Phase Diagram under Rotation and Magnetic Field

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The University of Tokyo

— The 8th International Conference on Chirality, Vorticity and Magnetic Field in Quantum Matter —

Talk Plan

Magnetic Field

* $B + T$ well understood (inverse magnetic catalysis)
* $B + \mu_B$ under intensive discussions
* B toward detection in HIC / magnetars

Rotation

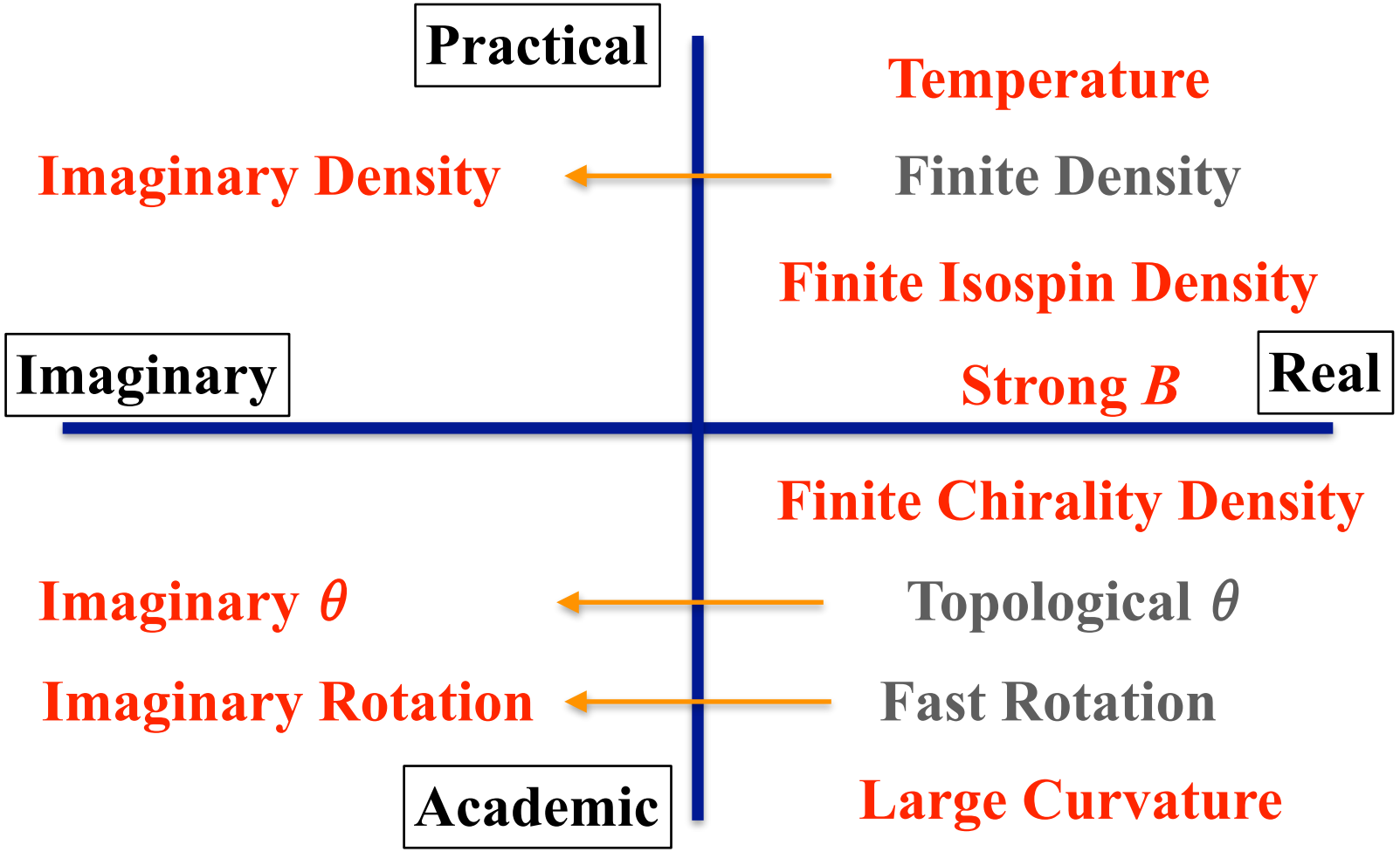
* Tension between LQCD and pQCD half resolved
* Unnatural observation in LQCD (personal opinion)

Magnetic Field + Rotation

* We must apologize; we seem to have been wrong…

Motivation

Real / Imag. / Academic / Practical



Magnetic Field

$\boldsymbol{B} + \mu_B$

Spatially inhomogeneous phase may appear.

Basar-Dunne-Kharzeev (2010)

Strong-$\boldsymbol{B}$ → Reduction to (1+1)D → Spiral Condensate

In (1+1)D, finite density originates from anomaly.

Brauner-Yamamoto (2016)

Chiral anomaly → $\dfrac{1}{4\pi^2 f_\pi}\mu_B \boldsymbol{B} \cdot \nabla \pi^0$

Source to generate spatially modulated condensate.

$B + \mu_B$

ChiEFT ~ Chiral Magnet **Brauner-Yamamoto (2016)**

$$H = -J\sum_n \boldsymbol{S}\cdot\boldsymbol{S}_{n+1} - 2\mu H\sum_n S_n^x + \boldsymbol{D}\cdot\sum_n \boldsymbol{S}_n\times\boldsymbol{S}_{n+1}$$

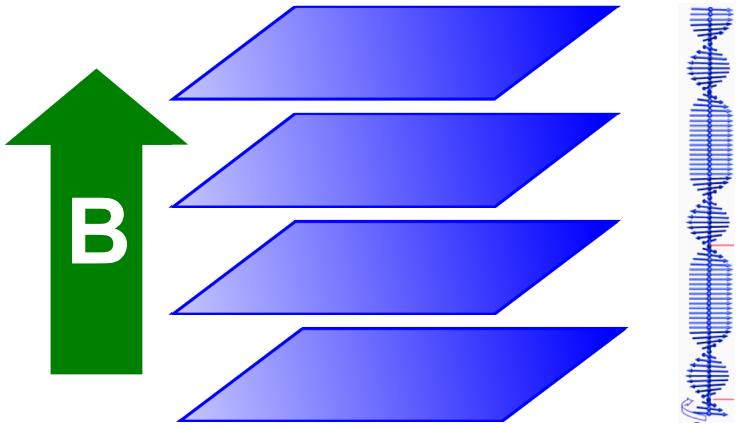
$$\simeq \underbrace{\tilde{J}S^2}_{f_\pi^2}\int dz\left[\frac{1}{2}(\partial_z\phi(z))^2 - \underbrace{\beta}_{m_\pi^2}\cos\phi(z) - \underbrace{\alpha}_{\mu_q B/(4\pi^2 f_\pi^2)}\partial_z\phi(z)\right]$$

$$\left(\frac{\alpha}{\sqrt{\beta}}\right) \geq \frac{4}{\pi} \quad\Rightarrow\quad \frac{\mu_q B}{4\pi^2 f_\pi^2 m_\pi} \geq \frac{4}{\pi}$$

Model-indep. and robust prediction!

Kishine et al. (2012)

$B + \mu_B$

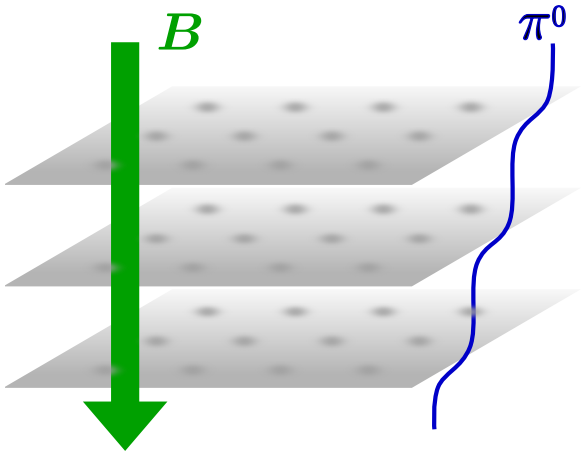


Brauner-Yamamoto (2016)

$$n_B = \frac{\partial \langle \mathcal{H} \rangle}{\partial \mu} = \frac{B_z}{4\pi^2 f_\pi} \partial_z \pi^0$$

π^0 domain-walls

$$\pi_1(\mathrm{U}(1)) = \mathbb{Z}$$

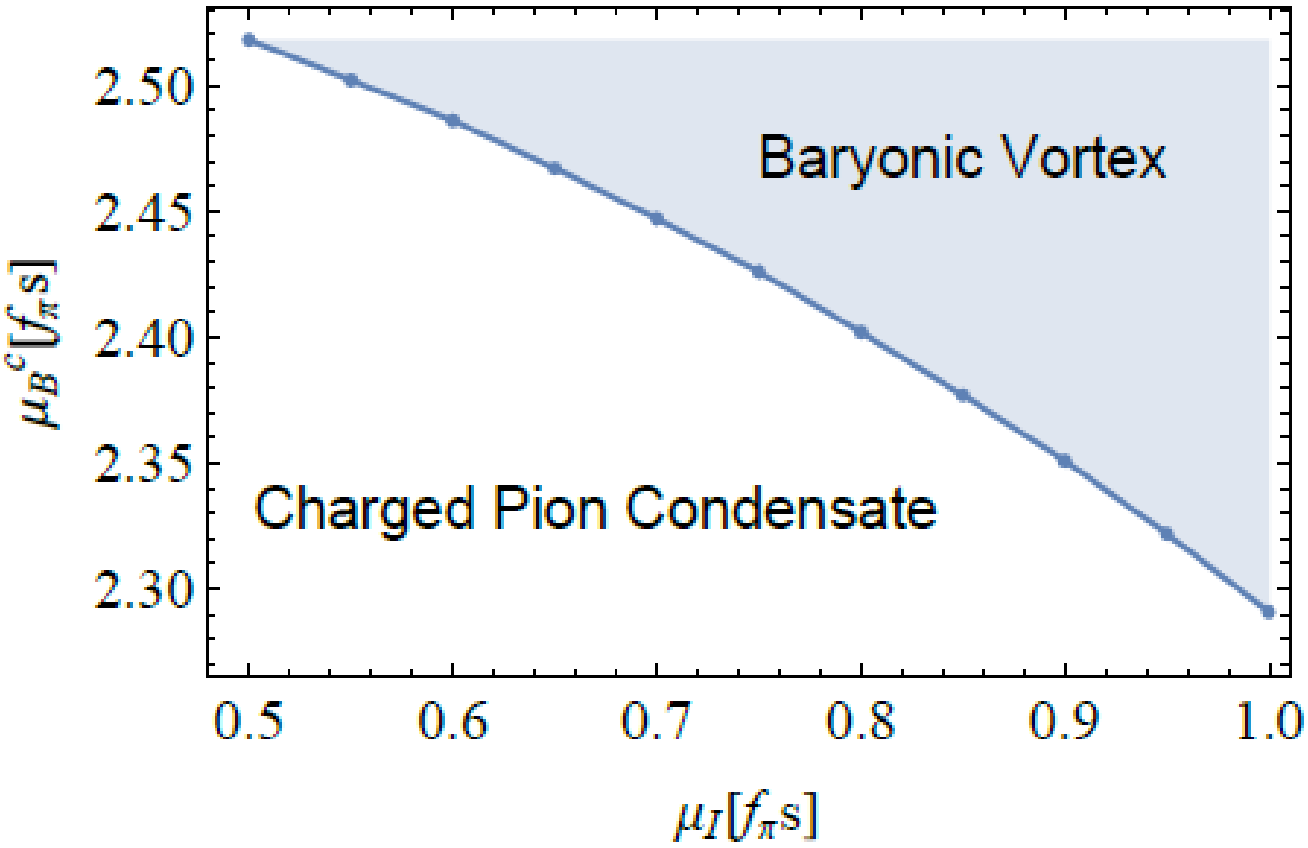


Skyrmion Crystal (Nuclear Matter)
→ π^0 domain-walls

Chen-Fukushima-Qiu (2021)

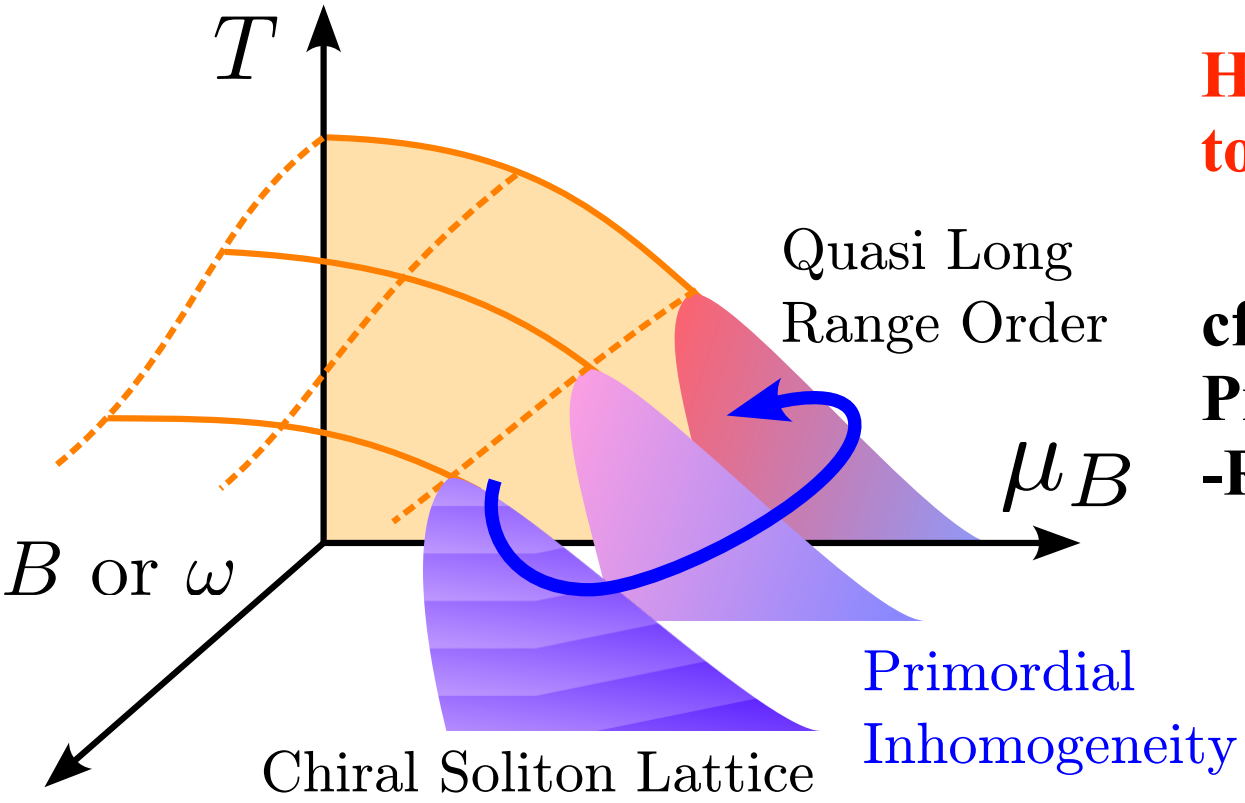
$B + \mu_B$

Nitta-Qiu (2024) **Vortex Skyrmions** **Talk by Evans**



Can we see it?

Fukushima-Hidaka-Inoue-Shigaki-Yamaguchi (2023)



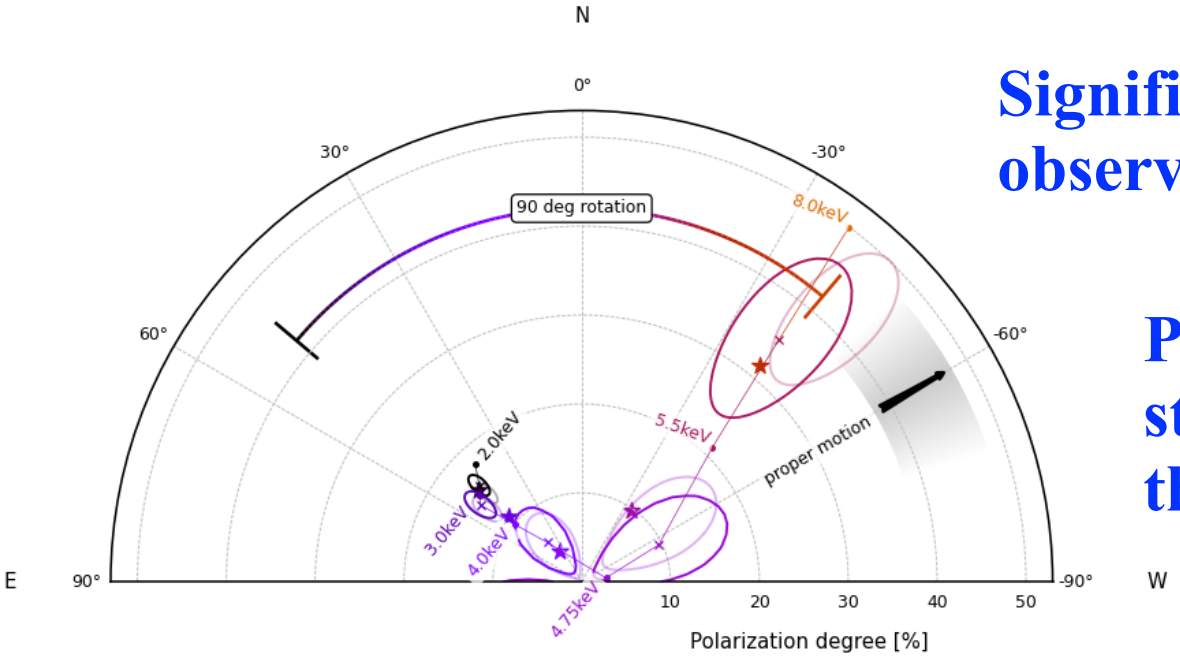
HBT sensitive to the cluster?

cf. HBT by Pisarski-Rennecke--Rischke (2023)

Magnetar Physics

Science 378, 646-650 (2022)

Imaging-X-ray Polarimetry Explorer (IXPE)

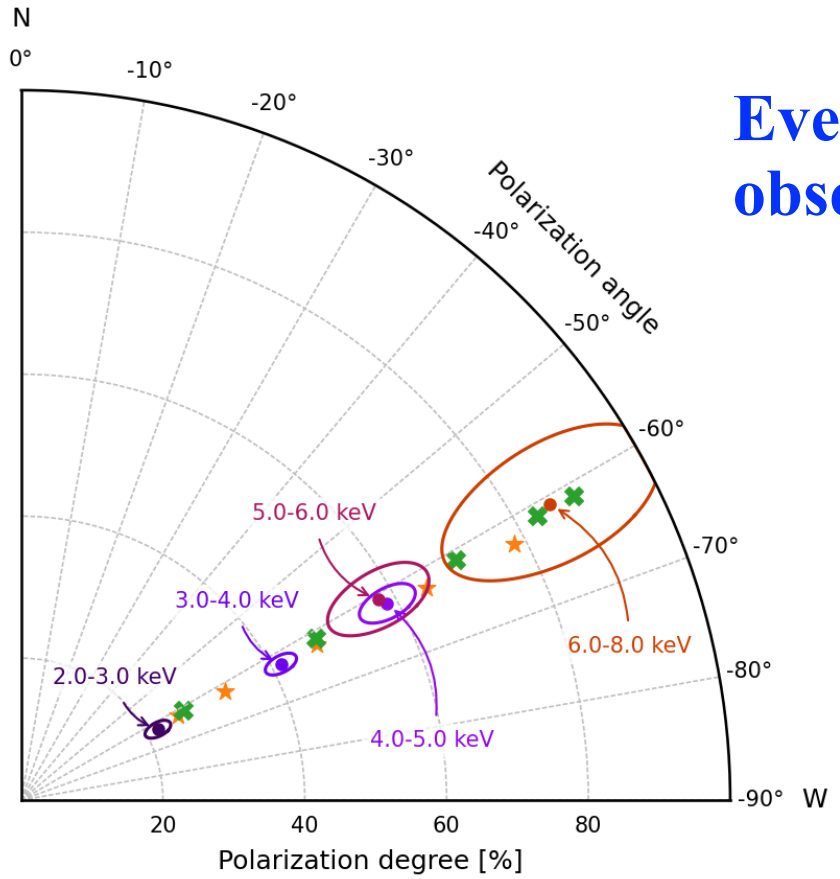


Significant polarization was observed — how!?

Polarization angle has strong dependence on the photon energy.

Magnetar Physics

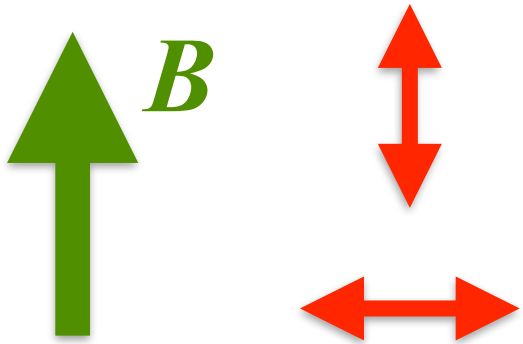
Astrophys.J.Lett.944, L27 (2023)



Even 80% polarization was observed — surprise!

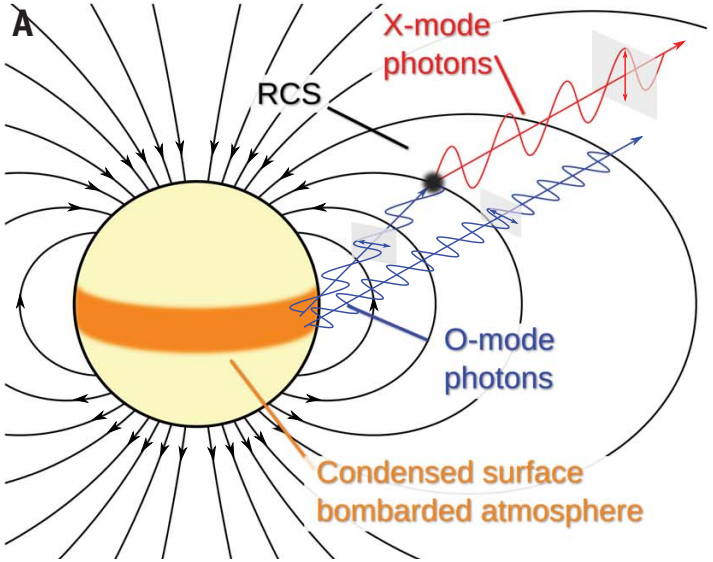
Polarization angle has no dependence on the photon energy???

Magnetar Physics



O-mode (ordinary mode)
Parallel to the magnetic field

X-mode (extraordinary mode)
Perpendicular to the magnetic field

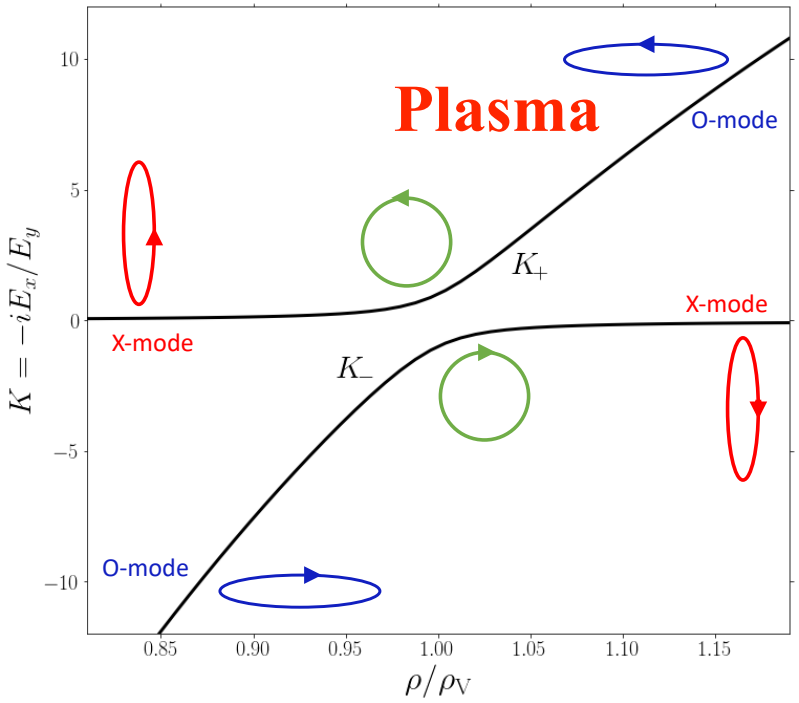


Common terminology in *their* community…

Science 378, 646-650 (2022)

Magnetar Physics

Don Lai (2022)



Vacuum

Assume:

No mode conversion for
$E < E_{\rm ad}$

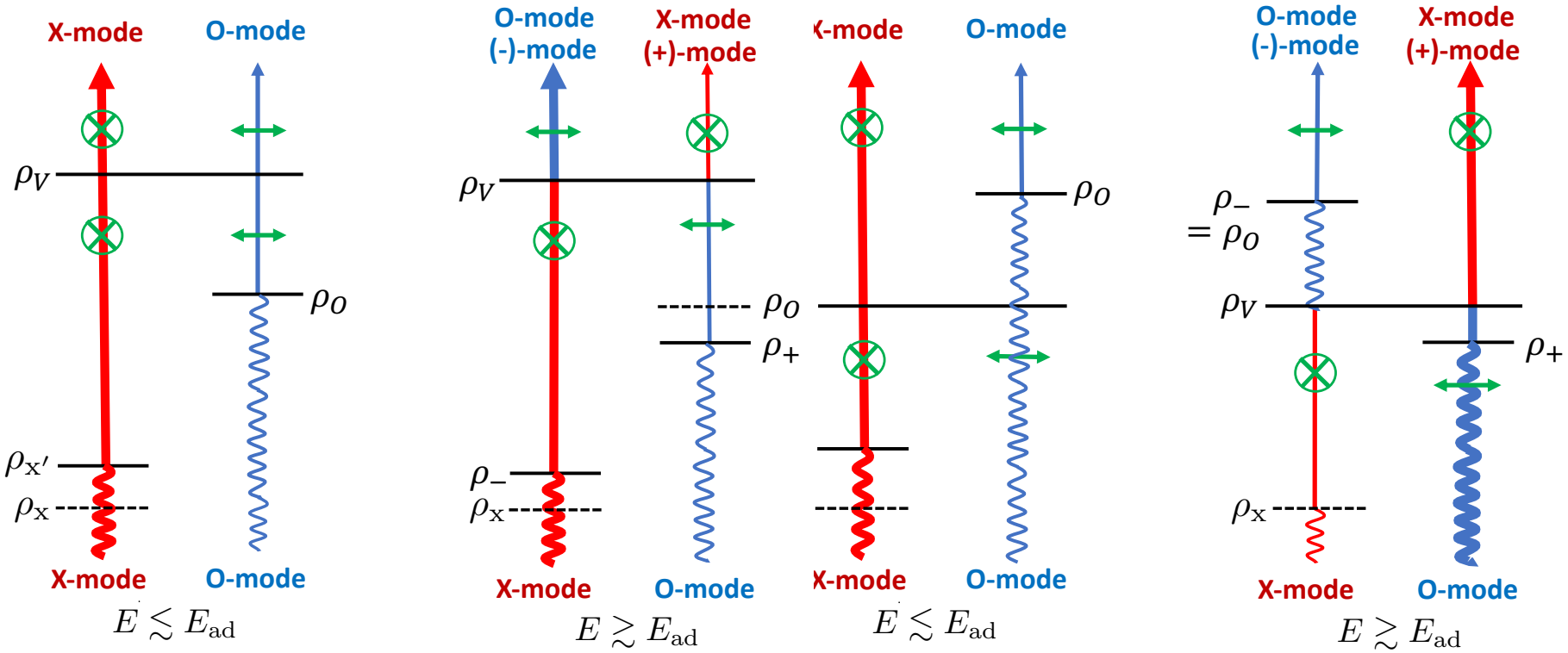
Mode conversion for
$E > E_{\rm ad}$

Density of atmosphere (*e* + ions)

Magnetar Physics

Don Lai (2022)

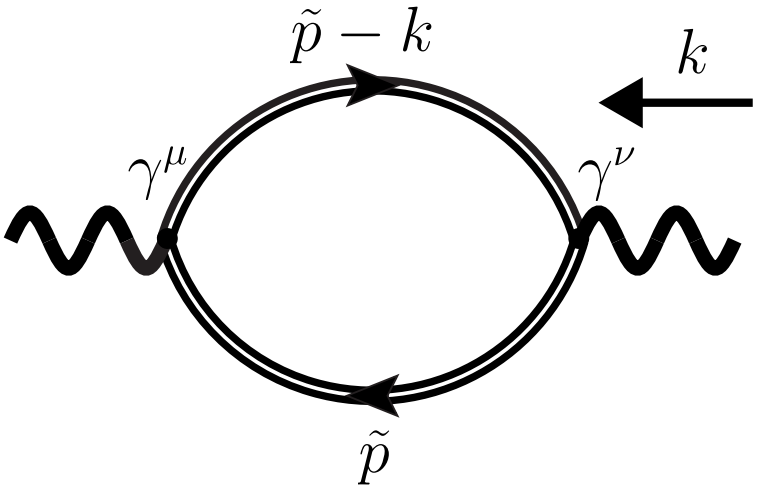
Small magnetic field case **Strong magnetic field case**



Magnetar Physics

In our community, this type of calculation is very familiar…

The polarization (or the pair annihilation/creation) is:



Ghosh, Shovkovy, Wang (2024)

Fukushima-Hidaka-Uji (comming very soon)

We can (should) apply our technology to their physics!

Rotation

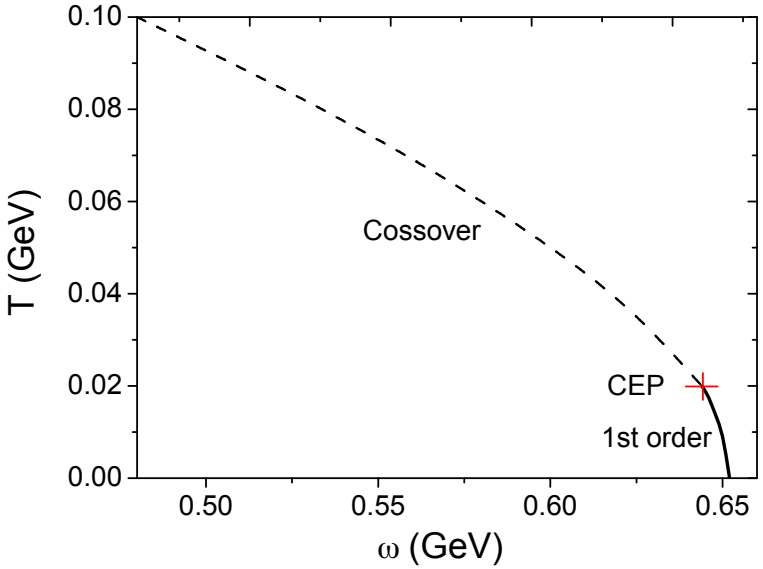
Rotation

Angular Velocity ~ Finite Density

Chen-Fukushima-Huang-Mameda (2015)

$$H \rightarrow H - \boldsymbol{J}\cdot\boldsymbol{\Omega} \Leftrightarrow H - N\mu$$

Phase Diagram at Finite Angular Velocity



Jiang-Liao (2016)

This is a phase diagram at zero distance (at the rotation center).

No orbital angular mom.!

Talk by Zhu

Imaginary Rotation

Finite Imaginary Angular Velocity

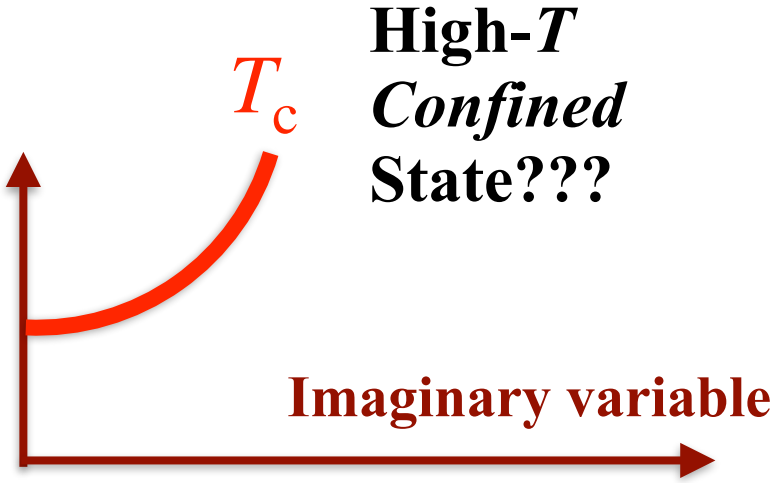
Angular velocity breaks Hermiticity of the Dirac operator and the sign problem is turned on… ($\boldsymbol{J}\cdot\boldsymbol{\Omega}\sim N\mu$)

Expected behavior of phase transition

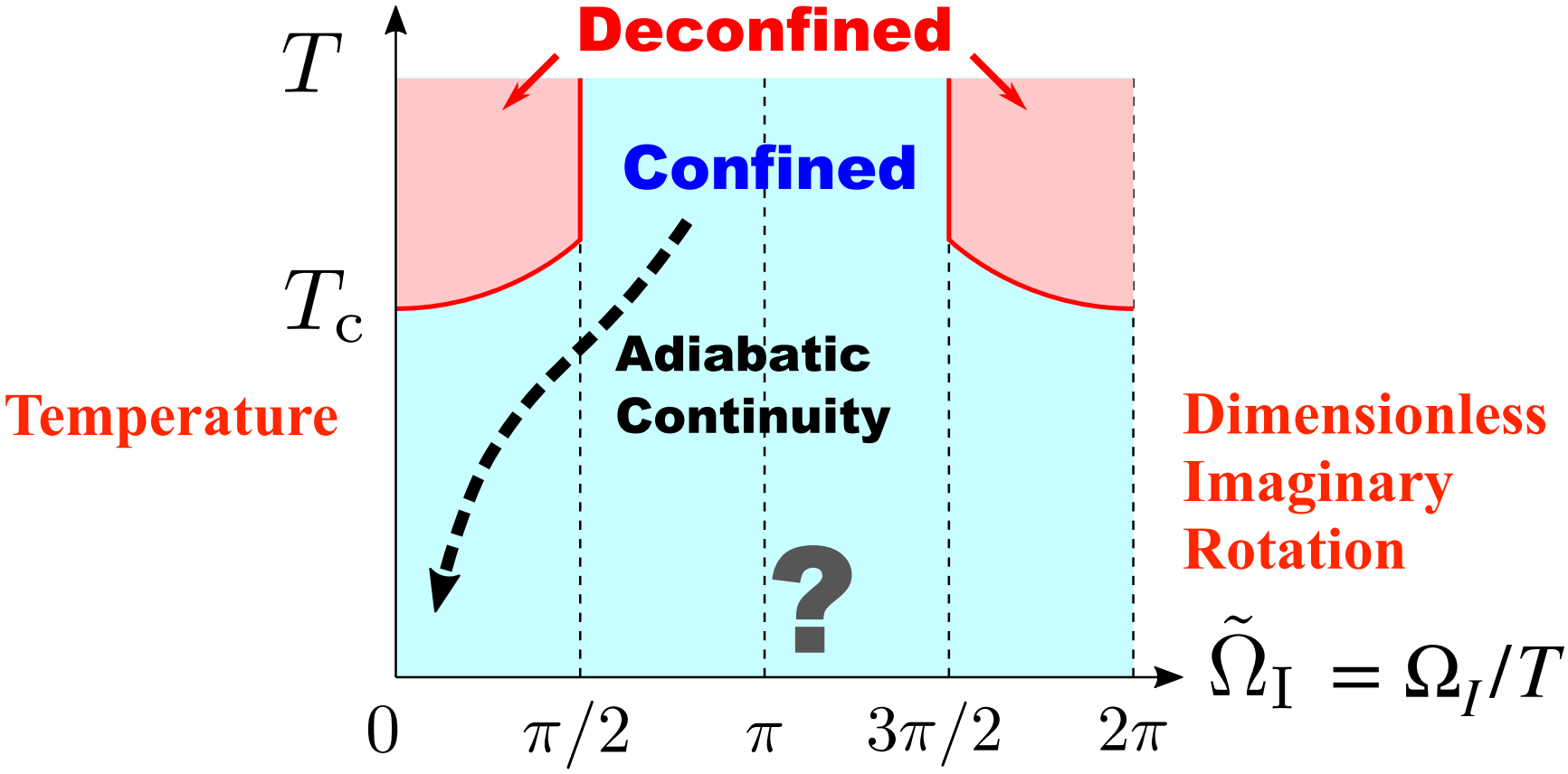
$$T(\Omega_I^2) \;\rightarrow\; T(-\Omega^2)$$

Increasing Function

Decreasing Function



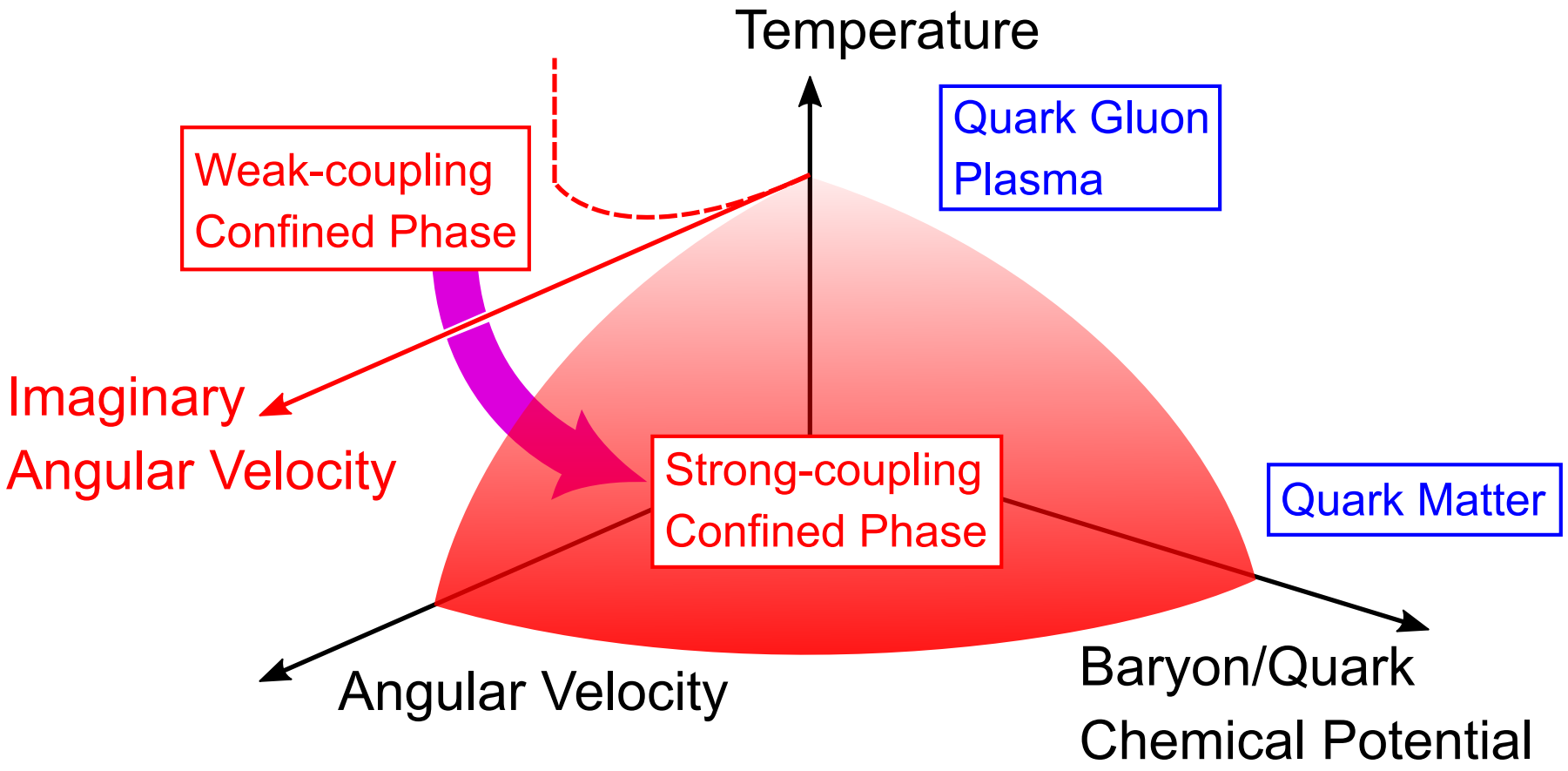
Imaginary Rotation



Chen-Fukushima-Shimada (2022)

Imaginary Rotation

New approach to confinement physics



Polyakov Loop Potential

Rotating GPY-Weiss potential

Chen-Fukushima-Shimada (2022)

$$V_g(\boldsymbol{\phi};\tilde{\Omega}_{\rm I}) = -\frac{2T^4}{\pi^2}\sum_{\alpha\in\Phi}\sum_{n=1}^{\infty}\frac{\cos(n\boldsymbol{\phi}\cdot\boldsymbol{\alpha})\cos\left(n\tilde{\Omega}_{\rm I}\right)}{\left\{n^2+2\tilde{r}^2\left[1-\cos\left(n\tilde{\Omega}_{\rm I}\right)\right]\right\}^2}$$

Singular for analytical cont.

The singularity physically represents the violation of causality if the boundary is not imposed.

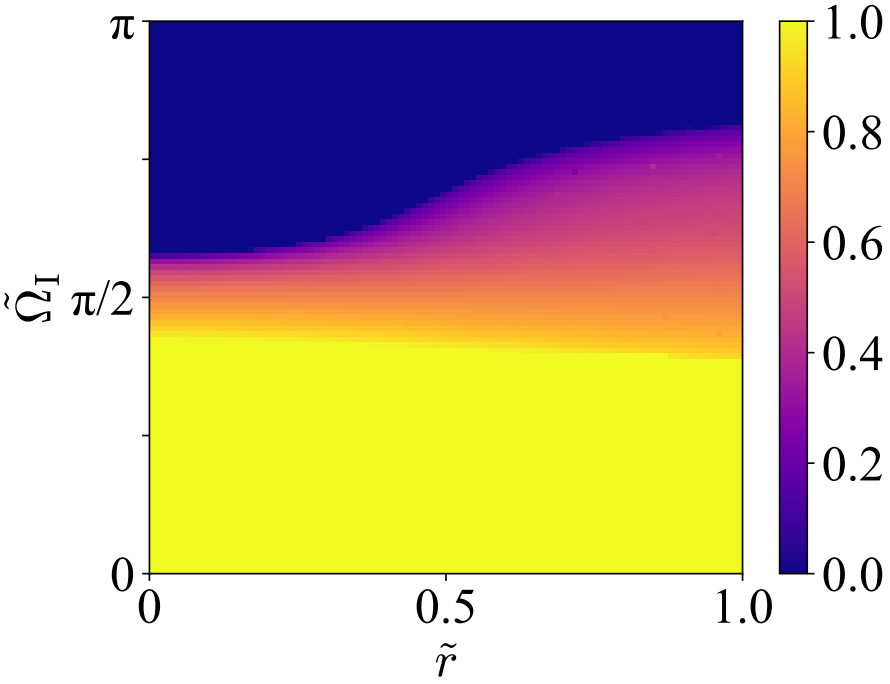
Ghosts (confining potential) are not affected at $r=0$ because they are spin-0 particles.

Talk by Singha

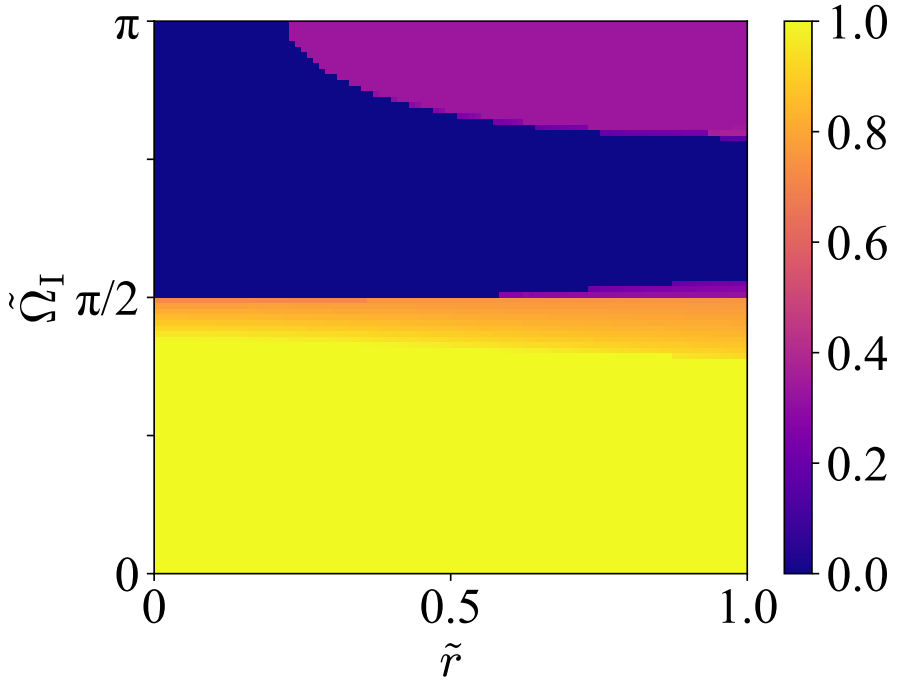
Polyakov Loop Potential

Chen-Fukushima-Shimada (2024)

SU(2) Pure YM



SU(3) Pure YM

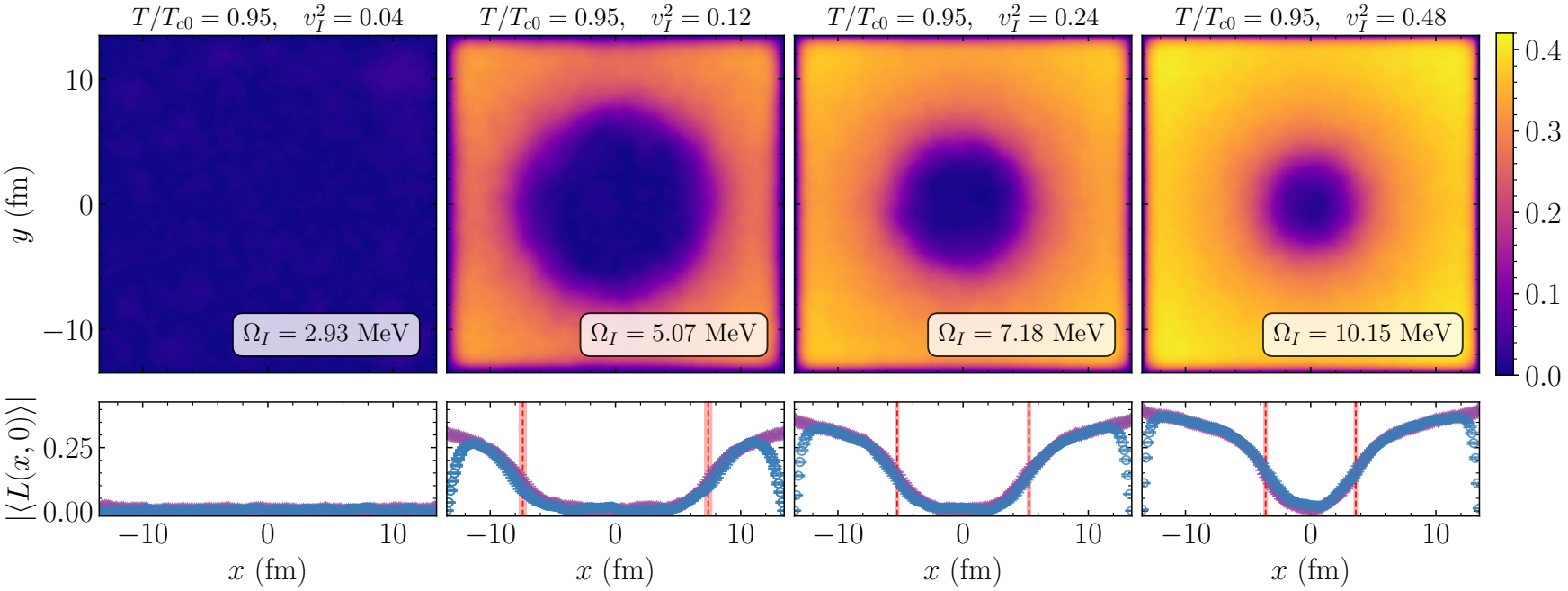


More "deconfined" for farer from the center

→ Real rotation would favor "confinement" ?

vs. Lattice

Braguta-Chernodub-Roenko (2023) **Talks by Braguta/Roenko**



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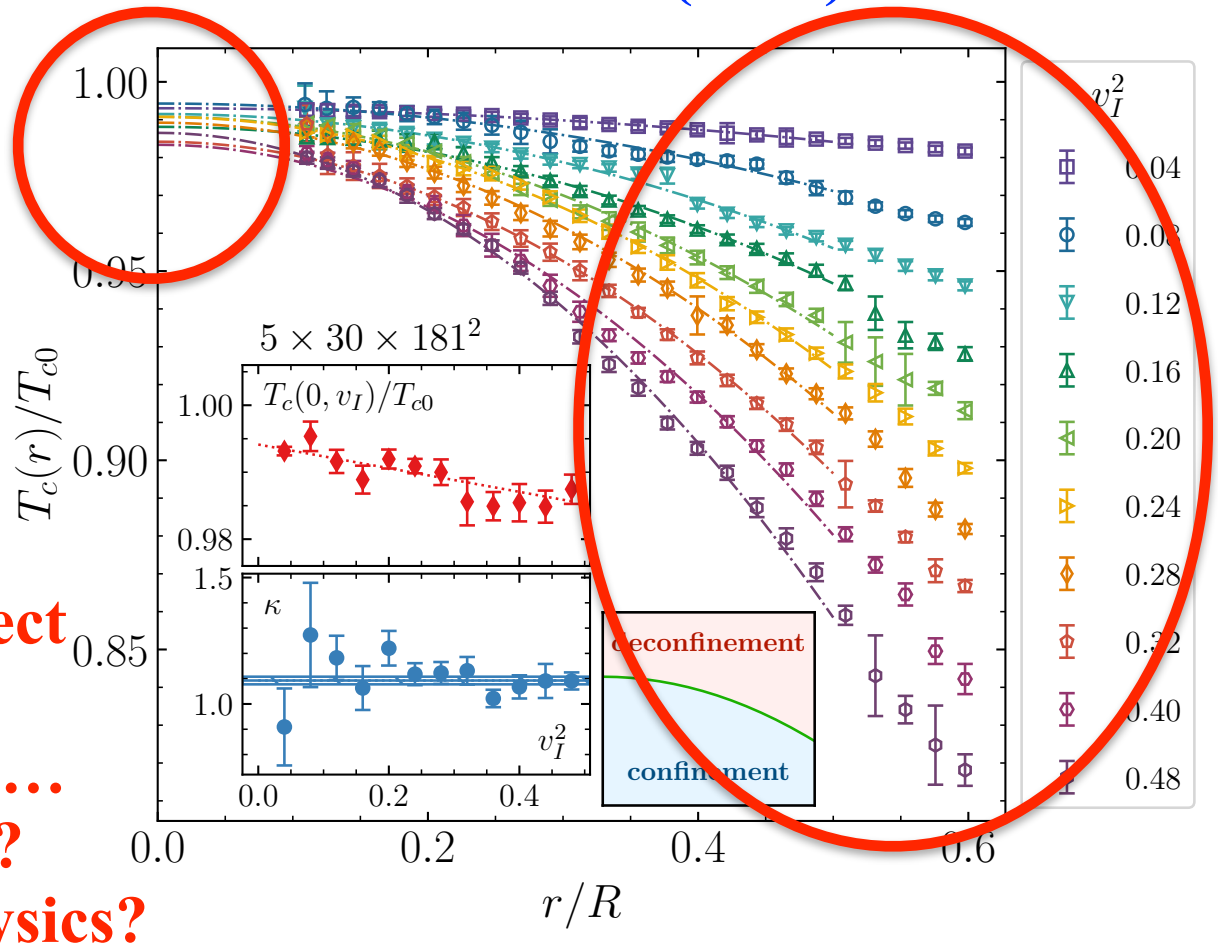


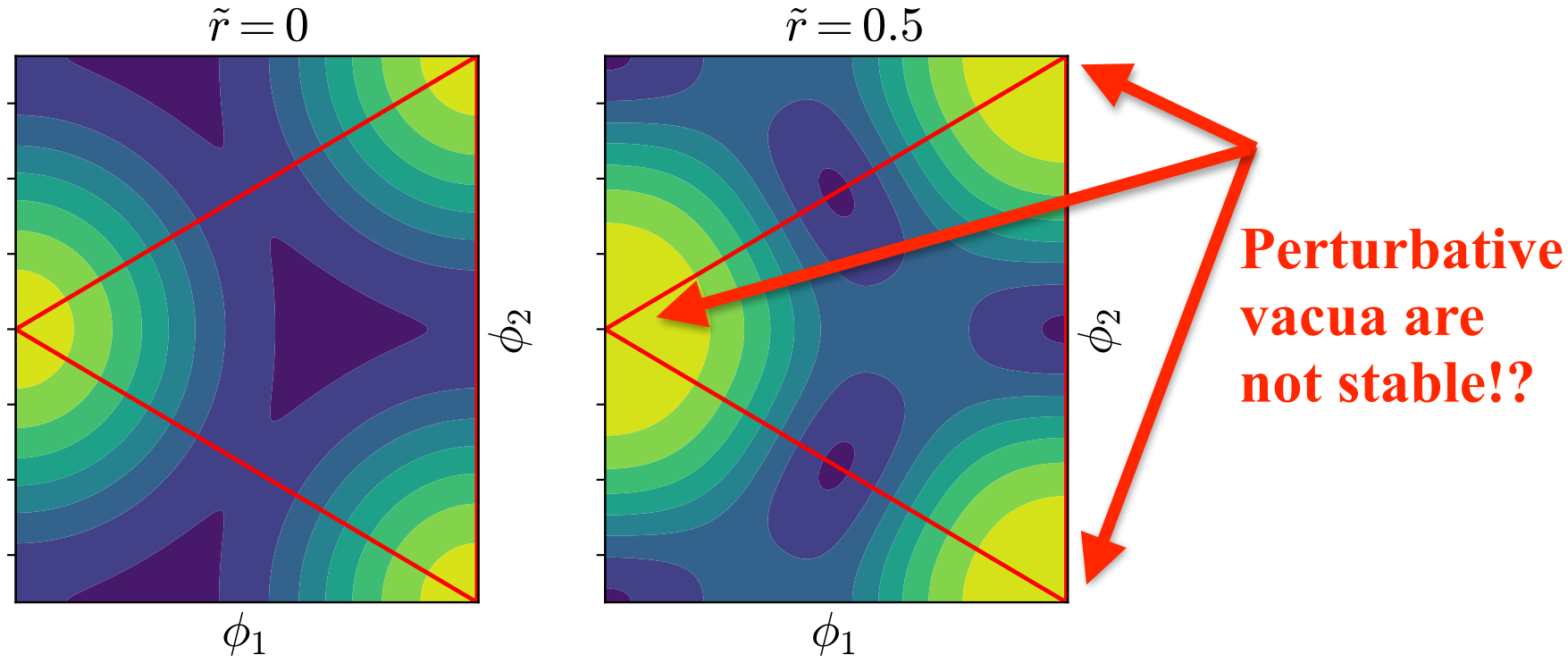
Spin effect does not show up… Artifact? New physics?

This trend agrees with pQCD !!!

Polyakov Loop Potential

Chen-Fukushima-Shimada (2024)

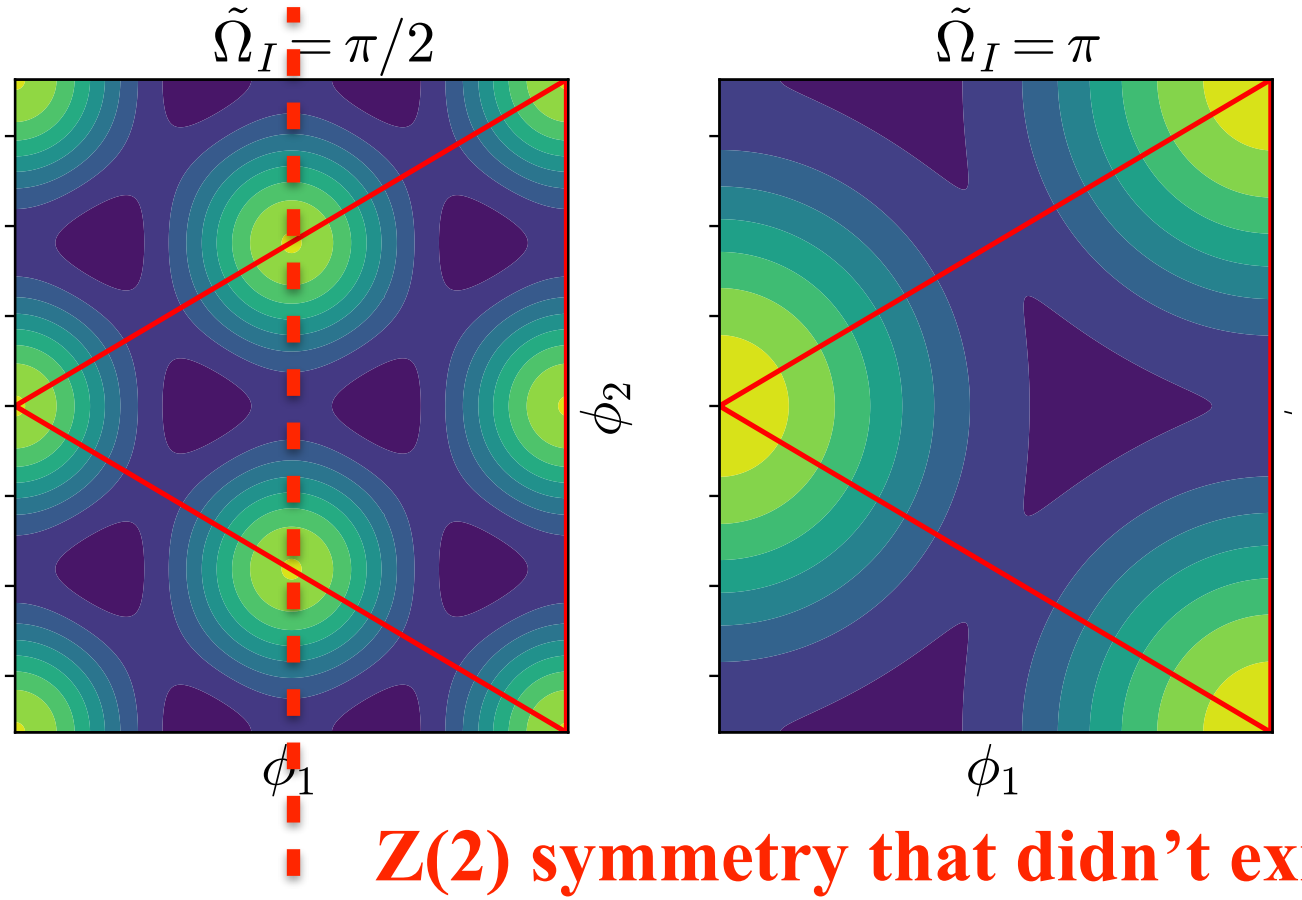
Not a standard "deconfined" phase



This is a testable prediction for lattice!

Polyakov Loop Potential

Accidental "emergent symmetry" ???



Z(2) symmetry that didn't exist…?

Including Fermions

Chen-Fukushima-Shimada (2024)

Adding "free" fermions with dynamical mass

$$\mathcal{Z}_{\mathrm{f}T,\omega} = \mathrm{Det}(\gamma^{\mu} G_{\mathrm{B}\,\mu} + m)$$

Search for the potential minimum of the Polyakov loop and the dynamical mass.

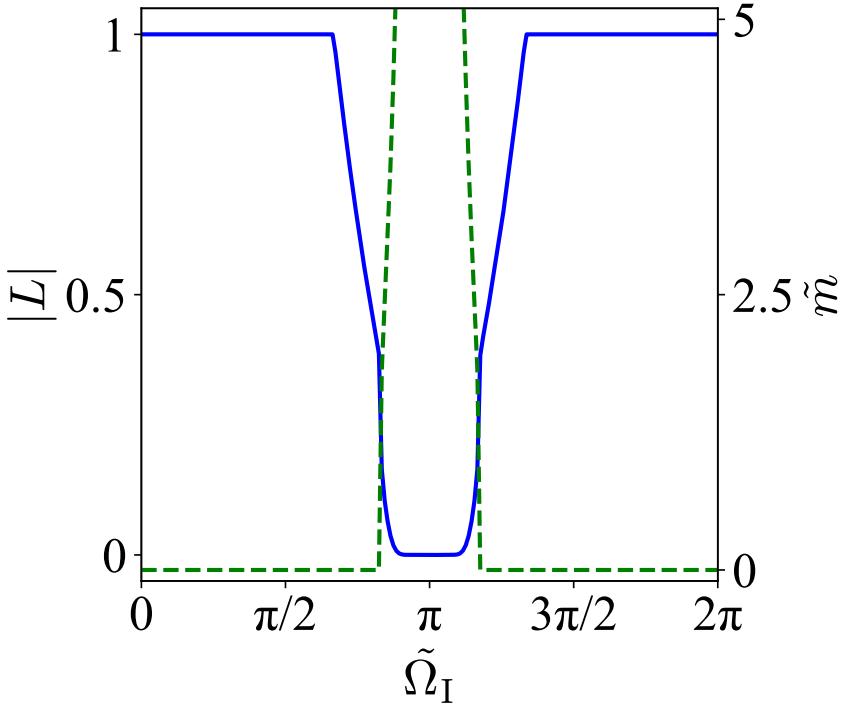
Once symmetry breaking is turned on, the mass blows up.

We may introduce a model such as NJL, but this is the model-independent analysis based on QCD!

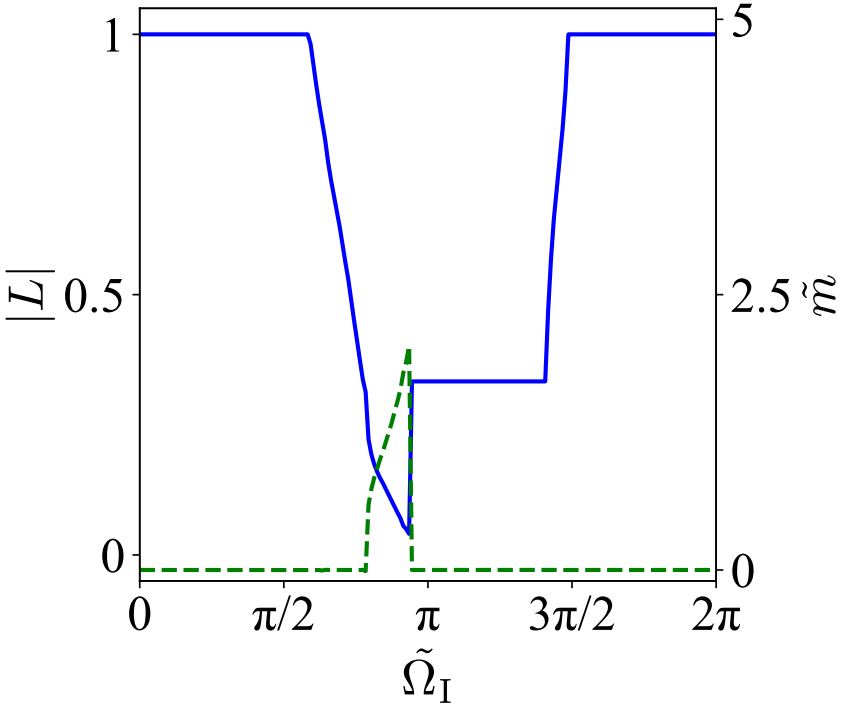
Including Fermions

Chen-Fukushima-Shimada (2024)

SU(2) full (2 flavor)



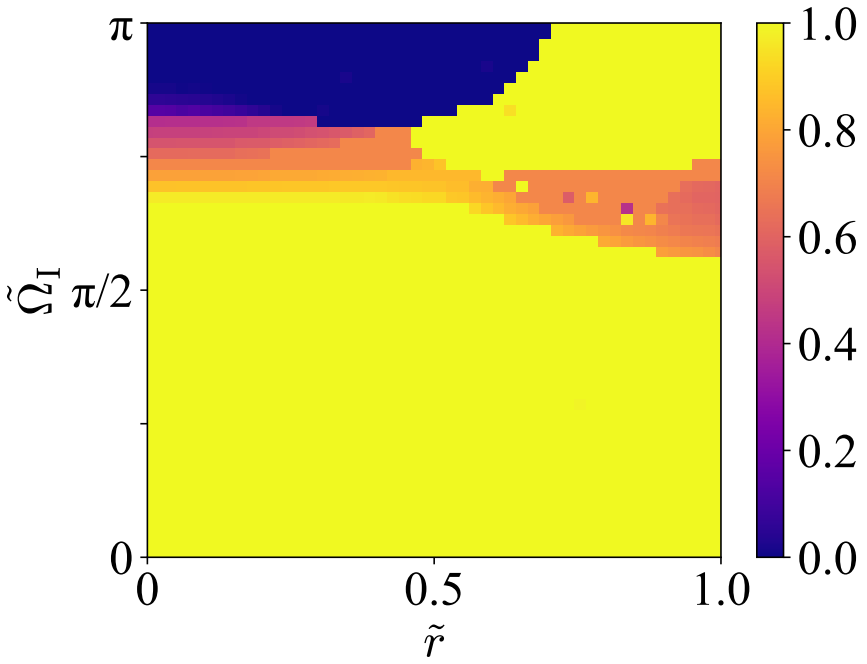
SU(3) full (2 flavor)



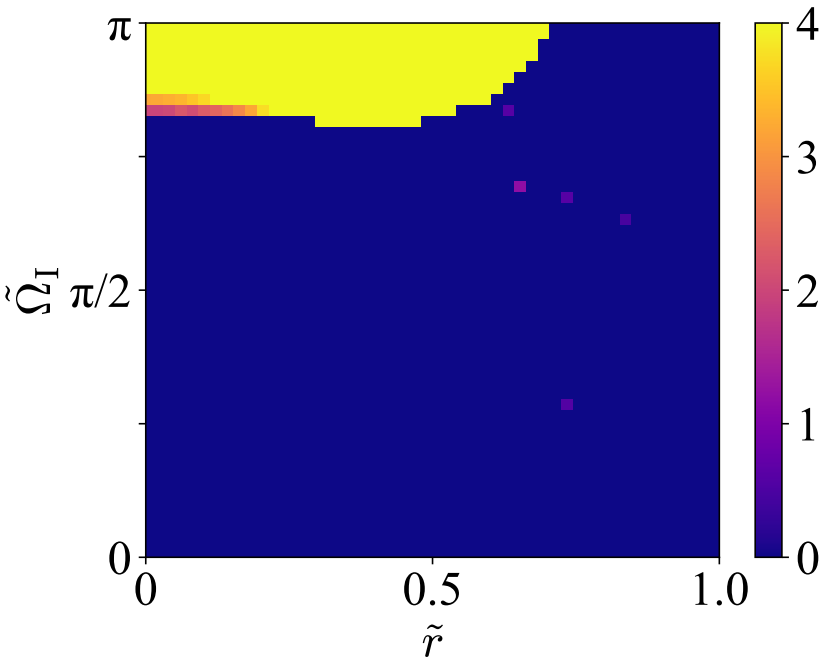
Almost correlated… but SU(3) is terribly complicated!

Including Fermions

Polyakov loop



Chiral condensate



Chen-Fukushima-Shimada (2024)

It seems that fermion mass dictates the Polyakov loop.

Talk by Singha

Magnetic Field + Rotation

Chen-Fukushima-Huang-Mameda (2015)

(I) First, we make a comment on the Lorentz force in a rotating frame. The gauge fields are transformed in a rotating frame into the following form:

$$A_\mu = A_i e^i_\mu = (-B\Omega r^2/2, By/2, -Bx/2, 0) \ , \qquad (9)$$

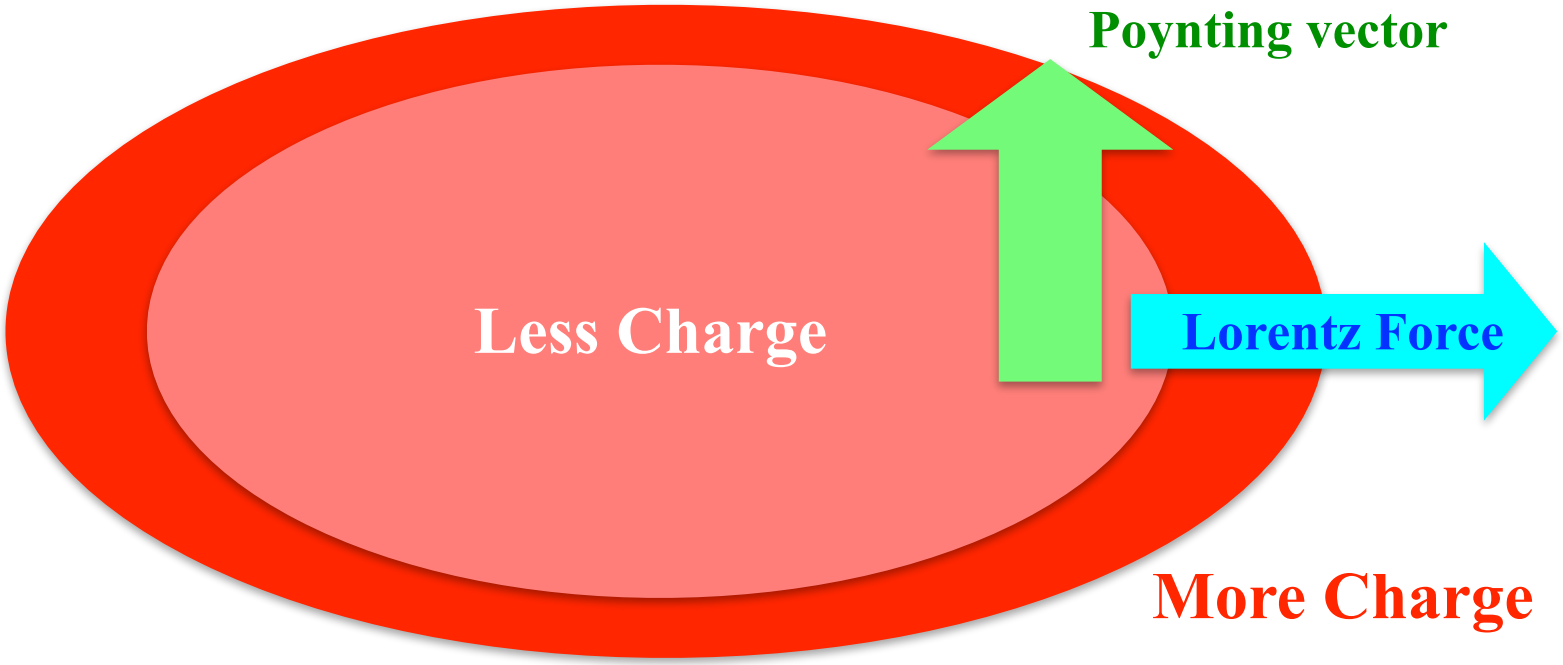
which leads to an electric field; $\boldsymbol{E} = -\boldsymbol{\nabla} A_0 = B\Omega(x, y, 0)$. Hence, naïvely, one may want to identify this $\boldsymbol{E}$ as the Lorentz force:

$$\boldsymbol{F} = e\boldsymbol{v} \times \boldsymbol{B} = eB\Omega(x, y, 0) \ , \qquad (10)$$

where $\boldsymbol{v} = \Omega(-y, x, 0)$ is the velocity vector at $(x, y, 0)$ caused by rotation. However, $A_0 = -B\Omega r^2/2$ does not appear in Eq. (5) because the gamma matrix $\gamma^t = \gamma^i e^t_i$ cancels it out. Therefore, rotation does not induce any electromagnetic effect. This is an important point that ensures our later discussion on the similarity between rotation and finite density for relativistic theories.

$B + \Omega$

$A_0 = -B\Omega r^2/2$ **is PHYSICAL!!!**



B and induced ***E*** makes a finite $\boldsymbol{J}_{\mathrm{EM}}$

$B + \Omega$

Fukushima-Hattori-Mameda (appear soon) **Talk by Mameda**

$$Z = \mathrm{tr}\exp\Big[-\beta(H - \Omega\mathcal{J})\Big]$$

Total angular momentum is the conserved quantity, but this corresponds to the off-equilibrium situation!

The partition function in equiliburium should be:

$$Z = \mathrm{tr}\exp\Big[-\beta(H_0 - \Omega\mathcal{J}_{\mathrm{kin}})\Big]$$ **Gauge Invariant!**

$$\mathcal{J}_{\mathrm{kin}} = \mathcal{J} - \mathcal{J}_{\mathrm{EM}}$$

Consistent with the covariant density operator.

Summary

Magnetic field → Precision science

- □ Inhomogeneous phase could be triggered.
- □ Hadron spectra changed by the magnetic field.

Partial agreement between pQCD and LQCD

- □ Radial dependence is qualitatively consistent.
- □ The remaining puzzle is the behavior at the center.
- □ At the center, only the spin makes the contribution…?

Rotating *B* system offers us a test example

- □ What is the principle to use either canonical/kinetic?