

Effects of the QCD critical point on spin polarization of Λ hyperons

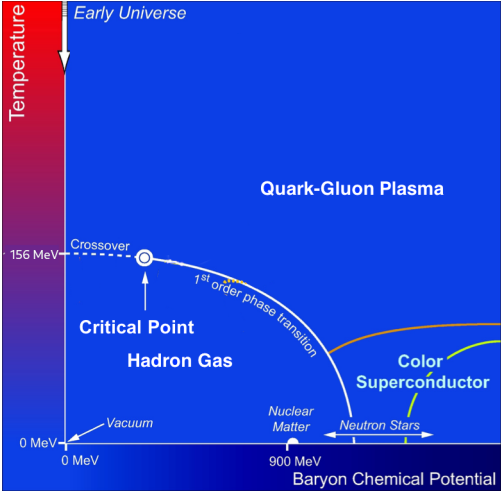
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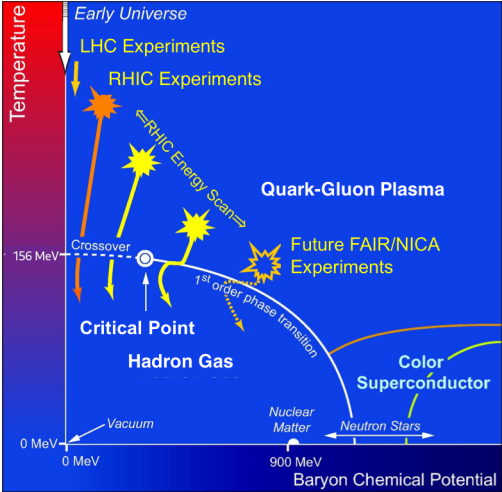
8th International Conference on Chirality, Vorticity, and Magnetic Field in
Quantum Matter



Conjectured phase diagram of QCD



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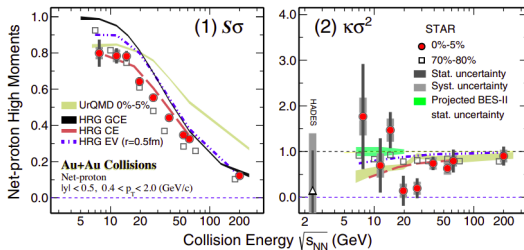


Signatures of QCD critical point

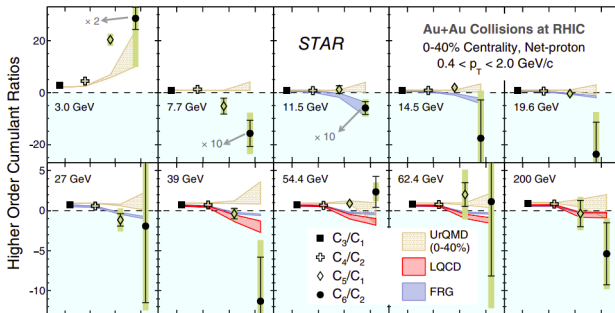
- Correlation length (ξ) diverges near critical point.
- Moments of fluctuations of conserved quantities are related to correlation length as, [PRL 102, 032301 \(2009\)](#),

$$\langle(\delta N)^2\rangle \sim \xi^2 \quad , \quad \langle(\delta N)^3\rangle \sim \xi^{4.5} \quad , \quad \langle(\delta N)^4\rangle_c \sim \xi^7 \dots$$

- Negative sign of the kurtosis, $\kappa = \langle(\delta N)^4\rangle/\sigma^4 - 3$, as the critical point is approached from the crossover side, [PRL 107, 052301 \(2011\)](#).



STAR Collaboration, [PRL 126, 092301 \(2021\)](#)



STAR Collaboration, PRL 130, 082301 (2023)

Can the critical point affect spin polarization measurements?

Model description

- Hydrodynamics rests on the assumption of local thermodynamic equilibrium.
- Near a critical point, the relaxation time diverges, $\tau \sim \xi^z$ (where z is the dynamic critical exponent, ≈ 3 for dynamic universality class model H).
- Non-equilibrium effects can be included by introducing a slow, non-hydrodynamic scalar mode, ϕ . [PRD 98, 036006 \(2018\)](#).
- The mode ϕ will affect hydrodynamic evolution by modifying EoS i.e. $p(\varepsilon, n) \rightarrow p(\varepsilon, n, \phi)$, which in turn will influence the evolution of ϕ .
- If we are not too close to the critical point, the back reaction of critical fluctuations on hydrodynamic variables may be neglected. Any residual effects appear in the scaling laws of transport coefficients (to be discussed later).

Relativistic Hydrodynamics

- Hydrodynamic equations that we solve are

$$D_\mu T^{\mu\nu} = 0$$

$$D_\mu N_B^\mu = 0$$

where

$$\Delta_{\alpha\beta}^{\mu\nu} u^\gamma D_\gamma \pi^{\alpha\beta} = -\frac{\pi^{\mu\nu} - \pi_{NS}^{\mu\nu}}{\tau_\pi} - \frac{4}{3} \pi^{\mu\nu} D_\gamma u^\gamma$$

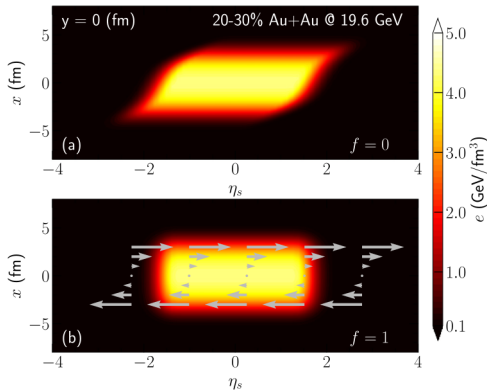
$$u^\gamma D_\gamma \Pi = -\frac{\Pi - \Pi_{NS}}{\tau_\Pi} - \frac{4}{3} \Pi D_\gamma u^\gamma$$

$$T^{\mu\nu} = \varepsilon u^\mu u^\nu - (p + \Pi) \Delta^{\mu\nu} + \pi^{\mu\nu}$$

$$N_B^\mu = n u^\mu$$

Initial condition

We use the initial condition of [PRC 102 \(2020\) 014909](#), [PRC 104, 054908 \(2021\)](#)



$$T^{\tau\tau} = e(x, y, \eta_s) \cosh(f y_{\text{CM}})$$

$$T^{\tau\eta} = \frac{1}{\tau_0} e(x, y, \eta_s) \sinh(f y_{\text{CM}})$$

f controls the fraction of longitudinal momentum attributed to fluid velocity.

Image from [PRC 104, 054908 \(2021\)](#)

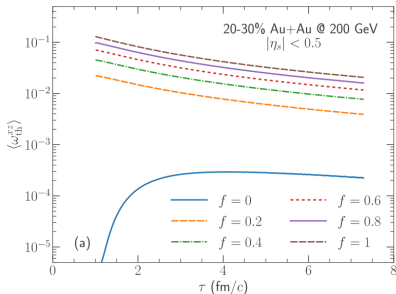


Image from [PRC 104, 054908 \(2021\)](#)

$$e(x, y, \eta_s) = \mathcal{N}_e(x, y) \exp \left[-\frac{(|\eta_s - y_{\text{CM}}| - \eta_0)^2}{2\sigma_\eta^2} \theta(|\eta_s - y_{\text{CM}}| - \eta_0) \right]$$

$$M(x, y) = m_N \sqrt{n_A^2 + n_B^2 + 2n_A n_B \cosh(y_{\text{beam}})}$$

$$y_{\text{CM}} = \tanh^{-1} \left[\frac{n_A - n_B}{n_A + n_B} \cosh(y_{\text{beam}}) \right]$$

We compute n_A and n_B from optical Glauber model.

Equation of state

We use the model from [PRC 101, 034901 \(2020\)](#)

- The pressure at non-zero T and μ_B can be obtained through a Taylor series expansion about $\mu_B = 0$ as follows

$$P_{QCD}(\mu_B, T) = T^4 \sum_n c_{2n}(T) \left(\frac{\mu_B}{T}\right)^{2n} \quad \Big] \rightarrow \text{noCP}$$

- The presence of CP makes some of the coefficients non-analytic

$$T^4 c_n(T) \rightarrow T^4 c_n^{\text{Non-Ising}}(T) + T_c^4 c_n^{\text{Ising}}(T)$$

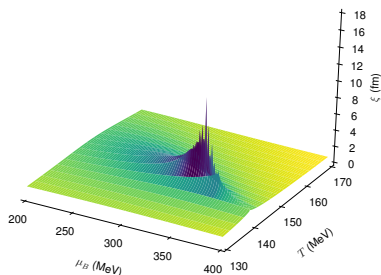
Equivalently,

$$P_{QCD}(\mu_B, T) = P^{\text{reg}}(\mu_B, T) + P^{\text{crit}}(\mu_B, T) \quad \Big] \rightarrow \text{CP}$$

- P^{crit} is obtained by mapping to 3D-Ising model in the “critical region”.

Choice of critical region

$$\xi^2 = \frac{1}{H_0} \left(\frac{\partial M(r, h)}{\partial h} \right)_r$$



Correlation length, ξ , plotted as a function of μ_B and T .

For the critical region, we take $\xi_0 = 1.75$ fm. Smaller value of ξ_0 would mean a larger critical region.

Transport coefficients

The dynamical universality class of the QCD critical point is Model H.
[PRD 70 \(2004\) 056001](#).

$$\zeta \sim \xi^3 \quad , \quad \eta \sim \xi^{0.05}$$

The critical behavior of these transport coefficients can be modeled as

$$\zeta = \zeta_0 \left(\frac{\xi}{\xi_0} \right)^3 \quad , \quad \eta = \eta_0 \left(\frac{\xi}{\xi_0} \right)^{0.05} \quad \text{for } \xi > \xi_0.$$

Similarly for τ_π and τ_Π , i.e.

$$\tau_\Pi = \tau_\Pi^0 \left(\frac{\xi}{\xi_0} \right)^3 \quad , \quad \tau_\pi = \tau_\pi^0 \left(\frac{\xi}{\xi_0} \right)^{0.05} \quad \text{for } \xi > \xi_0.$$

For $\xi < \xi_0$, $\zeta = \zeta_0$ and $\eta = \eta_0$,

$$\eta_0(\mu_B, T) = 0.08 \left(\frac{\varepsilon + p}{T} \right) \quad , \quad \zeta_0(\mu_B, T) = 15\eta_0(\mu_B, T) \left(\frac{1}{3} - c_s^2 \right)^2$$

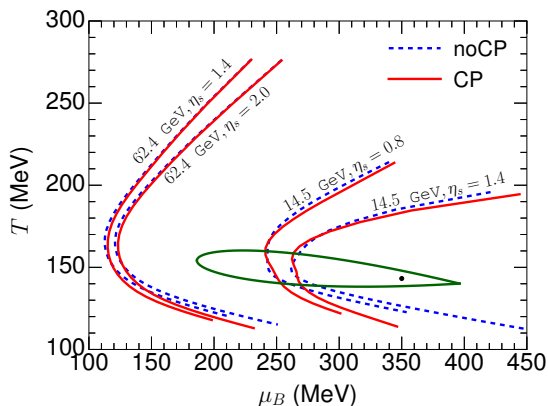
$$\frac{\tau_\pi^0}{5} = \tau_\Pi^0 = \frac{C_\eta}{T}$$

Other information

- Particlization surface is the constant energy density hypersurface, $\varepsilon_{sw} = 0.3 \text{ GeV}/\text{fm}^3$.
- The surface is found using the CORNELIUS code.
- The surface is input to the UrQMD for afterburning.
- The spin polarization analysis is done on ε_{sw} .

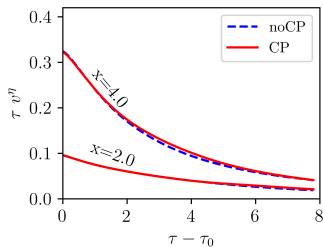
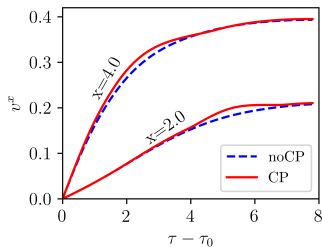
Results

Hydrodynamic trajectories in the phase diagram

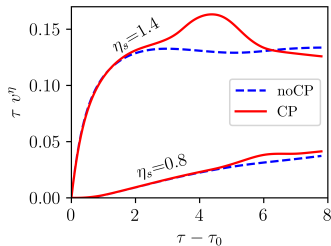
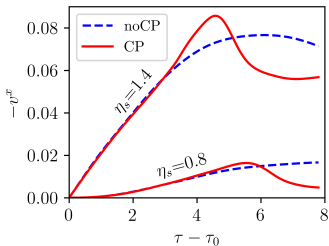


SKS and J. Alam, PRD 107, 074042 (2023)

Effects on flow velocity

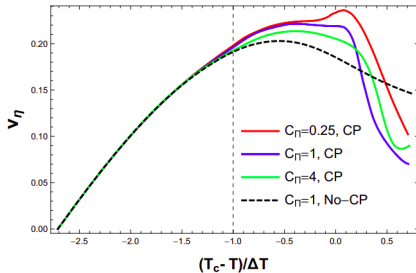
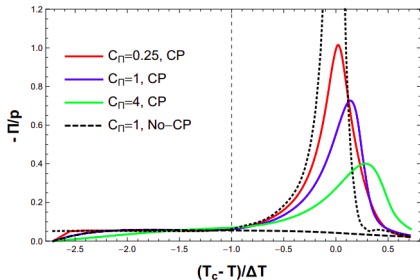


Time evolution of (left) v^x and, (right) τv^η for different values of x when $y = \eta_s = 0$.



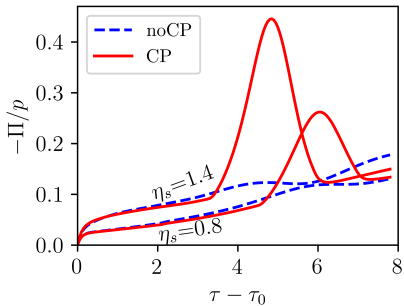
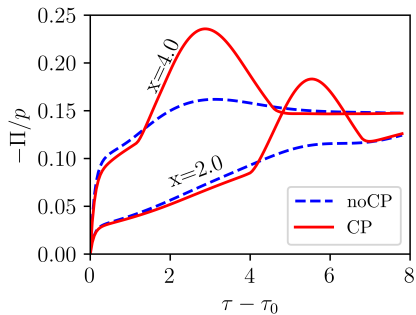
Time evolution of (left) v^x and, (right) τv^η for different values of η_s when $x = y = 0$.

Understanding the effects on flow velocity

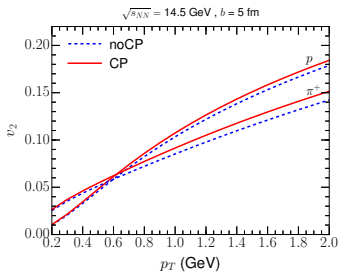
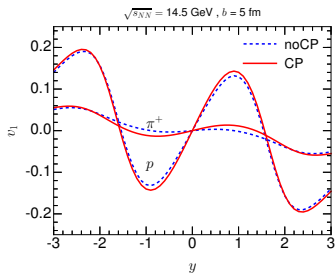
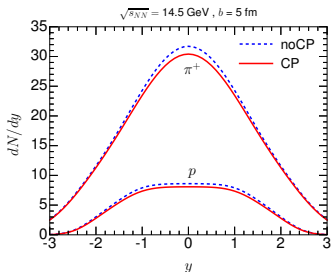
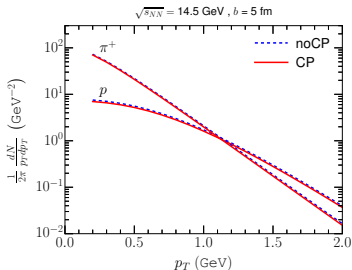


Hydrodynamic evolution trajectory corresponding to spatial rapidity $\eta = 1.5$.
Image from [PRC 95, 034902 \(2017\)](#)

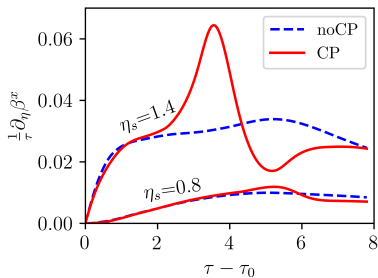
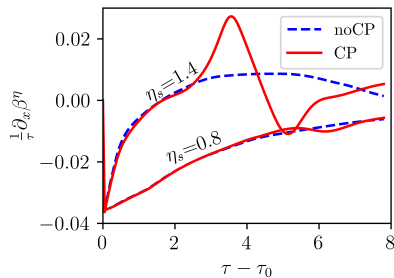
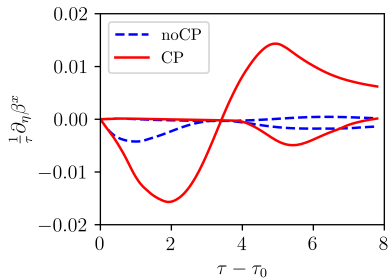
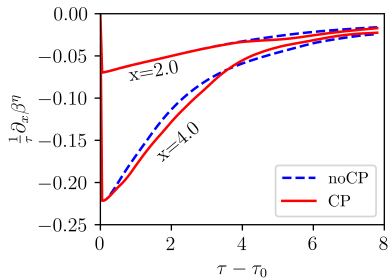
Evolution of bulk pressure



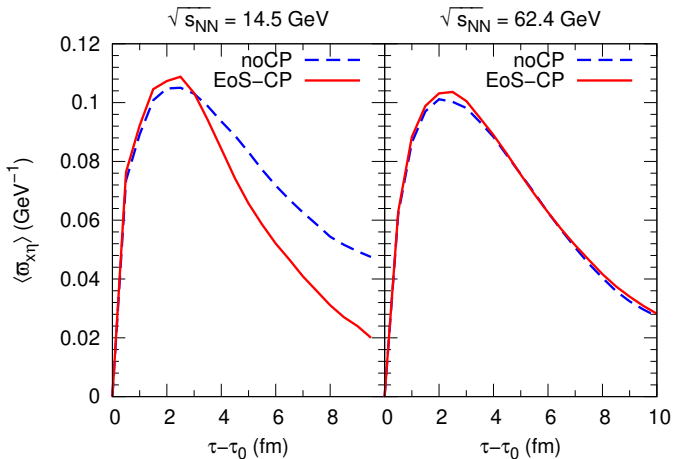
Effects on bulk observables



Effects on gradients of $\beta^\mu = u^\mu / T$



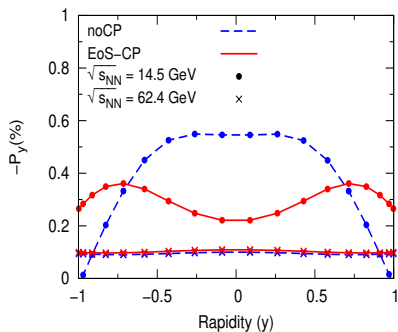
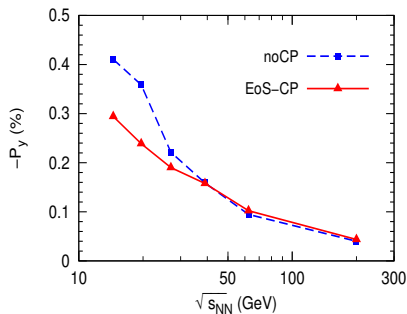
Effects on thermal vorticity ($f = 0$)



SKS and J. Alam, EPJC (2023) 83:585

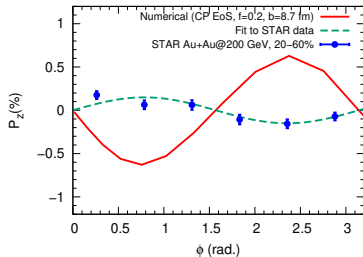
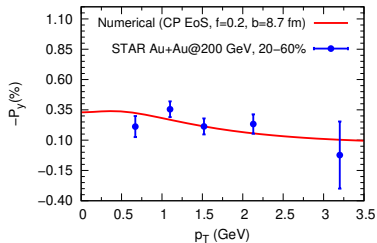
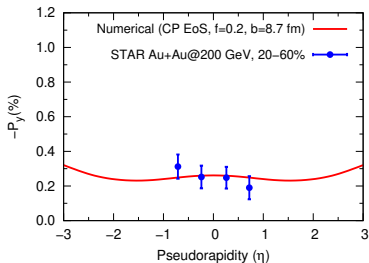
Effects on spin polarization of Λ -hyperons ($f = 0$)

Considering contribution from thermal vorticity alone !!!



(left) Global polarization plotted as function of collision energy. It gets suppressed as CP is approached, (right) rapidity distribution of spin polarization, large suppression around mid-rapidity. [SKS and J. Alam, EPJC \(2023\) 83:585](#)

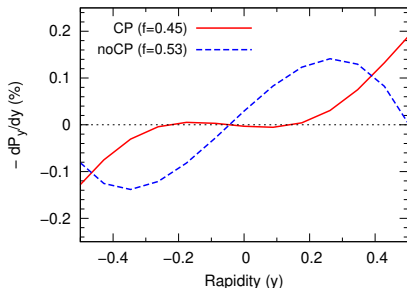
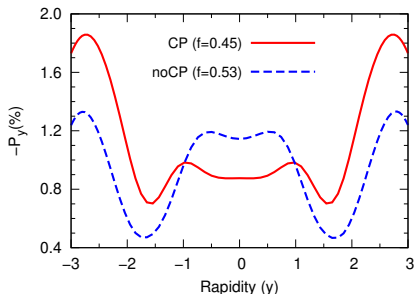
Comparison with experimental data at $\sqrt{s_{NN}} = 200$ GeV



Results with ϖ contribution only. SKS and J. Alam, EPJC (2023) 83:585

Effects on spin polarization near critical point

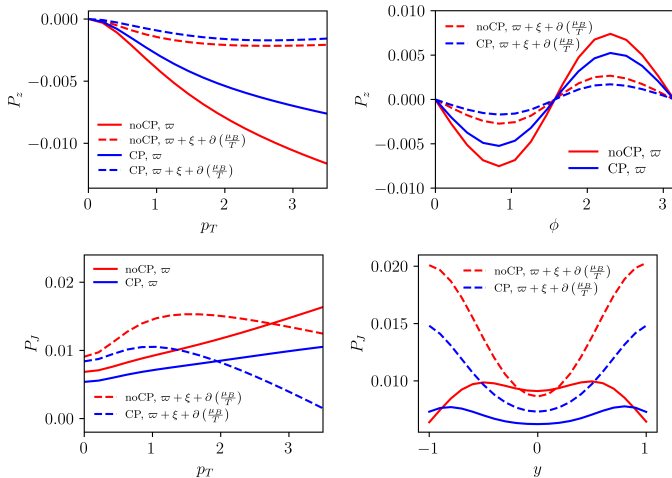
- Fix f to experimental data on global polarization for Au+Au collisions at $\sqrt{s_{NN}} = 14.5$ for CP and noCP cases.
- The suppression around mid-rapidity survives.
- A qualitative change in the behavior of rapidity profile observed with CP, as seen from the slopes.



SKS and J. Alam, EPJC (2023) 83:585

Results including shear and μ_B IP at $\sqrt{s_{NN}} = 14.5$ GeV (Preliminary)

Analysis using Andrea Palermo's "hydro-foil" after adding SHE
(<https://github.com/AndrePalermo/hydro-foil>)



Summary

- We analyze the effects of QCD critical point on the spin polarization of Λ hyperons.
- The model study suggests a qualitative change in the rapidity dependence of P_J .
- It might serve as an indicator of QCD CP. Further study is needed.
 - Validity of the assumption of smallness of back reaction of critical fluctuations on hydrodynamic variables. Valid in 1+1 dimensions [PRD 102, 094025 \(2020\)](#).
 - Spin polarization is an extremely sensitive observable.
 - P_J vs ϕ is sensitive to initial condition. [A. Palermo et al. arXiv:2404.14295](#)
 - Wrong sign in P_z vs ϕ without isothermal approximation at 200 GeV.
 - Analysis usually performed at particlization surface. Sensitive to ε_{sw} .
 - Sensitive to initial vorticity (parameter f).

Thank You !!!