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Charge transport in strongly magnetized relativistic matter

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July 25, 2024

[Ghosh, Shovkovy, Phys. Rev. D 109, 096018 (2024)]
[Ghosh, Shovkovy, arXiv:2404.01388]
[Ghosh, Shovkovy, arXiv:2407.13828]

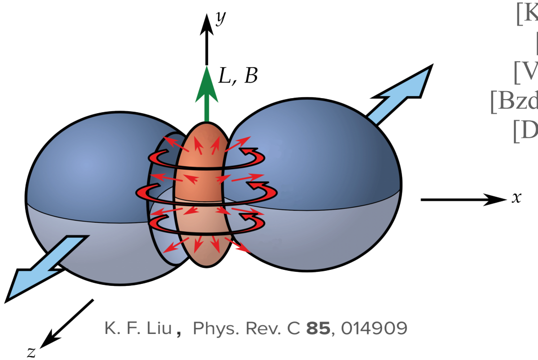
The 8th International Conference on Chirality, Vorticity and Magnetic Field in Quantum Matter

July 22 – 26, 2024, West University of Timisoara, Romania

Magnetized QGP @ RHICs

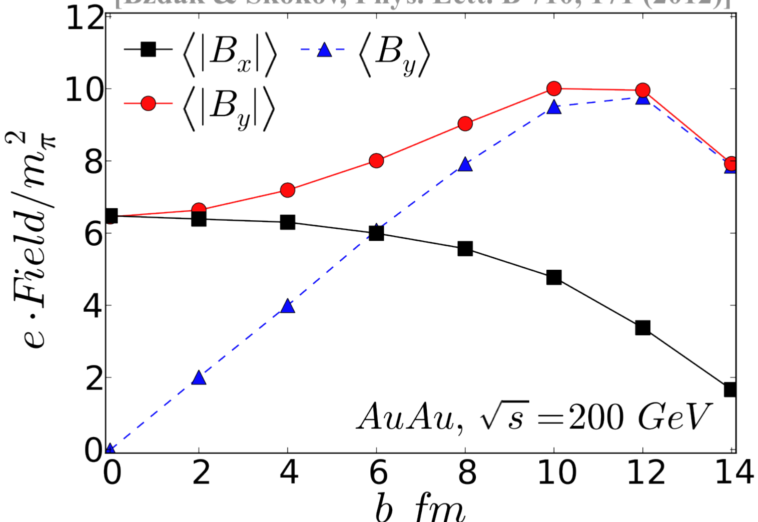
- Magnetic field in RHICs:
  - is strong in magnitude $|eB| \sim m_\pi^2$
  - is highly sensitive to the impact parameter (b)
  - fluctuates from event to event
  - is short lived, $t \sim R_{\text{Au}}/c$

[Rafelski & Müller, PRL, 36, 517 (1976)]
[Kharzeev et al., arXiv:0711.0950]
[Skokov et al., arXiv:0907.1396]
[Voronyuk et al., arXiv:1103.4239]
[Bzdak &. Skokov, arXiv:1111.1949]
[Deng & Huang, arXiv:1201.5108]
…

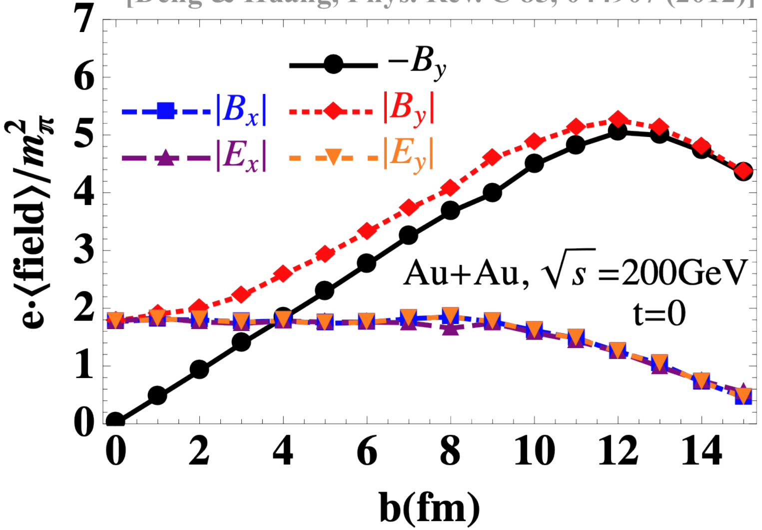


K. F. Liu , Phys. Rev. C **85**, 014909

[Bzdak & Skokov, Phys. Lett. B 710, 171 (2012)]

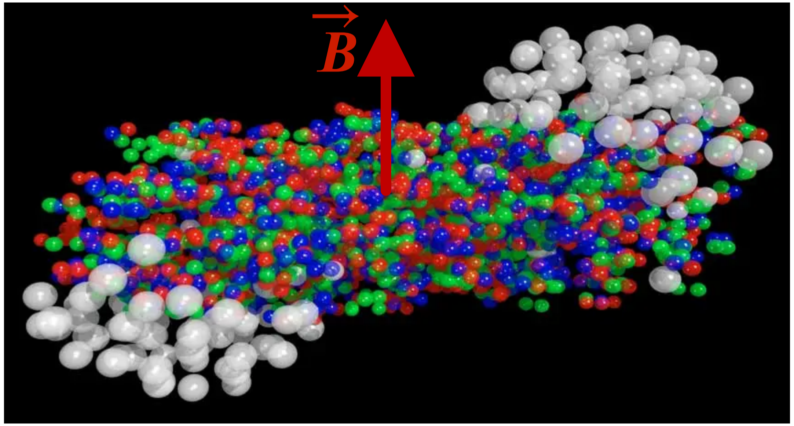


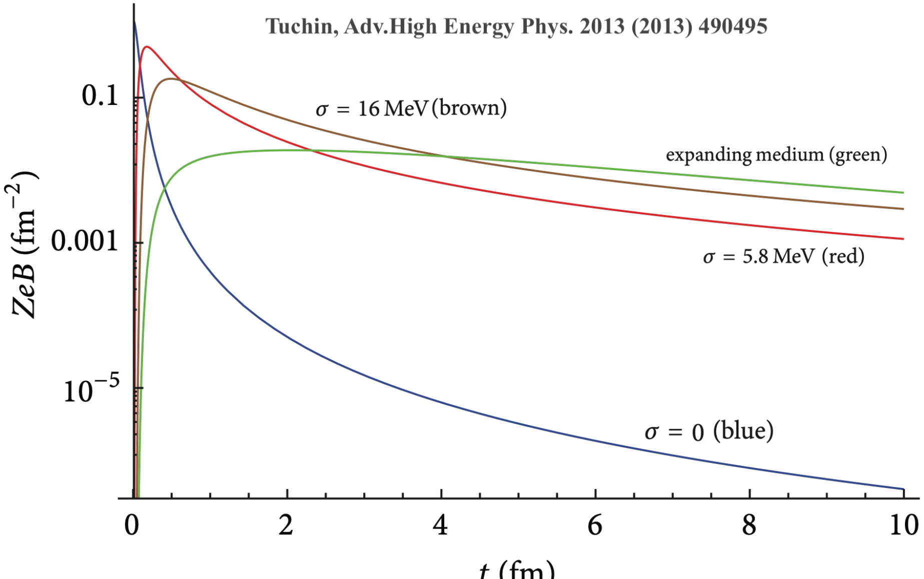
[Deng & Huang, Phys. Rev. C 85, 044907 (2012)]

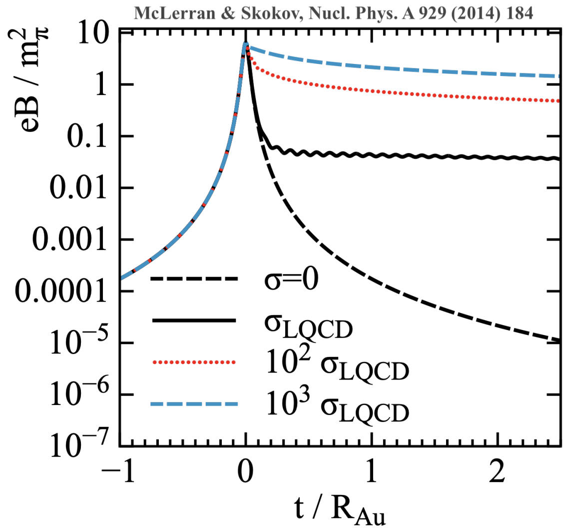


Conductivity

- How **strongly magnetized** is QGP in heavy-ion collisions?
- *Trapping* and *survival* of magnetic field depends on the **conductivity** of QGP
- **Conductivity** is **unknown** @ $B \neq 0$







Conductivity at $B \neq 0$

unreliable

– Phenomenological models (holographic models, NJL, etc.)

[Mamo, JHEP **08**, 083 (2013)]
[Fukushima & Okutsu, Phys. Rev. D **105**, 054016 (2022)]
[Kurian & Chandra, Phys. Rev. D **96**, 114026 (2017)]
[Das, Mishra, Mohapatra, Phys. Rev. D **101**, 034027 (2020)]
[Satapathy, Ghosh, Ghosh, Phys. Rev. D **104**, 056030 (2021)]
[Bandyopadhyay et al. EPJC **83**, 489 (2023)]
…

– Few analytical attempts within a gauge theory (*LLL approximation* or *an effective "longitudinal" kinetic theory*)

[Hattori & Satow, PRD **94**, 114032 (2016)]
[Hattori, Satow, Yee, Phys. Rev. D **95**, 076008 (2017)]
[Fukushima & Hidaka, Phys. Rev. Lett. **120**, 162301 (2018)]
[Fukushima & Hidaka, JHEP **04**, 162 (2020)]

– Lattice calculations

[Buividovich et al. Phys. Rev. Lett. **105**, 132001 (2010)]
[Astrakhantsev et al. Phys. Rev. D **102**, 054516 (2020)]
[Almirante et al. arXiv:2406.18504]

Conductivity of QGP

- Conductivity at $B=0$ (analytical, QED): $\sigma \simeq 15.7\, T/[e^2 \ln\left(2.5T/m_D\right)]$

  [Arnold, Moore, Yaffe, JHEP 05 (2003) 051]

- Conductivity of QGP at $B=0$ (lattice):

$$\sigma \simeq \left(5.8 \pm 2.9\right)\frac{T}{T_c}\ \text{MeV}$$

[Ding, et al. PRD83, 034504 (2011)]

$\sigma \simeq$ 1.1 MeV @ T=200 MeV to 5.6 MeV @ T=350 MeV

[Aarts et al. JHEP 1502, 186 (2015)]

- Conductivity at $B \neq 0$ (lattice):

$$\frac{\Delta\sigma_{\parallel}}{TC_{\text{em}}} = \frac{3\,|eB|}{4\pi^2 T^2} C_\tau(|eB|)$$

where $C_\tau\left(4\ \text{GeV}^2\right) \approx 0.134$ and $C_\tau\left(9\ \text{GeV}^2\right) \approx 0.142$

[Almirante et al. arXiv:2406.18504]

Preview of main results

- Use quantum field theory and Kubo's formula for the conductivity tensor

[Ghosh, Shovkovy, arXiv:2404.01388] & [Ghosh, Shovkovy, arXiv:2407.13828]

- Fermion damping rate $\Gamma_n(p_z)$ is obtained from first-principles in a **gauge theory** (exact amplitudes & full kinematics) in Landau-level representation

- $\Gamma_n(p_z) \sim \alpha|eB|/T$ is determined by **one-to-two** & **two-to-one** processes!

- Sub-leading corrections $(2 \to 2)$ to $\Gamma_n(p_z)$ are suppressed by $\alpha T^2/|eB|$

- Results are **reliable** for $|eB| \gg \alpha T^2$ (**QED**) or $|eB| \gg \alpha_s T^2$ (**QCD**)

- Transport mechanisms for $\sigma_\perp$ and $\sigma_\parallel$ are very different (with $\sigma_\perp/\sigma_\parallel \ll 1$)

- Generally, $\sigma_\perp$ is suppressed and $\sigma_\parallel$ is enhanced by a strong magnetic field*

Electrical conductivity tensor

- Kubo's formula

[Ghosh, Shovkovy, arXiv:2404.01388] & [Ghosh, Shovkovy, arXiv:2407.13828]

$$\sigma_{ij} = \lim_{\Omega\to 0} \frac{\mathrm{Im}\left[\Pi_{ij}(\Omega+i0;\mathbf{0})\right]}{\Omega} = -\frac{\alpha}{8\pi T}\sum_{f=1}^{N_f} q_f^2 \int \frac{dk_0 d^3\boldsymbol{k}}{\cosh^2\frac{k_0-\mu_f}{2T}} \mathrm{tr}\left[\gamma^i A_{\boldsymbol{k}}^f(k_0)\gamma^j A_{\boldsymbol{k}}^f(k_0)\right]$$

where $A_k^f(k_0)$ is the fermion spectral density, $A_k^f(k_0) \sim \Gamma_n(p_z)$

- When $\Gamma_n(p_z) \to 0$, the transverse and longitudinal conductivities read

$$\sigma_\perp \simeq \alpha \sum_{n=0}^{\infty} \int dp_z \Gamma_n(p_z)\, F_\perp(E_n, E_{n+1}, p_z)$$

$$\sigma_\parallel \simeq \alpha \sum_{n=0}^{\infty} \int dp_z \frac{F_\parallel(E_n,\ p_z)}{\Gamma_n(p_z)}$$

In the limit $\Gamma_n \to 0$:

$\sigma_\perp \to 0$

$\sigma_\parallel \to \infty$

- $\sigma_\perp$ is **nonzero** only because of interactions (via hopping between LLs)
- $\sigma_\parallel$ is **finite** only because of interactions (same as for $B = 0$)

Damping rate for $|eB| \gg \alpha T^2$

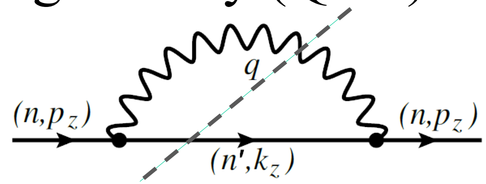
- Gauge invariant definition of damping rate @ $B \neq 0$

$$\Gamma_n(p_z) = \frac{1}{2p_0}\int d^4u' \int d^4u \,\mathrm{Tr}\left[\frac{2\pi\ell^2}{V_\perp}\int dp \sum_s \bar{\Psi}_{n,p,s}(u')\,\mathrm{Im}\Sigma(u',u)\Psi_{n,p,s}(u)\right]$$

[Ghosh, Shovkovy, Phys. Rev. D 109, 096018 (2024)]

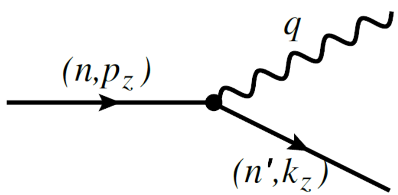
- Imaginary part of self-energy in gauge theory (QED) at leading order

$$\mathrm{Im}\,\Sigma_n(p_z) =$$

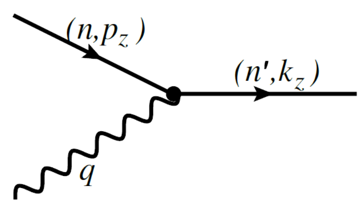


i.e., the underlying processes are

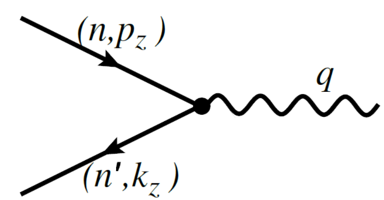
$n > n'$



$n < n'$



$\forall\ n\ \ n'$



The resulting $\Gamma_n(p_z)$ is of the order of $\alpha|eB|/T$. [Subleading $\Gamma_n(p_z) \sim \alpha^2 T$]

Damping rate, $|eB| \gg \alpha T^2$

- Analytical expression for the damping rate [Ghosh, Shovkovy, Phys. Rev. D 109, 096018 (2024)]

$$\Gamma_n(p_z) = \frac{\alpha|qB|}{4p_0} \sum_{n'=0}^{\infty} \sum_{\{s\}} \int d\xi \, \frac{\mathcal{M}_{n,n'}(\xi)\left[1 - n_F(s_1 E_{n',k_z^{s'}}) + n_B(s_2 E_q)\right]}{s_1 s_2 \sqrt{(\xi - \xi^-)(\xi - \xi^+)}}$$

where function $\mathscr{M}_{n,n'}(\xi)$ is determined by the squared amplitudes (QED)

[Ghosh, Shovkovy, arXiv:2407.13828]

$$\mathcal{M}_{n,n'}(\xi) = -\left(n + n' + \bar{m}_0^2 \ell^2\right)\left[\mathcal{I}_0^{n,n'}(\xi) + \mathcal{I}_0^{n-1,n'-1}(\xi)\right] + (n+n')\left[\mathcal{I}_0^{n,n'-1}(\xi) + \mathcal{I}_0^{n-1,n'}(\xi)\right]$$

and the Landau-orbit overlap function is $\mathcal{I}_0^{n,n'}(\xi) = \frac{(n')!}{n!} e^{-\xi} \xi^{n-n'} \left(L_{n'}^{n-n'}(\xi)\right)^2$

Same spin-averaged $\Gamma_n(p_z)$ is obtained from the poles of the propagator (!)

Poles of the propagator allow one to obtain both $\Gamma_n^{(+)}(p_z)$ & $\Gamma_n^{(-)}(p_z)$ (!)

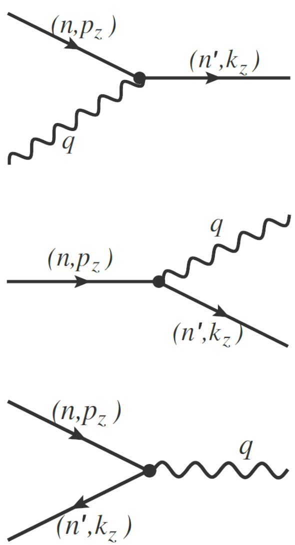
Fermion damping rate, $|eB| \gg \alpha T^2$

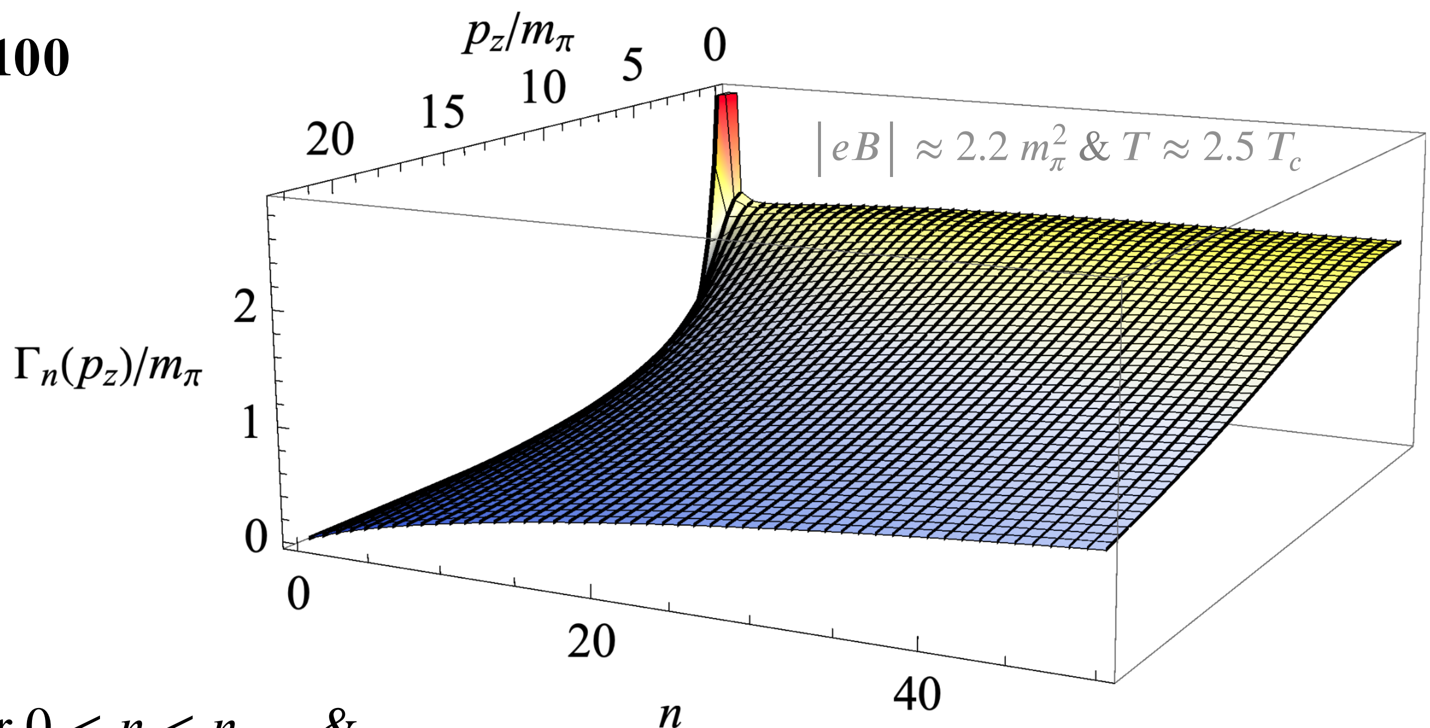
Calculation of damping rates $\Gamma_n(p_z)$ is the most costly numerical part

Many Landau levels: $\boldsymbol{n_{\max} = 50}$

[Ghosh, Shovkovy, Phys. Rev. D 109, 096018 (2024)]

$\boldsymbol{n'_{\max} = 2n_{max} = 100}$





Numerical data for $0 < n < n_{\max}$ &

a wide range of longitudinal momenta: $0 < p_z < p_{z,\max}$

Conductivity of QED plasma

- Parameters: $15m_e \le T \le 80m_e$ and $(15m_e)^2 \le |eB| \le (200m_e)^2$

[Ghosh, Shovkovy, arXiv:2404.01388], [Ghosh, Shovkovy, arXiv:2407.13828]

Note the hierarchy

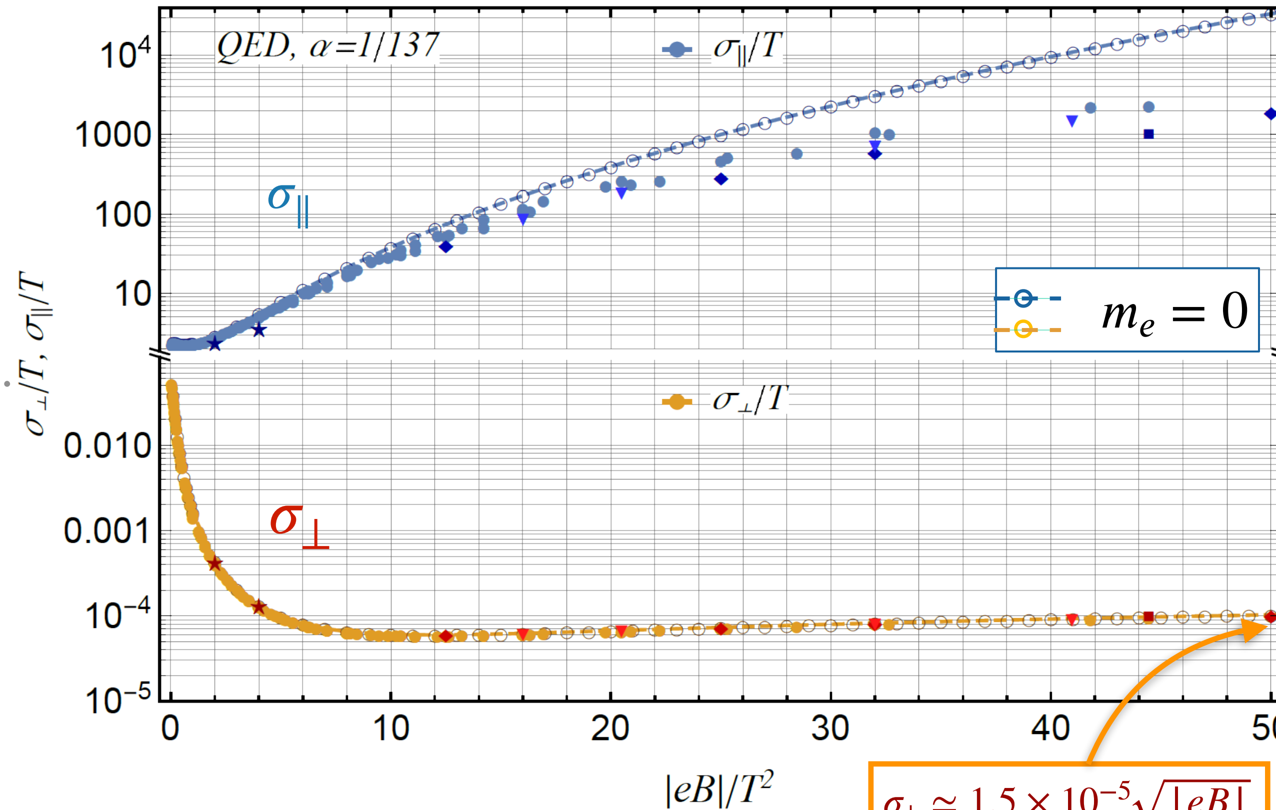
$$\frac{\sigma_\perp}{\sigma_\parallel} \propto \alpha^2 \ll 1$$

Ultra-relativistic regime:

$$\frac{\sigma_\perp}{T} \simeq \tilde{\sigma}_\perp\left(\frac{|eB|}{T^2}\right) + \ldots$$

$$\frac{\sigma_\parallel}{T} \simeq \tilde{\sigma}_\parallel\left(\frac{|eB|}{T^2}\right) + \ldots$$

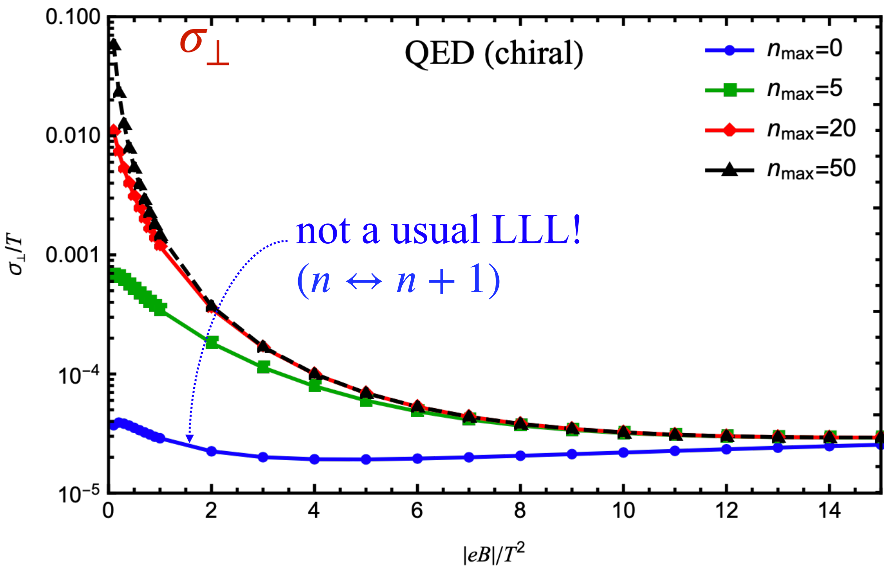
Chiral limit: $m_e = 0$

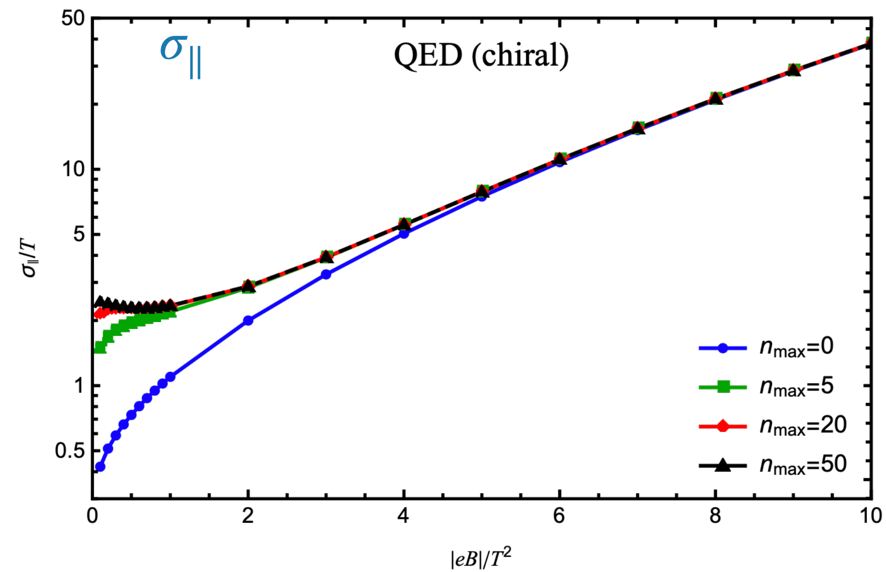


$\sigma_\perp \simeq 1.5 \times 10^{-5}\sqrt{|eB|}$

Convergence of Landau-level sum

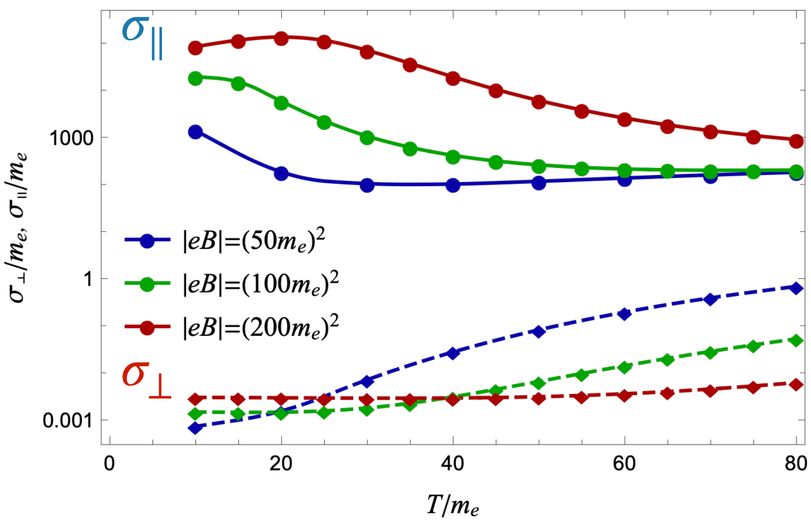
- In a wide range of parameters, a large number of Landau levels must be included
  - The Landau-level sum in $\sigma_\perp$ requires $n_{\max} \gtrsim 30T^2/|eB|$
  - The Landau-level sum in $\sigma_\parallel$ requires $n_{\max} \gtrsim 10T^2/|eB|$

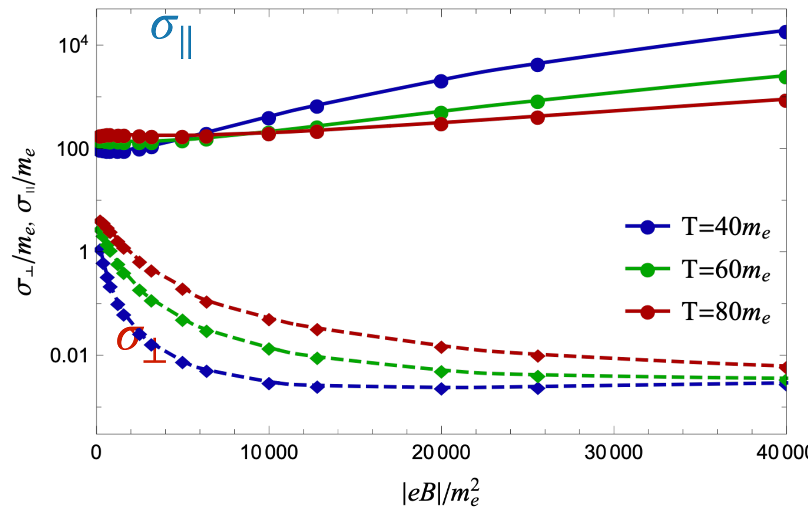




Dependence on T and $|eB|$

- T-dependence of $\sigma_\perp$ and $\sigma_\parallel$ resemble conductivity in semiconductors and metals, respectively
  - $\sigma_\perp \propto \Gamma_n(p_z)$ tends to increase with temperature (~ semiconductors)
  - $\sigma_\parallel \propto 1/\Gamma_n(p_z)$ tends to decrease with temperature (~ metals)
- B-dependence is nearly opposite ($\sigma_\perp$ decreases and $\sigma_\parallel$ increases with B)





Conductivity of QCD plasma

- Parameters: $15m \le T \le 80m$ and $(15m)^2 \le |eB| \le (200m)^2$

QCD coupling is large,

$\alpha_s = 1$
(also, $\alpha_s = 0.5, 2$)

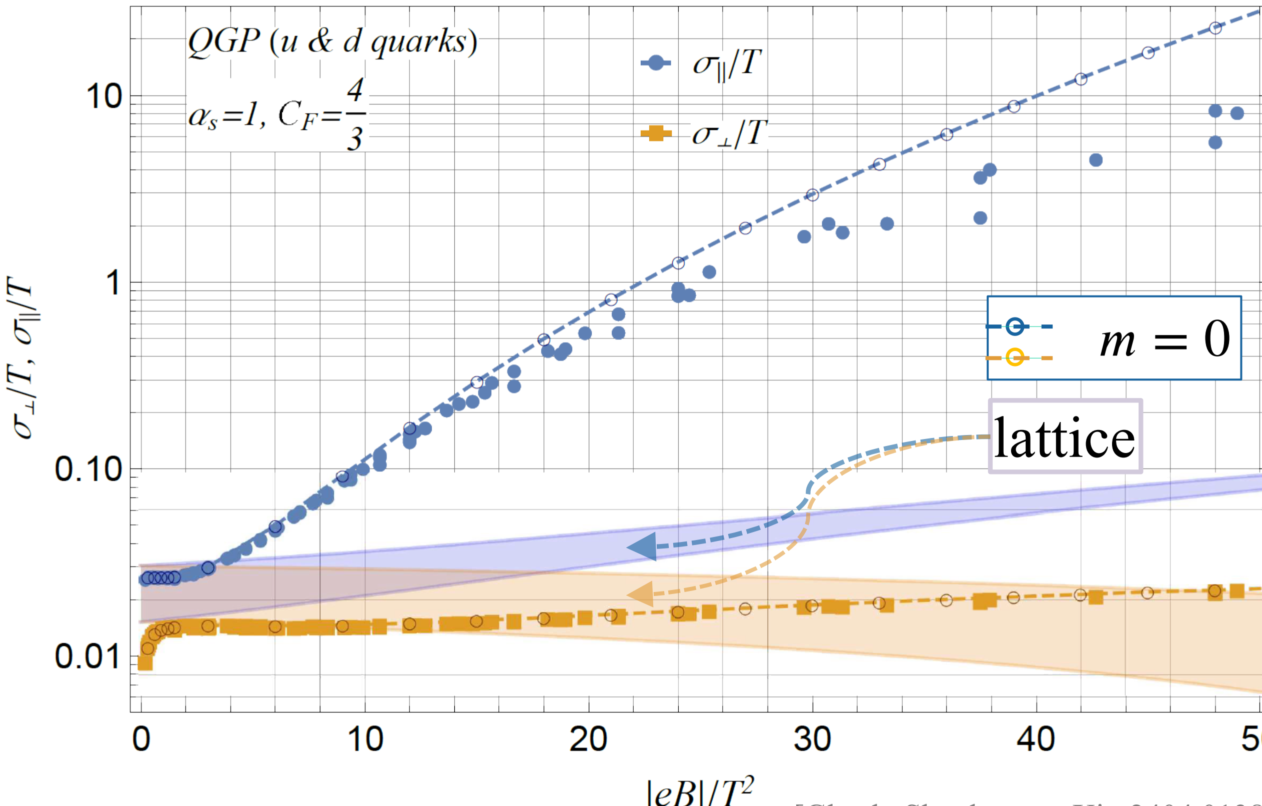
Compared to lattice,

$\sigma_\perp$ - is similar

$\sigma_\parallel$ - is much larger

Other processes may be important

$2 \to 2$ (?)



[Ghosh, Shovkovy, arXiv:2404.01388]
[Ghosh, Shovkovy, arXiv:2407.13828]

Transport mechanism

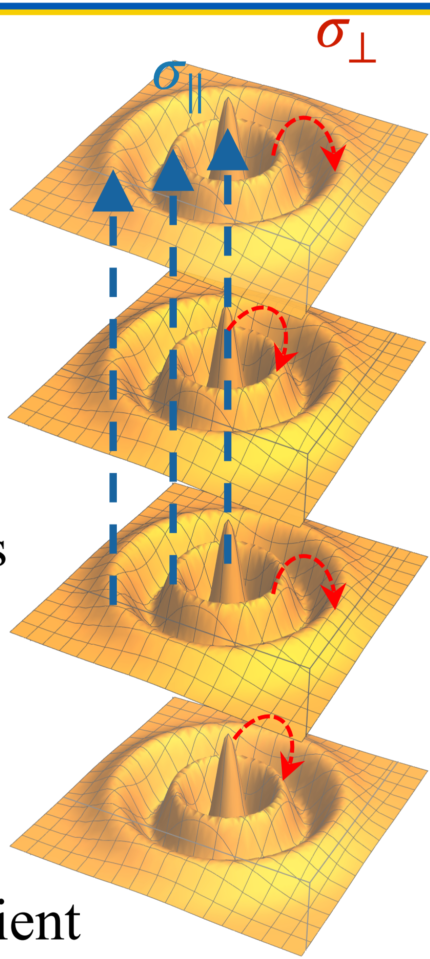
- Longitudinal conductivity
  - Conductivity is **disrupted** by transitions/scattering

$$\sigma_{\parallel} \simeq \alpha \sum_{n=0}^{\infty} \int dp_z \frac{F_{\parallel}(E_n,\ p_z)}{\Gamma_n(p_z)}$$

  - Individual LLs contribute like independent species*
- Transverse conductivity
  - Conductivity is **driven** by transitions (hopping) between LLs

$$\sigma_{\perp} \simeq \alpha \sum_{n=0}^{\infty} \int dp_z \Gamma_n(p_z)\ F_{\perp}(E_n, E_{n+1}, p_z)$$

  - At least, transitions between 0th and 1st LLs are required
- Unless $|eB|/T^2 \gg 10$, the LLL approximation is insufficient



*Their damping rates are determined by scattering and transitions to other LLs

Summary

- Conductivity is obtained for a plasma in a strong magnetic field from first principles within a gauge theory (QED/QCD)
- The mechanisms for longitudinal and transverse conductivities are revealed
- Results are **reliable** when $|eB| \gg \alpha T^2$ (**QED**) or $|eB| \gg \alpha_s T^2$ (**QCD**)
- The damping rates are determined by one-to-two and two-to-one processes
- Conductivity is highly anisotropic ($\sigma_\perp / \sigma_\parallel \ll 1$)
- Under realistic conditions, many Landau levels contribute
- The results are relevant for heavy-ion physics, astrophysics, etc.