



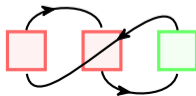
# Spin Alignment Induced by Curved Freeze-Out Hypersurface

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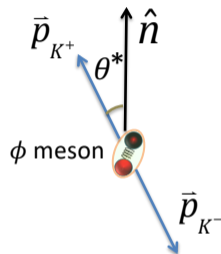
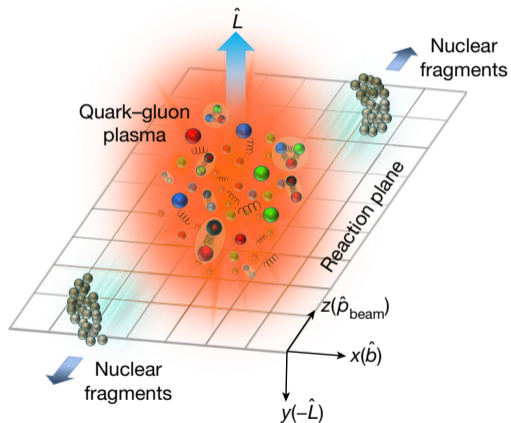
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July 22nd @ Chirality2024



# Spin Alignment



$$\frac{1}{N} \frac{dN}{d \cos \theta^*} = \frac{1}{2} + \frac{3}{4} (3 \cos^2 \theta^* - 1) \left( \Theta_{00} - \frac{1}{3} \right)$$

spin alignment

Heavy Ion Collision

Liang, Wang, PLB 629, 20 (2005)

STAR, Nature 614, 244-248 (2023)

# What is Tensor Polarization?

A massive vector boson's spin density matrix

$$\Theta_{rs} = \begin{pmatrix} \Theta_{1,1} & \Theta_{1,0} & \Theta_{1,-1} \\ \Theta_{0,1} & \Theta_{0,0} & \Theta_{0,-1} \\ \Theta_{-1,1} & \Theta_{-1,0} & \Theta_{-1,-1} \end{pmatrix}$$

$$\Theta = \frac{1}{3}\mathbb{1} + \frac{1}{2}\sum_{i=1}^3 P_i S_i + \sum_{m=-2}^2 (-1)^m T_{2,m} S_{2,-m} \quad (1)$$

$\langle S^i \rangle$ : Vector polarization (3 DoFs)

$\langle S_{2,m} \rangle$ : Tensor polarization (5 DoFs)

with

$$S^1 = S_x = \frac{i}{\sqrt{2}} \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}, \quad S^2 = S_y = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \quad S^3 = S_z = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}.$$

And  $S_{2,m} = \sum_{m_1, m_2} \langle 1, m_1; 1, m_2 | 2, m \rangle S_{1, m_1} S_{1, m_2}$  the rank-2 spherical irreducible tensor.

# How to Detect Tensor Polarization?

- Strong decay, parity even  $\Rightarrow T_{2,m}$  only:

$$\frac{1}{N} \frac{dN}{d\Omega}(\theta^*, \phi^*) = \frac{1}{4\pi} - \sqrt{\frac{3}{10\pi}} \sum_{m=-2}^2 (-1)^m T_{2,m} Y_{2,-m}(\theta^*, \phi^*)$$

- For now, *spin alignment* only:

$$\frac{1}{N} \frac{dN}{d \cos \theta^*} = \frac{1}{2} + \frac{3}{4} (3 \cos^2 \theta^* - 1) \left( \Theta_{00} - \frac{1}{3} \right)$$

- $T_{2,\pm 1}, T_{2,\pm 2} \rightarrow \phi^*$  distribution (off-diagonal polarization)

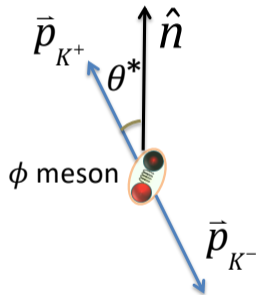
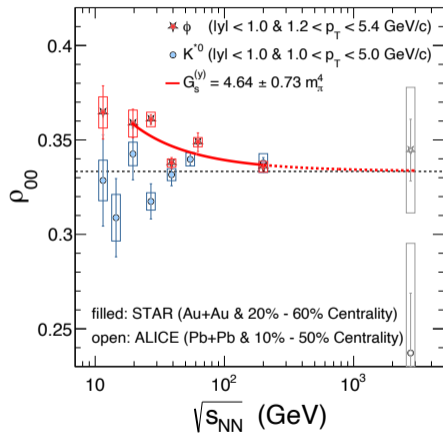


Figure: STAR, Nature.614.244-248 (2023)

# Global Spin Alignment

## Global spin alignment



- $\phi$  meson  $\Theta_{00} > 1/3$  and too big

$$\Theta_{00} - \frac{1}{3} \sim P_\Lambda^2 \sim 10^{-4}$$

- $K^{*0}$  different from  $\phi$

Figure: STAR, Nature 614, 244-248 (2023)

# Physical Mechanisms

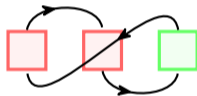
$$\phi \text{ meson: } \delta\Theta_{00} = \Theta_{00} - \frac{1}{3} \approx +c_{\Lambda} + c_B + c_s + c_F + c_L + c_H + c_{\phi} + c_g + \dots$$

Physical mechanism	$\delta\Theta_{00}$
$c_{\Lambda}$ : Quark coalescence + vorticity [1]	magnitude $\sim -10^{-4}$
$c_B$ : Quark coalescence + EM-field [1]	magnitude $\sim 10^{-4}$
$c_s$ : Spectrum splitting [2]	unclear
$c_F$ : Quark fragmentation [3]	magnitude $\sim 10^{-5}$
$c_L$ : Local spin alignment [4]	magnitude $\sim -10^{-2}$
$c_H$ : Second-order hydro fields [5]	unclear
$c_{\phi}$ : Vector meson field [6]	$> 0$ , fit to data
$c_g$ : Glasma fields [7]	$< 0$ , magnitude unclear

- [1]. Liang, Wang, PLB 629, 20 (2005); Yang *et al.* PRC 97, 034917 (2018); Xia *et al.* PLB 817, 136325 (2021); Becattini *et al.* PRC 88, 034905 (2013).
- [2]. Liu, Li, arXiv: 2206.11890; Sheng *et al.*, Eur.Phys.J.C 84, 299 (2024); Wei, Huang, Chin.Phys.C 47,104015 (2023).
- [3]. Liang, Wang PLB 629, 20 (2005);
- [4]. Xia *et al.* PLB 817, 136325 (2021); Gao, PRD 104, 076016 (2021).
- [5]. Kumar, Yang, Gubler, PRD 109, 054038(2024); Gao, Yang, Chin.Phys.C 48, 053114 (2024); ZZ, Huang, Becattini, Sheng, 2024.
- [6]. Sheng *et al.*, PRD 101, 096005 (2020); Sheng *et al.*, PRD 102, 056013 (2020); Sheng *et al.*, PRL 131, 042304 (2023).
- [7]. Muller, Yang, PRD 105, L011901 (2022); Kumar *et al.*, Phy. Rev. D108, 016020 (2023).

# Overview

- Flat freezeout hypersurface:  $\delta\Theta_{00} \propto (\partial\beta)(\partial\beta), (\partial\partial\beta), \dots$



- Curved freezeout hypersurface:  $\delta\Theta_{00} \propto (\partial\beta), \dots$

# Spin Density Matrix and Wigner function

- Free Lagrangian for neutral vector bosons

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m^2 A_\mu A^\mu$$

- Future time-like (particle) Wigner function:

$$\widehat{W}_+^{\mu\nu}(x, k) = \frac{1}{2\pi} \int d^4s e^{ik \cdot s} \widehat{A}^\nu(x - \frac{s}{2}) \widehat{A}^\mu(x + \frac{s}{2}) \theta(k^2) \theta(k^0)$$

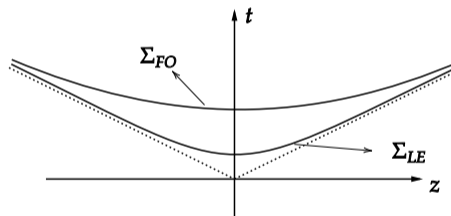
- Phase space distribution:

$$f(x, k) = \sum_r \epsilon_r^{\mu*}(k) \epsilon_r^\nu(k) W_{\mu\nu}(x, k)$$

$$\Theta_{rs}(x, k) = \epsilon_r^{\mu*}(k) \epsilon_s^\nu(k) W_{\mu\nu}(x, k) / f(x, k)$$

- Spin density matrix:

$$\Theta_{rs}(\mathbf{k}) \equiv \frac{\text{Tr}(\widehat{\rho} \widehat{a}_{\mathbf{k}}^{\dagger+} \widehat{a}_{\mathbf{k}}^r)}{\sum_r \text{Tr}(\widehat{\rho} \widehat{a}_{\mathbf{k}}^{r\dagger} \widehat{a}_{\mathbf{k}}^r)} = \frac{\int d\Sigma \cdot k \Theta_{rs}(x, k) f(x, k)}{\int d\Sigma \cdot k f(x, k)}$$



Freeze-out hypersurface



- Local equilibrium density operator (LEDO)

$$\hat{\rho}_{\text{LE}} = \frac{1}{Z_{\text{LE}}} \exp \left\{ - \int_{\Sigma} d\Sigma_{\mu}(y) \left[ \hat{T}^{\mu\nu}(y) \beta_{\nu}(y) - \frac{1}{2} \hat{S}^{\mu\rho\sigma}(y) \Omega_{\rho\sigma}(y) \right] \right\}$$

stress tensor
spin tensor

thermal current  $u_{\nu}/T \sim \mathcal{O}(1)$ 
spin potential  $\sim \mathcal{O}(\partial)$

Canonical currents:  $T^{\mu\nu} = -F^{\mu\rho}\partial^{\nu}A_{\rho} - g^{\mu\nu}\mathcal{L}$ ,  $S^{\mu\rho\sigma} = -F^{\mu\rho}A^{\sigma} + F^{\mu\sigma}A^{\rho}$

Integral measure:  $d\Sigma_{\mu}(y) = d\Sigma(y)n_{\mu}(y)$ , normal vector  $n_{\mu}(y)$

- LEDO maximizes the entropy of  $\hat{\rho}_{\text{LE}}$  on hypersurface  $\Sigma$ :

$$S[\hat{\rho}_{\text{LE}}] = -\text{Tr}(\hat{\rho}_{\text{LE}} \ln \hat{\rho}_{\text{LE}}),$$

under matching conditions  $n_{\mu}T^{\mu\nu}(x) = n_{\mu} \langle \hat{T}^{\mu\nu}(x) \rangle_{\text{LE}} [\beta, \Omega]$ ,  $n_{\mu}S^{\mu\rho\sigma}(x) = n_{\mu} \langle \hat{S}^{\mu\rho\sigma}(x) \rangle_{\text{LE}} [\beta, \Omega]$

Zubarev, Prozorkevich, Smolyanskii, Theo. and Math. Phys. 40, 821 (1979)

van Weert, Ann. of Phys. 140, 133 (1982)

Becattini, Buzzegoli, Palermo, Particles 2, 197 (2019)

# Cumulant Expansion

- LEDO  $\hat{\rho}_{\text{LE}} = \frac{1}{Z_{\text{LE}}} \exp \left\{ - \int_{\Sigma} d\Sigma_{\mu}(y) [\hat{T}^{\mu\nu}(y)\beta_{\nu}(y) - \frac{1}{2}\hat{S}^{\mu\rho\sigma}(y)\Omega_{\rho\sigma}(y)] \right\}$

"Gaussian" term  $\hat{A} = -\beta_{\nu}(x)\hat{P}^{\nu} = -\beta_{\nu}(x) \int_{\Sigma} d\Sigma_{\mu}(y)\hat{T}^{\mu\nu}(y)$

"Perturbative" terms  $\hat{B} = - \int_{\Sigma} d\Sigma_{\mu}(y) [\hat{T}^{\mu\nu}(y)(\beta_{\nu}(y) - \beta_{\nu}(x)) - \frac{1}{2}\hat{S}^{\mu,\rho\sigma}(y)\Omega_{\rho\sigma}(y)]$

- Cumulant expansion  $e^{\hat{A}+\hat{B}} = e^{\hat{A}} \sum_{n=0}^{\infty} \hat{B}_n$ ,

with  $\hat{B}_n \sim \mathcal{O}(\partial^n)$ ,  $\hat{B}_0 = 1$ ,  $\hat{B}_1 = \int_0^1 d\lambda e^{-\lambda\hat{A}}\hat{B}e^{\lambda\hat{A}}$ ,  $\hat{B}_2 = \dots$

$$W_{\mu\nu}(x, k) = \frac{\sum_{n=0}^{\infty} \langle \hat{B}_n \hat{W}_{\mu\nu}(x, k) \rangle_0}{\sum_{n=0}^{\infty} \langle \hat{B}_n \rangle_0} = \sum_{n=0}^{\infty} \langle \hat{B}_n \hat{W}_{\mu\nu}(x, k) \rangle_{0,c}$$

with  $\langle \hat{O} \rangle_0 = \text{Tr} (e^{\hat{A}} \hat{O}) / \text{Tr} (e^{\hat{A}})$ .

# Zeroth-Order Result

- Cumulant expansion at 0th order:  $W_{\mu\nu}^{(0)}(x, k) = \langle \widehat{W}_{\mu\nu}(x, k) \rangle_0$
- "Free" distribution:  $\langle \widehat{a}_{\mathbf{k}}^{s\dagger} \widehat{a}_{\mathbf{q}}^r \rangle_0 = (2\pi)^3 \delta^{rs} \delta^{(3)}(\mathbf{k} - \mathbf{q}) (2E_{\mathbf{k}}) n_B(\beta(x) \cdot k)$   
Bose-Einstein distribution:  $n_B(x) = 1/(e^x - 1)$

- Zeroth-order Wigner fn.: nothing polarized.

$$\begin{aligned}
 W_{\mu\nu}^{(0)}(x, k) &= \int \frac{d^3 \mathbf{p}_1}{(2\pi)^3} \frac{d^3 \mathbf{p}_2}{(2\pi)^3} (2\pi)^3 \frac{1}{2E_{\mathbf{p}_1}} \frac{1}{2E_{\mathbf{p}_2}} e^{-i(\mathbf{p}_1 - \mathbf{p}_2) \cdot x} \\
 &\quad \times \sum_{a_1, a_2} \delta^{(4)}\left(k - \frac{\mathbf{p}_1}{2} - \frac{\mathbf{p}_2}{2}\right) \langle \widehat{a}_{\mathbf{p}_2}^{a_2\dagger} \widehat{a}_{\mathbf{p}_1}^{a_1} \rangle_0 \epsilon_{a_1}^\mu(\mathbf{p}_1) \epsilon_{a_2}^{\nu*}(\mathbf{p}_2). \\
 &= -\delta(k^2 - m^2) \theta(k^0) \Delta_{\mu\nu}^{(k)} n_B(\beta(x) \cdot k)
 \end{aligned}$$

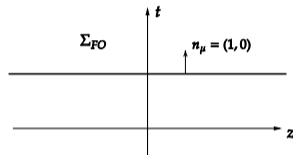
$$\Delta_{\mu\nu}^{(k)} = \eta_{\mu\nu} - k_\mu k_\nu / m^2$$

- Recall that  $\Theta_{rs}(x, k) \propto \epsilon_r^{\mu*}(k) \epsilon_s^\nu(k) W_{\mu\nu}(x, k)$ , which leads to  $\Theta_{rs}^{(0)} = \delta_{rs}$

# Flat Hypersurface: First-Order Gradient Expansion

Cumulant expansion:  $W_{\mu\nu}^{(1)}(x, k) = \langle \widehat{B}_1 \widehat{W}_{\mu\nu}(x, k) \rangle_{0,c} = \langle \widehat{B}_1 \widehat{W}_{\mu\nu}(x, k) \rangle_0 - \langle \widehat{B}_1 \rangle_0 \langle \widehat{W}_{\mu\nu}(x, k) \rangle_0$

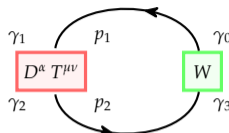
$$\widehat{B}_1 = -n_\mu \partial_\alpha \beta_\nu(x) \int_0^1 d\lambda \int_P d^3 \mathbf{y} (y-x)^\alpha \widehat{T}^{\mu\nu}(y - i\lambda\beta(x)) + \frac{1}{2} n_\mu \Omega_{\rho\sigma}(x) \int_0^1 d\lambda \int_P d^3 \mathbf{y} \widehat{S}^{\mu\rho\sigma}(y - i\lambda\beta(x))$$



$$\int_0^1 d\lambda \int_P d^3 \mathbf{y} (y-x)^\alpha \langle \widehat{T}^{\mu\nu}(y - i\lambda\beta(x)) \widehat{W}_{\xi\zeta}(x, k) \rangle_{0,c} = \boxed{D^\alpha T^{\mu\nu}} \boxed{W_{\xi\zeta}} + \boxed{D^\alpha T^{\mu\nu}} \boxed{W_{\xi\zeta}}$$

$$\int_0^1 d\lambda \int_P d^3 \mathbf{y} \langle \widehat{S}^{\mu\rho\sigma}(y - i\lambda\beta(x)) \widehat{W}_{\xi\zeta}(x, k) \rangle_{0,c} = \boxed{S^{\mu\rho\sigma}} \boxed{W_{\xi\zeta}} + \boxed{S^{\mu\rho\sigma}} \boxed{W_{\xi\zeta}}$$

# First-Order Gradient Expansion: Diagram Rules



$$\gamma_0 = \int_0^1 d\lambda \int_P d^3\mathbf{y} (y-x)^\alpha \prod_{i=0}^3 \left( \sum_{a_i} \int \frac{d^3\mathbf{p}_i}{(2\pi)^3} \frac{1}{2E_{\mathbf{p}_i}} \right) (2\pi)^3 \delta^{(4)} \left( k - \frac{p_1}{2} - \frac{p_2}{2} \right) \left[ t^{\mu\nu}_{\gamma_1\gamma_2}(p_1, -p_2) e^{i(p_0 - p_3) \cdot x} \right. \\ \left. \times \left\langle \hat{a}_{\mathbf{p}_1}^{a_1} \hat{a}_{\mathbf{p}_0}^{a_0 \dagger} \right\rangle_0 \left\langle \hat{a}_{\mathbf{p}_2}^{a_2 \dagger} \hat{a}_{\mathbf{p}_3}^{a_3} \right\rangle_0 \epsilon_{\gamma_0}^{a_0} (p_0) \epsilon_{\gamma_1}^{a_1} (p_1) \epsilon_{\gamma_2}^{a_2} (p_2) \epsilon_{\gamma_3}^{a_3} (p_3) e^{-i(p_1 - p_2) \cdot (y_1 - i\lambda_1 \beta(x))} \right]$$

$$= \delta(k^0 - E_{\mathbf{k}}) \int_0^1 d\lambda \quad \underbrace{D^\alpha t^{\mu\nu}_{\gamma_1\gamma_2}(p_1, -p_2)}_{\text{vertex}} \underbrace{(2n \cdot p_1)(n_B(\beta \cdot p_1) + 1) (2n \cdot p_2) n_B(\beta \cdot p_2)}_{\text{Bose-Einstein distributions from lines}} \underbrace{\begin{pmatrix} -\Delta_{(p_1)}^{\gamma_0\gamma_1} \\ -\Delta_{(p_2)}^{\gamma_2\gamma_3} \end{pmatrix}}_{\text{spins sums from lines}} \Big|_{p_1=p_2=k} \quad \text{momentum conservation}$$

$$t^{\mu\nu}_{\gamma_1\gamma_2}(p_1, p_2) = \frac{e^{-\lambda(p_1+p_2) \cdot \beta}}{(2n \cdot p_1)(2n \cdot p_2)} \left[ p_1^\mu p_2^\nu \eta_{\gamma_1\gamma_2} - p_{2,\gamma_2} p_2^\nu \eta_{\gamma_1}^\mu - \frac{1}{2} (p_1 \cdot p_2) \eta^{\mu\nu} \eta_{\gamma_1\gamma_2} + \frac{1}{2} p_{1,\gamma_2} p_{2,\gamma_1} \eta^{\mu\nu} - \frac{1}{2} m^2 \eta^{\mu\nu} \eta_{\gamma_1\gamma_2} \right]$$

$$D^\alpha = \left( -\frac{i}{2} \right) \left( \eta^{\alpha\zeta} - n^\alpha n^\zeta \right) \left( \frac{\partial}{\partial p_1^\zeta} - \frac{\partial}{\partial p_2^\zeta} \right)$$

ZZ, Huang, Becattini, Sheng, to appear

- Adaptability (higher-order results, additional operators in LEDO, fermions)

# Flat Hypersurface: First-Order Gradient Expansion

- Recall that:  $W_{\mu\nu}^{(1)}(x, k) = -n_\xi \partial_\alpha \beta_\lambda(x) \int_0^1 d\lambda \int_P d^3 \mathbf{y} (y-x)^\alpha \left\langle \widehat{T}^{\xi\lambda}(y - i\lambda\beta(x)) \widehat{W}_{\mu\nu}(x, k) \right\rangle_{0,c}$  + spin potential contribution

$$W_{\perp, \mu\nu}^{(1)}(x, k) = -i\delta(k^2 - m^2)\theta(k^0)n_B(1 + n_B)\Delta_{\mu\rho}^{(k)}\Delta_{\nu\sigma}^{(k)} \left[ \underbrace{\omega^{\rho\sigma}}_{\text{thermal vorticity}} - \Xi_\alpha^{[\rho} \left( \underbrace{\xi^{\sigma]\alpha}}_{\text{thermal shear}} + \underbrace{\delta\Omega^{\sigma]\alpha}}_{\text{net spin potential: } \Omega - \omega} \right) \right]$$

with:  $\Xi^{\mu\nu} = \eta^{\mu\nu} - \frac{k^\mu n^\nu}{(n \cdot k)}$

- Spin alignment at leading order:

$$\delta\Theta_{00}(x, k) \approx \frac{1}{3n_B\delta(k^2 - m^2)\theta(k^0)} \left( \epsilon_y^\mu(k)\epsilon_y^\nu(k) + \frac{1}{3}\Delta_{(k)}^{\mu\nu} \right) W_{\mu\nu}(x, k)$$

- Space-time reversal odd:  $W_{\mu\nu}^{(1)} = -W_{\nu\mu}^{(1)} \Rightarrow \delta\Theta_{00} = 0 + \mathcal{O}(\partial^2)$

ZZ, Huang, Becattini, Sheng, to appear

$$W_{\gamma_0\gamma_5}^{(2)}|_{TT}(x,k) = \hat{n}_{\mu_1}\hat{n}_{\mu_2}\partial_{\alpha_1}\beta_{\nu_1}(x)\partial_{\alpha_2}\beta_{\nu_2}(x) \left\{ \begin{array}{l} \boxed{D_{(1)}^{\alpha_1} T_{(1)}^{\mu_1\nu_1}} \quad \boxed{D_{(2)}^{\alpha_2} T_{(2)}^{\mu_2\nu_2}} \quad \boxed{W_{\gamma_0\gamma_5}} \\ + \text{ other 15 diagrams} \end{array} \right\}$$

$$W_{\gamma_0\gamma_5}^{(2)}|_{TS}(x,k) = \hat{n}_{\mu_1}\hat{n}_{\mu_2}\partial_{\alpha_1}\beta_{\nu_1}(x)\Omega_{\rho_2\sigma_2}(x) \left\{ \begin{array}{l} \boxed{D_{(1)}^{\alpha_1} T_{(1)}^{\mu_1\nu_1}} \quad \boxed{S_{(2)}^{\mu_2\rho_2\sigma_2}} \quad \boxed{W_{\gamma_0\gamma_5}} \\ + \text{ other 15 diagrams} \end{array} \right\}$$

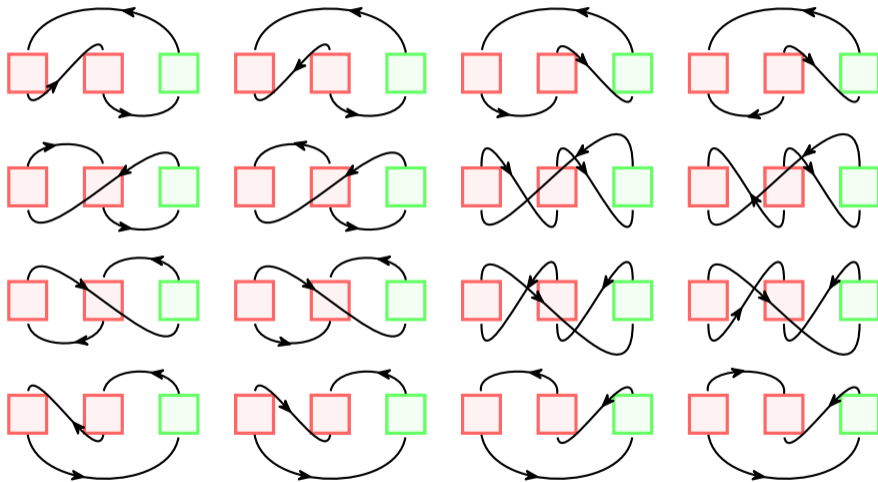
$$W_{\gamma_0\gamma_5}^{(2)}|_{ST}(x,k) = \hat{n}_{\mu_1}\hat{n}_{\mu_2}\Omega_{\rho_1\sigma_1}(x)\partial_{\alpha_2}\beta_{\nu_2}(x) \left\{ \begin{array}{l} \boxed{S_{(1)}^{\mu_1\rho_1\sigma_1}} \quad \boxed{D_{(2)}^{\alpha_2} T_{(2)}^{\mu_2\nu_2}} \quad \boxed{W_{\gamma_0\gamma_5}} \\ + \text{ other 15 diagrams} \end{array} \right\}$$

$$W_{\gamma_0\gamma_5}^{(2)}|_{SS}(x,k) = \hat{n}_{\mu_1}\hat{n}_{\mu_2}\Omega_{\rho_1\sigma_1}(x)\Omega_{\rho_2\sigma_2}(x) \left\{ \begin{array}{l} \boxed{S_{(1)}^{\mu_1\rho_1\sigma_1}} \quad \boxed{S_{(2)}^{\mu_2\rho_2\sigma_2}} \quad \boxed{W_{\gamma_0\gamma_5}} \\ + \text{ other 15 diagrams} \end{array} \right\}$$

$$W_{\gamma_0\gamma_5}^{(2)}|_T(x,k) = \hat{n}_{\mu_1}\partial_{\alpha_1}\partial_{\alpha_2}\beta_{\nu_1}(x) \left\{ \begin{array}{l} \boxed{D_{(1)}^{\alpha_1} D_{(1)}^{\alpha_2} T_{(1)}^{\mu_1\nu_1}} \quad \boxed{W_{\gamma_0\gamma_5}} \\ + \text{ the other diagram} \end{array} \right\}$$

$$W_{\gamma_0\gamma_5}^{(2)}|_S(x,k) = \hat{n}_{\mu_1}\partial_{\alpha_1}\Omega_{\rho_1\sigma_1}(x) \left\{ \begin{array}{l} \boxed{D_{(1)}^{\alpha_1} S_{(1)}^{\mu_1\rho_1\sigma_1}} \quad \boxed{W_{\gamma_0\gamma_5}} \\ + \text{ the other diagram} \end{array} \right\}$$

# 16 Diagrams





# Flat Hypersurface: Second-Order Gradient Expansion

- Following the cumulant expansion and the diagram rules:

$$W_{\perp, \mu\nu}^{(2)} | \omega^2 = \frac{1}{2} \delta(k^2 - m^2) \theta(k^0) n_B (1 + n_B) (1 + 2n_B) \left( \eta_{\rho\sigma} - \frac{k_\rho k_\sigma}{2m^2} \right) \omega_{\mu\rho}(x) \omega_{\sigma\nu}(x) + \Delta_{\mu\nu}^{(k)} \dots$$

$$W_{\perp, \mu\nu}^{(2)} | \partial\omega = \delta(k^2 - m^2) \theta(k^0) n_B (1 + n_B) (1 + 2n_B) \frac{1}{2(n \cdot k)} \partial_\alpha \omega_{\rho\sigma}(x) \\ \times \left\{ \eta^{\alpha\rho} \hat{k}^\sigma n_\mu n_\nu - n^\rho \hat{k}^\sigma \eta^\alpha_{(\mu} n_{\nu)} - 2\eta^{\alpha\rho} \eta^\sigma_{(\mu} n_{\nu)} + 2n^\rho \eta^\sigma_{(\mu} \eta_{\nu)}^\alpha \right\} + \Delta_{\mu\nu}^{(k)} \dots$$

⋮

- PT even:  $W_{\mu\nu}^{(2)}(x, k) = W_{\nu\mu}^{(2)}(x, k)$

ZZ, Huang, Becattini, Sheng, to appear

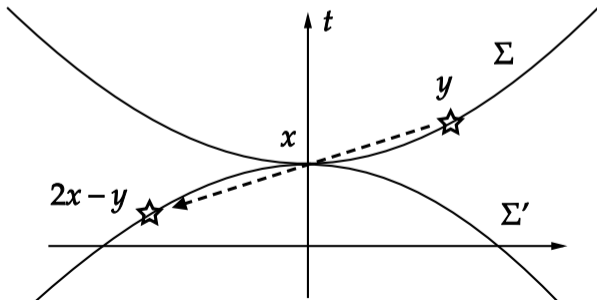


# Break PT Symmetry of the System

How to break  $W_{\mu\nu}^{(1)} = -W_{\nu\mu}^{(1)}$  to get  $\delta\Theta_{00} \propto \partial_\alpha\beta_\nu, \Omega_{\rho\sigma}$ ?

- Dissipative process  $\rightarrow$  Time reversal symmetry broken W. Dong *et al.*, PRD 109, 056025 (2024); ...
- Chiral symmetry breaking  $\rightarrow$  Parity symmetry broken not clear
- Freeze-out hypersurface curved this work

Recall that  $W_{\mu\nu}^{(1)} = -W_{\nu\mu}^{(1)}$   
based on  $\Sigma = \Sigma'$



# Geometry of Curved Hypersurface

- Assume  $\Sigma$  is a **space-like** 3D hypersurface:

$$F(x) = 0, \forall x \in \Sigma$$

Choose a proper  $F$ , s.t.  $v_0 > 0$ , with  $v_\mu = \partial F(x) / \partial x^\mu$

Normal vector of the hypersurface:  $n_\mu = v_\mu / \sqrt{v \cdot v}$

- Taylor expansion around  $x$  upto second order:

$$n \cdot (y - x) = \frac{1}{2} B_{\mu\nu} (y - x)^\mu (y - x)^\nu + \mathcal{O} \left( ((y - x)_\perp)^3 \right)$$

- Curvature tensor:

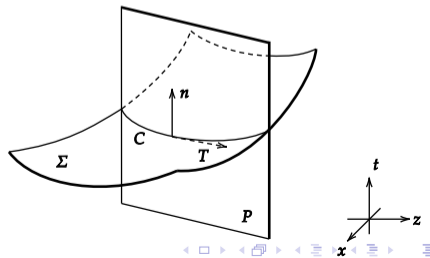
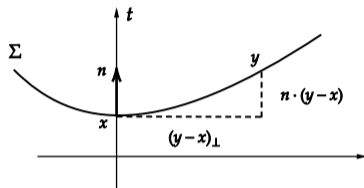
projection  $\perp$  to  $n$

$$B_{\mu\nu}(x) \equiv -\frac{1}{\sqrt{v \cdot v}} \Delta_{\mu\rho}^{(n)} \Delta_{\nu\sigma}^{(n)} \frac{\partial^2 F(x)}{\partial x_\rho \partial x_\sigma}$$

curvature of curve  $C$

$\Delta_{\mu\nu}^{(n)} T^\mu T^\nu$

$$B_{\mu\nu} T^\mu T^\nu = \kappa_C \left( -T_\perp^2 \right)$$



# Curved Hypersurface: First-Order Gradient Expansion

- Cumulant expansion:  $W_{\mu\nu}^{(1)}(x, k) = \langle \widehat{B}_1 \widehat{W}_{\mu\nu}(x, k) \rangle_{0,c}$

$$\widehat{B}_1 = -\partial_\alpha \beta_\nu(x) \int_0^1 d\lambda \int_{y \in \Sigma} d\Sigma_\mu(y) (y-x)^\alpha \widehat{T}^{\mu\nu}(y - i\lambda\beta(x)) + \frac{1}{2} \Omega_{\rho\sigma}(x) \int_0^1 d\lambda \int_{y \in \Sigma} d\Sigma_\mu(y) \widehat{S}^{\mu\rho\sigma}(y - i\lambda\beta(x))$$

- Taylor expansion at  $x$ : For  $y \in \Sigma$ ,  $y' \in P(x)$ ,  $(y - y')_\mu = \frac{1}{2} B_{\rho\sigma}(y' - x)^\rho (y' - x)^\sigma n_\mu + \dots$

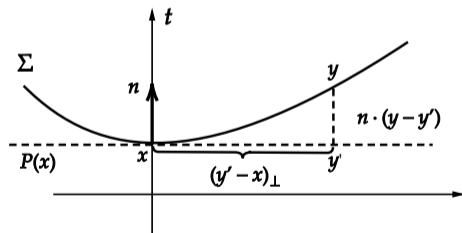
$$\int_{y \in \Sigma} \rightarrow \int_{y' \in P(x)}$$

$$y_\mu \rightarrow y'_\mu + n \cdot (y - y') n_\mu$$

$$d\Sigma_\mu(y) \rightarrow d^3 \mathbf{y}'_\perp [n_\mu(x) - B_{\mu\nu}(x)(y' - x)^\nu]$$

$$\widehat{T}^{\mu\nu}(y - i\lambda\beta(x)) \rightarrow \widehat{T}^{\mu\nu}(y' - i\lambda\beta(x))$$

$$+ n \cdot (y - y') (n \cdot \partial) \widehat{T}^{\mu\nu}(y' - i\lambda\beta(x))$$



- Valid when  $1/\kappa \gg \lambda$

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$$\begin{aligned} \widehat{B}_1 = & -\partial_\alpha \beta_\nu(x) n_\mu(x) \int_0^1 d\lambda \int_{y' \in P} d^3 \mathbf{y}'_\perp (y' - x)^\alpha \widehat{T}^{\mu\nu}(y' - i\lambda\beta(x)) \longrightarrow \text{calculated previously} \\ & + \partial_\alpha \beta_\nu(x) B_{\mu\rho}(x) \int_0^1 d\lambda \int_{y' \in P} d^3 \mathbf{y}'_\perp (y' - x)^\rho (y' - x)^\alpha \widehat{T}^{\mu\nu}(y' - i\lambda\beta(x)) \\ & - \frac{1}{2} \partial_\alpha \beta_\nu(x) B_{\rho\sigma}(x) n_\mu \int_0^1 d\lambda \int_{y' \in P} d^3 \mathbf{y}'_\perp (y' - x)^\alpha (y' - x)^\rho (y' - x)^\sigma (n \cdot \partial) \widehat{T}^{\mu\nu}(y' - i\lambda\beta(x)) \\ & - \frac{1}{2} \partial_\alpha \beta_\nu(x) B_{\rho\sigma}(x) n^\alpha n_\mu \int_0^1 d\lambda \int_{y' \in P} d^3 \mathbf{y}'_\perp (y' - x)^\rho (y' - x)^\sigma \widehat{T}^{\mu\nu}(y' - i\lambda\beta(x)) + \mathcal{O}(\partial, B^2) \\ & + \dots \longrightarrow \text{spin potential contributions} \end{aligned}$$

# Curved Hypersurface: Results

- New terms at first order of gradient:

$$\begin{aligned}
 \underbrace{W_{\perp}^{(1)\mu\nu}|_B(x,k)}_{\text{projected Wigner fn.}} &= \delta(k^2 - m^2)\theta(k^0)n_B(1 + n_B)\frac{B_{\alpha\beta}(x)}{2(n \cdot k)} \left\{ \underbrace{\xi_{\rho\sigma}(x)}_{\text{thermal shear}} \left( \eta^{\rho\sigma} + \frac{k^\rho k^\sigma}{2m^2} \right) \Xi^{\alpha(\mu} \Xi^{\beta\nu)} \right. \\
 &\quad \left. + \left( \underbrace{\delta\Omega_{\rho\sigma}(x)}_{\text{net spin potential: } \Omega - \omega} - \xi_{\rho\sigma}(x) \right) \Xi^{\alpha\rho} \Xi^{\beta(\mu} \Delta_{(k)}^{\nu)\sigma} + \underbrace{(\Delta_{(k)}^{\mu\nu} \dots)}_{\text{no spin polarization}} \right\}
 \end{aligned}$$

- Recall that:

$$\begin{aligned}
 \Xi^{\mu\nu} &= \left( \eta^{\mu\nu} - \frac{k^\mu n^\nu}{(n \cdot k)} \right) \\
 \delta\Theta_{00}(x,k) &\approx \frac{1}{3n_B\delta(k^2 - m^2)\theta(k^0)} \left( \epsilon_y^\mu(k)\epsilon_y^\nu(k) + \frac{1}{3}\Delta_{(k)}^{\mu\nu} \right) W_{\mu\nu}(x,k)
 \end{aligned}$$

- Spin alignment at  $\mathcal{O}(\partial)$  !

# Curved Hypersurface: Why Vorticity Disappears?

- The local equilibrium density operator upto  $\mathcal{O}(\partial)$ :

$$\hat{\rho}_{\text{LE}} \approx \frac{1}{Z} \exp \left\{ -\beta(x) \cdot \hat{P} + \frac{1}{2} \omega_{\rho\sigma}(x) \hat{J}^{\rho\sigma}(x) - \frac{1}{2} \zeta_{\lambda\nu}(x) \hat{Q}_{\Sigma}^{\lambda\nu}(x) + \frac{1}{2} \delta\Omega_{\lambda\nu}(x) \hat{R}_{\Sigma}^{\lambda\nu}(x) \right\}$$

vorticity  $\leftarrow \hat{J}^{\rho\sigma}(x) = \int_{\Sigma} d\Sigma_{\mu}(y) \left[ (y-x)^{\rho} \hat{T}^{\mu\sigma}(y) - (y-x)^{\sigma} \hat{T}^{\mu\rho}(y) + S^{\mu\rho\sigma}(y) \right]$

shear  $\leftarrow \hat{Q}_{\Sigma}^{\lambda\nu}(x) = \int_{\Sigma} d\Sigma_{\mu}(y) \left[ (y-x)^{\lambda} \hat{T}^{\mu\nu}(y) + (y-x)^{\nu} \hat{T}^{\mu\lambda}(y) \right]$

net spin potential  $\leftarrow \hat{R}_{\Sigma}^{\rho\sigma}(x) = \int_{\Sigma} d\Sigma_{\mu}(y) \left[ \hat{S}^{\mu\rho\sigma}(y) \right]$

- The total angular momentum (w.r.t.  $x$ )  $\hat{J}^{\rho\sigma}(x)$ : independent of hypersurface  $\Sigma$
- The leading order contribution from vorticity always be:

$$W_{\perp, \mu\nu}^{(1)}(x, k) = -i\delta(k^2 - m^2)\theta(k^0)n_B(1 + n_B)\Delta_{\mu\rho}^{(k)}\Delta_{\nu\sigma}^{(k)}\omega^{\rho\sigma}$$



# Example

- Assume:  $k_\mu = mn_\mu$ ,  $u_\mu = n_\mu$ , constant temperature on  $\Sigma$
- Decompose thermal shear tensor:

$$\xi_{\mu\nu} = u_\mu u_\nu (u \cdot \partial) \frac{1}{T} + \frac{1}{3T} \theta \Delta_{\mu\nu}^{(u)} + \frac{1}{T} A_{(\mu} u_{\nu)} + \frac{1}{T} \sigma_{\mu\nu}$$

fluid current
acceleration  $(u \cdot \partial)u_\mu$

scalar expansion  $\partial \cdot u$ 
shear tensor

$$\delta\Theta_{00}|_{\text{shear}}(x, k) = (1 + n_B) \left\{ \frac{(u \cdot \partial)(1/T)}{4m} \left( \kappa_y(x) - H(x) \right) + \frac{\theta}{9mT} \left( \kappa_y(x) - H(x) \right) - \frac{1}{6mT} \left( \epsilon_y^\alpha(k) B_{\alpha\beta}(x) \sigma^\beta_\nu \epsilon_y^\nu(k) + \frac{1}{3} B_{\alpha\beta}(x) \sigma^{\alpha\beta} \right) \right\}$$

curvature along direction y
average curvature

- Curvature along "y":  $\kappa_y(x) = B_{\alpha\beta}(x) \epsilon_y^\alpha(k) \epsilon_y^\beta(k)$ , average curvature:  $H(x) = -\frac{1}{3} B^\mu{}_\mu(x)$

# Example

fluid current



- Assume:  $k_\mu = mn_\mu$ ,  $u_\mu = n_\mu$ , constant temperature on  $\Sigma$

- Bjorken expansion:  $\sqrt{t^2 - z^2} = \tau_c \approx 5 \sim 10$  fm.

Curvature along "y" :  $\kappa_y = 0$ , average curvature:  $H = \frac{1}{3\tau_c}$ , scalar expansion  $\theta = \frac{1}{\tau_c}$

$$\delta\Theta_{00}|_{\text{expansion}}(x, k) = (1 + n_B) \left\{ \frac{\theta}{9mT} \left( \kappa_y(x) - H(x) \right) \right\} \sim -10^{-4}$$

# Discussion

Spin alignment more sensitive to the curvature of the freezeout hypersurface?

A massive spin-1/2 Dirac fermion:

$$S^\mu(x, k) = -\frac{1}{8m} \left\{ \epsilon^{\mu\nu\rho\sigma} \Omega_{\rho\sigma} k_\nu + 2 \frac{\epsilon^{\mu\nu\rho\sigma} k_\rho n_\sigma}{k \cdot n} \left[ (\xi_{\nu\lambda} + \delta\Omega_{\nu\lambda}) k^\lambda - \partial_\nu \alpha \right] \right\} \\ + \mathcal{O}(\partial^2, B)$$

Liu, Huang, Sci. China-Phys. Mech. Astron. 65, 272011 (2022)

Buzzegoli, PRC 105, 044907 (2021)

# Conclusion and Outlook

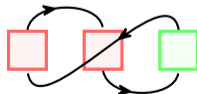
- Flat freezeout hypersurface  $\Rightarrow$  spin alignment  $\propto \mathcal{O}(\partial^2)$

$$(\partial\beta)(\partial\beta), (\partial\beta)\Omega, \Omega\Omega, \partial\partial\beta, \partial\Omega$$

$\partial_\mu\beta_\nu$  = thermal vorticity + thermal shear,  $\Omega_{\rho\sigma}$  – spin potential

- Curved freezeout hypersurface  $\Rightarrow$  spin alignment  $\propto$  thermal shear ...
- Spin alignment also at the first order of gradient:  
Dissipation;  
Chiral symmetry breaking...

# Thank you!



# Cumulant Expansion

Cumulant expansion:

$$e^{\hat{A}+x\hat{B}} = e^{\hat{A}} \sum_{n=0}^{\infty} x^n \hat{B}_n, \quad x \rightarrow 0$$

$$\hat{B}_0 = 1 \tag{2}$$

$$\hat{B}_1 = \int_0^1 d\lambda_1 \hat{B}(\lambda_1) \tag{3}$$

...

$$\hat{B}_n = \int_0^1 d\lambda_1 \int_0^{\lambda_1} d\lambda_2 \cdots \int_0^{\lambda_{n-1}} d\lambda_n \hat{B}(\lambda_1) \hat{B}(\lambda_2) \cdots \hat{B}(\lambda_n) \tag{4}$$

with  $\hat{B}(\lambda_i) = e^{-\lambda_i \hat{A}} \hat{B} e^{\lambda_i \hat{A}}$ .

# From MVSD to Tensor Polarization

- Space-time reversal property caused by power counting rules

$$\boxed{f_{r,s}^{(n)} = (-1)^{r+s+n} f_{-s,-r}^{(n)}} \quad (5)$$

with  $f_{r,s}^{(n)}$  the n-order MVSD.

- Spin density matrix in phase space  $\rho_{rs}(x, k)$

$$f_{rs}(x, k) = \Theta_{rs}(x, k) f(x, k) \quad (6)$$

with  $f(x, k) = \sum_r f_{rr}(x, k)$  the scalar distribution

- The leading order of polarization (y-axis as the spin axis)

$$\begin{aligned} \{P_x, P_y, P_z\} &= \frac{2}{3n_B} \left\{ \sqrt{2} \operatorname{Im} f_{01}^{(1)}, f_{11}^{(1)}, \sqrt{2} \operatorname{Re} f_{01}^{(1)} \right\} \\ \{T_{2,0}, T_{2,1}, T_{2,2}\} &= \frac{1}{3n_B} \left\{ \sqrt{\frac{2}{3}} (f_{11}^{(2)} - f_{00}^{(2)}), -\sqrt{2} f_{01}^{(2)}, f_{-11}^{(2)} \right\} \end{aligned}$$

$$\boxed{\Theta_{00} - \frac{1}{3} = \frac{2}{9n_B} (f_{00}^{(2)} - f_{11}^{(2)})} \quad (7)$$