Lattice study of rotating QCD properties

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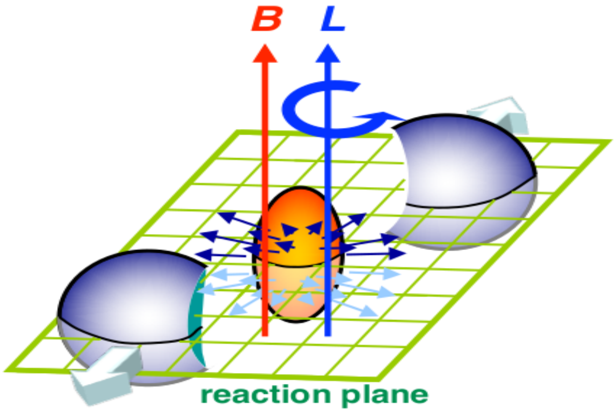
Outline:

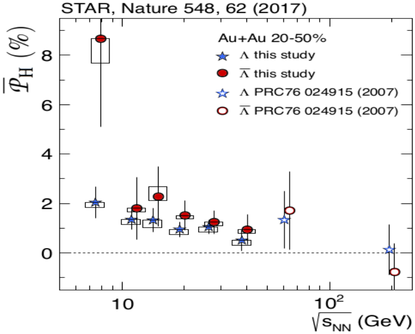
- Introduction
- Moment of inertia of QGP
- Inhomogeneous phase transitions in QGP
  - Local critical temperature
  - Decomposition of the action
  - Local thermalization
- Conclusion

Rotation of QGP in heavy ion collisions



- QGP is created with non-zero angular momentum in non-central collisions

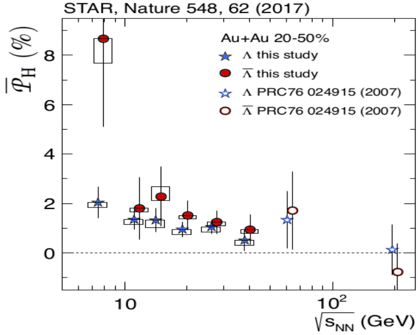
Rotation of QGP in heavy ion collisions



Angular velocity from STAR (Nature 548, 62 (2017))

- $\Omega = (P_\Lambda + P_{\bar{\Lambda}})\frac{k_B T}{\hbar}$ (Phys. Rev. C 95, 054902 (2017))
- $\Omega \sim 10$ MeV ($v \sim c$ at distances 10-20 fm, $\sim 9 \times 10^{21} s^{-1}$)
- Relativistic rotation of QGP

Rotation of QGP in heavy ion collisions



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- Relativistic rotation of QGP

How relativistic rotation influences QCD?

Theoretical studies

- ## Critical temperatures

  M.N. Chernodub, S. Gongyo, JHEP 01 (2017) 136
  Y. Jiang, J. Liao, Phys.Rev.Lett. 117 (2016) 19, 192302
  A. A. Golubtsova and N. S. Tsegelnik, Phys. Rev. D 107, 106017 (2023)
  X. Chen, L. Zhang, D. Li, D. Hou, and M. Huang, JHEP 07, 132
  X. Wang, M. Wei, Z. Li, and M. Huang, Phys. Rev. D 99, 016018 (2019)
  Y. Jiang, Eur. Phys. J. C 82, 949 (2022)
  K. Mameda and K. Takizawa, Phys. Lett. B 847, 138317 (2023)
  Y. Chen, X. Chen, D. Li, and M. Huang,arXiv:2405.06386
  P. Singha, V.E. Ambrus, M.N. Chernodub e-Print: 2407.07828
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- ## Moment of inertia

  M.N. Chernodub, S. Gongyo, Phys.Rev.D 95 (2017) 9, 096006
  Y. Fujimoto, K. Fukushima, Y. Hidaka, Phys.Lett.B 816 (2021) 136184
  V.E. Ambruş, M.N. Chernodub, Phys.Rev.D 108 (2023) 8, 085016
  E. Siri, N. Sadooghi, 2405.09481 [hep-ph]
  ...

- ## Inhomogeneous phase transition

  M.N. Chernodub, Phys.Rev.D 103 (2021) 5, 054027
  S. Chen, K. Fukushima, and Y. Shimada, (2024), arXiv:2404.00965 [hep-ph]
  Y. Jiang, (2024), arXiv:2406.03311 [nucl-th]
  N. R. F. Braga and O. C. Junqueira, Phys. Lett. B 848, 138330 (2024)
  ..

- ...

Lattice studies

- The first lattice study

  A. Yamamoto and Y. Hirono, Phys. Rev. Lett. 111, 081601 (2013)

- Critical temperature of gluodynamics

  V. Braguta, A. Kotov, D. Kuznedelev, A. Roenko, JETP Lett. 112 (2020) 1, 6
  V. Braguta, A. Kotov, D. Kuznedelev, A. Roenko, Phys.Rev.D 103 (2021) 9, 094515

- Critical temperatures in QCD

  V. Braguta, A. Kotov, A. Roenko, D. Sychev, PoS LATTICE2022 (2023) 190
  Ji-Chong Yang, Xu-Guang Huang, e-Print: 2307.05755

- Moment of inertia

  V. Braguta, M. Chernodub, A. Roenko, D. Sychev, Phys.Lett.B 852 (2024) 138604
  V. Braguta, M. Chernodub, I. Kudrov, A. Roenko, D. Sychev, JETP Lett. 117 (2023) 9
  V. Braguta, M. Chernodub, I. Kudrov, A. Roenko, D. Sychev, e-Print: 2310.16036

- Inhomogeneous phase transition

  V. Braguta, M. Chernodub, A. Roenko, Phys.Lett.B 855 (2024) 138783
  V. Braguta, M. Chernodub, A. Roenko, to be published

Study of rotating QGP

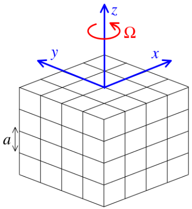
- Our aim: study rotating QCD within lattice simulations
- Rotating QCD at thermodynamic equilibrium
  - At the equilibrium the system rotates with some Ω
  - The study is conducted in **the reference frame which rotates with QCD matter**
  - QCD in external gravitational field
- **Boundary conditions are very important!**

Details of the simulations

- Gluodynamics is studied at thermodynamic equilibrium in external gravitational field
- The metric tensor

$$g_{\mu\nu} = \begin{pmatrix} 1 - r^2\Omega^2 & \Omega y & -\Omega x & 0 \\ \Omega y & -1 & 0 & 0 \\ -\Omega x & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

- Geometry of the system: $N_t \times N_z \times N_x \times N_y = N_t \times N_z \times N_s^2$



Details of the simulations

- Partition function ($\hat{H}$ is conserved)

$$Z = \mathrm{Tr}\, \exp\left[-\beta \hat{H}\right] = \int DA\ \exp\left[-S_G\right]$$

- Euclidean action(in the cylindrical coordinates)

$$\begin{aligned} S_G \quad = \quad & \frac{1}{2g^2} \int d^4x \Big[(F^a_{\tau r})^2 + (F^a_{\tau \hat{\varphi}})^2 + (F^a_{\tau z})^2 + \\ & + (F^a_{rz})^2 + \left(1 - (\Omega r)^2\right)(F^a_{\hat{\varphi} z})^2 + \left(1 - (\Omega r)^2\right)(F^a_{r \hat{\varphi}})^2 + \\ & + 2ir\Omega (F^a_{r\hat{\varphi}} F^a_{\tau r} - F^a_{\hat{\varphi} z} F^a_{\tau z}) \Big] \end{aligned}$$

- Decomposition of the action

$$\begin{aligned} S_G \quad &= \quad S_0 + S_1 \Omega + S_2 \Omega^2 \\ S_1 \quad &= \quad \frac{i}{g^2} \int d^4x\ r \left[F^a_{r\hat{\varphi}} F^a_{\tau r} - F^a_{\hat{\varphi} z} F^a_{\tau z} \right] \\ S_2 \quad &= \quad -\frac{1}{2g^2} \int d^4x\ r^2 \left[(F^a_{\hat{\varphi} z})^2 + (F^a_{r\hat{\varphi}})^2 \right] \end{aligned}$$

Details of the simulations

- *Ehrenfest–Tolman effect:* **In gravitational field the temperature is not constant in space at thermal equilibrium**

$$T(r)\sqrt{g_{00}} = const = 1/\beta$$

$$T(r)\sqrt{1 - r^2\Omega^2} = 1/\beta$$

- Rotation effectively heats the system from the rotation axis to the boundaries $T(r) > T(r = 0)$
- **One could expect that rotation decreases the critical temperature**
- We use the designation $T = T(r = 0) = 1/\beta$

Details of the simulations

Boundary conditions

- **Periodic b.c.:**
  - $U_{x,\mu} = U_{x+N_i,\mu}$
  - Not appropriate for the field of velocities of rotating body
- **Dirichlet b.c.:**
  - $U_{x,\mu}\big|_{x\in\Gamma} = 1, \quad A_\mu\big|_{x\in\Gamma} = 0$
  - Violate Z_3 symmetry
- **Neumann b.c.:**
  - Outside the volume $U_P = 1, \quad F_{\mu\nu} = 0$
- The dependence on boundary conditions is the property of all approaches
- One can expect that boundary conditions influence our results considerably, but their influence is restricted due to the screening

Details of the simulations

Sign problem

$$
\begin{aligned}
S_G \quad = \quad & \frac{1}{2g^2}\int d^4x \Big[(F^a_{\tau r})^2 + (F^a_{\tau\hat{\varphi}})^2 + (F^a_{\tau z})^2 + \\
& + (F^a_{rz})^2 + \big(1-(\Omega r)^2\big)(F^a_{\hat{\varphi}z})^2 + \big(1-(\Omega r)^2\big)(F^a_{r\hat{\varphi}})^2 + \\
& + 2ir\Omega(F^a_{r\hat{\varphi}}F^a_{\tau r} - F^a_{\hat{\varphi}z}F^a_{\tau z}) \Big]
\end{aligned}
$$

- The Euclidean action has imaginary part (**sign problem**)
- Simulations are carried out at imaginary angular velocities $\Omega \to i\Omega_I$
- The results are analytically continued to real angular velocities
- This approach works up to sufficiently large Ω

EoS of rotating gluodynamics

- Free energy of rotating QGP

$$F(T,R,\Omega) = F_0(T,R) + C_2\Omega^2 + ...$$

- The moment of inertia

$$C_2 = -\frac{1}{2} I_0(T,R), \quad I_0(T,\Omega) = -\frac{1}{\Omega}\left(\frac{\partial F}{\partial \Omega}\right)_{T,\Omega\to 0}$$

- Instead of $I_0(T,R)$ we calculate $K_2 = -\frac{I_0(T,R)}{F_0(T,R)R^2}$
- Sign of K_2 coincides with the sign of $I_0(T,R)$
- Sometimes instead of Ω^2 we use $v^2 = (\Omega R)^2$ and $v_I^2 = (\Omega_I R)^2$

EoS of rotating gluodynamics

- Classical moment of inertia

$$I_0(R) = \int_V d^3x x_\perp^2 \rho_0(x_\perp)$$

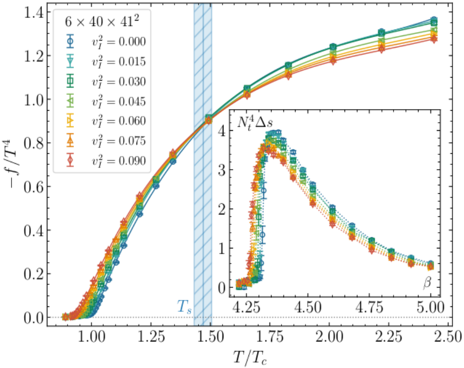
- Related to the trace of EMT $T^\mu_\mu = \rho_0(x_\perp)c^2$
- Generation of mass scale in QCD and scale anomaly

$$T^\mu_\mu \sim \langle G^2 \rangle \sim \langle H^2 + E^2 \rangle$$

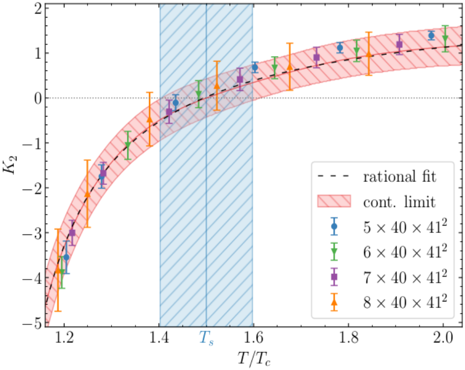
- In QCD the gluon condensate $\langle G^2 \rangle \neq 0$
- *One could anticipate:* $\rho_0 \sim \langle H^2 + E^2 \rangle$?
- $I_0 = I_{mech} + I_{magn}$ *valid for QCD!*
  $I_{mech} = \langle J_z^2 \rangle - (\langle J_z \rangle)^2 \sim \langle {S_1}^2 \rangle$
  $I_{magn} = \frac{1}{3} \int d^3x r^2 \langle H^2 \rangle \sim \langle S_2 \rangle$

Calculation of free energy on the lattice

- $F = -T \log Z$ impossible to calculate on the lattice
- $\frac{\partial F}{\partial \beta} \sim \langle \Delta s(\beta) \rangle = s(\beta)_T - s(\beta)_{T=0}, \quad \beta = \frac{6}{g^2}$
- $\frac{F(T)}{T^4} \sim \int_{\beta_0}^{\beta_1} d\beta' \langle \Delta s(\beta') \rangle$

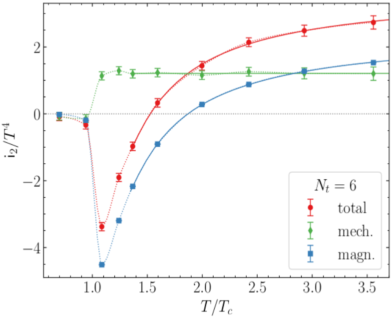


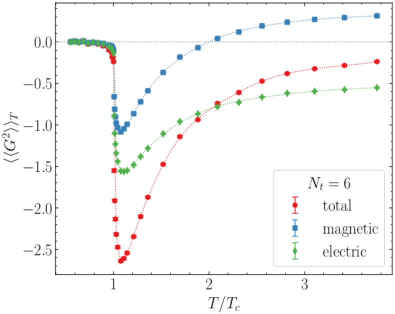
Moment of inertia of gluon plasma



- $I(T,R) = -F_0(T,R)K_2R^2$
- $I < 0$ for $T < 1.5T_c$ and $I > 0$ for $T > 1.5T_c$
- $I < 0$ is related to magnetic condensate and the scale anomaly
- We believe that the same is true for QCD

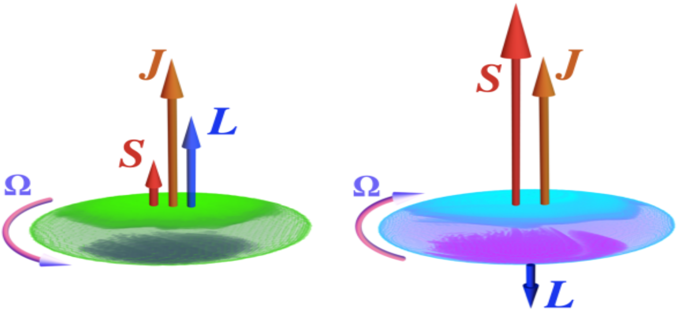
Moment of inertia of gluon plasma





- $i_2 = \frac{I_0}{VR_\perp^2}, \quad I_0 = I_{mech} + I_{magn}$
  $I_{mech} = \langle J_z^2\rangle - (\langle J_z\rangle)^2$
  $I_{magn} = \frac{1}{3}\int d^3xr^2\langle H^2\rangle$
- Gluon condensate: $\langle G^2\rangle = \langle E^2\rangle + \langle H^2\rangle$

Negative Barnett effect(?)



- $J = I_2\Omega = -\left(\frac{\partial F}{\partial \Omega}\right)_T, \quad \mathbf{J} = \mathbf{L} + \mathbf{S}$
- $\mathbf{L} \uparrow\uparrow \mathbf{\Omega}, \quad \mathbf{S} \uparrow\downarrow \mathbf{\Omega}$ might lead to $\mathbf{J} \uparrow\downarrow \mathbf{\Omega}$ and $I_2 < 0$

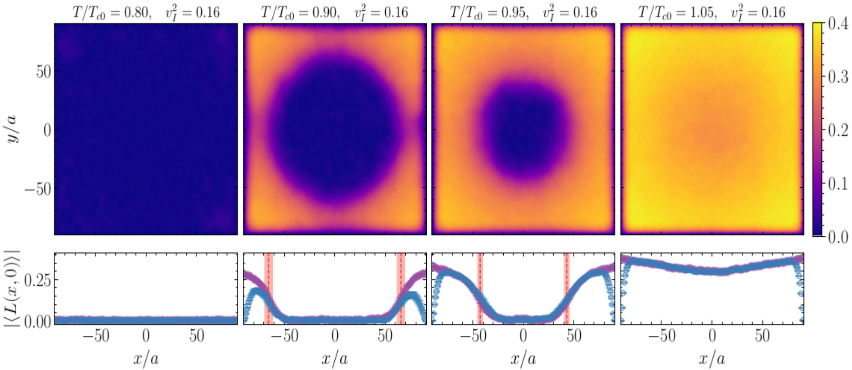
Inhomogeneous phase transition in QGP

- Ehrenfest–Tolman law

$$T(r) = \frac{T_0}{\sqrt{1-(\Omega r)^2}} = \frac{T_0}{\sqrt{1+(\Omega_I r)^2}}$$

- Rotation effectively heats the system: $T(r) > T(r=0)$
- Inhomogeneous phase: confinement in the center and deconfinement in the periphery
  (M. Chernodub, Phys. Rev. D 103, 054027 (2021))
- For imaginary rotation: deconfinement/confinement in the center/periphery

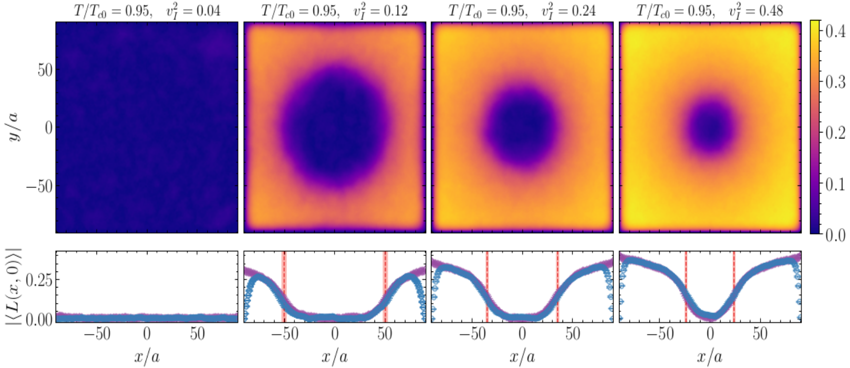
Inhomogeneous phase transition in QGP



- Huge lattices are required for simulations
- Cylindrical Symmetry is restored
- The results for PBC and OBC coincides in the bulk
- Confinement in the center and deconfinement in the periphery

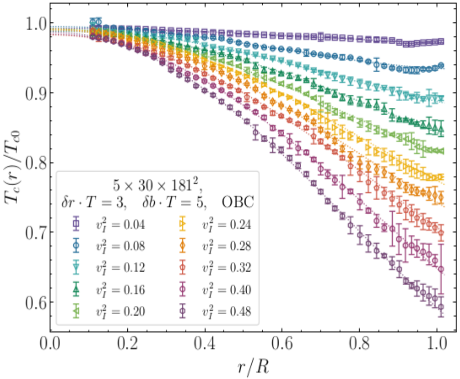
  In disagreement with Ehrenfest–Tolman law
- Inhomogeneous phase takes place below T_c

Inhomogeneous phase transition in QGP



- The phase transition is induced by rotation

Local critical temperature $T_c(r, \Omega_I)$

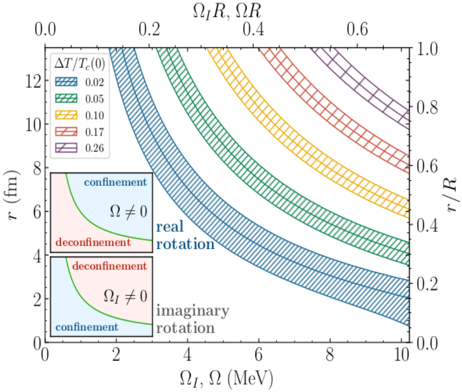


- Our results can be well described by the formula

$$\frac{T_c(r,\Omega_I)}{T_{c0}} = 1 - \kappa_2(\Omega_I r)^2 + \kappa_4(\Omega_I r)^2 \left(\frac{r}{R}\right)^2 + \chi_4(\Omega_I r)^4 + \ldots$$

- Within the uncertainty $\frac{T_c(r=0,\Omega_I)}{T_{c0}} = 1$
- Weak dependence on the simulation parameters

Analytical continuation to real rotation



- Analytical continuation $\Omega_I^2 \to -\Omega^2$:

$$\frac{T_c(r,\Omega)}{T_{c0}} = 1 + \kappa_2 (\Omega r)^2$$

- Inhomogeneous phase can be realised for $T > T_{c0}$
- Deconfinement in the center and confinement in the periphery

Decomposition of the action

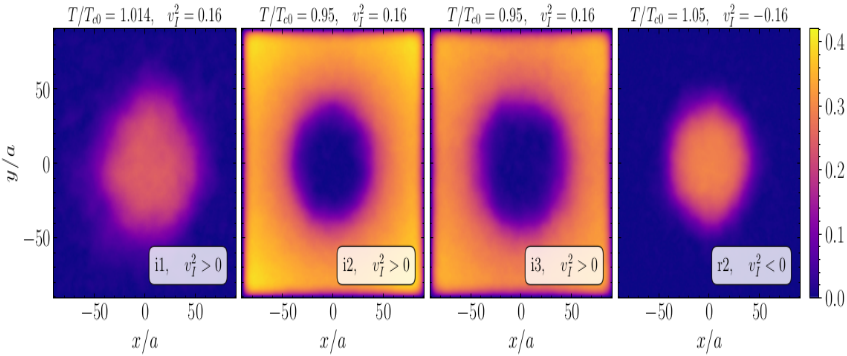
- Rotating action in the cylindrical coordinates

$$S = S_0 + S_1 \Omega_I + S_2 \Omega_I^2$$

  - $S_1 = -\frac{1}{g^2} \int d^4x \; r \left[F^a_{r\hat{\varphi}} F^a_{\tau r} - F^a_{\hat{\varphi} z} F^a_{\tau z} \right]$
  - $S_2 = \frac{1}{2g^2} \int d^4x \; r^2 \left[(F^a_{\hat{\varphi} z})^2 + (F^a_{r\hat{\varphi}})^2 \right]$
- S_1 is the total angular momentum and gives $I > 0$
- S_2 is the centrifugal force and gives $I < 0$

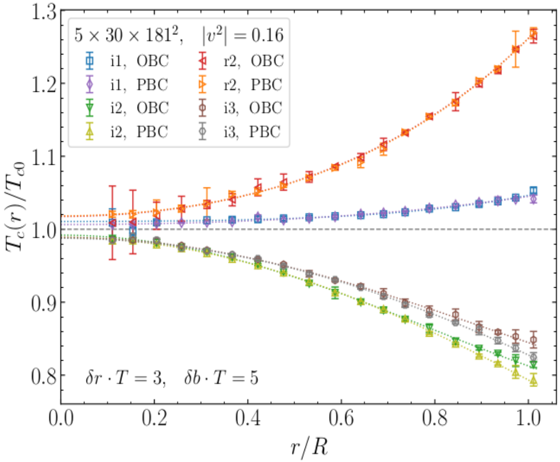
How S_1 and S_2 influence on the inhomogeneous phase transition?

Decomposition of the action



- S_2 is similar to the total acton and gives the dominant contribution
- S_1 effect is the opposite to the the total acton

Decomposition of the action



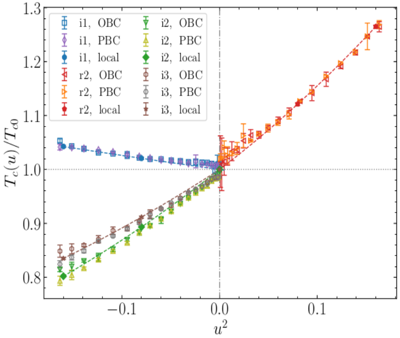
- S_1 increases the local critical temperature
- S_2 decreases the local critical temperature
- The contribution of S_2 is dominant

Local thermalization hypothesis

$$S = \frac{1}{2g^2}\int d^4x \Big[(F^a_{\tau r})^2 + (F^a_{\tau\hat{\varphi}})^2 + (F^a_{\tau z})^2 + (F^a_{rz})^2 + \\ + \left(1-(\Omega r)^2\right)(F^a_{\hat{\varphi}z})^2 + \left(1-(\Omega r)^2\right)(F^a_{r\hat{\varphi}})^2 + \\ + 2ir\Omega(F^a_{r\hat{\varphi}}F^a_{\tau r} - F^a_{\hat{\varphi}z}F^a_{\tau z}) \Big]$$

- For slow rotation $\Omega\zeta \ll 1$ the coefficients vary slowly
- Local thermalization approximation: study the action with the coefficients freezed at $r = r_0$

Local thermalization hypothesis



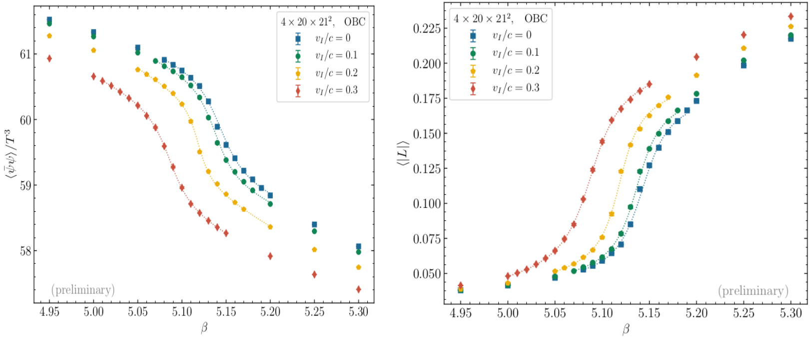
- Good agreement with the full action for sufficiently small Ω
- A lot of advantages
  - The higher order coefficients can be found
    $T_c(r,\Omega)/T_{c0} = 1 + \sum_n c_n(\Omega r)^{2n}$, $\quad T_c(r=0,\Omega)/T_{c0} = 1$
  - Weak dependence on the BC
  - One can study small lattices
  - Allows to understand inhomogeneous phase transition

Origin of the inhomogeneous phase transition

$$S_G = \int d^4x \, \Big[\beta \left((F^a_{x\tau})^2 + (F^a_{y\tau})^2 + (F^a_{z\tau})^2 + (F^a_{xz})^2 \right) + \\ + \tilde{\beta} \left((F^a_{yz})^2 + (F^a_{xy})^2 \right) \Big]$$

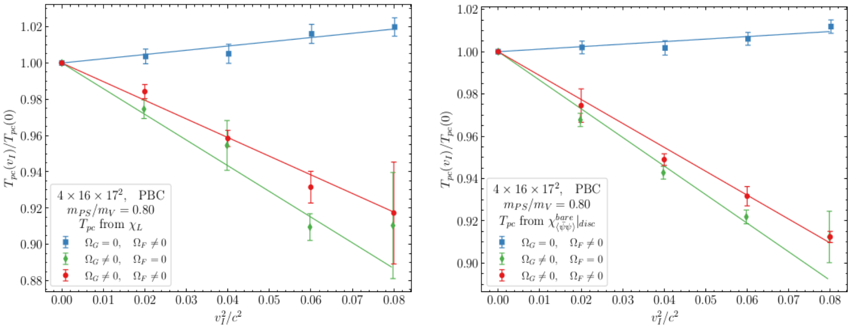
- Linear in Ω term can be neglected
- External gravitational field leads to the asymmetric action
  $\beta = \frac{1}{2g^2}, \quad \tilde{\beta} = \frac{1}{2\tilde{g}^2}, \quad \frac{\tilde{\beta}}{\beta} = 1 - (\Omega r)^2$
- The asymmetry $\tilde{\beta}/\beta$ is larger in the periphery region leading to the shift of the critical temperature

Simulation with fermions



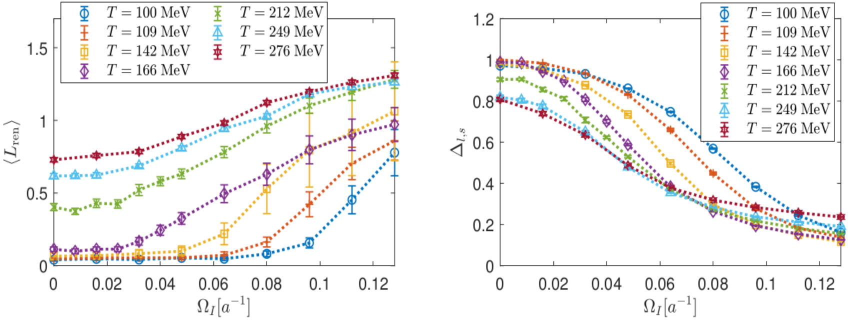
- Lattice simulation with Wilson fermions
- Critical couplings of both transitions coincide
- Critical temperatures are increased

Simulation with fermions



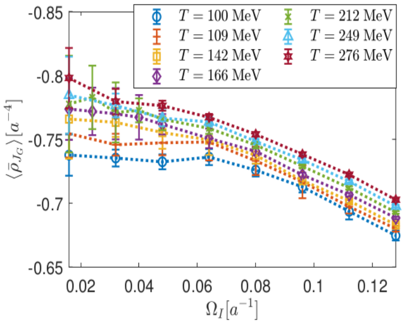
- QCD action: $S = S_f(\Omega_F) + S_g(\Omega_G)$
- One can introduce velocities for gluons Ω_G and fermions Ω_F
- $\Omega_F \neq 0, \Omega_G = 0$ decreases critical temperatures
- $\Omega_F = 0, \Omega_G \neq 0$ increases critical temperatures
- The gluon sector gives the dominant contribution

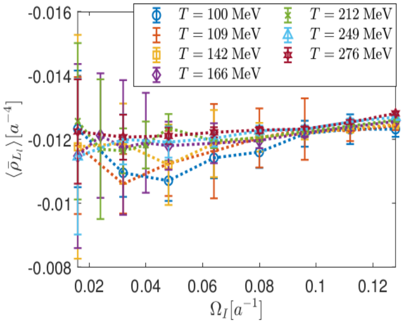
Simulation with fermions (e-Print: 2307.05755)



- Increase of the bulk average critical temperatures of both transitions

Simulation with fermions (e-Print: 2307.05755)





- Rotational rigidities: $\rho_{J_G} = \frac{J_G}{\Omega R^2}, \quad \rho_{L_f} = \frac{L_f}{\Omega R^2}$
- Spin susceptibility: $\zeta_f = \frac{s}{\Omega}$
- Negative moment of inertia

Conclusion

- Lattice study of rotating gluodynamics and QCD have been carried out
- We calculated the moment of inertia of GP. It is negative at temperatures $T < 1.5T_c$ and positive at larger temperatures
- We observed inhomogeneous phase transition in GP: deconfinement in the central and confinement in the periphery regions
- External gravitational field leads to asymmetryc action and shift of the critical temperature in the periphery regions
- We believe that all observed effects remain in QCD

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THANK YOU!