Magnetic fields in pre-equilibrium: a faster road towards isotropization

ANA JULIA MIZHER*

* Unicid/IFT-Unesp, Brazil.

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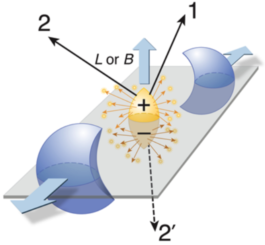
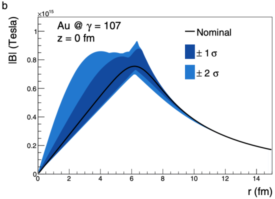
Timișoara

Magnetic fields in HIC

Non-central collisions: the strongest magnetic fields observed in the laboratory (4 orders higher than the strongest magnetic field observed in nature - magnetars).

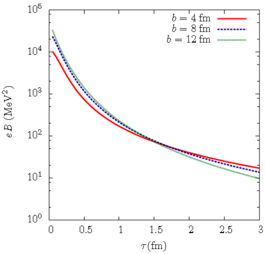


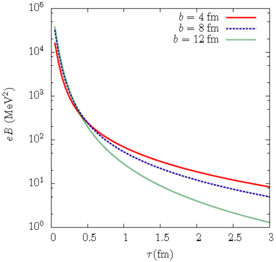


D. Brandenburg et al, Eur. Phys. J. A, 57, 299 (2021).

First experimental observation corroborates value predicted 12 years ago $eB \approx 10^{19}$G.

First estimate of the strength of magnetic fields





$$
\begin{aligned}
eB_s &\approx Z\alpha_{EM} exp(-2Y_0)\frac{4b}{\tau^3} \\
eB_p &\approx cZ_{EM} exp(-Y_0/2)\frac{1}{R^{1/2}\tau^{3/2}} f(b/R)
\end{aligned}
$$

Nucl.Phys.A 803 (2008) 227-253

Other estimates

- *Estimate of the magnetic field strength in heavy-ion collisions, V. Skokov, A. Illarionov, V. Toneev, Int.J.Mod.Phys.A 24 (2009) 5925-5932;*
- *Time and space dependence of the electromagnetic field in relativistic heavy-ion collisions*, K. Tuchin, Phys.Rev.C 88 (2013) 2, 024911;
- *Initial value problem for magnetic field in heavy ion collisions, K. Tuchin, Phys.Rev.C 93 (2016) 1, 014905;*
- *Magnetic field in expanding quark-gluon plasma, K. Tuchin and E. Stewart, Phys.Rev.C 97 (2018) 4, 044906*
- *Centrality dependence of photon yield and elliptic flow from gluon fusion and splitting induced by magnetic fields in relativistic heavy-ion collisions, A. Ayala, J. D. Castaño-Yepes,1, I. Dominguez, J. Salinas and M. E. Tejeda-Yeomans, Eur.Phys.J.A 56 (2020) 2, 53.*

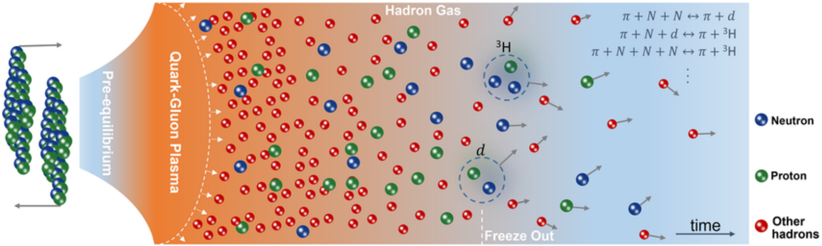
$$\vdots$$

- *Incomplete electromagnetic response of hot QCD matter, Z. Wang, J. Zhao, C. Greiner, Z. Xu, P. Zhuang, Phys.Rev.C 105 (2022) 4, L041901.*

Magnetic field relevant during the first instants after the collision

QGP evolution

Evolution of the system formed after a heavy-ion collision



Pre-equilibrium must be affected by the magnetic field

Pre-equilibrium

- During pre-equilibrium the energy is deposited in strong color fields which are liberated after the Glasma - dominate from $\tau \sim 1/Q_s$.
- Once the system has reached a local thermal equilibrium, or at least approximately isotropized, the matter in the mid-rapidity region (at sufficiently low pT) is described by relativistic fluid dynamics.
- Classical Yang-Mills theory does not isotropize when the system is subject to a rapid longitudinal expansion
- The gap between YM and hydrodynamic evolution has been bridged by effective kinetic theory (EKT).
- Descriptions using anisotropic hydro have also been proposed.

Effective kinetic theory

A.Kurkela and Y.Zhu, PRL 115, 182301 (2015)

An appropriate set of Boltzmann equations which will, on sufficiently long time and distance scales, correctly describe the dynamics of typical ultrarelativistic excitations,

$$(\partial_t + \mathbf{v} \cdot \nabla_{\mathbf{x}})f = -C[f]$$

where $f(\mathbf{x}, \mathbf{p}, t)$ is the phase space density of (quasi-)particles and $C[f]$ is a spatially-local collision term that represents the rate at which particles get scattered out of the momentum state p minus the rate at which they get scattered into this state. To leading order

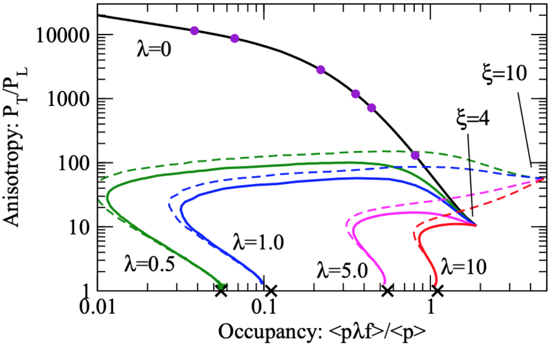
$$-\frac{df_{\mathbf{p}}}{d\tau} = \mathcal{C}_{1\leftrightarrow 2}[f_{\mathbf{p}}] + \mathcal{C}_{2\leftrightarrow 2}[f_{\mathbf{p}}] + \mathcal{C}_{exp}[f_{\mathbf{p}}]$$

Initial conditions,

$$f(p_z, p_t) = \frac{2}{\lambda} A f_0(p_z \xi / \langle p_T \rangle \rangle, p_\perp / \langle p_T \rangle)$$

$$f_0(\hat{p}_z, \hat{p}_\perp) = \frac{1}{\sqrt{\hat{p}_\perp^2 + \hat{p}_z^2}} e^{-2(\hat{p}_\perp^2 + \hat{p}_z^2)/3}$$

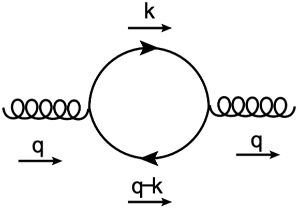
Ratio of the transverse and parallel pressure from EKT



A. Kurkela and Y. Zhu, Phys. Rev. Lett. 115, 182301 (2015)

The gluon polarization tensor

The medium in pre-equilibrium is saturated by gluons $\rightarrow$ indirect effect of the magnetic field



$$\Pi^{\mu\nu} = g^2 \left(g_{\parallel}^{\mu\nu} - \frac{q_{\parallel}^{\mu} q_{\parallel}^{\nu}}{q_{\parallel}^2} \right) \sum_f \frac{|q_f B|}{8\pi^2} e^{-q_{\perp}^2/(2|q_f B|)}$$

Calculating the pressure

In the strong field limit the propagator including the 1-loop fermion corrections is

$$iG^{\mu\nu} = \frac{\left(g_{\parallel}^{\mu\nu} - q_{\parallel}^{\mu} q_{\parallel}^{\nu} / q_{\parallel}^2\right)}{q_{\parallel}^2 - q_{\perp}^2 - g^2 \sum_f \frac{|q_f B|}{8\pi^2} e^{-q_{\perp}^2/(2|q_f B|)}}.$$

The pressure is given by $P = -V$

$$\begin{aligned} V &= -\frac{i}{2} \int \frac{d^2 q_{\parallel}}{(2\pi)^2} \frac{d^2 q_{\perp}}{(2\pi)^2} \\ &\times \ln\left(-q^2 + g^2 \sum_f \frac{|q_f B|}{8\pi^2} e^{-q_{\perp}^2/(2|q_f B|)}\right), \end{aligned}$$

with $q^2 = q_{\parallel}^2 - q_{\perp}^2$.

Calculating the pressure

Expanding the exponential to first order in B:

$$e^{-q_\perp^2/(2|q_f B|)} \to 1 - q_\perp^2/(2|q_f B|)$$

we get

$$\frac{i}{2a}\int db \int \frac{d^2 q_\parallel}{(2\pi)^2}\frac{d^2 \tilde{q}_\perp}{(2\pi)^2}\frac{1}{(q_\parallel^2 - \tilde{q}_\perp^2 - b)}$$

where we have defined

$$\begin{aligned} a &\equiv \left[1 - \frac{g^2}{8\pi^2}\right] = \left[1 - \frac{\alpha_s}{2\pi}\right] \\ b &\equiv g^2 \sum_f \frac{|q_f B|}{8\pi^2} = \frac{\alpha_s}{2\pi}|eB| \\ \tilde{q}_\perp^2 &\equiv a\, q_\perp^2, \end{aligned}$$

and used $\alpha_s = g^2/4\pi$, $q_u = 2/3\ e$, $|q_d| = 1/3\ e$ and $\alpha_s = 0.3$.

Calculating the pressure

We regularize separately the integral in $q_{\|}$ and $q_{\perp}$. For $q_{\|}$ getting,

$$V = \frac{1}{(8\pi)a}\int db \int \frac{d^2\tilde{q}_{\perp}}{(2\pi)^2}\left[\frac{1}{\varepsilon} + \ln\left(\frac{\mu^2}{\tilde{q}_{\perp}^2 + b}\right)\right].$$

We employ $\bar{MS}$ and absorb the divergence in the coupling. For the integral in $q_{\perp}$ we apply a sharp cutoff $|eB|_{\max} \gtrsim m_{\pi}^2$. Taking the scale $\mu = \Lambda_{QCD}$, we obtain,

$$V = \frac{1}{(64\pi^2)a}\left[3a\Lambda^2 b + b^2 \ln\left(\frac{b}{(\Lambda_{QCD}/2)^2}\right) + \left(b + a\Lambda^2\right)^2 \ln\left(\frac{(\Lambda_{QCD}/2)^2}{b + a\Lambda^2}\right)\right]$$

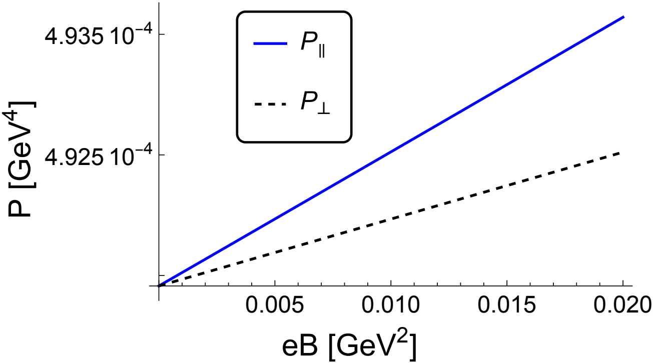
The parallel pressure is given by $P_{\perp} = P_{\|} + Mx_i\partial|eB|\partial x_i$. For simplicity we consider

$$P_{\perp} = P_{\|} - \eta M|eB|,$$

and take $\eta = 0.5$ and $M = -\frac{\partial V}{\partial eB}$.

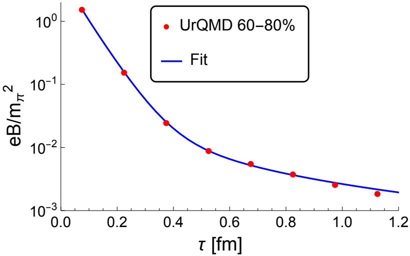
Parallel and transverse pressure

Parallel and transverse pressure as a function of eB.



Magnetic field profile

We adopt the magnetic field profile calculated using UrQMD, including participants and spectators in Au+Au semi-central collisions, 60-80% centrality, at $\sqrt{s_{NN}} = 200$ GeV **Eur.Phys.J.A 56 (2020) 2, 53**.



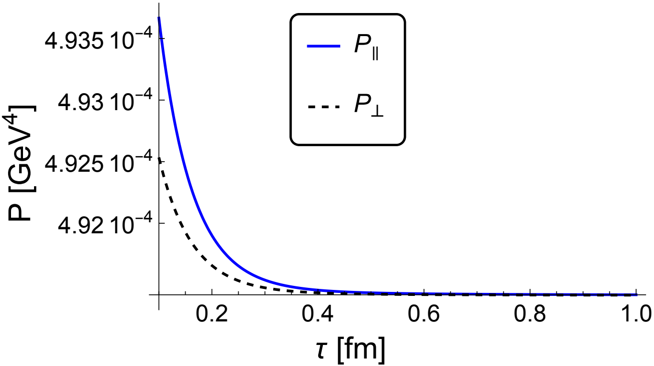
The time dependence of the field strength can be parametrized as

$$\frac{|eB|}{m_\pi^2} = Ae^{-B\tau} + \frac{C}{\tau^D},$$

with $A = 4.432$, $B = 15.895\ \text{fm}^{-1}$, $C = 0.003\ \text{fm}^D$ and $D = 1.682$.

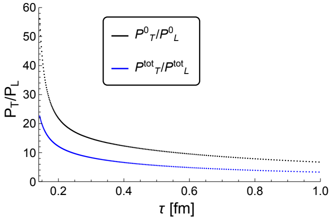
Parallel and transverse pressure

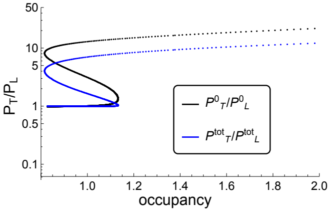
Parallel and transverse pressure as a function of the proper time τ,



Magnetic field as a catalyst of isotropy

Comparing the results from pure EKT (PRL 115, 182301 (2015)) and our results summed up to EKT, we see that when the magnetic field is taken into account the isotropization is reached faster.





The ratio of P_T / P_L is also lower for all the values of occupancy along the evolution of the system and it reaches 1 for a value slightly higher than in the case of pure EKT. The results for EKT were taken considering $\varepsilon = 10$ and the coupling $\lambda = 10$.

Summarizing

- Magnetic fields may affect gluon fields via quantum fluctuations involving quarks.
- We calculated the parallel and perpendicular pressure in a regime satured of gluon fields in the presence of a magnetic field. The calculation was performed in a regime of strong field.
- We showed that comparing to calculations that use EKT, the effect of the magnetic field is to accelerate the isotropization.
- Future improvements of our approach include to relax the strong fi-eld approximation and to estimate the magnetization using a space profile for the magnetic field rather than considering a field that is homogeneous in space.

Thank you!