Mixed inhomogeneous phase in vortical gluon plasma from lattice simulation

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in collaboration with

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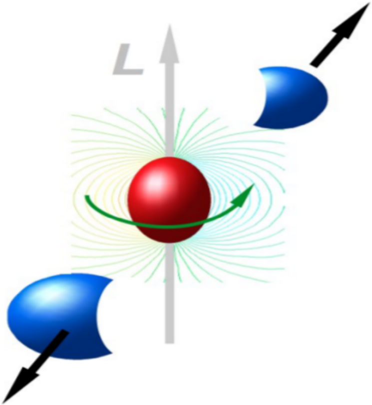
JOINT INSTITUTE FOR NUCLEAR RESEARCH

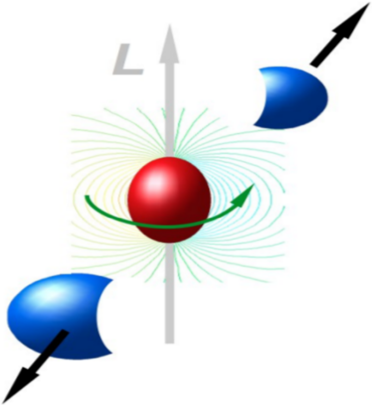
Outline

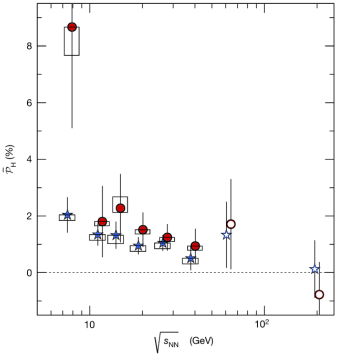
- Introduction
- Mixed inhomogeneous phase in rotating gluon system and local critical temperature
- Decomposition of rotating action
- Results for local action approximation
- Conclusions

- In non-central heavy ion collisions the creation of QGP with angular momentum is expected.



- In non-central heavy ion collisions the creation of QGP with angular momentum is expected.
- The rotation occurs with relativistic velocities.





[L. Adamczyk et al. (STAR), Nature **548**, 62–65 (2017), arXiv:1701.06657 [nucl-ex]]

$\langle \omega \rangle \sim 7$ MeV ($\sqrt{s_{NN}}$-averaged)

Critical temperature in rotating QCD

All* theoretical models assume rigid rotation, $\Omega \neq 0$.

- P. Singha, V. E. Ambrus, and M. N. Chernodub, (2024), arXiv:2407.07828 [hep-ph]
- Y. Chen, X. Chen, D. Li, and M. Huang, (2024), arXiv:2405.06386 [hep-ph]
- K. Mameda and K. Takizawa, Phys. Lett. B **847**, 138317 (2023), arXiv:2308.07310 [hep-ph]
- F. Sun, K. Xu, and M. Huang, Phys. Rev. D **108**, 096007 (2023), arXiv:2307.14402 [hep-ph]
- H.-L. Chen, Z.-B. Zhu, and X.-G. Huang, Phys. Rev. D **108**, 054006 (2023), arXiv:2306.08362 [hep-ph]
- and many others ...

Lattice results for gluodynamics: the confinement critical temperature increases with rotation

- V. Braguta, A. Kotov, D. Kuznedelev, and A. Roenko, JETP Lett. **112**, 6–12 (2020)
- V. Braguta, A. Kotov, D. Kuznedelev, and A. Roenko, Phys. Rev. D **103**, 094515 (2021), arXiv:2102.05084 [hep-lat]

Lattice results for QCD: the chiral and deconfinement critical temperatures both increase with rotation; fermions and gluons have opposite influence on T_c. [See Talk by V. Braguta]

- V. Braguta, A. Kotov, A. Roenko, and D. Sychev, PoS **LATTICE2022**, 190 (2023), arXiv:2212.03224 [hep-lat]
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Mostly the global T_c is measured, but action is spatially inhomogeneous.

Inhomogeneity in rotating (Q)GP

Tolman-Ehrenfest effect: In gravitational field the temperature isn't a constant in space at thermal equilibrium. In the co-rotating reference frame:

$$T(r) = \frac{T_0}{\sqrt{1-\Omega^2 r^2}} = \frac{T_0}{\sqrt{1+\Omega_I^2 r^2}} \,. \tag{1}$$

TE law suggests that the rotation effectively heats the periphery, the local critical temperature decreases:

$$T_c^{TE}(r,\Omega) = T_{c0}\sqrt{1-\Omega^2 r^2} \qquad \Rightarrow \qquad [\text{ inhomogeneous phase in rotating (Q)GP }]$$

In the result, confinement is in the center, deconfinement is at the periphery (for *real* rotation):

- 2+1 cQED: M. N. Chernodub, Phys. Rev. D **103**, 054027 (2021), arXiv:2012.04924 [hep-ph]
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Qualitatively consistent results:

- S. Chen, K. Fukushima, and Y. Shimada, (2024), arXiv:2404.00965 [hep-ph] [See Talk by K. Fukushima]
- Y. Jiang, (2024), arXiv:2406.03311 [nucl-th]

Observables are calculated on the lattice from first principles:

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int DA\, \mathcal{O}[A] \exp\big(- S_G[A] \big) , \qquad \text{where} \qquad Z = \int DA \, \exp\big(- S_G[A] \big) . \tag{2}$$

The Euclidean action S_G in co-rotating reference frame is formulated in curved space,
[A. Yamamoto and Y. Hirono, Phys. Rev. Lett. **111**, 081601 (2013), arXiv:1303.6292 [hep-lat]]

$$g^E_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & y\Omega_I \\ 0 & 1 & 0 & -x\Omega_I \\ 0 & 0 & 1 & 0 \\ y\Omega_I & -x\Omega_I & 0 & 1 + r^2\Omega_I^2 \end{pmatrix} , \tag{3}$$

where $r^2 = x^2 + y^2$, and the angular velocity is imaginary, $\Omega_I = -i\Omega$, to avoid the sign problem.

Lattice formulation of rotating gluodynamics

The gluon action is a quadratic function in angular velocity

$$S = \frac{1}{4g_0^2}\int d^4x\,\sqrt{g_E}\,g_E^{\mu\nu}g_E^{\alpha\beta}F^a_{\mu\alpha}F^a_{\nu\beta} \equiv S_0 + S_1\Omega_I + S_2\frac{\Omega_I^2}{2}\,, \tag{4}$$

where

$$S_0 = \frac{1}{4g_0^2}\int d^4x F^a_{\mu\nu}F^a_{\mu\nu}\,, \tag{5}$$

$$S_1 = \frac{1}{g_0^2}\int d^4x\left[yF^a_{xy}F^a_{y\tau} + yF^a_{xz}F^a_{z\tau} - xF^a_{yx}F^a_{x\tau} - xF^a_{yz}F^a_{z\tau}\right], \tag{6}$$

$$S_2 = \frac{1}{g_0^2}\int d^4x\left[r^2(F^a_{xy})^2 + y^2(F^a_{xz})^2 + x^2(F^a_{yz})^2 + 2xyF^a_{xz}F^a_{zy}\right], \tag{7}$$

Sign problem

- The sign problem is due to the linear terms ($S_1 \neq 0$)
- The Monte–Carlo simulation is conducted with imaginary angular velocity $\Omega_I = -i\Omega$.
- The results are analytically continued to real angular velocity, $\Omega^2 \leftrightarrow -\Omega_I^2$.

Lattice setup and observables

Causality restriction

- Analytic continuation is allowed only for bounded system with $\Omega r < 1$, i.e. $v_I^2 = (\Omega_I R)^2 < 1/2$
- Boundary conditions are important! (two different types of b.c.: open/periodic)

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Observables

The Polyakov loop is an order parameter,

$$L(x,y) = \frac{1}{N_z} \sum_z \mathrm{Tr} \left[\prod_{\tau=0}^{N_t-1} U_4(\vec{r},\tau) \right], \qquad L = \frac{1}{N_s^2} \sum_{x,y} L(x,y) \,. \tag{8}$$

In confinement $\langle L \rangle = 0$; in deconfinement $\langle L \rangle \neq 0$. $\langle L \rangle = e^{-F_Q/T}$
The local critical temperature is associated with the peak of the local Polyakov loop susceptibility

$$\chi_L(r) = \langle |L(r)|^2 \rangle - \langle |L(r)| \rangle^2 \,. \tag{9}$$

We use tree-level improved (Symanzik) lattice action; lattice size $N_t \times N_z \times N_s^2$; $R \equiv a(N_s - 1)/2$;
The temperature is $T = 1/N_t a$. It coincides with the temperature on the rotation axis T_0.

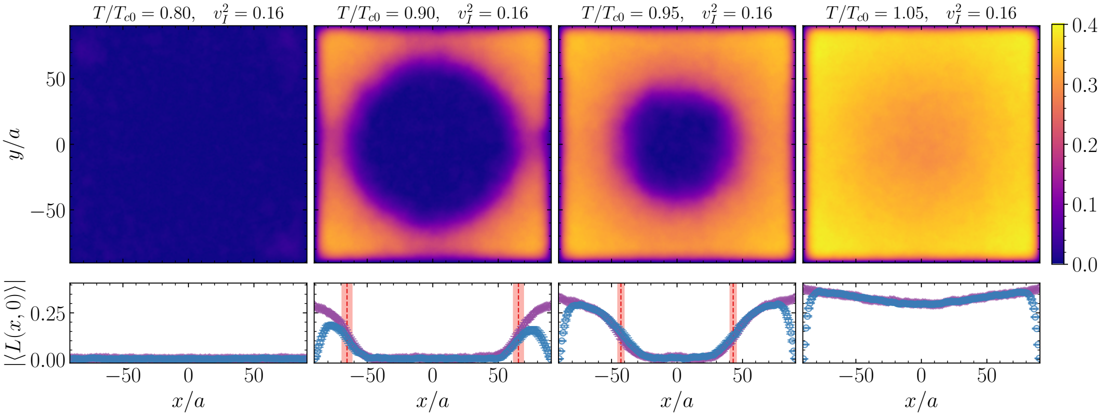


Figure: The distribution of the local Polyakov loop in x, y-plane for the lattice of size $5 \times 30 \times 181^2$ at the fixed imaginary velocity at the boundary $v_I^2 \equiv (\Omega_I R)^2 = 0.16$ and different on-axis temperatures, $T = 1/N_t a$.

- As the (on-axis) temperature increases, the radius of the inner confining region shrinks.
- Boundary is screened.
- Local thermalization takes place; Phase transition occurs as a vortex evolution.

Inhomogeneous phases for imaginary rotation

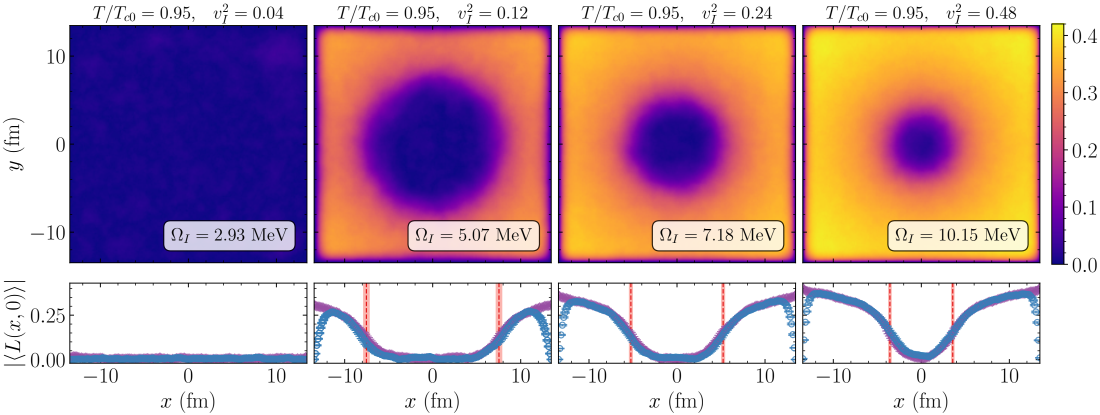
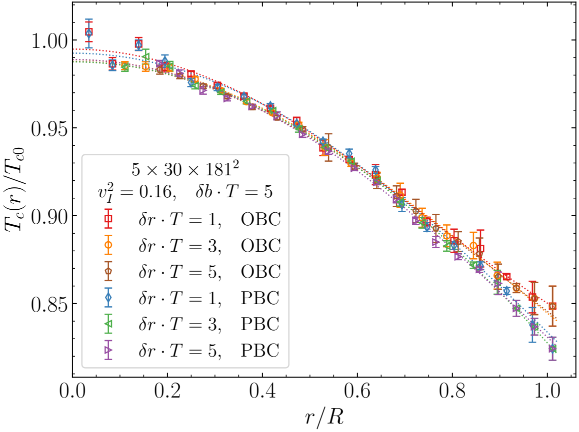


Figure: The distribution of the local Polyakov loop in x, y-plane for the lattice of size $5 \times 30 \times 181^2$ at the fixed temperature $T = 0.95\, T_{c0}$ and different Ω_I; System size $R = 13.5$ fm.

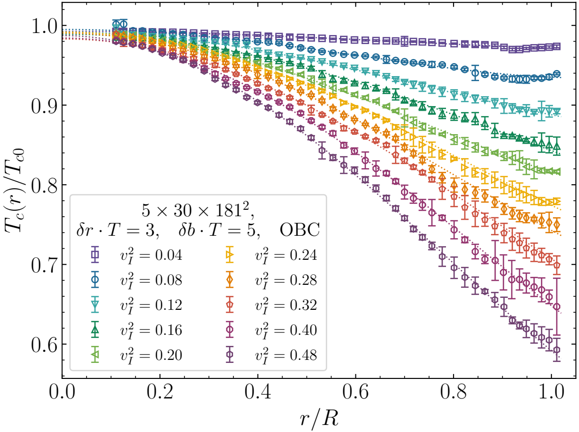
- Mixed inhomogeneous phase may be observed for $T \lesssim T_{c0}$. For imaginary rotation, deconfinement appears at the periphery; confinement is in the central regions.
- The confinement region shrinks with the increase in Ω_I;

Local critical temperature

We split the system into thin cylinders of width δr and measure local critical temperature



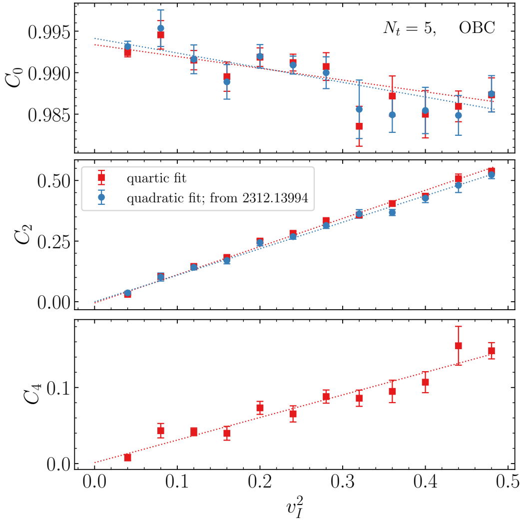
- Results for different width $\delta r \cdot T = 1, \ldots, 5$ are in agreement.
- We discard δb layers adjacent to boundary.
- There is a minor difference on b.c. only at $r/R \sim 1$



- The results in the whole region are well described by the quartic formula

$$\frac{T_c(r)}{T_{c0}} = C_0 - C_2 \left(\frac{r}{R}\right)^2 + C_4 \left(\frac{r}{R}\right)^4 . \quad (10)$$

- In the central region, $r/R \lesssim 0.5$, quadratic fit is sufficient ($C_4 = 0$).



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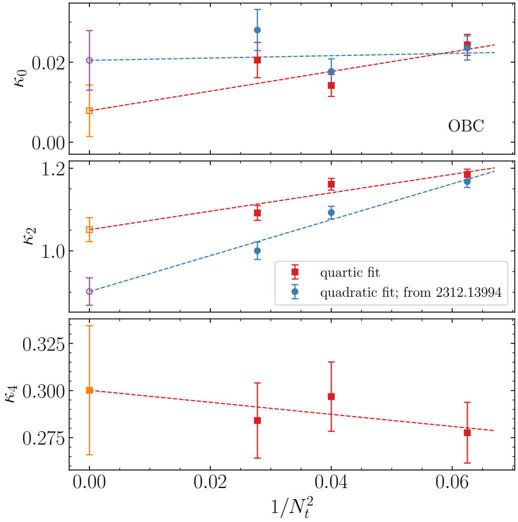
- In the central region, $r/R \lesssim 0.5$, quadratic fit is sufficient ($C_4 = 0$).
- We found numerically that

$$C_i(v_I^2) = a_i + \kappa_i v_I^2 . \quad (11)$$

- $T_c(0) \approx T_{c0}$ with few percent accuracy:
  - Effects of finite radius R.
  - Effects of averaging in layers of width δr.



The local critical temperature decreases with imaginary angular velocity.

$$\frac{T_c(r,\Omega_I)}{T_{c0}} = 1 - (\Omega_I r)^2 \left(\kappa_2 - \kappa_4 \left(\frac{r}{R}\right)^2\right) . \quad (12)$$

- The vortical curvature in continuum limit from quadratic fit ($r/R \lesssim 0.5$) is universal

$$\kappa_2 = 0.902(33)\,, \quad (13)$$

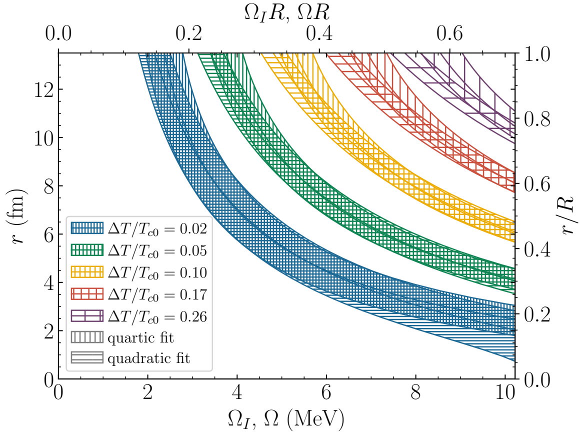
- And from quartic fit (for OBC) there is

$$\kappa_2 = 1.051(29)\,, \qquad \kappa_4 = 0.300(34)\,, \quad (14)$$

where κ_4 term is a finite volume correction;

- We can not distinguish $\sim \Omega^4$ term.

Phase diagram



The critical distance r may be found from the following conditions:

- for imaginary angular velocity

$$T_{c0} - \Delta T = T_c(r, \Omega_I)\,,$$

(confinement in the center;
deconfinement at the periphery)

- for real angular velocity

$$T_{c0} + \Delta T = T_c(r, \Omega)\,.$$

(deconfinement in the center;
confinement at the periphery)

The diagram has the same shape for a given $\Delta T > 0$ (plot for $R = 13.5$ fm).

Decomposition of rotating action

The action of rotating gluons is a quadratic function in Ω_I,

$$S_G = S_0 + \quad S_1\Omega_I + \quad S_2\Omega_I^2\,, \tag{15}$$

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$$S_G = S_0 + \lambda_1 S_1 \Omega_I + \lambda_2 S_2 \Omega_I^2 \,, \tag{15}$$

where we introduce switching factors λ_1, λ_2.

- The first operator S_1 is an angular momentum of gluons (in laboratory frame).
- The second operator S_2 is related to the chromomagnetic fields F_{ij}^2.

The mechanical/magnetic contributions to the moment of inertia originate from these operators.

[See Talk by V. Braguta]

The following regimes of the rotation are possible:

i1) $\lambda_1 = 1, \quad \lambda_2 = 0; \quad v_I^2 > 0$

i2) $\lambda_1 = 0, \quad \lambda_2 = 1; \quad v_I^2 > 0$

i3) $\lambda_1 = 1, \quad \lambda_2 = 1; \quad v_I^2 > 0$ (physical regime; it is already considered above)

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i3) $\lambda_1 = 1, \quad \lambda_2 = 1; \quad v_I^2 > 0$ (physical regime; it is already considered above)

Note that in case i2) there is, actually, no sign problem:

r2) $\lambda_1 = 0, \quad \lambda_2 = 1; \quad v_I^2 < 0$ (real rotation)

Imaginary vs real rotation for different regimes

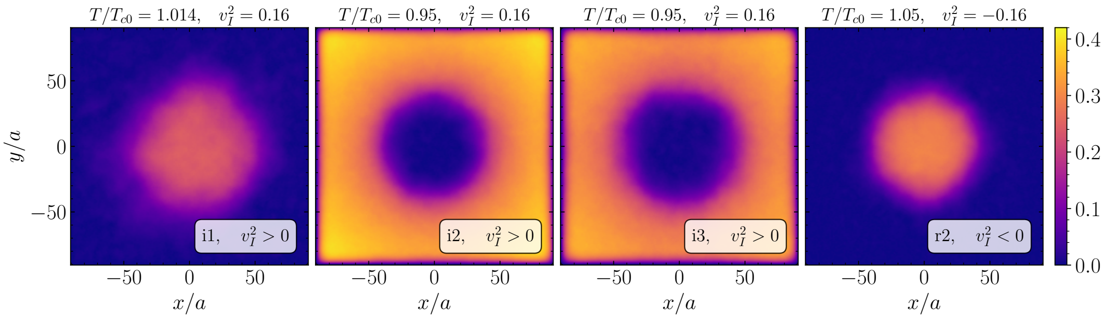
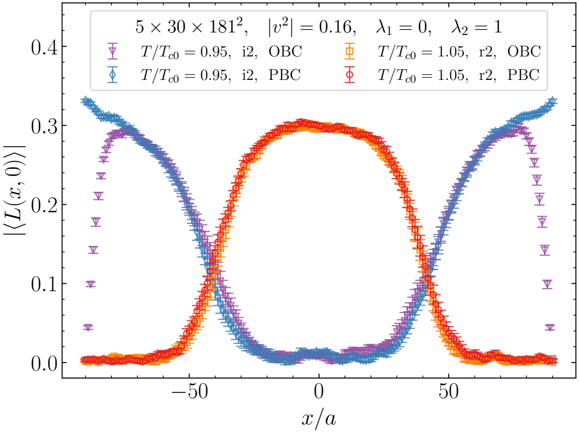


Figure: The distribution of the local Polyakov loop in x, y-plane for lattice size $5 \times 30 \times 181^2$, open boundary conditions (OBC) at fixed velocity $|v_I^2| = 0.16$ and different regimes.

- In the regimes i1 and r2, the rotation produces confinement phase in the outer region at $T > T_{c0}$.
  Regime r2 realizes real rotation for S_2 system.
- Phase arrangement is the same in i2- and i3-regimes.
  The radius of the inner region in regime i2 is slightly smaller, than in regime i3.

Imaginary vs real rotation for different regimes

The distributions of the Polyakov loop for real and imaginary rotation (S_1 term is omitted).

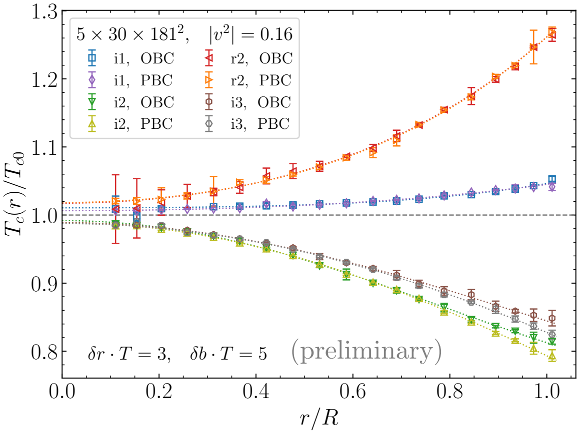


- (r2): $T = T_{c0} + \Delta T$
  for real rotation $v^2 = 0.16$
- (i2): $T = T_{c0} - \Delta T$
  for imaginary rotation $v_I^2 = 0.16$

Confinement $\leftrightarrow$ deconfinement
with approximately the same boundary.

Imaginary vs real rotation for different regimes

The local critical temperature in these regimes has different behaviour.



- In the i1-regime, $T_c(r) \nearrow$
- In the i2-regime, $T_c(r) \searrow$
  the vortical curvature $\kappa_2^{(\mathrm{i2})} > \kappa_2^{(\mathrm{i3})}$
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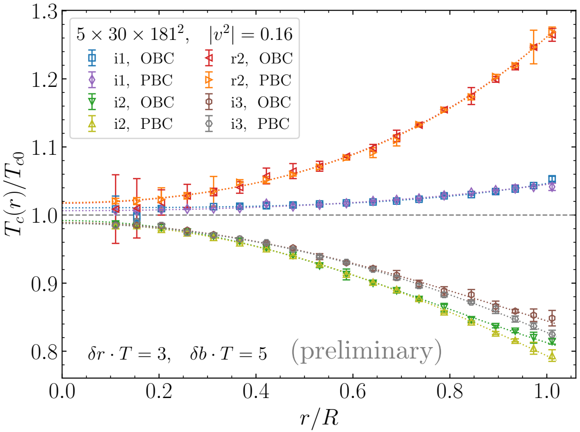

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  [See Talk by V. Braguta]
- The r2-regime are in agreement with a.c. of the i2-results

Local approximation for inhomogeneous action

We consider the system at distance r_0 from the rotation axis.
In the vicinity of the point $(x, y) = (r_0, 0)$ the coefficients in action is approximately the same.

The homogeneous local action is

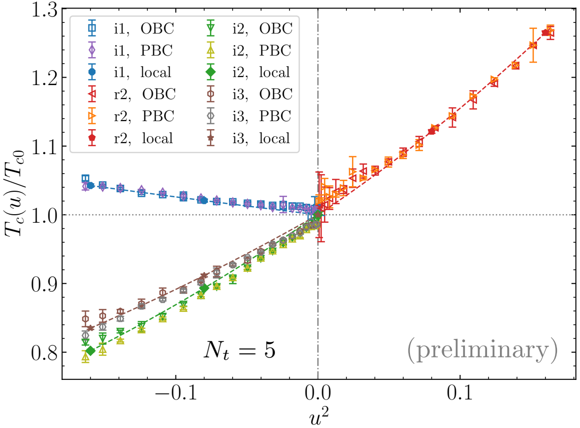
$$S_G = \frac{1}{2g^2} \int d^4x \left[F^a_{x\tau} F^a_{x\tau} + F^a_{y\tau} F^a_{y\tau} + F^a_{z\tau} F^a_{z\tau} + F^a_{xz} F^a_{xz} + \right. \\ \left. + \left(1 + u_I^2\right) F^a_{yz} F^a_{yz} + \left(1 + u_I^2\right) F^a_{xy} F^a_{xy} - 2u_I \left(F^a_{yx} F^a_{x\tau} + F^a_{yz} F^a_{z\tau} \right) \right], \quad (16)$$

where $u_I = \Omega_I r_0$ is a local velocity.

Local thermalization approximation

- The system (16) is simulated using standard lattice methods with PBC.
- Local approximation is free from the effects of finite R and the influence of boundary conditions.

Results for local action



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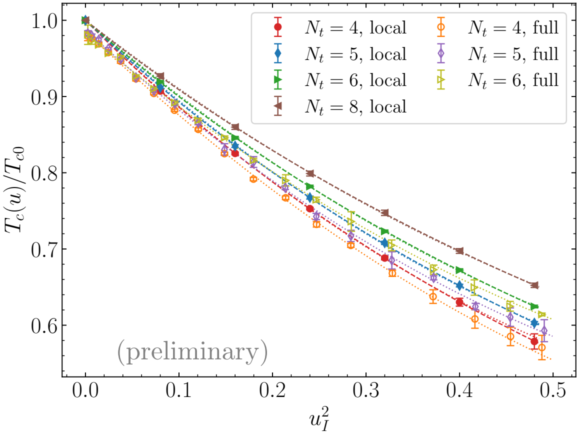


- The results for *local* action and for full system are in good agreement with each other in all regimes (at least for $|u^2| \lesssim 0.2$).
- The data are well described by the polynomial:

$$\frac{T_c(u_I)}{T_{c0}} = 1 - k_2 u_I^2 + k_4 u_I^4 \,. \qquad (17)$$

- In continuum limit the coefficients are

$$k_2 = 0.869(31)\,, \qquad k_4 = 0.388(53)\,. \qquad (18)$$

- The local critical temperature increases with real velocity $u = \Omega r$.

Asymmetry in local action

The local action without the linear term is

$$S_G = \int d^4x \left[\beta\left((F^a_{x\tau})^2 + (F^a_{y\tau})^2 + (F^a_{z\tau})^2 + (F^a_{xz})^2\right) + \tilde{\beta}\left((F^a_{yz})^2 + (F^a_{xy})^2\right)\right], \tag{19}$$

where $\beta = 1/2g^2$ and $\tilde{\beta}/\beta = 1 - (\Omega r_0)^2 = 1 + (\Omega_I r_0)^2$.

External gravitational field generates asymmetry in the coupling constants of different components of the fields $(F_{\mu\nu})^2$, which influences the dynamics of gluons.

- $\tilde{\beta}/\beta > 1$ (imaginary rotation) $\Rightarrow$ T_c decreases.
- $\tilde{\beta}/\beta < 1$ (real rotation) $\Rightarrow$ T_c increases.

The rotation influences the dynamics of gluons, it is not TE.

Conclusions

- Using lattice simulation, we found the mixed confinement-deconfinement phase in rotating SU(3) gluodynamics at thermal equilibrium.
- The local critical temperature increases for real rotation, and it is determined mainly by the local velocity of rotation $u = \Omega r$:

$$\frac{T_c(r,\Omega)}{T_{c0}} = 1 + (\Omega r)^2 \left(\kappa_2 - \kappa_4 \left(\frac{r}{R}\right)^2\right), \qquad \text{[full system with OBC]}, \qquad (20)$$

$$\frac{T_c(u)}{T_{c0}} = 1 + k_2 u^2 + k_4 u^4, \qquad \text{[local action]}, \qquad (21)$$

where κ_4 is sensitive to boundary effects.

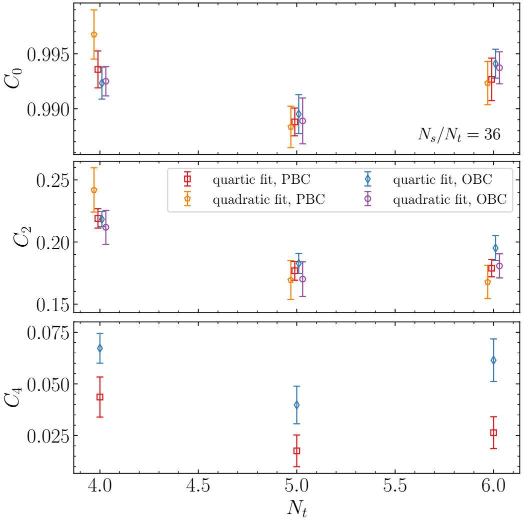
- The local critical temperature on the axis $T_c(0)$ is T_{c0} with few percent accuracy. It is not TE.
- The rotation generates asymmetry in the action for chromomagnetic fields. The results in different regimes resemble decomposition of $I = I_{\text{mech}} + I_{\text{magn}}$. [See Talk by V. Braguta]
- Phase transition occurs as a evolution of vortex of new phase. We expect similar picture for QCD.
- Previous results for T_c should be understood as bulk-averaged values.

V. V. Braguta, M. N. Chernodub, and A. A. Roenko, Phys. Lett. B **855**, 138783 (2024), arXiv:2312.13994 [hep-lat]

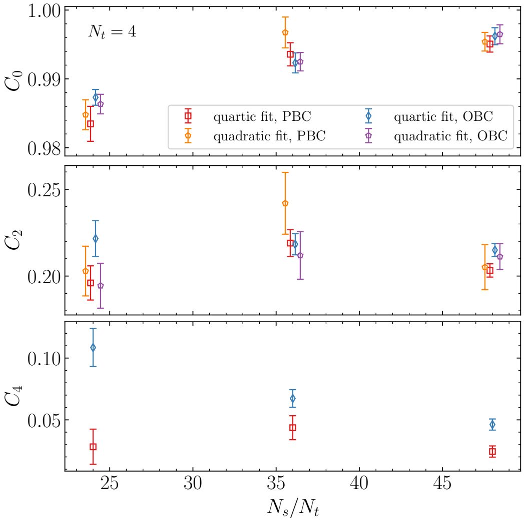
Another details coming soon: 2408.XXXXX

Thank you for your attention!

Backup



C_0
0.995
0.990
$N_s/N_t = 36$
C_2
0.25
0.20
0.15
quartic fit, PBC
quadratic fit, PBC
quartic fit, OBC
quadratic fit, OBC
C_4
0.075
0.050
0.025
4.0
4.5
5.0
5.5
6.0
N_t



C_0
1.00
0.99
0.98
$N_t = 4$
quartic fit, PBC
quadratic fit, PBC
quartic fit, OBC
quadratic fit, OBC
C_2
0.25
0.20
C_4
0.10
0.05
25
30
35
40
45
N_s/N_t