Gauge invariance and thermodynamic stability of rotating magnetized systems

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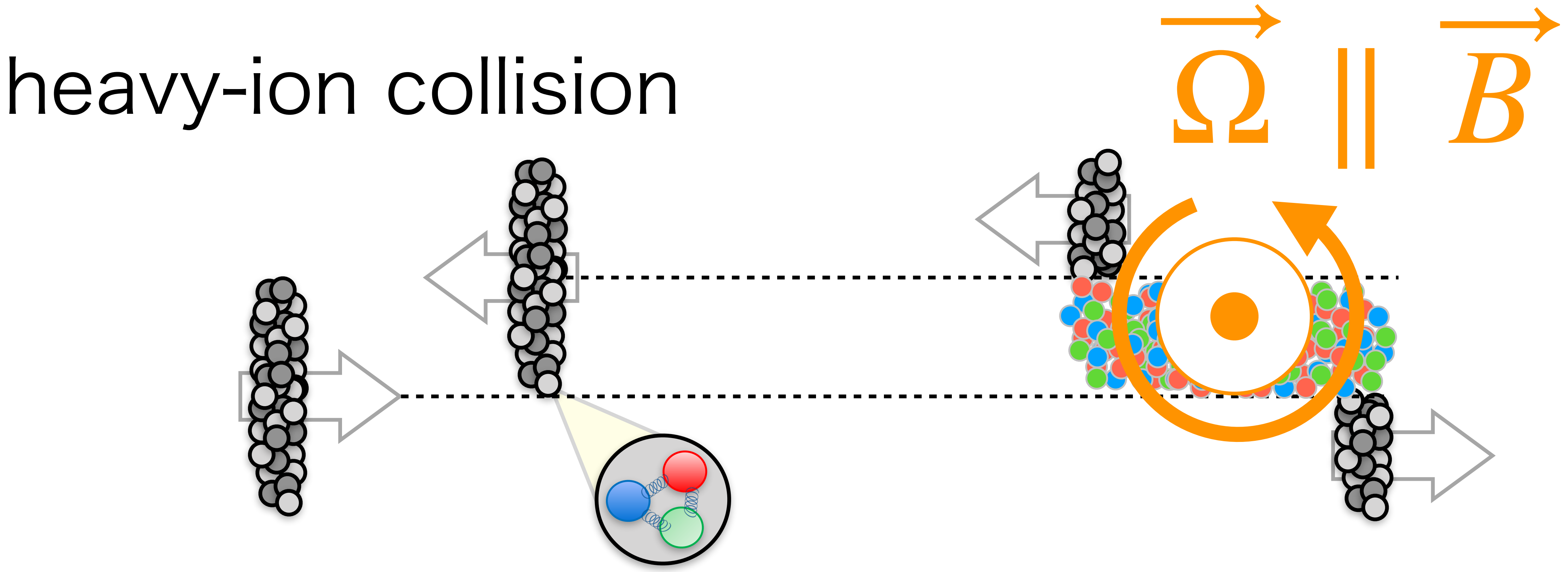
K. Fukushima, K. Hattori and KM, arXiv:2407.******* [hep-ph]

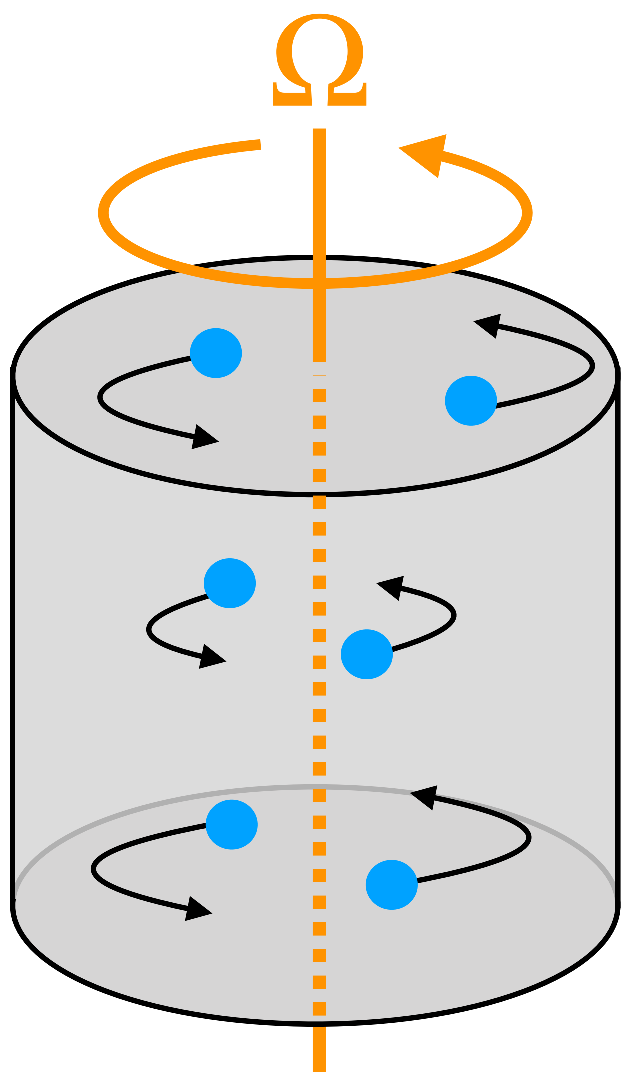
QCD matter under rotation



The Fastest Fluid
by Sylvia Morrow
Superhot material spins at an incredible rate.

- ✓ elementary particles affected by $\vec{\Omega}$ (= source of angular momentum)
- ✓ $\vec{\Omega} \parallel \vec{B}$ is more crucial than either $\vec{\Omega}$ or $\vec{B}$

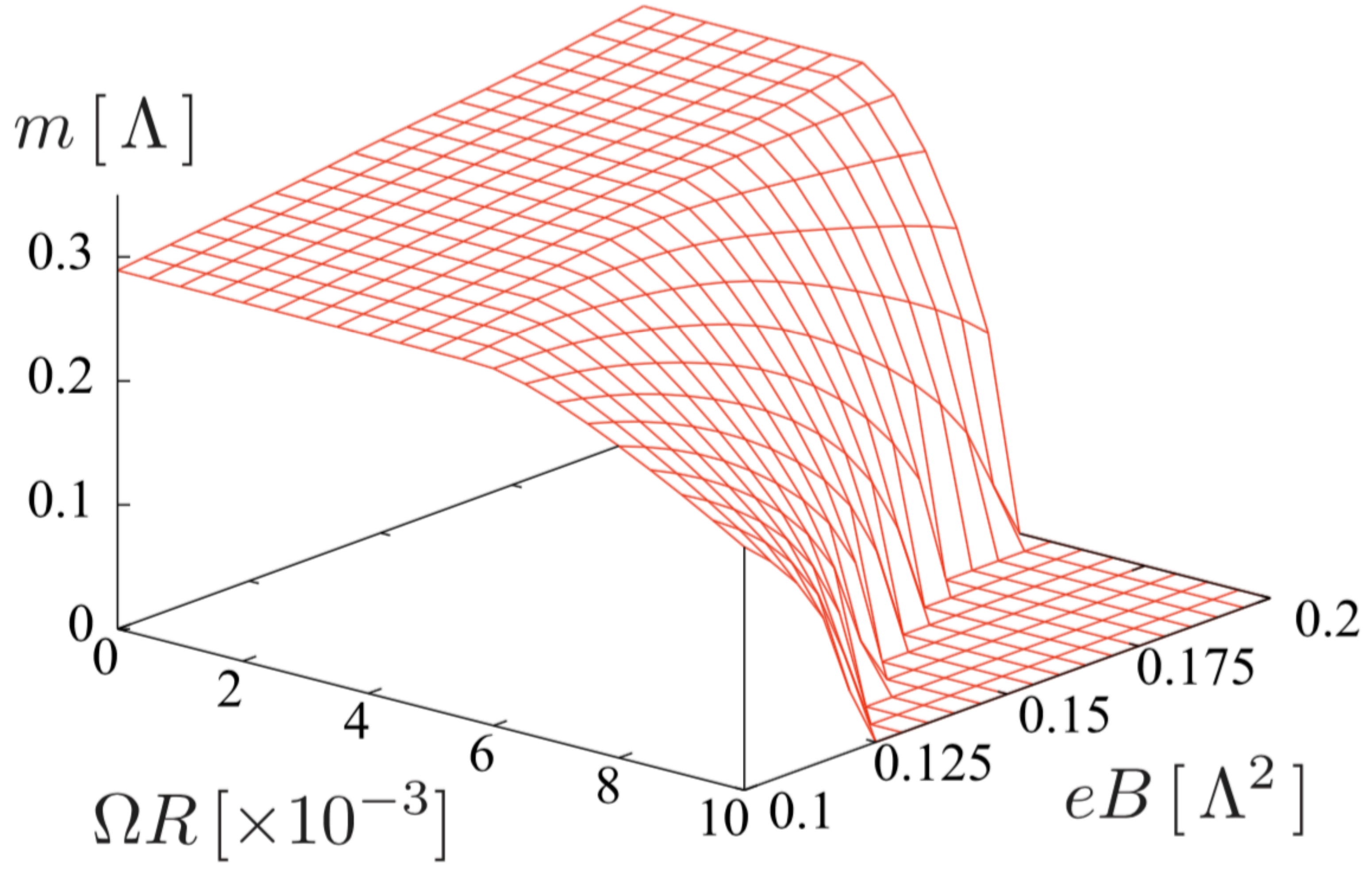
Early attempt : Thermodynamics



Landau-Lifshitz (1958)　Vilenkin (1979)

$$Z = \mathrm{tr}\exp[-\beta(H - \Omega\mathcal{J})] \longleftrightarrow H - \mu N$$

inverse magnetic catalysis



Chen-Fukushima-Huang-Mameda (2016)

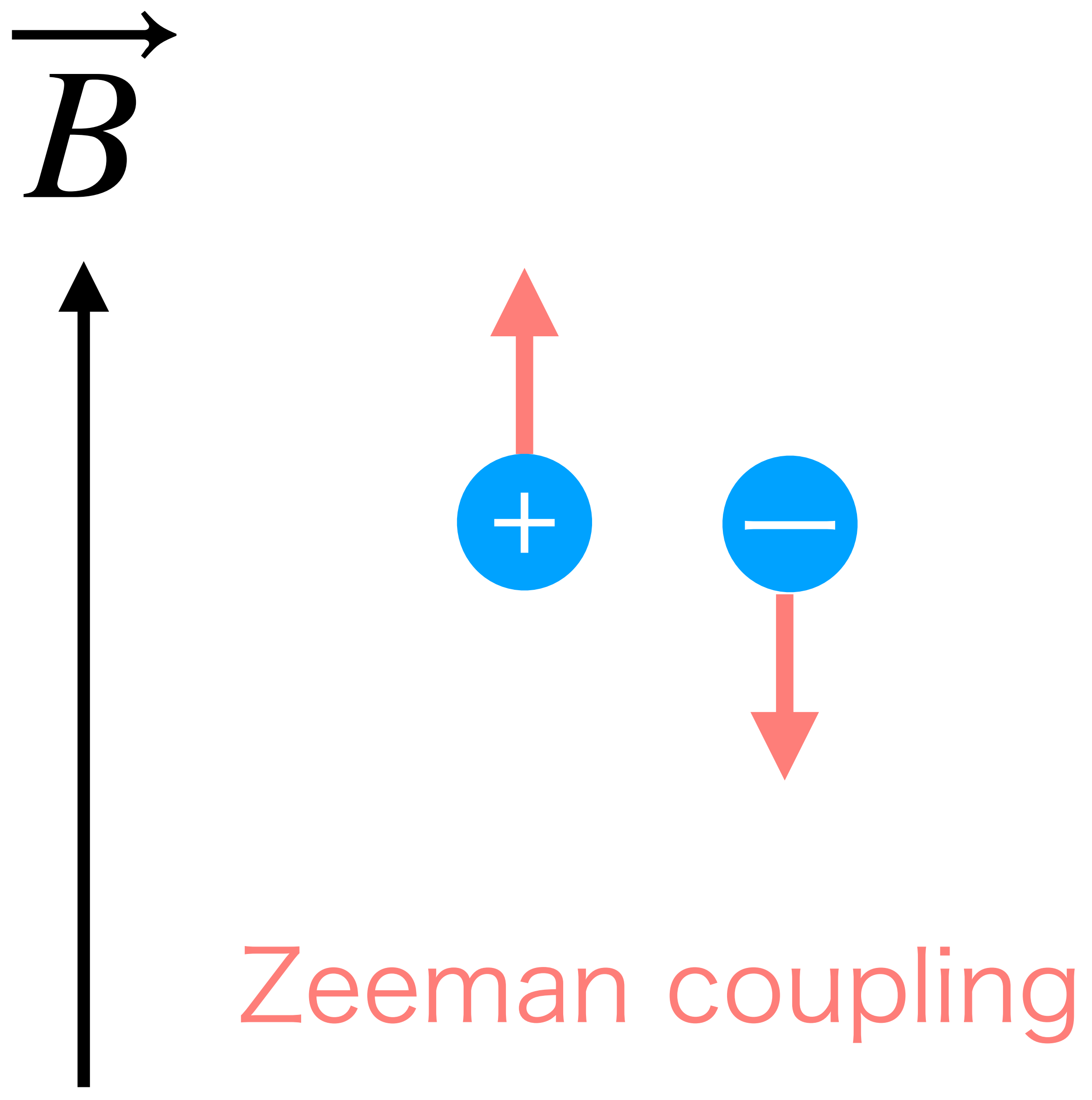
$$Z = \det\left[\mathcal{D}_B - \gamma^0\Omega(L+S)\right]$$

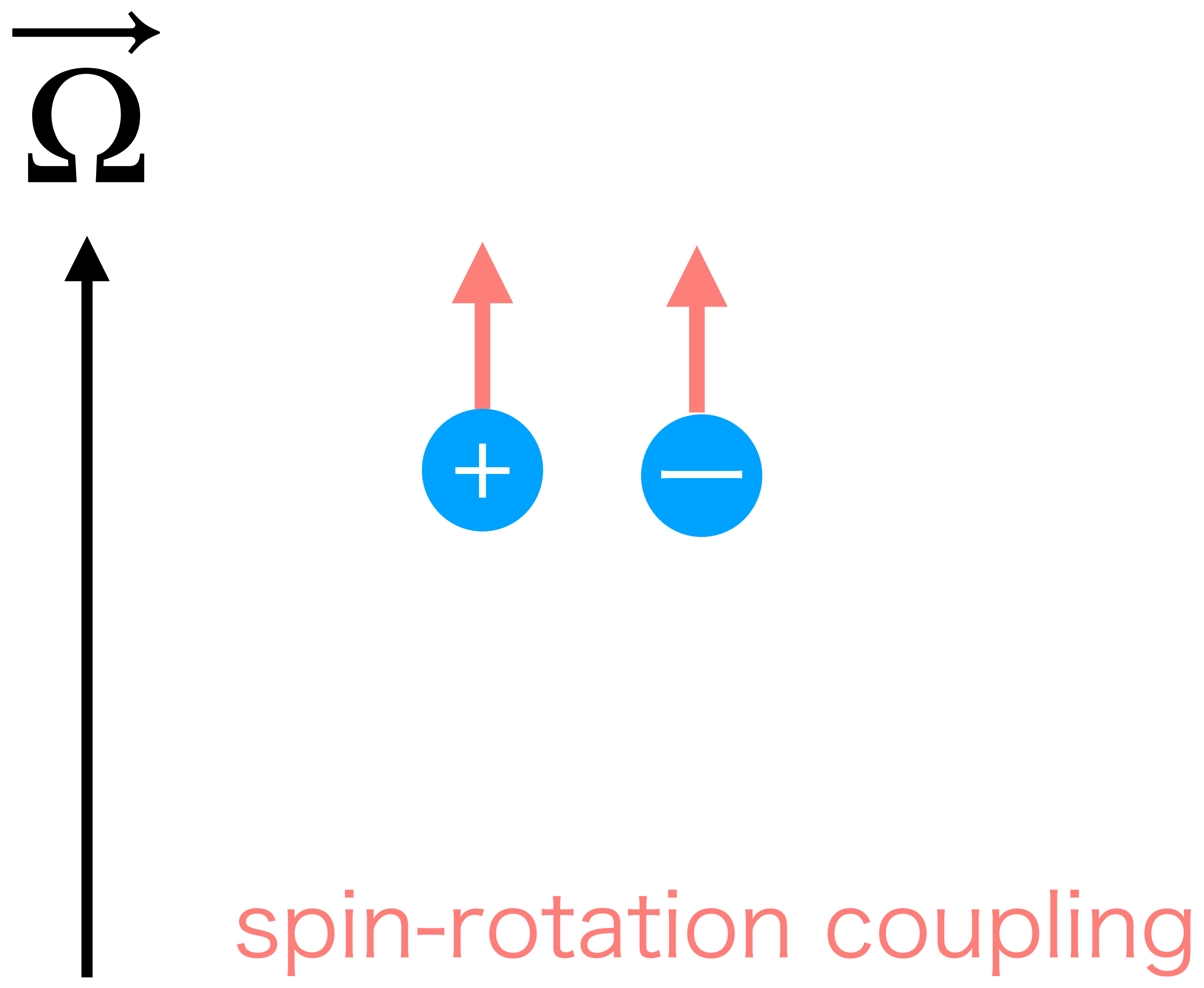
purely magnetic part

Early attempt : Transport

Hattori-Yin (2016)
Kubo formula

$$\rho = \frac{eB\Omega}{4\pi^2}$$





Early attempt : Transport

Hattori-Yin (2016)
Kubo formula

$$\rho = \frac{eB\Omega}{4\pi^2}$$

$$\vec{B} \parallel \vec{\Omega}$$

Puzzle on magneto-vortical charge

Kubo formula

$$\rho = \frac{eB\Omega}{4\pi^2}$$

Hattori-Yin (2016)

free energy

$$\rho = \frac{eB\Omega}{4\pi^2} + (\text{divergence w.r.t. AM})$$

Chen-Fukushima-Huang-Mameda (2016)

Ebihara-Fukushima-Mameda (2017)

Answer (as of June)

Kubo formula

$$\rho = \frac{eB\Omega}{4\pi^2}$$ Hattori-Yin (2016)

free energy

$$\rho = \frac{eB\Omega}{4\pi^2} + (\text{divergence w.r.t. AM})$$

Finial answer

Fukushima-Hattori-Mameda (in prep.)

correct Kubo formula

$$\rho = -\frac{eB\Omega}{4\pi^2}$$

correct free energy

$$\rho = -\frac{eB\Omega}{4\pi^2}$$

I will convince you!

Choice of angular momenta

$$Z = \det\left[\mathcal{D}_B - \gamma^0 \Omega(L + S)\right]$$

Chen-Fukushima-Huang-Mameda (2016)

$$L_{\rm can} = -\mathrm{i}(x\partial_y - y\partial_x)$$

conserved AM

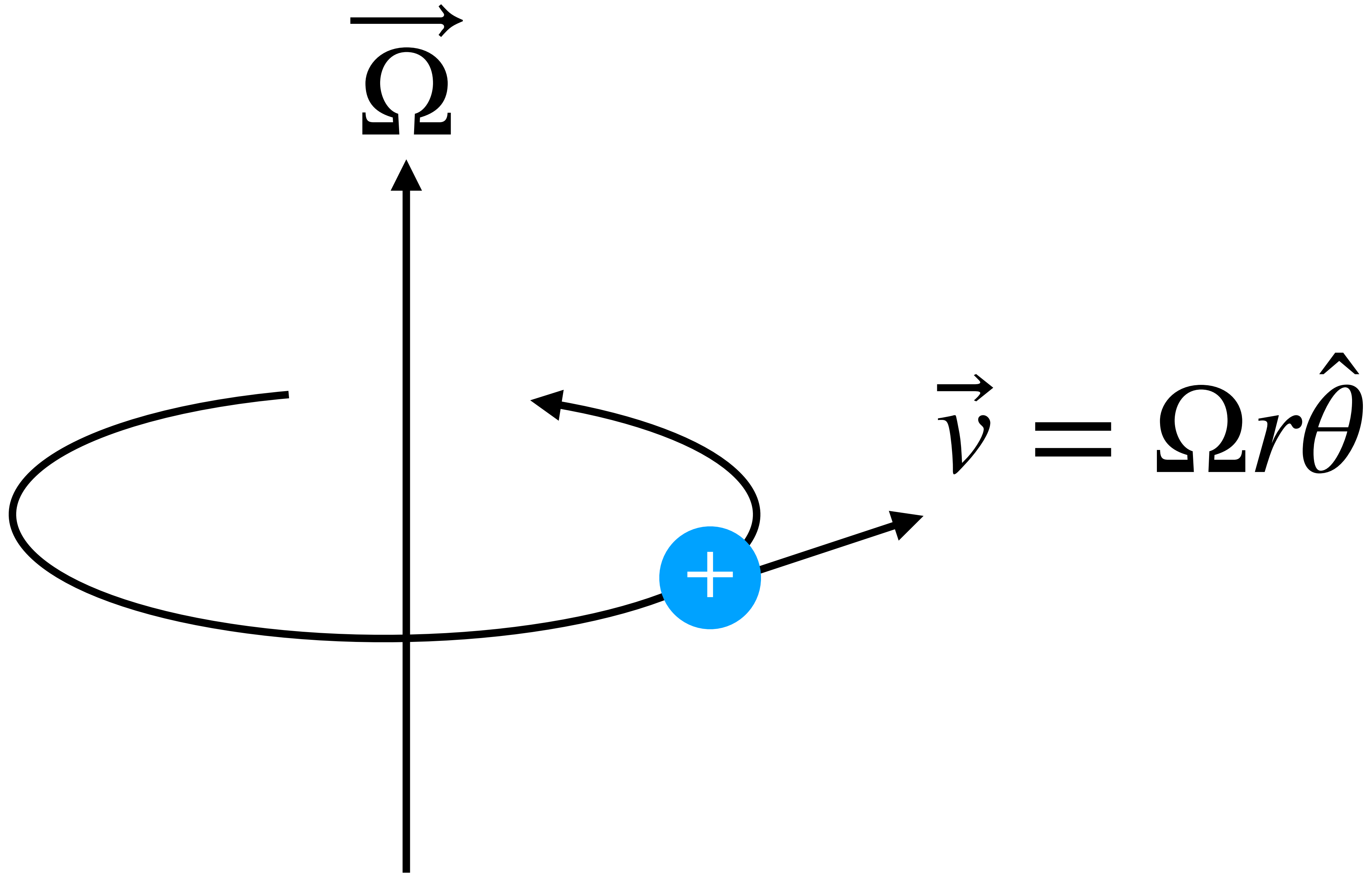
Fukushima-Hattori-Mameda (in prep.)

$$L_{\rm kin} = -\mathrm{i}(xD_y - yD_x)$$

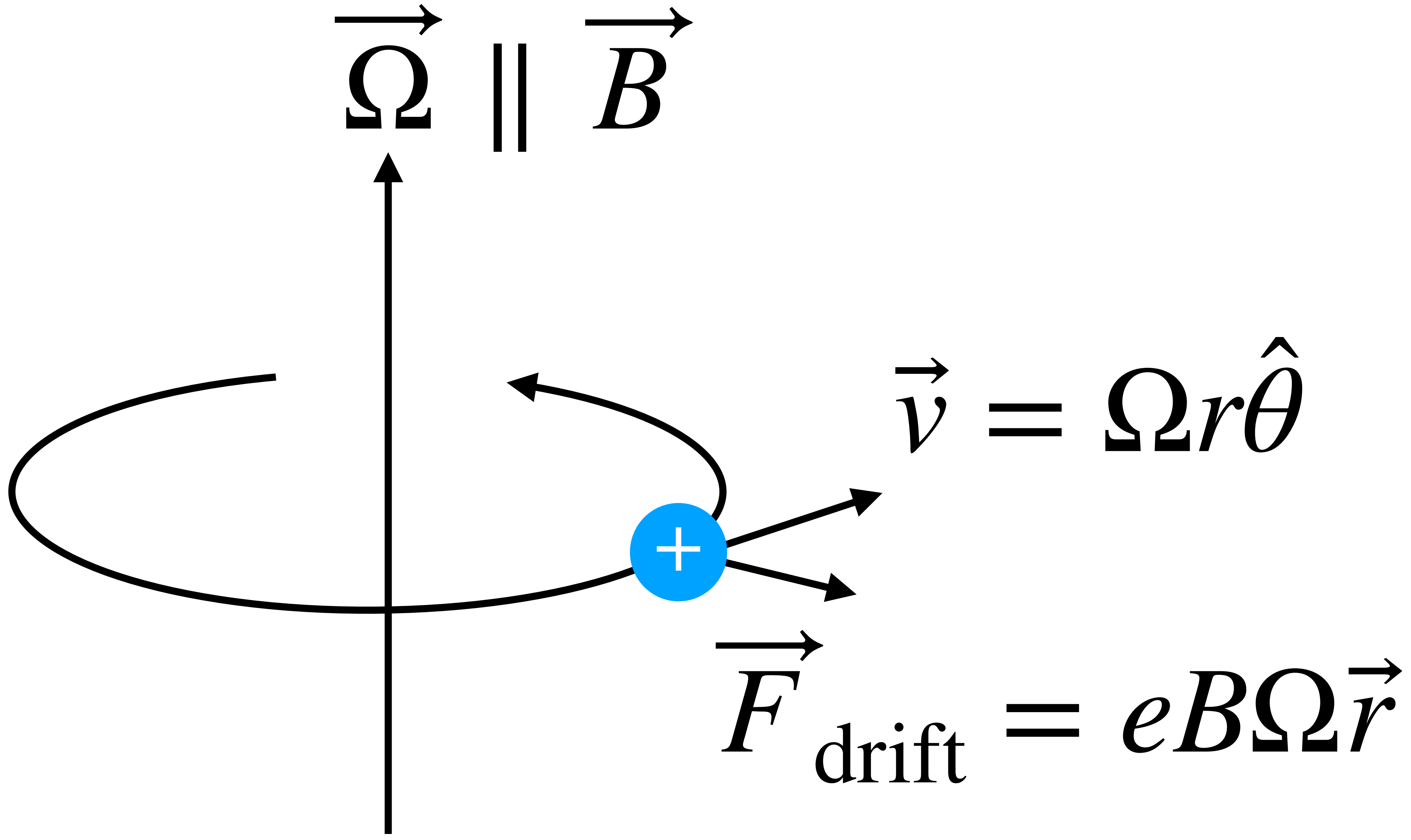
gauge invariant AM

cf. Buzzegoli (2020)

Classical interpretation



Classical interpretation

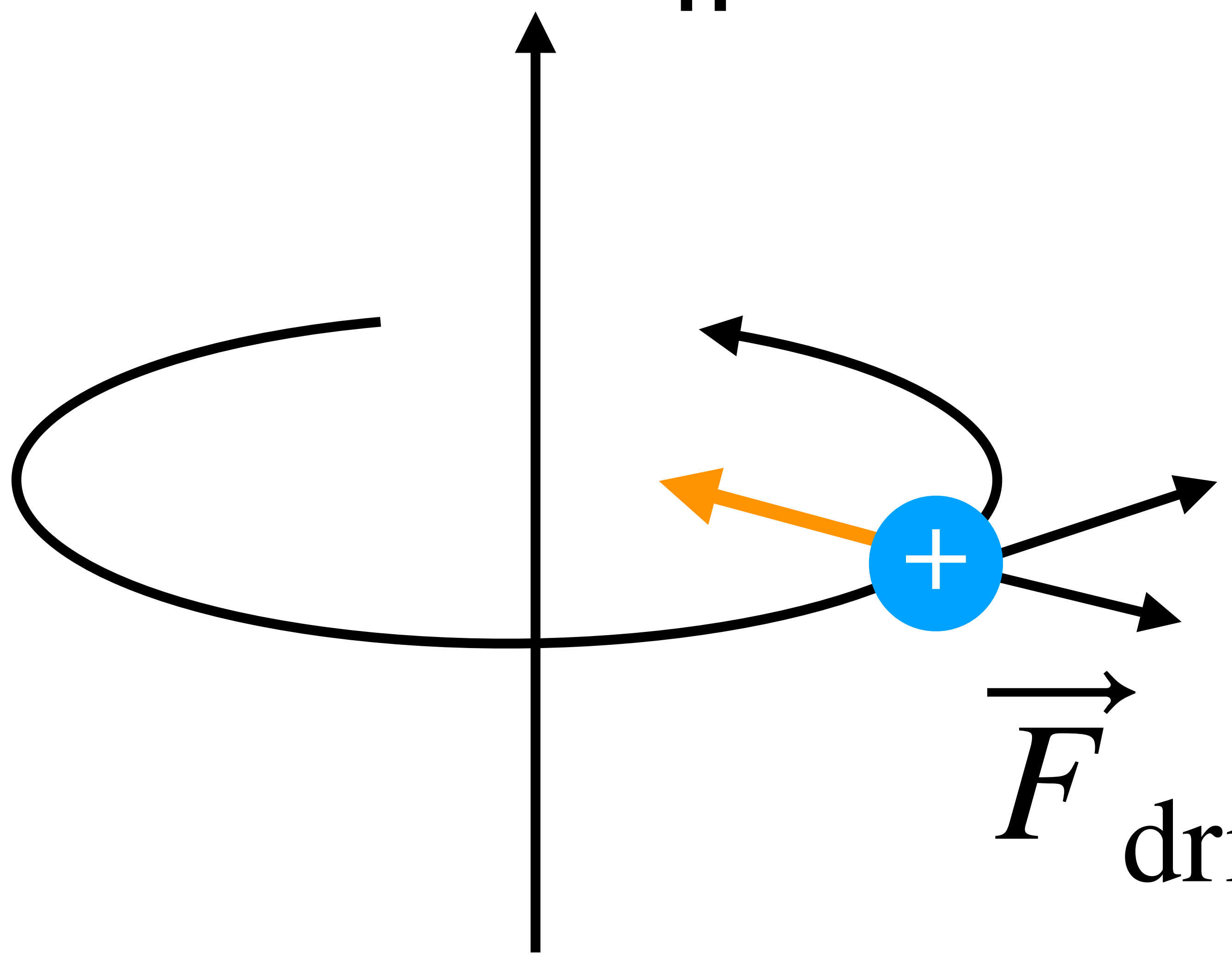


no longer circular

$H - \Omega L_{\text{can}}$ unstable

Classical interpretation

$$\vec{\Omega} \parallel \vec{B}$$



$$\vec{v} = \Omega r \hat{\theta}$$

$$\vec{F}_{\text{drift}} = eB\Omega\vec{r}$$

$$e\vec{E} = -\,eB\Omega\vec{r}$$
$$= -\,\vec{\nabla}\left[\Omega(L_{\text{can}} - L_{\text{kin}})\right]$$

for symmetric gauge

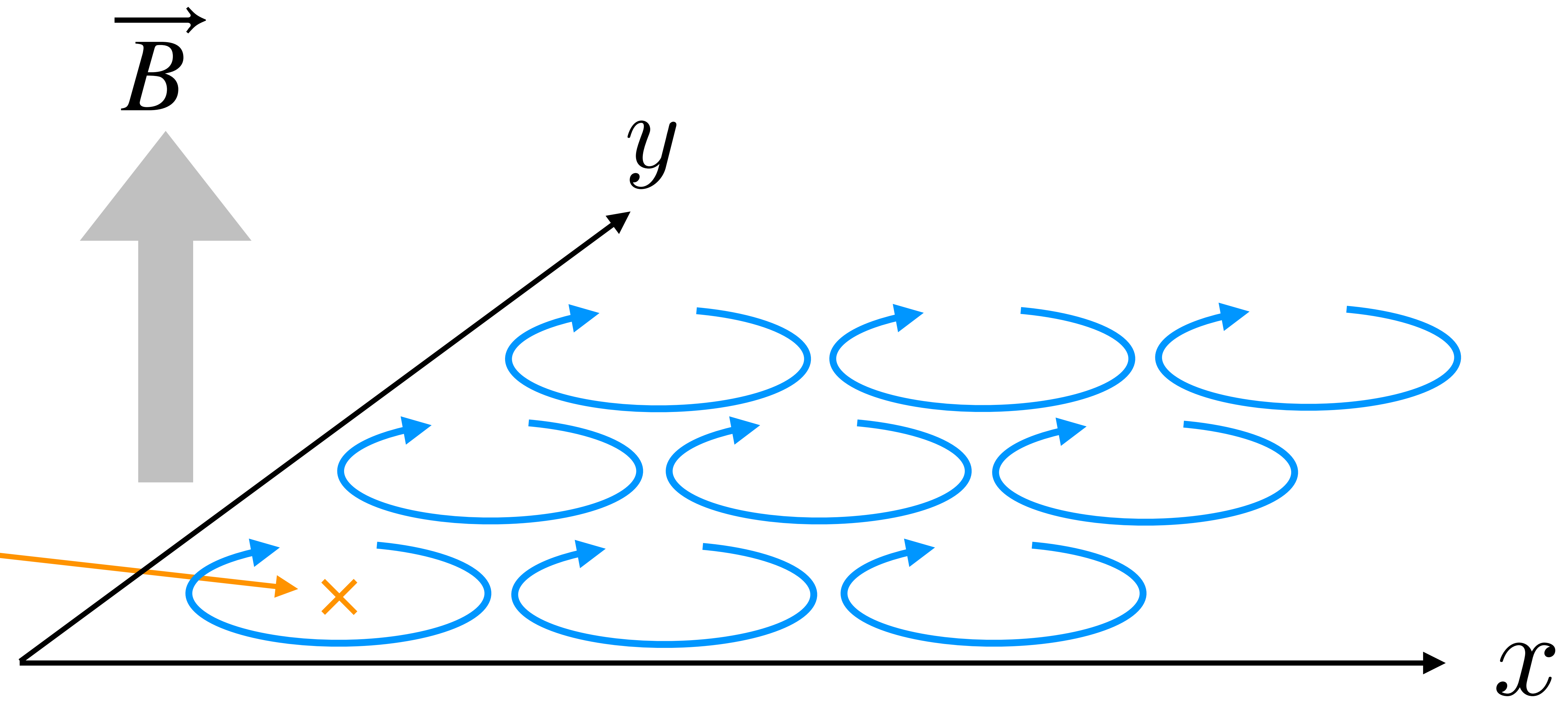
$$H + \Omega(L_{\text{can}} - L_{\text{kin}}) - \Omega L_{\text{can}} = H - \Omega L_{\text{kin}}$$ stable

gauge invariance ⟷ thermodynamic stability

Quantum mechanics

kinetic momentum $\overrightarrow{\Pi} = \vec{p} - e\overrightarrow{A}$

guiding center $\overrightarrow{X}$



Landau level basis $|\, n \,, m \,\rangle \propto (a^\dagger)^n \, (b^\dagger)^m \,|\, 0{,}0\rangle$

kinetic energy

$$\overrightarrow{\Pi}^2 = eB(2a^\dagger a + 1)$$

distance from origin

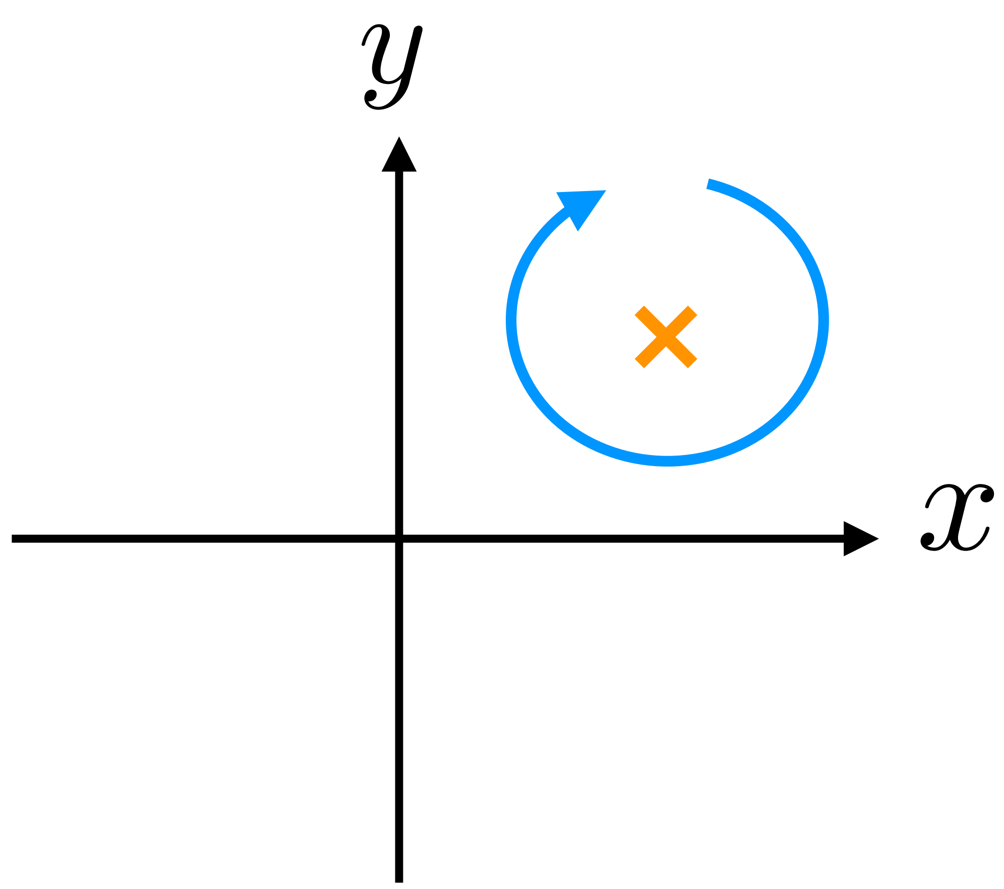
$$\overrightarrow{X}^2 = (2b^\dagger b + 1)/eB$$

Angular momenta

$$L_{\mathrm{kin}} = x\Pi_y - y\Pi_x = \Lambda + \Delta$$

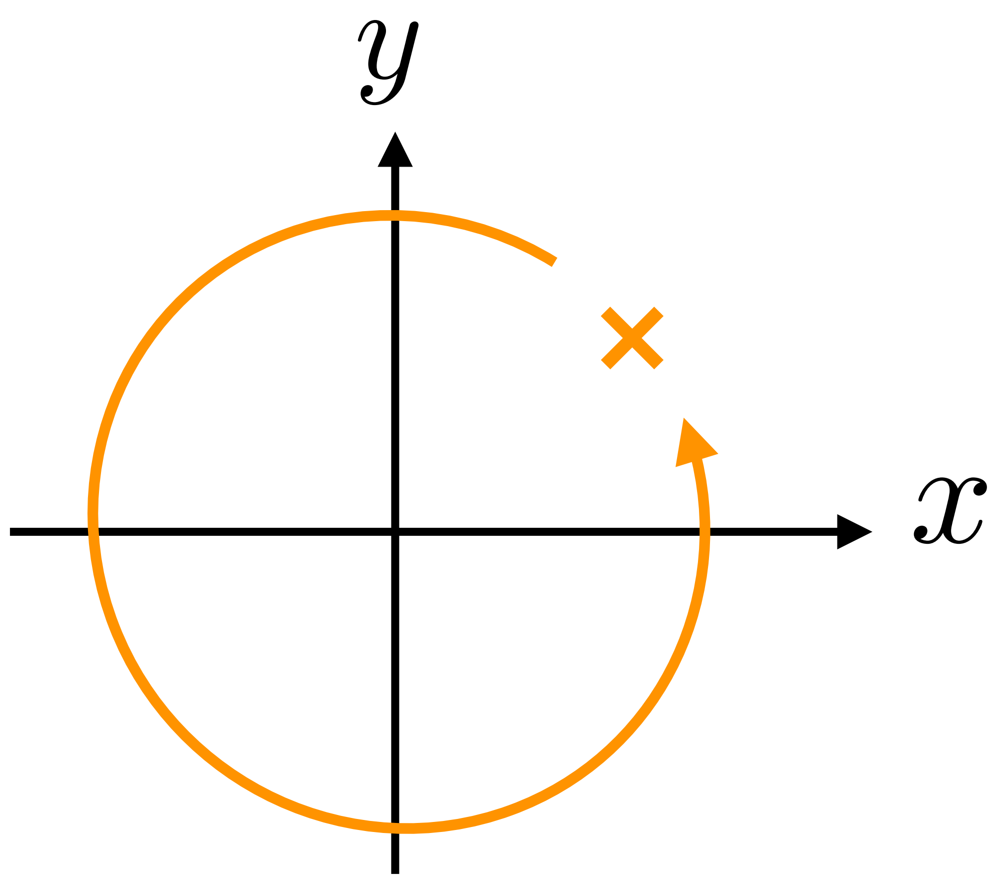
$$\Lambda = (x - X)\Pi_y - (y - Y)\Pi_y$$

$$\Delta = X\Pi_y - Y\Pi_x$$



cyclotron motion

$$= -(2a^\dagger a + 1)$$



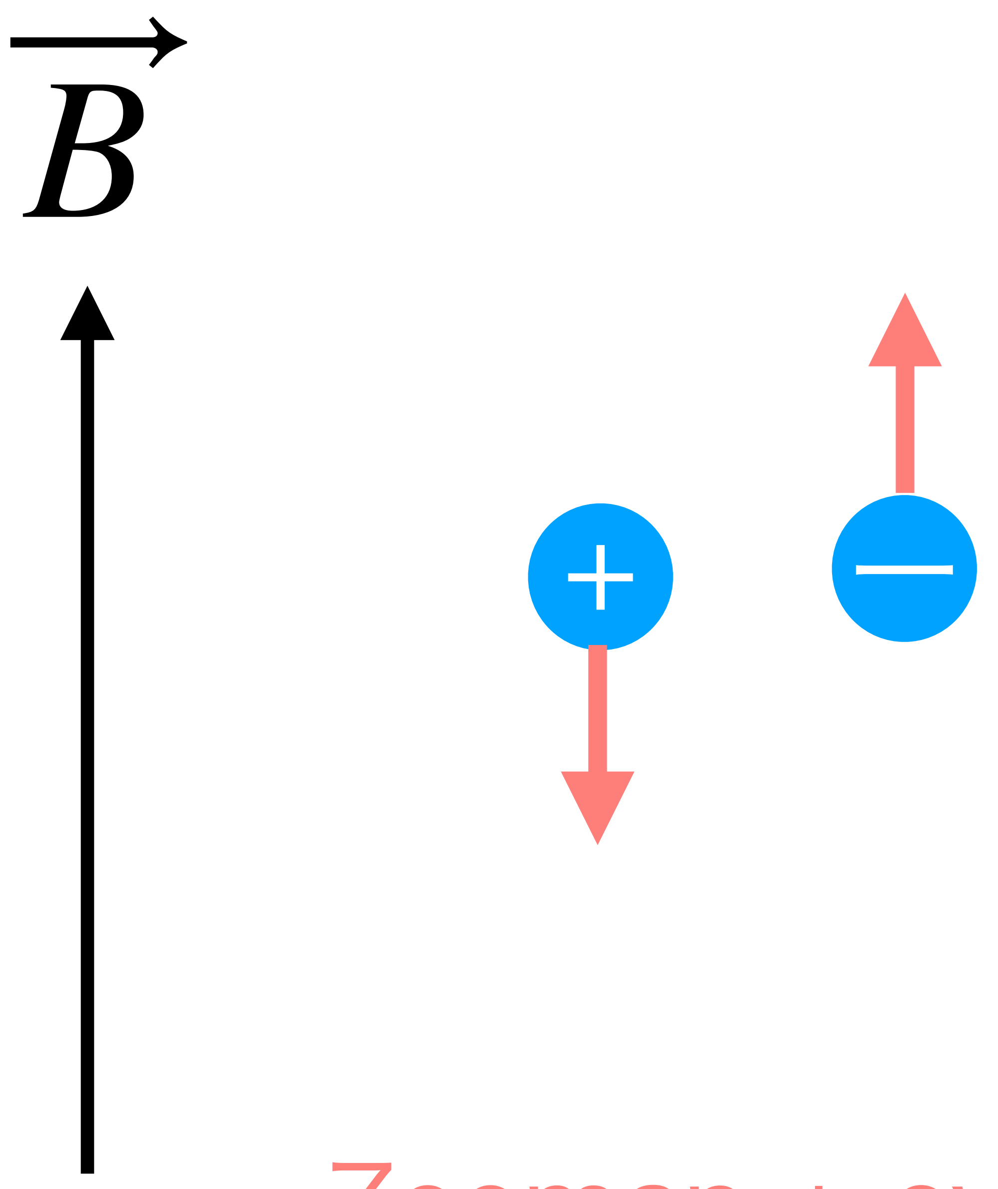
circular motion of guiding center

$$= \mathrm{i}(a^\dagger b^\dagger - ab)$$

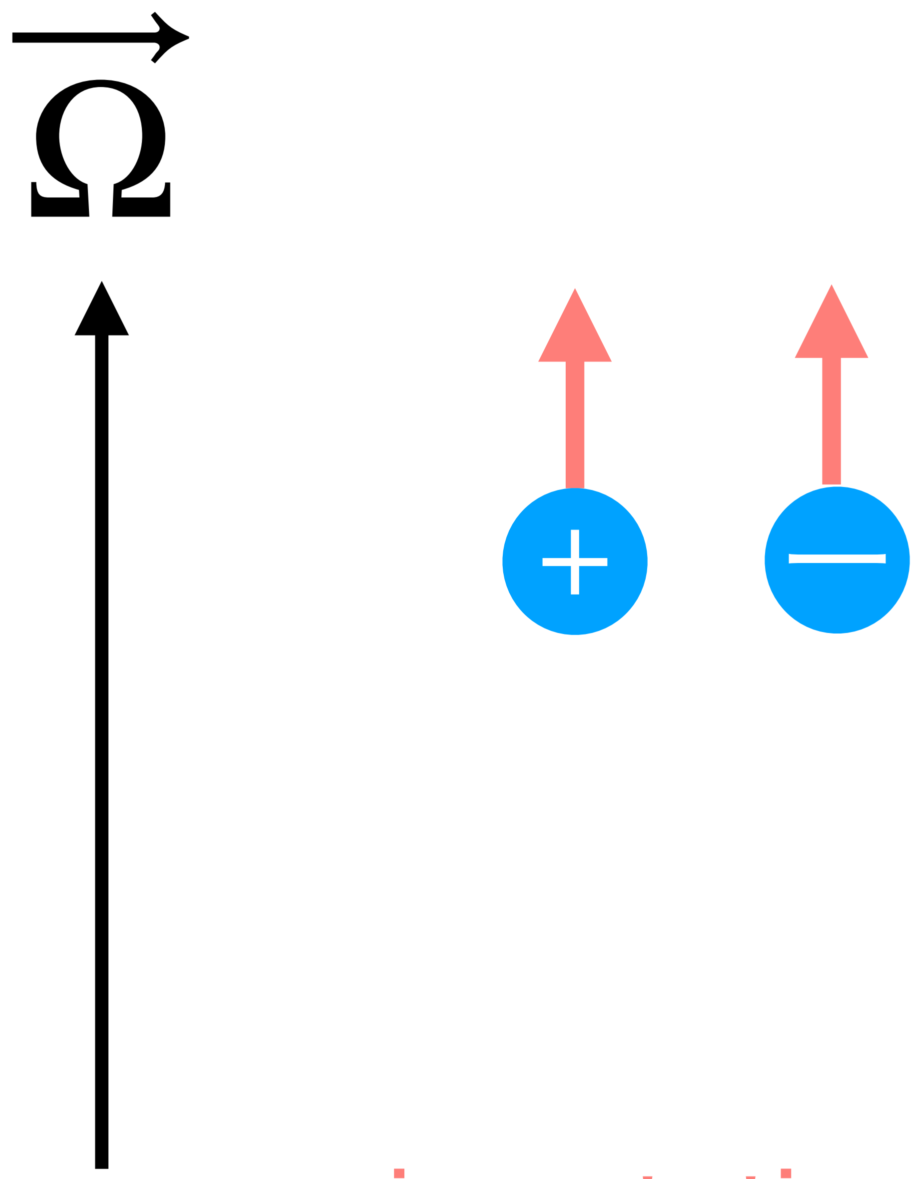
Lowest Landau Level (LLL)

$$\langle J_{\rm kin}\rangle_{\rm LLL} = \langle \Lambda + \Delta + S\rangle_{\rm LLL} = -1/2$$

-1 $\quad 0 \quad$ $+1/2$



Zeeman + cyclotron



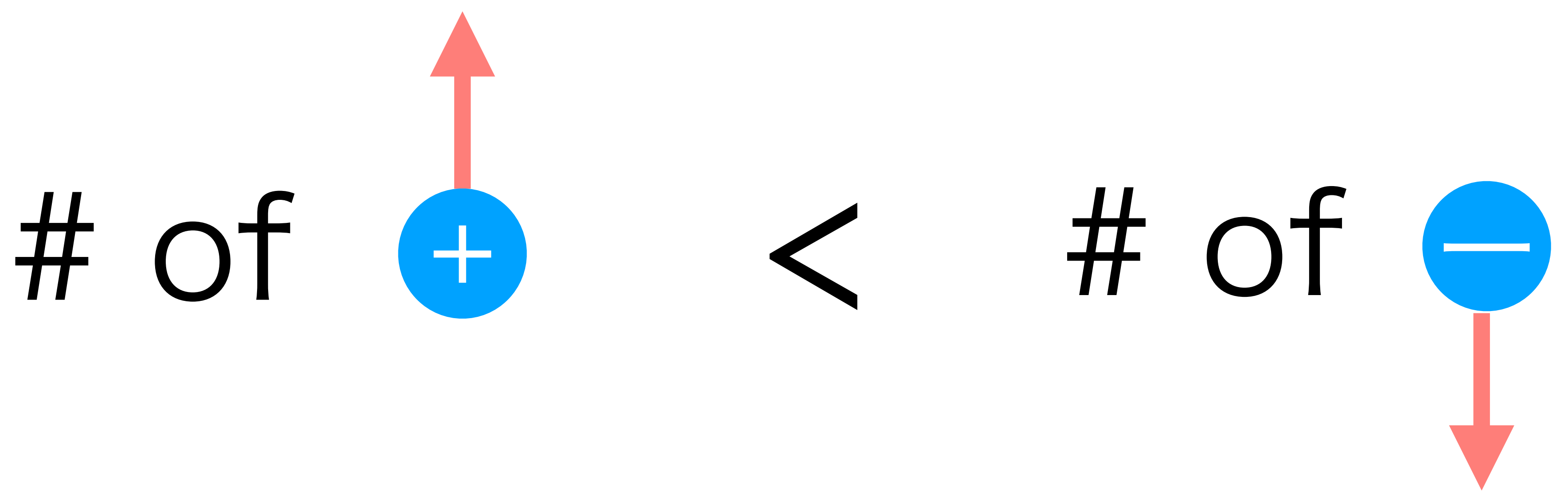
spin-rotation coupling

Lowest Landau Level (LLL)

$$\langle J_{\rm kin} \rangle_{\rm LLL} = \langle \Lambda + \Delta + S \rangle_{\rm LLL} = -1/2$$

$$\Lambda: -1 \qquad \Delta: 0 \qquad S: +1/2$$

$$\vec{B} \parallel \vec{\Omega}$$



This suggests

$$\rho = -\frac{eB\Omega}{4\pi^2}$$

LLL Pressure

Fukushima-Hattori-KM (in prep.)

$$Z = \det\Big[\mathcal{D}_B - \gamma^0 \Omega(\Lambda + \Delta + S)\Big]$$

in general, not diagonalized analytically

LLL Pressure

Fukushima-Hattori-KM (in prep.)

$$Z = \det\left[\mathcal{D}_B - \gamma^0 \underline{\Omega(\Lambda + \Delta + S)}\right]$$

$\nu = -\Omega/2$ (LLL)

$$P_{\mathrm{LLL}} = \frac{eB}{2\pi}\int\frac{\mathrm{d}p_z}{2\pi}\left[\ln\left(1+\mathrm{e}^{-\beta(\epsilon-\nu)}\right) + \ln\left(1+\mathrm{e}^{-\beta(\epsilon+\nu)}\right)\right]$$

massless limit $\rho = \frac{\partial P_{\mathrm{LLL}}}{\partial \nu} = -\frac{eB\Omega}{4\pi^2}$ (T-independent)

Comparisons

Fukushima-Hattori-Mameda (in prep.)
free energy (LLL)

$$\rho = \underbrace{\frac{eB\Omega}{4\pi^2}}_{\text{spin}} - \underbrace{\frac{eB\Omega}{2\pi^2}}_{\text{orbital}}$$

Ebihara-Fukushima-Mameda (2017)
incorrect

$$\rho = \frac{eB\Omega}{4\pi^2} + (\text{divergence w.r.t. AM})$$

due to $\vec{F}_{\text{drift}} = eB\Omega\vec{r}$

Hattori-Yin (2016)
linear responce (LLL)
incorrect

$$\rho = \frac{eB\Omega}{4\pi^2}$$

a mistake found

Yang et. al (2020) Mameda(2023)
chiral kinetic theory
correct

$$\rho = \frac{eB\Omega}{4\pi^2}$$

no Landau level formed by weak B

Anomaly-protected?

for any B only for strong B

$$\rho = \underbrace{\frac{eB\Omega}{4\pi^2}}_{\text{spin}} - \underbrace{\frac{eB\Omega}{2\pi^2}}_{\text{orbital}}$$

Only the spin part is due to chiral anomaly?

Hattori-Yin (2016)	YES	This is T-independent
Yang-Yamamoto (2021)	YES	This is derived as a Chern-Simons current

Anomaly-protected?

charge

$$\rho = \frac{\partial P_{\mathrm{LLL}}}{\partial \mu} = \frac{eB\Omega}{4\pi^2} - \frac{eB\Omega}{2\pi^2} + \frac{eB\mu}{2\pi^2}$$

spin orbital

angular momentum

$$J = \frac{\partial P_{\mathrm{LLL}}}{\partial \Omega} = \frac{eB\mu}{4\pi^2} - \frac{eB\mu}{2\pi^2} + \frac{eB\Omega}{8\pi^2}$$

$$\frac{\partial \rho}{\partial \Omega} = \frac{\partial J}{\partial \mu} = \frac{\partial^2 P_{\mathrm{LLL}}}{\partial \mu\, \partial \Omega}$$ → same coefficients shared

Anomaly-protected?

charge $$\rho = \frac{\partial P_{\rm LLL}}{\partial \mu} = \underbrace{\frac{eB\Omega}{4\pi^2}}_{\text{spin}} - \underbrace{\frac{eB\Omega}{2\pi^2}}_{\text{orbital}} + \frac{eB\mu}{2\pi^2}$$

angular momentum $$J = \frac{\partial P_{\rm LLL}}{\partial \Omega} = \frac{eB\mu}{4\pi^2} - \frac{eB\mu}{2\pi^2} + \frac{eB\Omega}{8\pi^2}$$

$$= j^5_{\rm CSE}/2$$

$$\frac{\partial \rho}{\partial \Omega} = \frac{\partial J}{\partial \mu} = \frac{\partial^2 P_{\rm LLL}}{\partial \mu\, \partial \Omega}$$

→ If $j^5_{\rm CSE}$ is anomaly-(un)protected, so is ρ

Moment of inertia of QCD vacuum

$$J = \frac{\partial P_{\mathrm{LLL}}}{\partial \Omega} = \frac{eB\mu}{4\pi^2} - \frac{eB\mu}{2\pi^2} + \frac{eB\Omega}{8\pi^2}$$

moment of inertia $$I = \frac{\partial J}{\partial \Omega} = \frac{eB}{8\pi^2}$$ (T-independent)

QCD vacuum = color magnetic media Savvidy (1977)

$$I_{\mathrm{QCD\,vacuum}} = \frac{N_f g\mathcal{B}}{8\pi^2} + I_{\mathrm{gluon}} \overset{?}{\neq} 0$$

cf. bag model analysis Mameda-Takizawa (2023)

Summary

- ✓ reformulate gauge-invariant and stable thermodynamics
- ✓ discover cyclotron contribution to magneto-vortical charge
- ✓ The charge is anomaly-unprotected part?
- ✓ kinder to lattice than pure rotation : no boundary effect, homogeneity
- ✓ applicability to various fields

  HIC : spin polarization under strong B

  neutron stars, electron systems, cold atoms, quantum optics