms with $1 \le r < q$ vanish when $l \to \infty$.

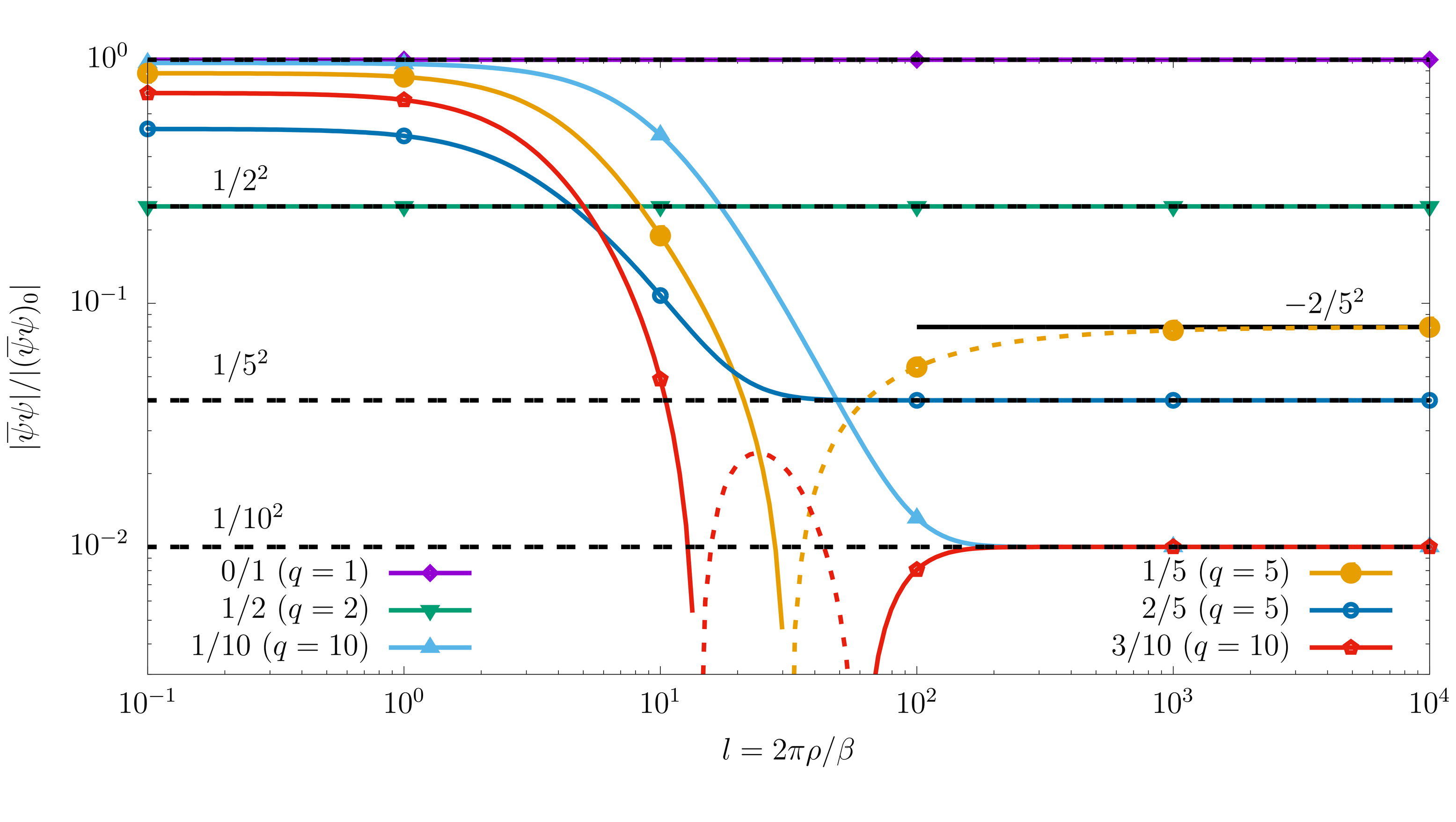
$k = p + q$ **odd case**

$$\left.\frac{FC^{i\Omega_I}}{M}\right|_{k=\mathrm{odd}} = \frac{\mu^2}{2\pi^2} + \frac{1}{6\beta^2q^2} - \sum_{r=1}^{q-1}\frac{(-1)^{r+1}}{2\pi^2\beta^2q^2}\frac{c_r}{x_r}\cos\left(\beta\mu q x_r\right)\mathrm{Im}\left[\Delta\psi_r\right],$$
where $\Delta\psi_r = \psi\left(\frac{\frac{r}{q}+ix_r+1}{2}\right) - \psi\left(\frac{\frac{r}{q}+ix_r}{2}\right)$ with ψ the digamma function.

$k = p + q$ **even case**

$$\left.\frac{FC^{i\Omega_I}}{M}\right|_{k=\mathrm{even}} = \frac{\mu^2}{2\pi^2} - \frac{1}{3\beta^2q^2} + \sum_{r=1}^{q-1}\frac{(-1)^{r+1}}{\pi^2\beta^2q^2}\frac{c_r}{x_r}\cos\left(\beta\mu q x_r\right)\mathrm{Im}\,\psi_r,$$
where $\psi_r = \psi(r/q + ix_r)$ with ψ the digamma function.

- The chemical potential contribution to the asymptotic term is independent of $\nu = \beta\Omega_I/2\pi$!
- The temperature dependence of the asymptotic term reveals the fractalized effective temperature $T_q = T/q$ [4]!



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