Shear viscosity of rotating, hot, and dense spin-half fermionic systems using Kubo formalism

Sarthak Satapathy[1], **Rajeev Singh**[2*], Pushpa Panday[1], Salman Ahamad Khan[1], Debarshi Dey[1]

[1]Department of Physics, Indian Institute of Technology Roorkee, Roorkee - 247667, India

[2]Center for Nuclear Theory, Department of Physics & Astronomy, Stony Brook University, 11794-3800, USA

[*]rajeevofficial24@gmail.com

In this study, we calculate the shear viscosity for rotating fermions with spin-half under conditions of high temperature and density. We employ the Kubo formalism, rooted in finite-temperature quantum field theory, to compute the field correlation functions essential for this evaluation. The one-loop diagram pertinent to shear viscosity is analyzed within the context of curved space, utilizing tetrad formalism as an effective approach in cylindrical coordinates. Our findings focus on high angular velocities, ranging from 0.1 to 1 GeV, which align with experimental expectations. Furthermore, we explore the inter-relationship between the chemical potential and angular velocity in this study.

1 Motivation

The motivation is to study how finite rotation impacts transport coefficients, especially shear viscosity in a fermion-dominated medium subject to high angular velocity. Employing the Kubo formalism, we incorporate the influence of rotation in our calculations by using the momentum space propagator for fermions, derived via the Fock-Schwinger method. The role of angular velocity is notably apparent in two areas:

- it modifies the distribution function through the summation of Matsubara frequencies, and
- it is present in the numerator of the shear viscosity expression via trace evaluations.

2 Ingredients

The metric tensor in a rotating setting at an angular velocity Ω is

$$g_{\mu\nu} = \begin{pmatrix} 1-\left(x^2+y^2\right)\Omega^2 & y\Omega & -x\Omega & 0 \\ y\Omega & -1 & 0 & 0 \\ -x\Omega & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}. \tag{1}$$

And, in curved spacetime the energy-momentum tensor is expressed as

$$T^{\mu\nu} = \frac{i}{4}\left(\overline{\psi}\widetilde{\gamma}^{\mu}\widetilde{D}^{\nu}\psi + \overline{\psi}\widetilde{\gamma}^{\nu}\widetilde{D}^{\mu}\psi\right) + \text{h.c}, \tag{2}$$

where ψ and $\overline{\psi}$ are the Dirac field operator and its conjugate, respectively, and h.c denotes the Hermitian conjugate. In a uniformly rotating frame, $\widetilde{D}^{\mu}$ is

$$\widetilde{D}_{\mu} = \left(\partial_t - i\frac{\Omega\Sigma_3}{2}, -\partial_x, -\partial_y, -\partial_z\right), \tag{3}$$

where $\Sigma_3 = (i/2)\left[\gamma^1,\gamma^2\right]$. The spacetime dependent gamma matrices $\widetilde{\gamma}^{\mu}$ in tetrad system are

$$\begin{aligned} &\widetilde{\gamma}^0 = \gamma^0, \quad \widetilde{\gamma}_0 = \gamma^0 - x\Omega\gamma^2 + y\Omega\gamma^1, \quad \widetilde{\gamma}^1 = \gamma^1 + y\Omega\gamma^0, \quad \widetilde{\gamma}_1 = -\gamma^1, \\ &\widetilde{\gamma}^2 = \gamma^2 - x\Omega\gamma^0, \quad \widetilde{\gamma}_2 = -\gamma^2, \quad \widetilde{\gamma}^3 = \gamma^3, \quad \widetilde{\gamma}_3 = -\gamma^3, \end{aligned} \tag{4}$$

where γ^{μ} are the gamma matrices in Minkowski space. The final piece is the Kubo formula for shear viscosity

$$\boxed{\eta = -\frac{1}{10}\lim_{q_0\to 0}\frac{\rho_\eta(q_0)}{q_0}}, \tag{5}$$

where $\rho_\eta(q)$ is the spectral function of shear viscosity calculated from the two-point correlation function of the shear stress tensor, $\pi^{\mu\nu} = \Delta^{\mu\nu}{}_{\alpha\beta}T^{\alpha\beta}$, and q_0 is the temporal component of the four-momentum.

3 Shear viscosity expression

Formula for calculating the shear viscosity in a rotating, hot, and dense fermionic system is

$$\begin{aligned} \eta(T,\mu,\Omega) = \frac{1}{240}\int\frac{d^3p}{(2\pi)^3}\left(\frac{1}{4E_p^2\Gamma T}\right)\times\Bigg\{&\Big\{160p_z^4 + 4p_\perp^2\Omega^2 + \left(\Omega - E_p - 3p_z\right)\Omega^3 + 8p_\perp^2 p_z\left(\Omega - 4E_p\right) + 32p_z^3\left(\Omega + 5E_p\right) - 8p_z^2\Omega\left(2E_p + 3\Omega\right)\Big\}\mathcal{N}_{\pm\mu,\mp\Omega/2} \\ &+ \Big\{160p_z^4 + 4p_\perp^2\Omega^2 + \left(\Omega - E_p - 3p_z\right)\Omega^3 + 8p_\perp^2 p_z\left(\Omega - 4E_p\right) + 32p_z^3\left(\Omega - 5E_p\right) - 8p_z^2\Omega\left(2E_p - 3\Omega\right)\Big\}\mathcal{N}_{\pm\mu,\pm\Omega/2}\Bigg\}, \end{aligned} \tag{6}$$

where $\mathcal{N}_{\pm\mu,\pm\Omega/2} = \sum_{s=\pm1} n_F(E_p + s\mu + s\Omega/2)\left\{1 - n_F(E_p + s\mu + s\Omega/2)\right\}$, $\mathcal{N}_{\pm\mu,\mp\Omega/2} = \sum_{\lambda=\pm1} n_F(E_p + \lambda\mu - \lambda\Omega/2)\left\{1 - n_F(E_p + \lambda\mu - \lambda\Omega/2)\right\}$.

This formula is particularly relevant for a hadronic medium consisting of spin-$1/2$ particles. However, it's crucial to maintain the magnitude of Ω at or above 0.1 GeV to apply this result effectively. The influence of Ω is incorporated both in the Fermi-Dirac distribution function and the numerator of the expression. This indicates that Ω functions similarly to an effective chemical potential within the medium.

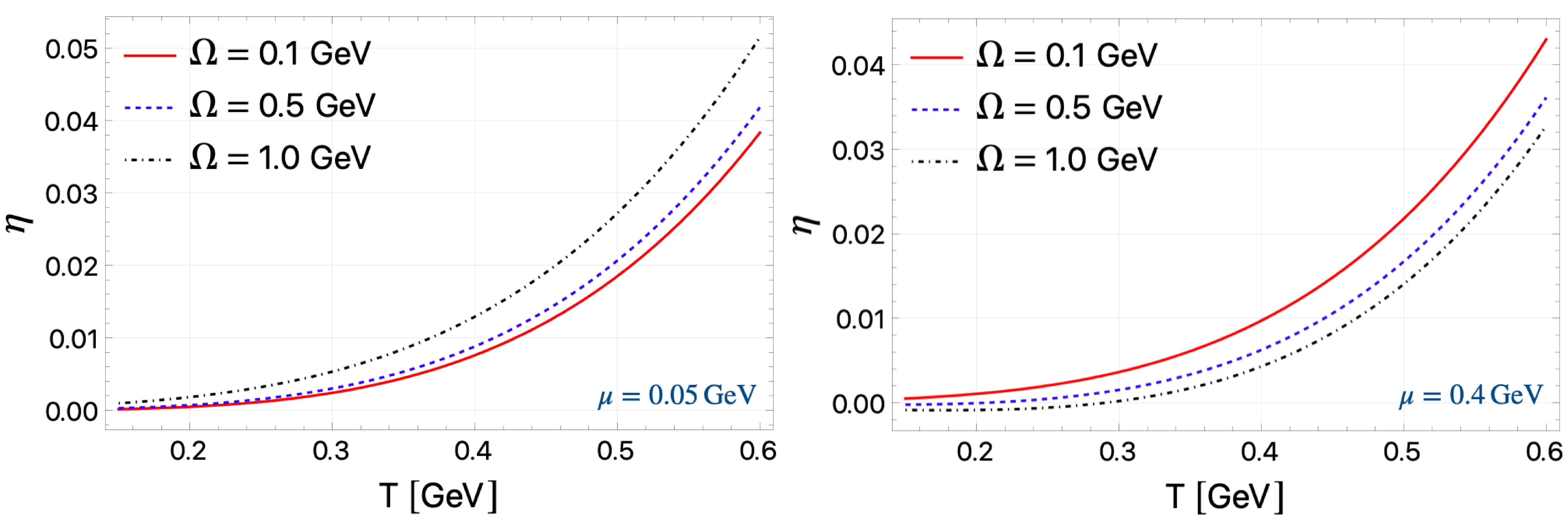

Figura 1: Behaviour of η with T for Ω = 0.1, 0.5, 1.0 GeV at μ = 0.05 GeV and 0.4 GeV.

References

S. Satapathy, R. Singh, P. Panday, S. A. Khan and D. Dey, [arXiv:2309.05284 [hep-ph]].