Eigenvalue distributions of the lattice fermion operators and the Chiral Magnetic Effect

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Introduction

- CME current on the lattice is underestimated. The staggered fermion may potentially have a problem in the $\mathrm{U_A}(1)$ sector. The Wilson (WL) and the overlap-Wilson (OW) fermions are more preferable.
- Intriguing to study the eigenvalue distributions of the modified Dirac operators associated with the observables such as the chirality and the CME current.
- The Dirac operator in the WL fermion is non-hermitian. The low-lying modes are sensitive to the boundary condition at infinity (Skin Effect).

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Methods

We modify the Dirac operator D as:

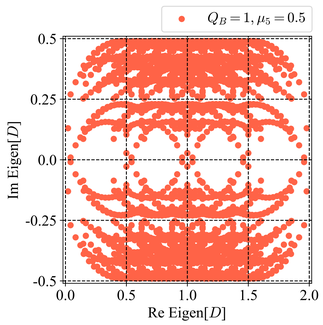
$$H_{45} := (\gamma_4\gamma_5)D\,, \qquad H_3 := (i\gamma_3)D\,.$$
$$N_5 = \frac{1}{V}\mathrm{Tr}[H_{45}^{-1}]\,, \qquad J_3 = \frac{1}{V}\mathrm{Tr}[H_3^{-1}]\,.$$

We impose the external parameters on the *free* fermions:

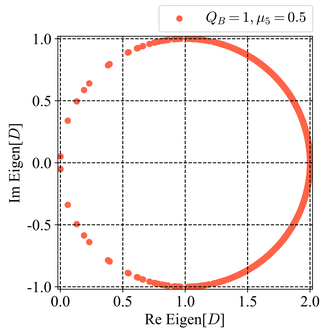
$$\mu_5\,, \qquad B = \frac{2\pi}{L_s^2}Q_B \quad (Q_B \in \mathbb{Z})\,.$$

D spectra at finite μ_5 and Q_B

WL fermion operator

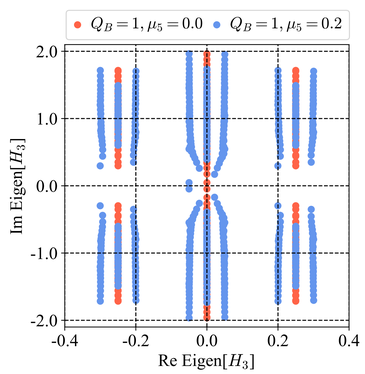


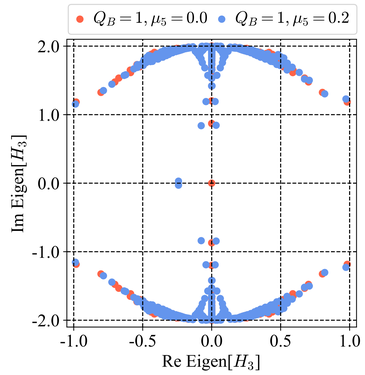
OW fermion operator



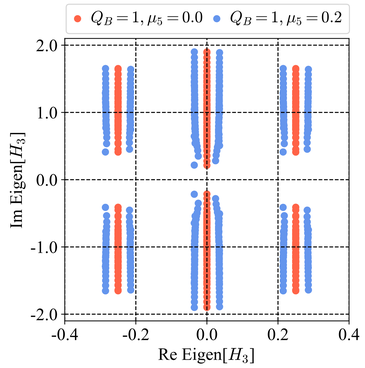
Results

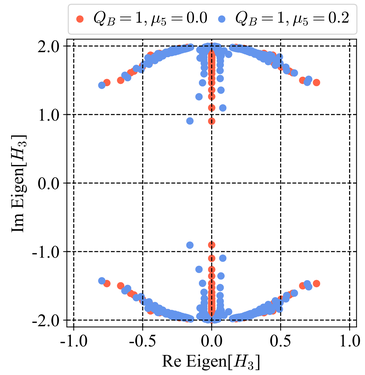
H_3 spectra under the PBC along x_4 (4^4 Lattice)





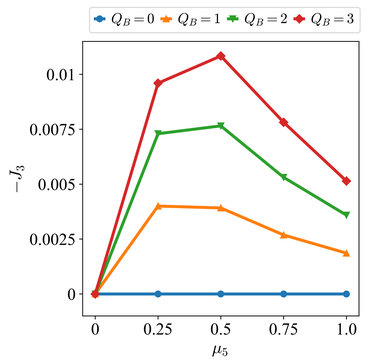
H_3 spectra under the APBC along x_4 (4^4 Lattice)

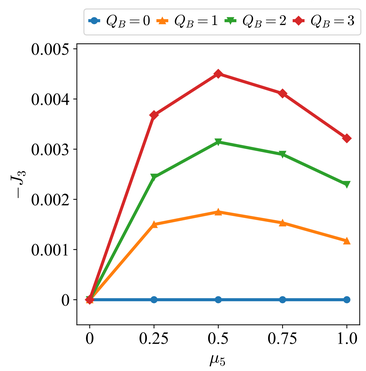




★ PBC is problematic due to zeromodes (singular J_3).
★ APBC is better but zeromodes are taken away!?

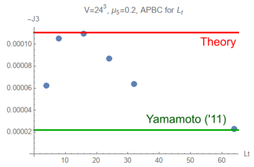
J_3 from the spectra under the APBC (6^4 Lattice)





Discussions

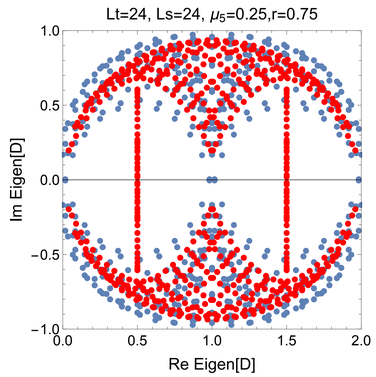
- With increasing L_t, the CME current J_3 increases and then *decreases* toward what was observed in the lattice simulation.



- H_{45} is more smooth and N_5 behaves much better.
- We find that the WL (and the OW) fermion operator which is non-Hermitian exhibits the Skin Effect.

Skin Effect

Eigenvalue distributions can be sensitive to the spatial boundary condition at infinity (whether the PBC or the OBC – open boundary condition). (1+1)D WL (in the $B \to \infty$ limit) the spectra show:



Significantly different low-lying modes appear between the OBC and the PBC and they do NOT vanish even in the thermodynamic limit.

This is well-investigated in non-Hermitian quantum mechanics recently, but more studies are needed in our field especially in the *finite-density* case (future task).

References

A. Yamamoto, *Chiral magnetic effect in lattice QCD with a chiral chemical potential*, PRL107, 031601 (2011).

B.B. Brandt, G. Endrodi, E. Garnacho-Velasco, G. Marko, *On the absence of the Chiral Magnetic Effect in equilibrium QCD*, 2405.09484 [hep-lat].