

# $\Lambda$ - $\bar{\Lambda}$ Spin Correlation in Heavy-ion Collisions

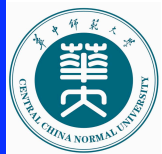


8<sup>th</sup> International Workshop on Chirality, Vorticity and Magnetic Field in Quantum Matter,  
July 22-26, 2024, West University of Timisoara, Romania

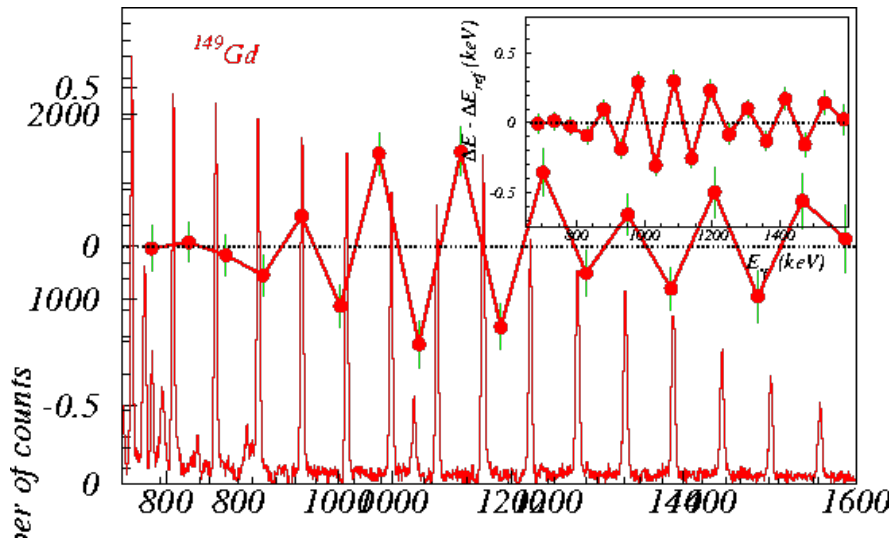
Xin-Nian Wang  
CCNU/LBNL

In collaboration with Xin-li Sheng and Xiangyu Wu

# Global Orbital Angular Momentum



Spins of superdeformed nuclei in low-energy nuclear collisions:



Janssens and Khoo, Ann. Rev. Nucl. Part. Sci. 41, 321 (1991).

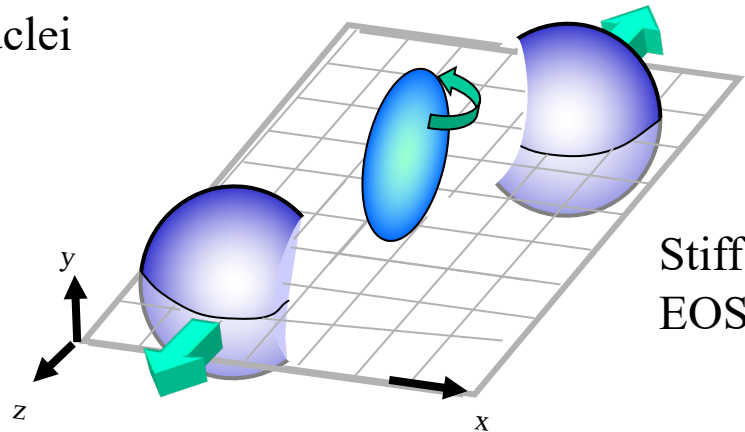
moments of inertial

I was a Changjiang visiting professor in Shandong University 2000-2005

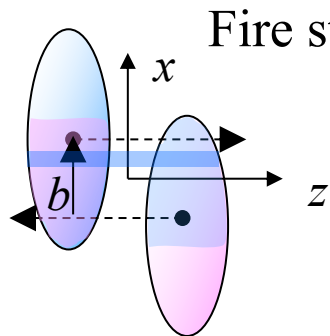
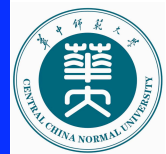


Zuo-tang Liang

Rotation of deformed nuclei



# Transverse gradient of fluid velocity & vorticity



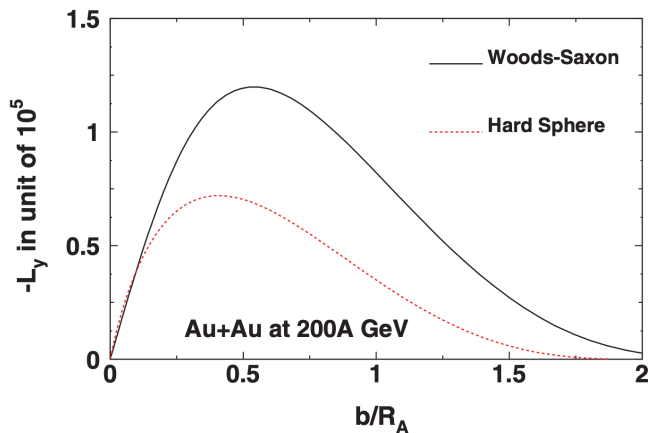
Fire streak model

$$p_z(x, b) = \frac{\sqrt{s}}{2} \frac{\frac{dN_{part}^P}{dx} - \frac{dN_{part}^T}{dx}}{c(s) \left( \frac{dN_{part}^P}{dx} + \frac{dN_{part}^T}{dx} \right)}$$

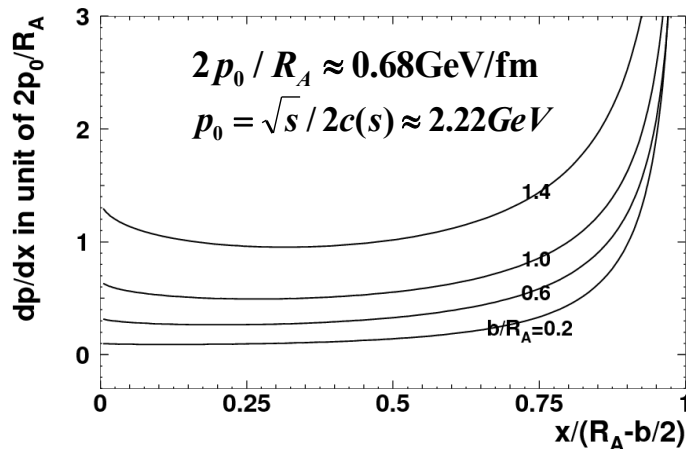
$$\vec{\omega} = \vec{\nabla} \times \vec{v} \sim -\hat{y} \left\langle \frac{1}{E} \frac{dp_z}{dx} \right\rangle$$

Collective longitudinal momentum  
per produced parton

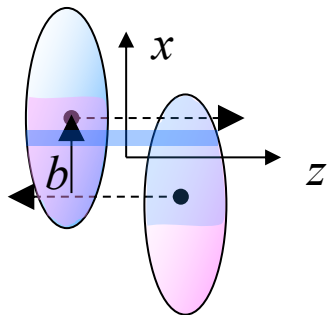
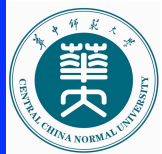
Liang & XNW, PRL 94 (2005) 102301



Total angular momentum carried by QGP



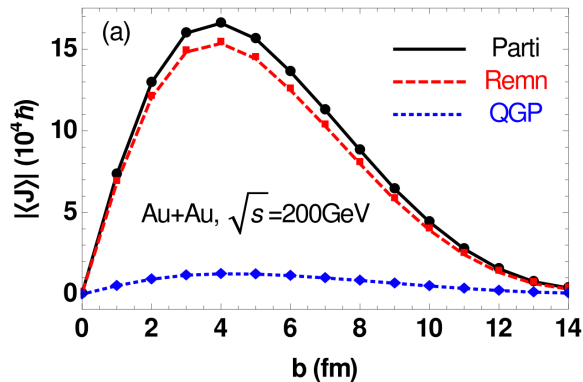
# Fluid velocity & vorticity in HIJING



No complete stopping but approximate Bjorken scaling.

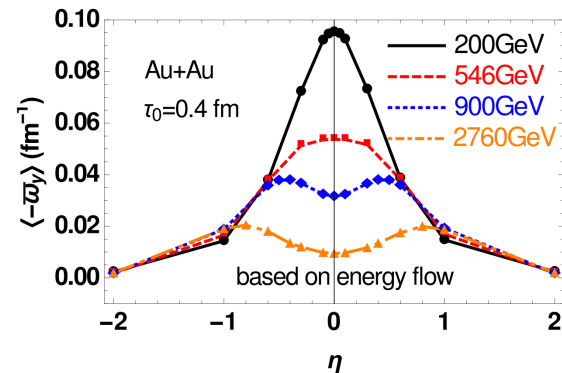
Small violation of BJ scaling at large  $y \rightarrow$  local angular momentum or vorticity

BJ scaling violation and vorticity increase at lower colliding energies



Gao, Chen, Deng, Liang, Q. Wang,  
*XNW, PRC 77 (2008)044902*

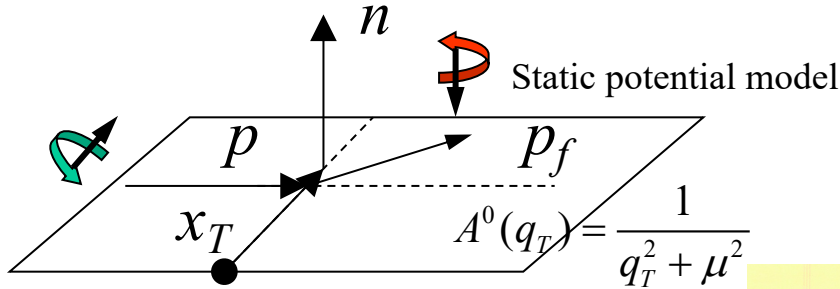
Pang, Petersen, Q. Wang and XNW,  
*PRL 117 (2016) 19, 192301*



Deng, Huang,  
*PRC 93 (2016) 6, 064907*

# Global spin polarization in A+A

Liang & XNW, PRL 94 (2005) 102301



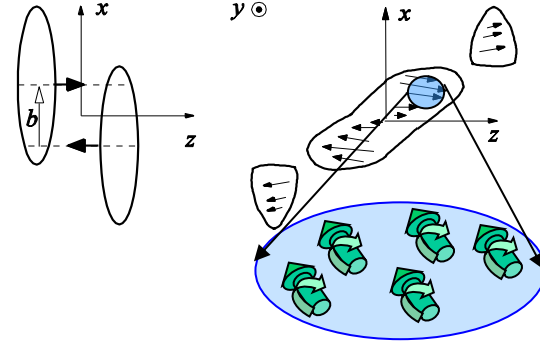
$$P_q \equiv \frac{\Delta\sigma}{\sigma} = -\pi \frac{\mu p}{4E(E + m_q)}$$

$p$ : relative momentum of parton scattering with impact parameter  $b \sim 1/\mu$

## spin-vorticity (orbital) coupling

nonrelativistic limit: ( $m_q \gg p, \mu$ )

$$P_q \approx -\pi \frac{\mu p}{4m_q^2} \sim -\omega/m \quad \text{or} \quad -\omega/T$$



Summary of gluon polarization

In a static potential model

$$M = \int \frac{d^3x}{2E} \bar{U}_N(\mathbf{q}, z) A^0(\mathbf{q}, z) U_N(\mathbf{q}, z)$$

$$A^0 = \frac{g}{(q^2 + \mu^2)}$$

$$\frac{d^2\sigma}{d^2q_T} = \frac{1}{(2\pi)^2} |M(\mathbf{q}, \mathbf{p})|^2$$

$$|M|^2 = \int \frac{d^3x}{2E} \int \frac{d^3x'}{2E} |\bar{U}_N(\mathbf{q}, z) A^0(\mathbf{q}, z) U_N(\mathbf{q}, z) \bar{U}_N(\mathbf{q}, z') A^0(\mathbf{q}, z') U_N(\mathbf{q}, z')|^2$$

$$= \int \text{Tr} [A^0(\mathbf{q}, z) A^0(\mathbf{q}, z')] \left( \frac{g}{(q^2 + \mu^2)} \right)^2$$

$$= g^4 \frac{G_T}{(q^2 + \mu^2)^2} \quad G_T \text{ - color factor}$$

Unpolarized cross section

$$\frac{d\sigma}{dq_T^2} = \frac{4\pi \alpha_s^2 G_T}{(q^2 + \mu^2)^2} = -\frac{d\sigma}{dt}$$

Averaging over  $\phi \in (-\frac{\pi}{2}, \frac{\pi}{2})$

$$\frac{1}{\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\phi \int db b K_0^2(\mu b) = \frac{1}{2\mu^2}$$

$$\frac{1}{\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\phi \cos\phi \int db b K_0(\mu b) K_1(\mu b) = 0$$

$$= \frac{2}{\pi} \frac{1}{\mu^2} \frac{\pi^2}{8} = \frac{1}{4} \frac{\pi}{\mu^2}$$

$$\langle P \rangle = \frac{\Delta\sigma}{\sigma} = \frac{-\mu p}{E(E+m)} \frac{\pi}{4}$$

$$= \begin{cases} -\frac{\mu}{E} \frac{\pi}{4} & p \gg m \\ -\frac{\pi p}{4} \frac{1}{m^2} & p \ll m \end{cases}$$

# Spin polarization in equilibrium & CME, CVE



Dirac Eq.  $[\gamma^\mu (i\partial_\mu + e_q A_\mu) - m] \psi(x) = 0$

Pu, Gao, Liang, Wang & XNW,  
PRL 109 (2012) 232301

Spin: vorticity coupling      Magnetic coupling

$$\delta E_s = \frac{\hbar}{2} \mathbf{n} \cdot \boldsymbol{\omega} + e_q \hbar \frac{\mathbf{n} \cdot \mathbf{B}}{E_p}$$

$$\Pi = \frac{1}{2} \int \frac{d^3 p}{(2\pi)^3} [f(E_p - \delta E_s) - f(E_p + \delta E_s)] \approx \int \frac{d^3 p}{(2\pi)^3} \delta E_s \frac{\partial f(E_p)}{\partial E_p}$$

Polarization on the freeze-out surface:

$$\frac{d\Pi^\alpha(p)/d^3 p}{d\rho(p)/d^3 p} = \frac{\hbar}{4m} \frac{\int d\Sigma_\lambda p^\lambda \tilde{\Omega}^{\alpha\sigma} p_\sigma f_{\text{FD}}(x, p) [1 - f_{\text{FD}}(x, p)]}{\int d\Sigma_\lambda p^\lambda f_{\text{FD}}(x, p)}$$

Becattini & Ferroni, EJPC 52 (2007) 597, Betz, Gyulassy & Torrieri, PRC 76 (2007) 044901, Becattini, Piccinini & Rizzo, PRC 77 (2008) 024906, Becattini, Csernai & Wang, PRC 87 (2013) 034905, Xie, Glastad & Csernai, PRC 92 (2015) 064901, Deng & Huang, arXiv 1603.06117

# Spin polarization and CME, CVE

$$[\gamma^\mu (i\partial_\mu + e_q A_\mu) - m] \psi(x) = 0$$

$$\gamma_\mu \left( p^\mu + \frac{1}{2} i \nabla^\mu \right) W(x, p) = 0,$$

Pu, Gao, Liang, Wang & XNW,  
PRL 109 (2012) 232301

Quantum kinetic equation

$$j^\mu = \int d^4 p \mathcal{V}^\mu = n u^\mu + \xi \omega^\mu + \xi_B B^\mu,$$

$$j_5^\mu = \int d^4 p \mathcal{A}^\mu = n_5 u^\mu + \xi_5 \omega^\mu + \xi_{B5} B^\mu.$$

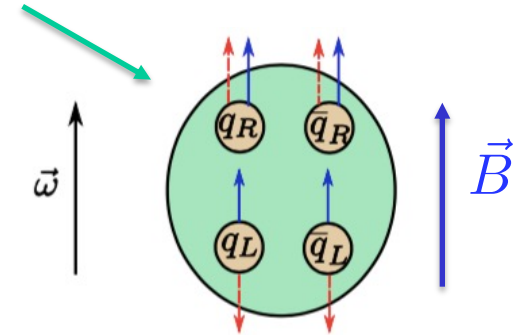
$$\partial_\mu j^\mu = 0, \quad \partial_\mu j_5^\mu = -\frac{Q^2}{2\pi^2} E \cdot B,$$

Chiral Magnetic Effect

Kharzeev, McLerran, Warringa (2008),  
Fukushima, Kharzeev, Warringa (2008)

Chiral Vorticity Effect

Son, Surowka (2009), Kharzeev, Son (2011)



Local spin polarization

# The most vortical fluid in nature

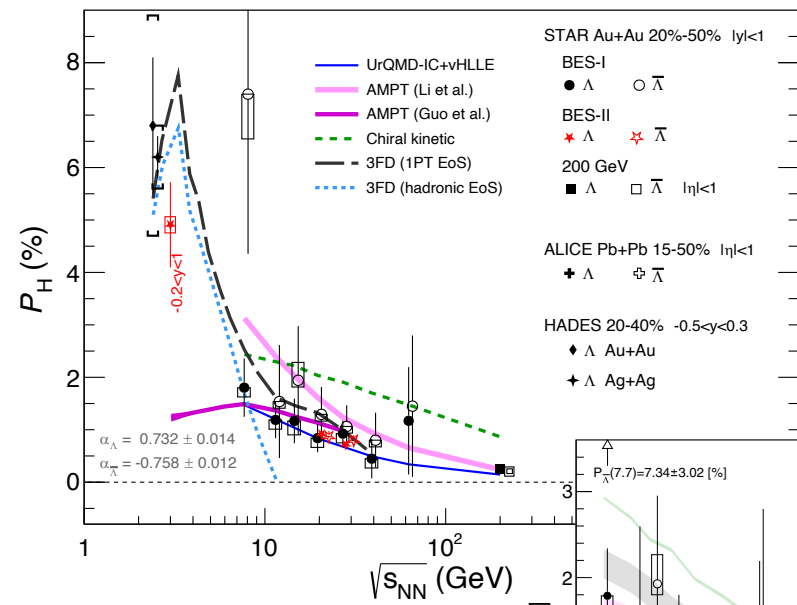
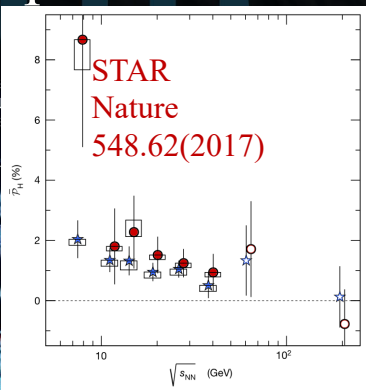
## Global hyperon polarization

THE INTERNATIONAL WEEK

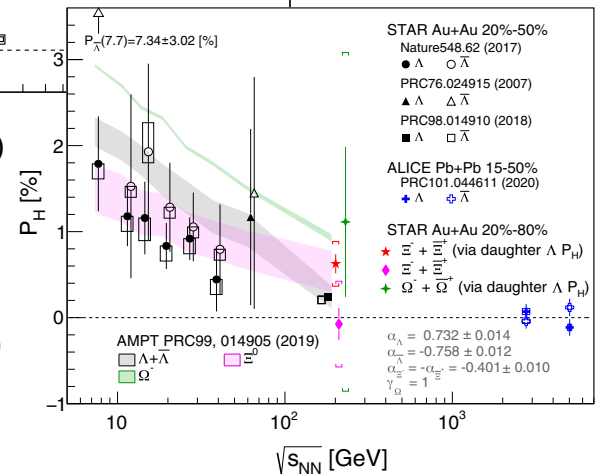
First observation of fluid vortices formed by heavy-ion collisions  
PAGES 34 & 62

### SUBATOMIC SWIRLS

$\omega \approx (9 \pm 1) * 10^{21}$ /second

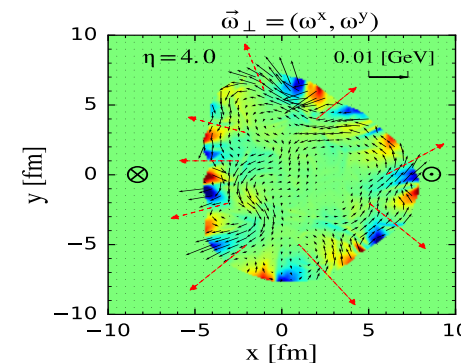
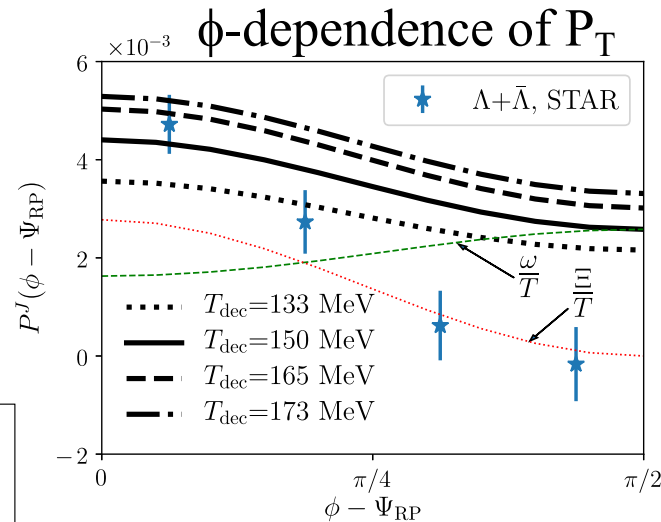
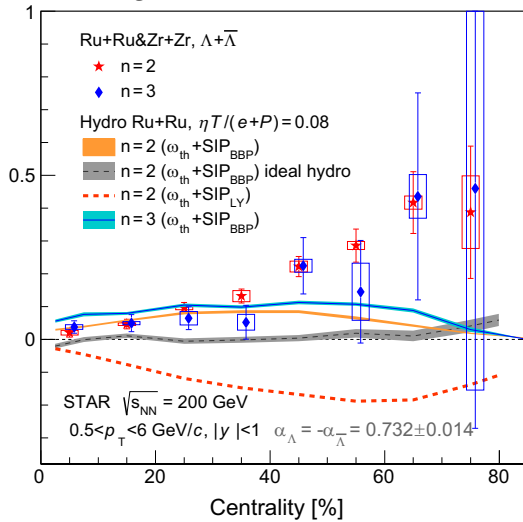
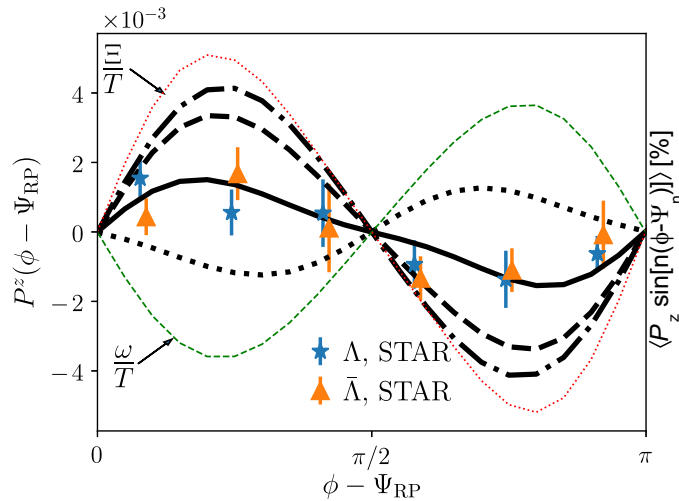
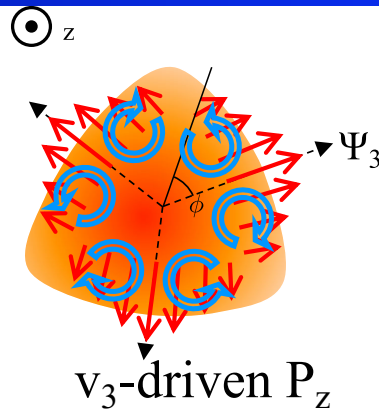
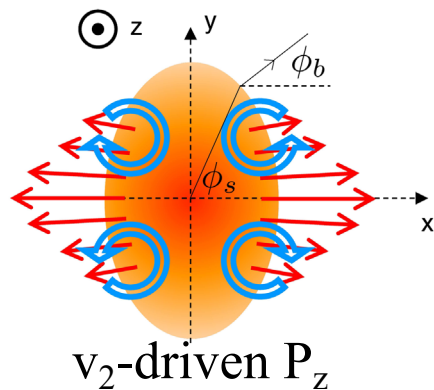
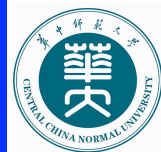


Becattini, Buzzegoli, Niida, Pu, Tang and Wang, arXiv:2402.04540





# Local spin polarization



# Vector meson spin alignment



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Physics Letters B 629 (2005) 20–26

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## Spin alignment of vector mesons in non-central A + A collisions

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Available online 3 October 2005

### Simple recombination model

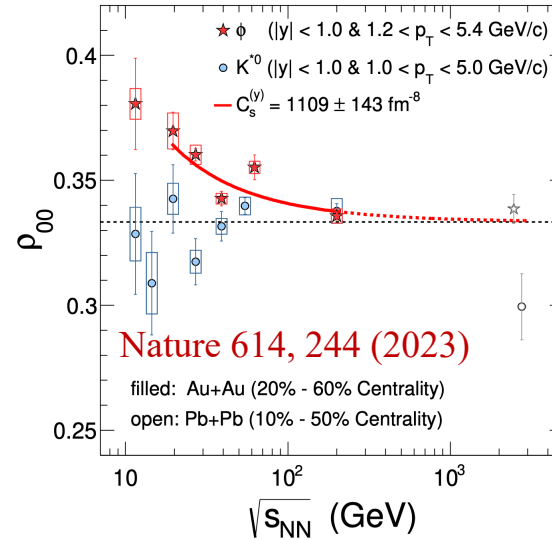
$$\rho_{00} = \frac{1 - P_q P_{\bar{q}}}{3 + P_q P_{\bar{q}}} \leq \frac{1}{3}$$

### More sophisticated recombination model

$$\rho^{00} \approx \frac{1}{3} - \frac{4}{9} \langle P_q(\mathbf{x}_q, \mathbf{p}_q) P_{\bar{q}}(\mathbf{x}_{\bar{q}}, \mathbf{p}_{\bar{q}}) \rangle$$

Sheng, Wang and XNW, Phys. Rev. D 102, 056013 (2020)

## STAR: Large $\phi$ meson spin alignment



Too big to be explained by vorticity, EM, etc

$$P_q(\bar{q}) \approx \frac{1}{4m_q} \epsilon^{\mu\nu\rho\sigma} \left[ \omega \pm \frac{e_q}{(u \cdot p)T} F_{\rho\sigma} \right] p_\nu$$

# Polarization via strong interaction force



Chiral quark model: **Manohar and Georgi (1984)**

Effective interaction between quarks, gluon and Goldstone boson between  $\Lambda_\chi$  and  $\Lambda_{QCD}$

$$\mathcal{L} = \bar{\psi} \gamma^\mu (iD_\mu + V_\mu + g_A A_\mu \gamma_5) \psi - m \bar{\psi} \psi + \frac{1}{4} f^2 \text{tr} \partial_\mu \Sigma^\dagger \partial^\mu \Sigma - \frac{1}{2} \text{tr} F_{\mu\nu} F^{\mu\nu} + \dots$$

$$V_\mu = \frac{1}{2} (\xi^\dagger \partial_\mu \xi + \xi \partial_\mu \xi^\dagger) \quad A_\mu = \frac{i}{2} (\xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger)$$

$$\xi = e^{iM/f} \quad M = \frac{1}{\sqrt{2}} \begin{bmatrix} \sqrt{\frac{1}{2}} \pi^0 + \sqrt{\frac{1}{6}} \eta & \pi^+ & K^+ \\ \pi^- & -\sqrt{\frac{1}{2}} \pi^0 + \sqrt{\frac{1}{6}} \eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}} \eta \end{bmatrix}$$

$$P_{q(\bar{q})}^\mu \approx \frac{1}{4m_q} \epsilon^{\mu\nu\rho\sigma} \left[ \omega_{\rho\sigma} \pm \frac{e_q}{(u \cdot p)T} F_{\rho\sigma} \pm \frac{g_V}{(u \cdot p)T} F_{\rho\sigma}^V \right] p_\nu$$

↑  
Strong interaction

# Polarization via strong interaction force



Spin Boltzmann transport equation with quark coalescence

Sheng, Oliva, Liang, Wang and XNW, PRL 131, 042304 (2023)

$$k \cdot \partial_x f_{\lambda_1 \lambda_2}^V(x, \mathbf{k}) = \frac{1}{8} \left[ \epsilon_\mu^*(\lambda_1, \mathbf{k}) \epsilon_\nu(\lambda_2, \mathbf{k}) \mathcal{C}_{\text{coal}}^{\mu\nu}(x, \mathbf{k}) - \mathcal{C}_{\text{diss}}(\mathbf{k}) f_{\lambda_1 \lambda_2}^V(x, \mathbf{k}) \right],$$

$$f_{\lambda_1 \lambda_2}^V(x, \mathbf{k}) \sim \frac{1}{\mathcal{C}_{\text{diss}}(\mathbf{k})} \left[ 1 - e^{-\mathcal{C}_{\text{diss}}(\mathbf{k}) \Delta t} \right] \epsilon_\mu^*(\lambda_1, \mathbf{k}) \epsilon_\nu(\lambda_2, \mathbf{k}) \mathcal{C}_{\text{coal}}^{\mu\nu}(x, \mathbf{k})$$

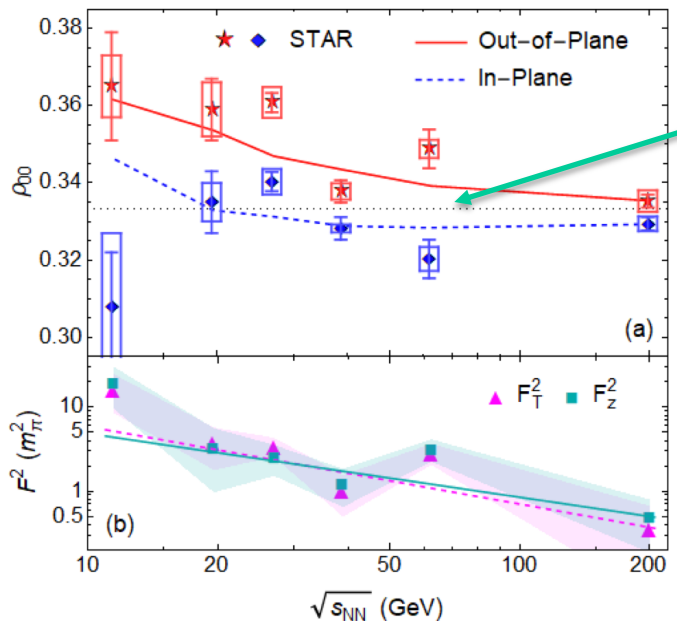
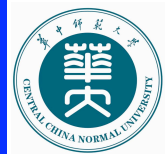
Spin alignment on the hadronization hyper surface

$$\rho_{00} \approx \frac{1}{3} + \frac{g_\phi^2}{m_\phi^2 T_{\text{eff}}^2} (C_1 B_\phi^2 + C_2 E_\phi^2) + \dots$$

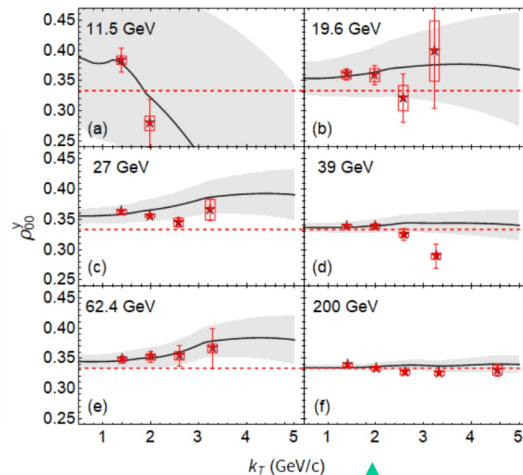
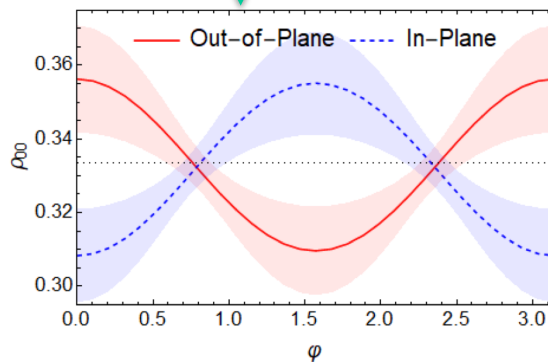
Momentum-dependence

$B_\phi^2, E_\phi^2$  rest frame  $\rightarrow$  collisions frame

# Barometer of strong force field fluctuations



In and out-plane splitting caused by  $v_2$  of the vector meson



$k_T$  dependence of  $\rho_{00}$  dictated by vector meson's spectra

$$\langle (g_\phi \mathbf{B}_{x,y}^\phi / T_h)^2 \rangle = \langle (g_\phi \mathbf{E}_{x,y}^\phi / T_h)^2 \rangle \equiv F_T^2$$

$$\langle (g_\phi \mathbf{B}_z^\phi / T_h)^2 \rangle = \langle (g_\phi \mathbf{E}_z^\phi / T_h)^2 \rangle \equiv F_z^2$$

# Hyperon spin correlations



Simple quark model of hyperon spin  $P_H^\mu(x, p) \approx P_s^\mu(x, R_s p), p_s = R_s p$

Hyperon spin correlation (H:  $\Lambda, \bar{\Lambda}$ )

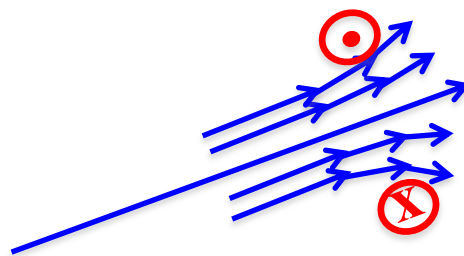
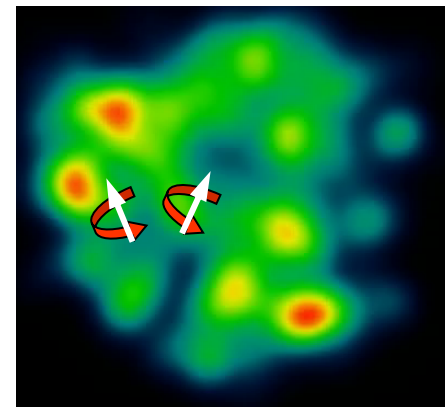
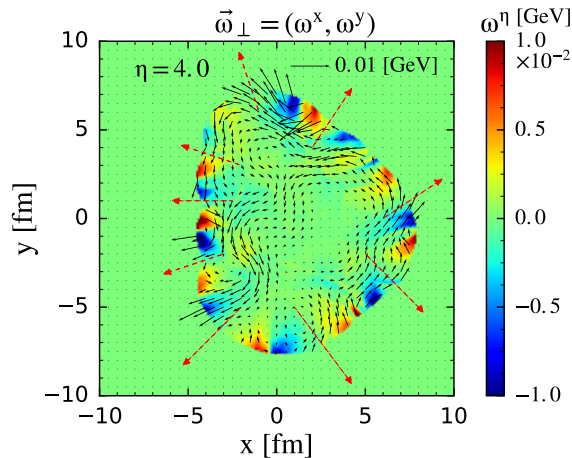
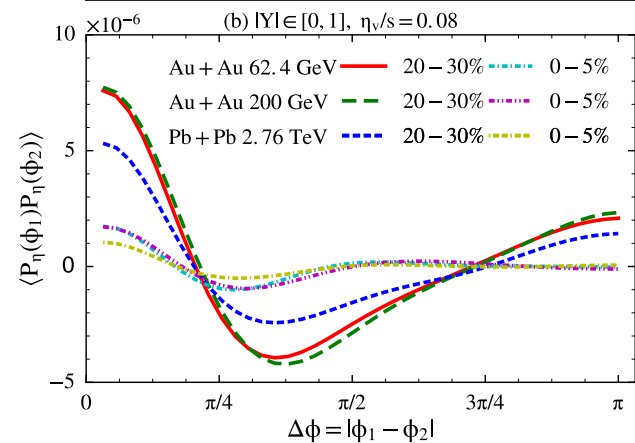
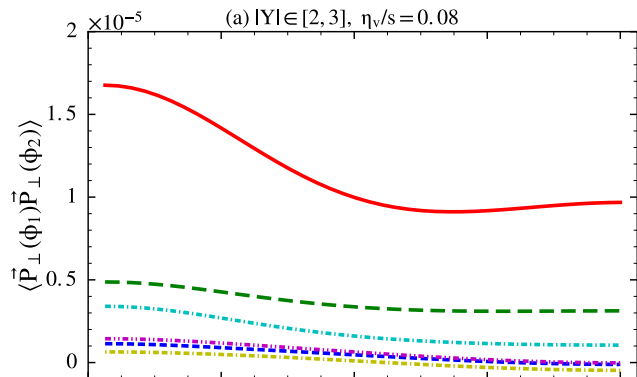
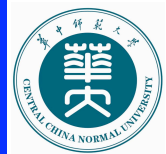
$$C_{H_1 H_2}^{\mu\nu}(p_1, p_2) \equiv \langle \mathcal{P}_{H_1}^\mu(p_1) \mathcal{P}_{H_2}^\nu(p_2) \rangle$$

$\langle \dots \rangle$  average over freeze-out hypersurface

Short-distance correlation of the  $\phi$  field

$$\langle g_\phi^2 F^\phi(x) F^\phi(y) / [T(x)T(y)] \rangle = F^2 G(x - y)$$
$$G(x - y) \equiv \exp \left[ -\frac{(x^0 - y^0)^2}{\sigma_t^2} - \frac{(\mathbf{x} - \mathbf{y})^2}{\sigma_x^2} \right]$$

# Hyperon spin correlations



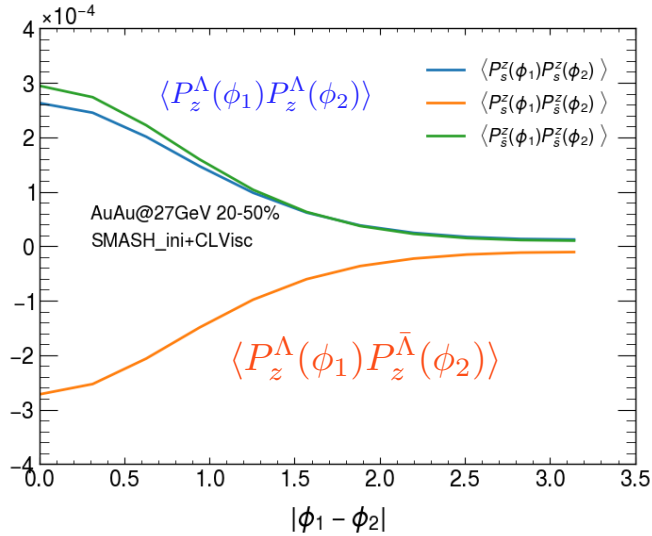
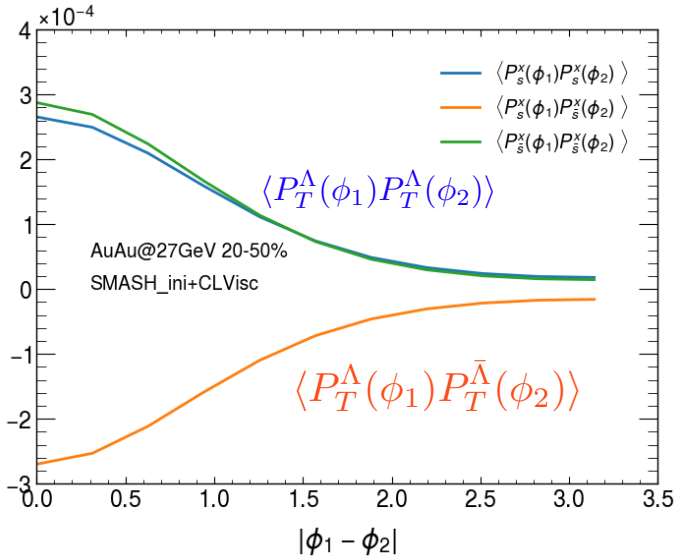
Pang, Petersen, Wang & XNW,  
PRL 117(2016) 192301

Vorticity ring:

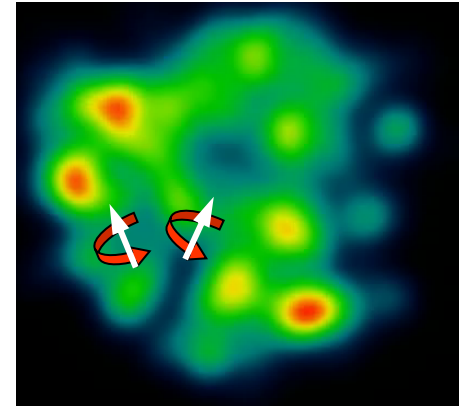
Lisa, et al, PRC 104 (2021) 1, 011901

$$\mathcal{R} = \left\langle \frac{\vec{\omega} \cdot (\hat{t} \times \vec{v})}{|\hat{t} \times \vec{v}|} \right\rangle$$

# Hyperon spin correlations

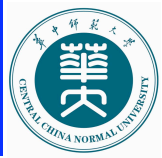


strong-field  
induced hyperon  
spin correlation

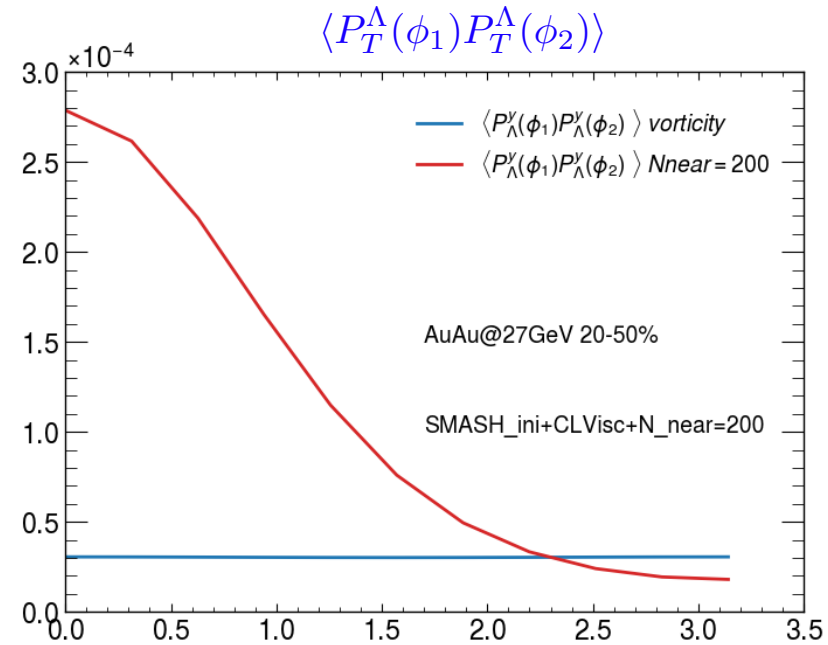
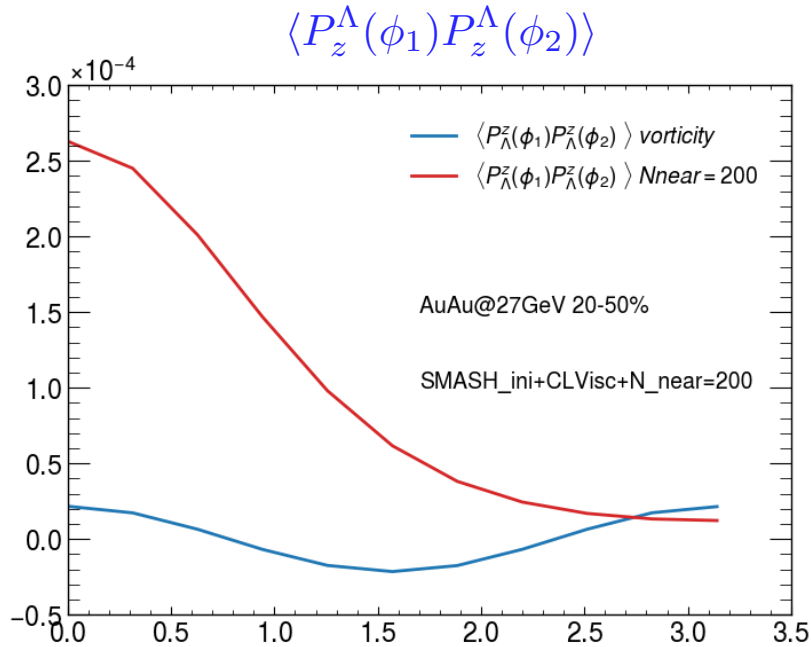


$$P_{q(\bar{q})}^\mu \approx \frac{1}{4m_q} \epsilon^{\mu\nu\rho\sigma} \left[ \omega_{\rho\sigma} \pm \frac{e_q}{(u \cdot p)T} F_{\rho\sigma} \pm \frac{g_V}{(u \cdot p)T} F_{\rho\sigma}^V \right] p_\nu$$





# Hyperon spin correlation



Dominated by the correlation due to strong force field!  
Sensitive to the correlation length  $\sigma$

# Summary and Future perspective



Spin dynamics opens up a new window for the study of QGP matter with many unexpected phenomena

- Spin alignment of  $K^*$ : correlation of strong force field of different flavor?
- Correlation of  $\Lambda$  spin polarization
- Spin alignment of  $J/\Psi$ : fluctuation of gluonic field at shorter distance?
- Effect of hadronic interaction?

