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Helicity conservation in electron-positron production in de Sitter space-time

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Abstract

Fermion production in an external Coulomb field on de Sitter expanding universe is studied. The amplitude and probability of pair production in an external Coulomb field are computed and the cases of large/small values of the expansion factor comparatively with the particle mass are studied. We obtain from our calculations that the modulus of the momentum is no longer a conserved quantity. We find that in the de Sitter space there are probabilities for production processes where the helicity is no longer conserved.

1 Amplitude and probability

The line element for the de Sitter universe is:

$$ds^2 = dt^2 - e^{2\omega t} d\vec{x}^{\,2} = \frac{1}{(\omega t_c)^2}(dt_c^2 - d\vec{x}^{\,2}) \tag{1}$$

where ω is the expansion factor and $\omega > 0$, while the conformal time $t_c = -\frac{1}{\omega} e^{-\omega t}$. To define half-integer spin fields on curved spacetime it is required to use the tetrad fields [2] $e_{\widehat{\mu}}(x)$ and $\widehat{e}^{\widehat{\mu}}(x)$, which fix the local frames and corresponding coframes and which are labeled by the local indices $\widehat{\mu}, \widehat{\nu}, ... = 0, 1, 2, 3$. For the line element (1), we can choose the Cartesian gauge with the nonvanishing tetrad components,

$$e^0_{\widehat{0}} = e^{-\omega t}; \;\; e^i_{\widehat{j}} = \delta^i_{\widehat{j}}\, e^{-\omega t}. \tag{2}$$

The positive/negative frequency modes of momentum $\vec{p}$ and helicity λ, derived in [1], are:

$$U_{\vec{p},\lambda}(t, \vec{x}\,) = \frac{\sqrt{\pi p/\omega}}{(2\pi)^{3/2}} \begin{pmatrix} \frac{1}{2} e^{\pi k/2} H^{(1)}_{\nu_-}(\frac{p}{\omega} e^{-\omega t}) \xi_\lambda(\vec{p}\,) \\ \lambda e^{-\pi k/2} H^{(1)}_{\nu_+}(\frac{p}{\omega} e^{-\omega t}) \xi_\lambda(\vec{p}\,) \end{pmatrix} e^{i\vec{p}\cdot\vec{x} - 2\omega t}; \tag{3}$$

where $H^{(1)}_\nu(z), H^{(2)}_\nu(z)$ are Hankel functions of the first and second kinds and $k = \frac{m}{\omega}, \nu_\pm = \frac{1}{2} \pm ik$, while the negative frequency solution is obtained by charge conjugation .

The Coulomb field in Minkowski space $A^0 = \frac{Ze}{|\vec{x}|}$. Then one finds for the corresponding de Sitter potential by using $A^\mu = \Omega^{-1} A^\mu_M$ [2, 4]:

$$A^{\widehat{0}}(x) = \frac{Ze}{|\vec{x}|} e^{-\omega t}, A^{\widehat{j}}(x) = 0, \tag{4}$$

where the hatted indices indicate that we refer to the components in the local Minkowski frames $A^{\widehat{\mu}} = e^{\widehat{\mu}}_\nu A^\nu$.

The amplitude contains the temporal part of the vector potential [2, 3],

$$\mathcal{A}_{e^-e^+} = -ie \int d^4x \, [-g(x)]^{1/2}\, \bar{U}_{\vec{p},\lambda}(x) \gamma_\mu A^{\widehat{\mu}}(x) V_{\vec{p}\,',\lambda'}(x) \tag{5}$$

Related to the above QED perturbative approach to the problem of particle production in de Sitter space, we must point out that we work in the chart with conformal time $t_c \in (-\infty, 0)$, which covers only half of the whole de Sitter manifold.

The final form of the transition amplitude for fermion pair production in the Coulomb field in de Sitter space is [2]:

$$\begin{aligned} \mathcal{A}_{e^-e^+} = i\frac{e^2 Z}{16\pi|\vec{p} + \vec{p}\,'|^2} \Bigg[&-sgn(\lambda') \left(p^{-1}\theta(p - p') f_{-k}\left(\frac{p'}{p}\right) + p'^{-1}\theta(p' - p) f_k\left(\frac{p}{p'}\right)\right) \\ &+ sgn(\lambda) \left(p^{-1}\theta(p - p') f_k\left(\frac{p'}{p}\right) + p'^{-1}\theta(p' - p) f_{-k}\left(\frac{p}{p'}\right)\right)\Bigg] \xi^+_\lambda(\vec{p}\,)\eta_{\lambda'}(\vec{p}\,'). \end{aligned} \tag{6}$$

introduced functions $f_k\left(\frac{p}{p'}\right)$, are defined as [2]:

$$\begin{aligned} f_k\left(\frac{p}{p'}\right) = {} & ie^{-\pi k} \frac{\left(\frac{p}{p'}\right)^{1+ik}}{ch^2(\pi k)} \frac{{}_2F_1\left(\frac{3}{2}, 1 + ik; \frac{3}{2} + ik; \left(\frac{p}{p'}\right)^2\right)}{B\left(-\frac{1}{2}, \frac{3}{2} + ik\right)} \\ & + \frac{\left(\frac{p}{p'}\right)^{1+ik}}{ch^2(\pi k)} \frac{{}_2F_1\left(\frac{3}{2}, 1 + ik; \frac{3}{2} + ik; \left(\frac{p}{p'}\right)^2\right)}{B\left(-ik, \frac{3}{2} + ik\right)} - \frac{\left(\frac{p}{p'}\right)^{-ik}}{ch^2(\pi k)} \frac{{}_2F_1\left(\frac{1}{2}, 1 - ik; \frac{1}{2} - ik; \left(\frac{p}{p'}\right)^2\right)}{B\left(ik, \frac{1}{2} - ik\right)} \\ & + ie^{\pi k} \frac{\left(\frac{p}{p'}\right)^{-ik}}{ch^2(\pi k)} \frac{{}_2F_1\left(\frac{1}{2}, 1 - ik; \frac{1}{2} - ik; \left(\frac{p}{p'}\right)^2\right)}{B\left(\frac{1}{2}, \frac{1}{2} - ik\right)}, \end{aligned} \tag{7}$$

From the final result given in (6) and (7), it is obvious that the transition amplitude is nonvanishing only for $p \neq p'$ and from here we conclude that the law of conservation for the modulus of the momentum is lost for the process of pair production in Coulomb field in de Sitter space.

The transition probability, which is obtained summing the square modulus of the amplitude after the final helicities:

$$\mathcal{P}_{e^-e^+} = \frac{1}{2} \sum_{\lambda\lambda'} |\mathcal{A}_{e^-e^+}|^2. \tag{8}$$

2 Helicity conservation

Our numerical and graphical analysis prove that the probability for pair production with opposite helicities $\lambda = -\lambda'$ (helicity conservation) is sensibly bigger than the probability for production of pair with equal helicities $\lambda = \lambda'$ (helicity non-conserving case) in the case $\omega > m$. For $m = 0$, the probability of pair production in the helicity non-conserving case is zero, while the probability for a conserving helicity process is finite . Summarizing our result for $\omega > m$, if the two momenta are close as moduli $p' \sim p$ but not equal, the production processes which preserve the helicity conservation law will be favored.

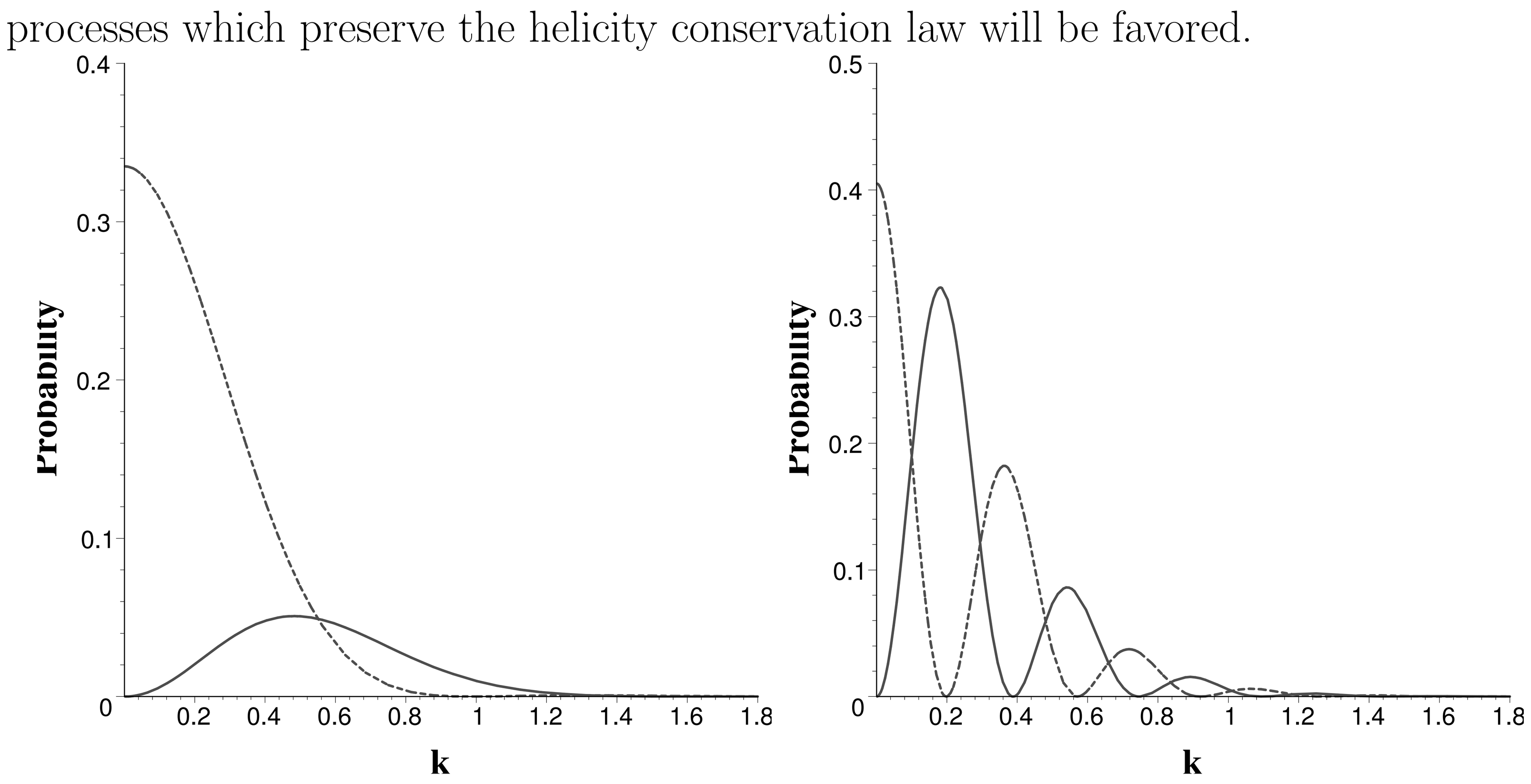

Fig. 1. Probability as a function of parameter k, for $p/p' = 0.1$ in the left figure and $p/p' = 0.0001$ in the right figure. The point line represents the case of helicity conservation and the solid line the case when helicity is not conserved.

Let us study the square modulus of the amplitude for the case $p < p'$. In an orthogonal local frame $\{\vec{e}_i\}$ we take the electron and positron momenta in the plane $(1, 2)$, denoting their spherical coordinates as $\vec{p} = (p, \alpha, \beta)$ and $\vec{p}\,' = (p', \gamma, \varphi)$, where $\alpha, \gamma \in (0, \pi)$; $\beta, \varphi \in (0, 2\pi)$. Then for $\beta = \pi$, $\varphi = 0$ and $p < p'$ the final result for the square amplitude is [2]:

$$|\mathcal{A}_{e^-e^+}|^2 \propto \begin{cases} \cos^2\left(\frac{\alpha+\gamma}{2}\right) & \text{for } \lambda = -\lambda' \\ \sin^2\left(\frac{\alpha+\gamma}{2}\right) & \text{for } \lambda = \lambda' \end{cases} \tag{9}$$

From this formula, if we set $\alpha = \pi$, $\gamma = 0$, corresponding to the case when the momenta of the electron and positron are parallel but move in opposite directions, we obtain zero probability for a helicity conservation process, while the probability for a process where helicity is not conserved, becomes maximum. All the particular cases where the momenta are oriented on different directions can now be obtained from (9). We can conclude that for $k \ll 1$, the helicity non-conserving processes in which the electrons and positrons have parallel momenta and move in the opposite directions, will be favored. This result shows that only in the helicity non-conserving processes the chances of separation between electrons and positrons in the early universe become important. Thus it seems that for $\alpha = 0$, $\gamma = 0$ the situation is opposite, conserving processes being dominant, with the specification that this case corresponds to the pair having parallel momenta and moving in the same direction, which increases the probability of annihilation of the pair.

References

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