Chiral condensate for accelerated and rotated observer

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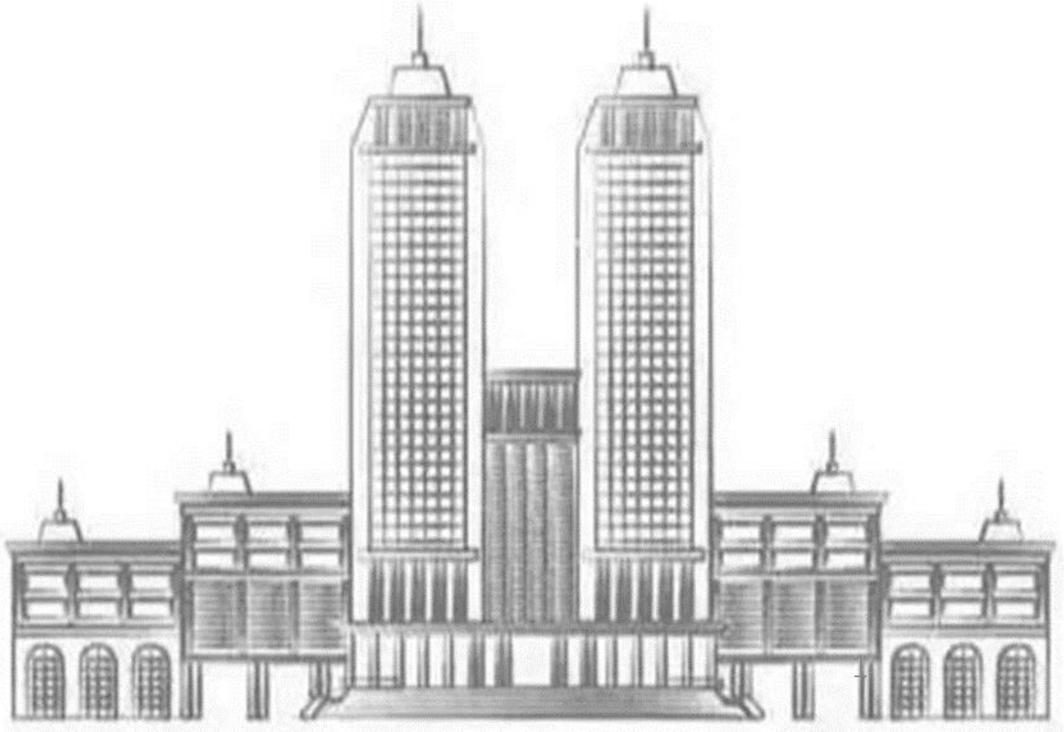
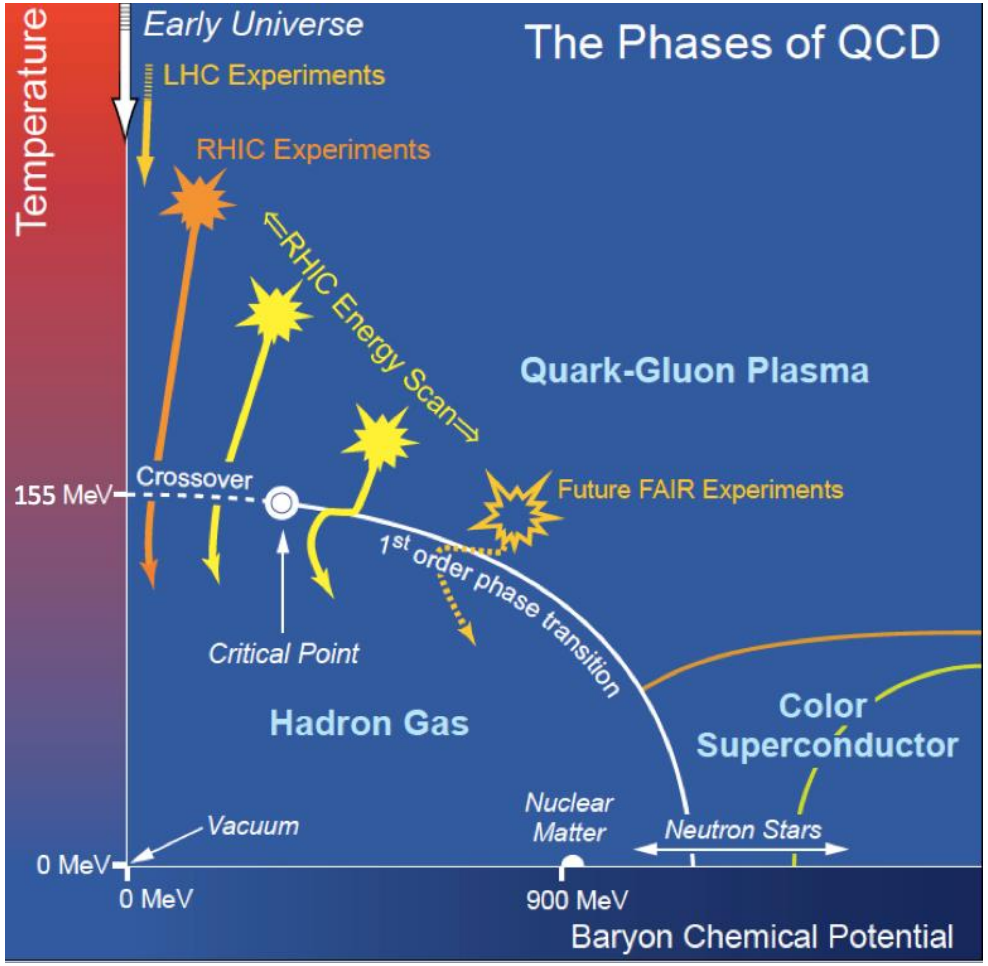


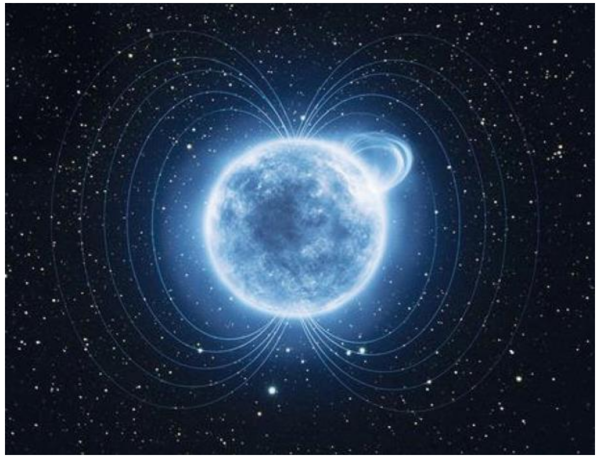
Outline

- Introduction：QCD phase transition under rotation and other condition
- NJL model study: Chiral condensate under rotation and acceleration
- Summary

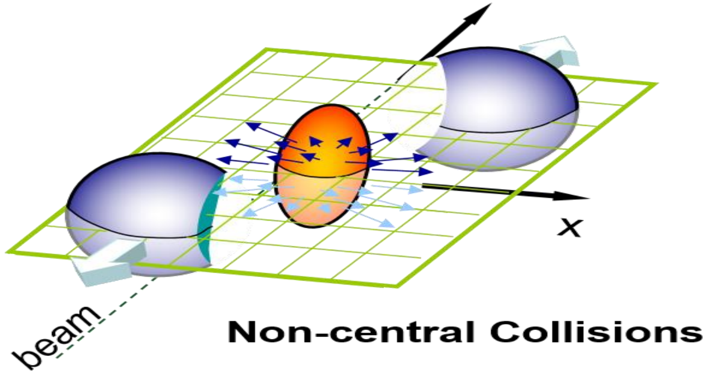
QCD phase transition



The Hot QCD White Paper (2015)



neutron star



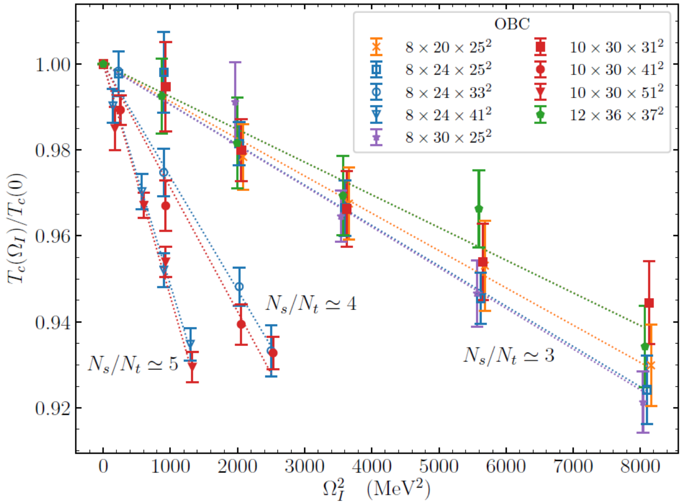
$\Omega \sim 0.01 - 0.1\ GeV$

Chiral condensate $< \bar{\psi}\psi >$

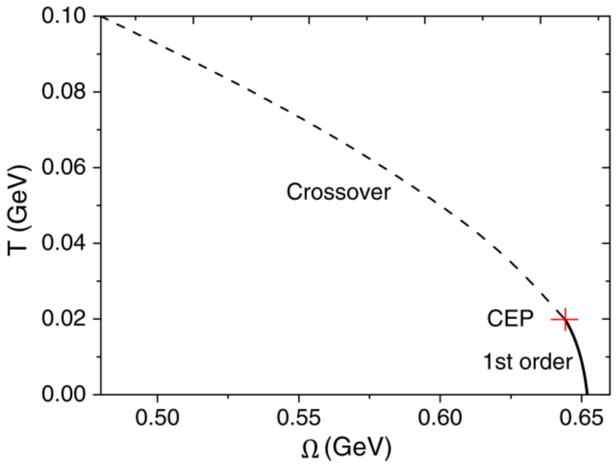
QGP phase $< \bar{\psi}\psi >$=0

Hadronic Phase $< \bar{\psi}\psi >$=finite

QCD under rotation

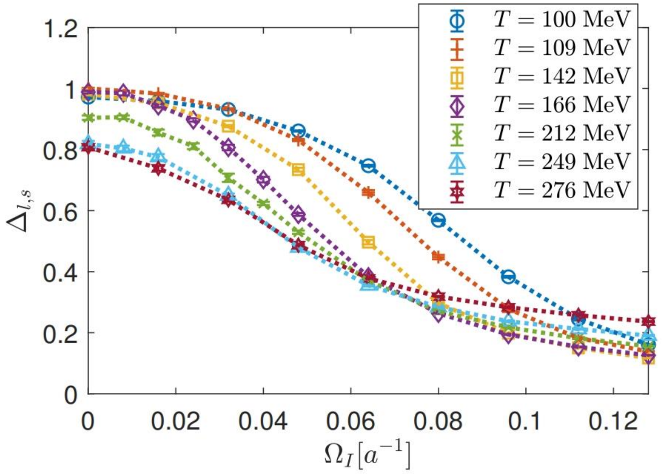


Braguta V V, Kotov A Y, Kuznedelev D D, et al. arXiv:2110.12302, 2021.

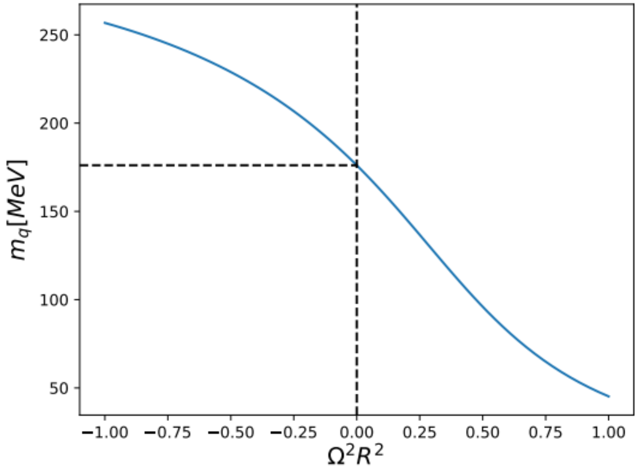


Y. Jiang and J. Liao, Phys. Rev. Lett. 117, 192302 (2016) arXiv:1606.03808

The lattice and model calculations yield opposite results!



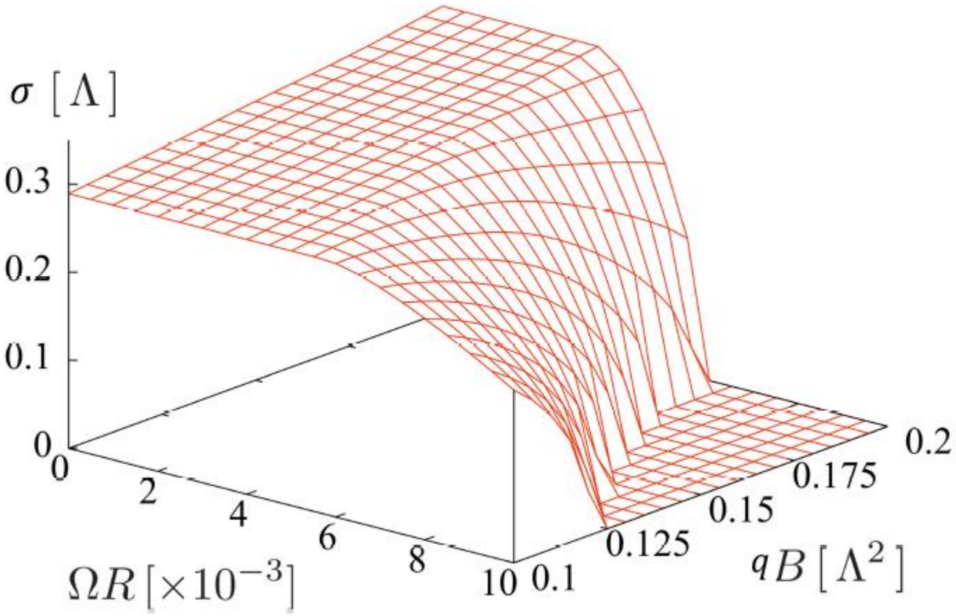
Ji-Chong Yang and Xu-Guang Huang arxiv:2307.05755



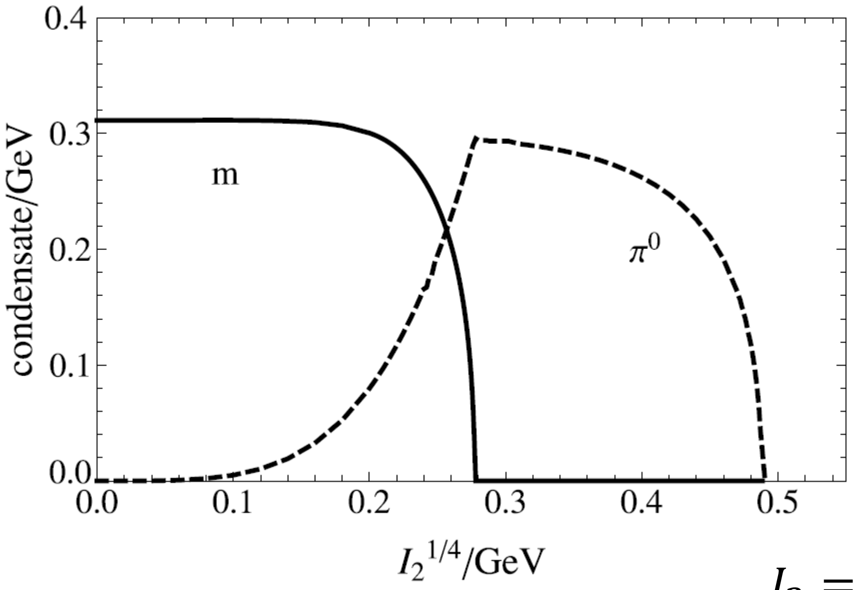
HL Chen, ZB Zhu, XG Huang arXiv:2306.08362

The properties under rotation remain unclear.

QCD phase transition



H. L. Chen, K. Fukushima, X. G. Huang and K. Mameda, Phys. Rev. D 93, 104052 (2016)



$I_2 = E \cdot B$

Cao, G., & Huang, X. G. (2016). Physics Letters B, 757, 1-5.

Recently, we have also been paying attention to QCD phase transitions under other conditions such as magnetic field, electric field.

What about the effective under acceleration?

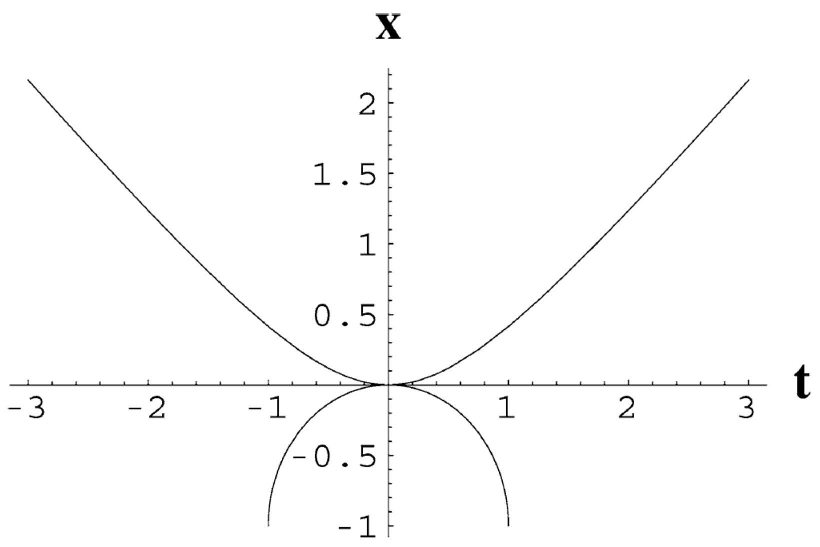
Uniform acceleration in relativity case

The equation of motion for a uniform acceleration particle:

$$\boldsymbol{v(t)} = \frac{\boldsymbol{at}}{\sqrt{\boldsymbol{1 + a^2t^2}}}, \xrightarrow{a \to 0} v(t) = at$$

$$\boldsymbol{z(t)} = \boldsymbol{a^{-1}}\left(\sqrt{\boldsymbol{1 + a^2t^2}} - \boldsymbol{1}\right)$$

$$\boldsymbol{a} = \frac{\boldsymbol{d}}{\boldsymbol{dt}}\frac{\boldsymbol{v}}{\sqrt{\boldsymbol{1 - v^2}}}$$



Taken from Kharzeev D, Tuchin K. Nuclear Physics A, 2005, 753(3-4): 316-334.

The trajectory is hyperbola in Minkowski coordinates :

$$\left(\boldsymbol{z} + \frac{\boldsymbol{1}}{\boldsymbol{a}}\right)^{\boldsymbol{2}} - \boldsymbol{t^2} = \frac{\boldsymbol{1}}{\boldsymbol{a^2}}$$

$\boldsymbol{a}$ is called proper acceleration

Rindler spacetime

Minkowski coordinates (T,X,Y,Z)

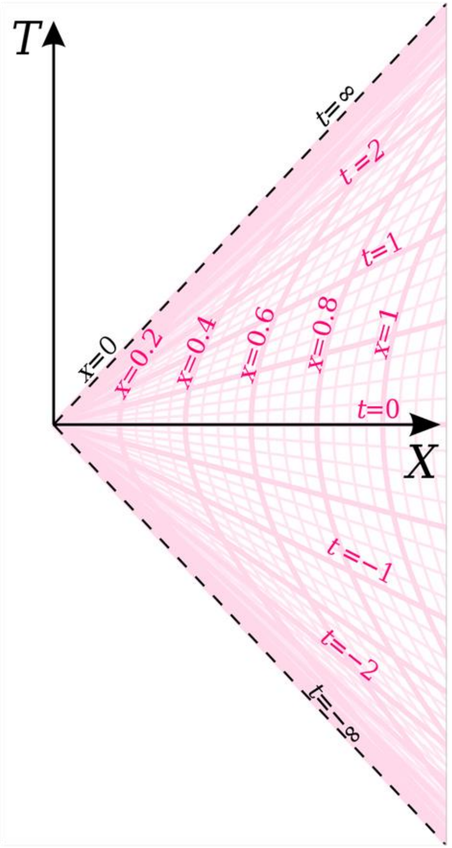
$$ds^2 = -dT^2 + dX^2 + dY^2 + dZ^2$$

Coordinates transformation :

$$T = x\sinh(at)\ , X = x\cosh(at), Y = y, Z = z$$

Rindler coordinates (t,x,y,z)

$$\boldsymbol{ds^2 = -(ax)^2 dt^2 + dx^2 + dy^2 + dz^2}$$



The world line in Minkowski coordinates: $T = x\sinh(a\tau)\ , X = x\cosh(a\tau)$

The world line in Rindler coordinates: $x = \dfrac{1}{a}\ , t = \tau$

Rindler spacetime

Kottler-Moller coordinates:

$$T = \left(x + \frac{1}{\alpha}\right)\sinh(\alpha t)$$
$$X = \left(x + \frac{1}{\alpha}\right)\cosh(\alpha t) - \frac{1}{\alpha}$$
$$Y = y$$
$$Z = z$$

$$t = \frac{1}{\alpha}\operatorname{artanh}\left(\frac{T}{X + \frac{1}{\alpha}}\right)$$
$$x = \sqrt{\left(X + \frac{1}{\alpha}\right)^2 - T^2} - \frac{1}{\alpha}$$
$$y = Y$$
$$z = Z$$

$$ds^2 = -(1 + \alpha x)^2 dt^2 + dx^2 + dy^2 + dz^2$$

Radar coordinates:

$$T = \frac{1}{\alpha}e^{\alpha x}\sinh(\alpha t)$$
$$X = \frac{1}{\alpha}e^{\alpha x}\cosh(\alpha t)$$
$$Y = y$$
$$Z = z$$

$$t = \frac{1}{\alpha}\operatorname{artanh}\frac{T}{X}$$
$$x = \frac{1}{2\alpha}\ln\left[\alpha^2\left(X^2 - T^2\right)\right]$$
$$y = Y$$
$$z = Z$$

$$ds^2 = e^{2\alpha x}\left(-dt^2 + dx^2\right) + dy^2 + dz^2$$

Unruh effect

The most famous effect induced by acceleration is Unruh effect.

The Hawking–Unruh effect predicts that the accelerated observer sees Minkowski vacuum state as a thermal bath of particles with temperature $T = a/2\pi$.

Define the annihilation and creation operator in acceleration frame: $a_R(\omega)$and $a_R^\dagger(\omega)$

We have $a_R|0\rangle_R=0$, where $|0\rangle_R$ is Rindler vacuum

According to the Unruh effect we have

$$ {}_M\langle 0|a_R a_R^\dagger\,|0\rangle_M \sim \left(exp\left(\frac{2\pi\omega}{a}\right) \pm 1\right)^{-1} $$

Unruh effect in heavy ion collisions

Acceleration will provide a temperature $T = a/2\pi$. Temperature will surely effect the QCD phase transition

So is this important in heavy-ion collisions?

According to Dmitri work[1], the Unruh effect under strong color fields should be observable.

As color glass condensate picture say that the strength of the color-electric field $E \sim Q_s^2/g$, where Q_s is the saturation scale and g is the strong coupling and the typical acceleration is $a \sim Q_S \sim 1GeV$ $(T = \frac{a}{2\pi} \sim 200MeV > T_c \cong 150MeV)$

[1] Kharzeev D, Tuchin K. From color glass condensate to quark–gluon plasma through the event horizon[J]. Nuclear Physics A, 2005, 753(3-4): 316-334.

Motivation

- Unruh effect may play an important role in QCD phase transition.

- QCD under rotation have attracted many attention.

- There are many interesting phenomena in the presence of parallel electromagnetic filed, which can be easily analogous to acceleration and rotation.

- The above points give us an academic interest to discuss QCD phase transition under both acceleration and rotation.

About this work

- We introduce the effects of acceleration and rotation through an accelerated-rotated spacetime background.

- We employ the Nambu-Jona-Lasinio model in an accelerated-rotated spacetime and apply the mean-field approximation.

- We solve the gap equation both numerically and analytically and work out the chiral condensate as a function of rotation and acceleration.

- Most of the results were obtained in $T = T_U$.

NJL model in general spacetime

$$\mathcal{L}_{NJL} = \bar{\psi}[i\gamma^{\mu}\nabla_{\mu} - m_0]\psi + \frac{G}{2}\left[\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi}i\gamma^5\psi\right)^2\right]$$

$$\{\gamma_{\mu}(x), \gamma_{\nu}(x)\} = 2g_{\mu\nu}(x), \{\gamma_{\hat{m}}, \gamma_{\hat{n}}\} = 2\eta_{\hat{m}\hat{n}}$$

$$g_{\mu\nu}g^{\nu\rho} = \delta_{\mu}^{\rho}, g^{\mu\nu}(x) = e_{\hat{m}}^{\mu}(x)e^{\nu\hat{m}}(x), \gamma_{\mu}(x) = e_{\mu}^{\hat{m}}(x)\gamma_{\hat{m}}.$$

Covariant derivative : $\nabla_{\mu} = \partial_{\mu} + \Gamma_{\mu}, \Gamma_{\mu} = -\frac{i}{4}\omega_{\mu\hat{i}\hat{j}}\sigma^{\hat{i}\hat{j}}, \sigma^{ij} = \frac{i}{2}[\gamma^{\hat{i}}, \gamma^{\hat{j}}], \omega_{\mu\hat{i}\hat{j}} = g_{ab}e_{\hat{i}}^{a}\nabla_{\mu}e_{\hat{j}}^{b}$

$$g_{\mu\nu} = \begin{pmatrix} (1+az)^2 - \omega^2 r^2 & \omega y & -\omega x & 0 \\ \omega y & -1 & 0 & 0 \\ -\omega x & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$\gamma^0(x) = \frac{1}{1+\boldsymbol{a}\cdot\boldsymbol{x}}\gamma^{\hat{0}}, \gamma^i(x) = \frac{(\boldsymbol{\omega}\times\boldsymbol{x})^i}{1+\boldsymbol{a}\cdot\boldsymbol{x}}\gamma^{\hat{0}} + \gamma^i$$

$$\Gamma_0 = -\frac{i}{2}\boldsymbol{\omega}\cdot\sigma + \frac{1}{2}\boldsymbol{a}\cdot\boldsymbol{\alpha},$$

NJL model action in rotation and acceleration frame:

$$S = \int d^4x \left\{\bar{\psi}\left[i\gamma^{\hat{\mu}}\partial_{\mu} + ia\cdot x\gamma^{\hat{i}}\partial_i + \frac{i}{2}a\cdot\gamma + \gamma^{\hat{0}}\omega\cdot J - m_0\phi\right]\psi + \frac{G}{2}\phi\left[\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi}i\gamma_5\psi\right)^2\right]\right\}$$

Where $\phi = 1 + \boldsymbol{a}\cdot\boldsymbol{x}$

The Dirac equation:

$$\left(\frac{i\gamma^{\hat{\mu}}\partial_{\mu} + ia\cdot x\gamma^{\hat{i}}\partial_i + \frac{i}{2}a\cdot\gamma + \gamma^{\hat{0}}\omega\cdot J}{\phi} - m\right)\psi = 0$$

Green's functions

To simplify the formalism, both acceleration and rotation are chosen along the z-direction.

$$\frac{\gamma^{\hat{0}}}{\phi}\left(i\partial_0+\omega\left(\frac{\sigma_3}{2}+\hat{L}_z\right)+\frac{i}{2}a\gamma^{\hat{0}}\gamma^{\hat{3}}\right)+i\gamma^{\hat{\imath}}\partial_i\equiv\hat{A}\qquad(\hat{A}-m)\psi=0$$

Solve the Dirac equation directly maybe is practicable, but it would be easier to simplify the calculation by introduce the Green's functions $S(x,y,s)$ and $G(x,y,s)$:

$$(\hat{A}-s)S(x,y,s)=\frac{1}{\sqrt{-g}}\delta(x,y),$$

$$(\hat{A}+s^{\dagger})G(x,y,s)=S(x,y,s),$$

$$(\hat{A}^2-s^{\dagger}s)G(x,y,s)=\frac{1}{\sqrt{-g}}\delta(x,y).$$

$$\hat{A}\equiv\frac{\gamma^{\hat{0}}}{\phi}\left(i\partial_0+\omega\left(\frac{\sigma_3}{2}+\hat{L}_z\right)+\frac{i}{2}a\gamma^{\hat{0}}\gamma^{\hat{3}}\right)+i\gamma^{\hat{\imath}}\partial_i$$

$$A^2=\frac{1}{\phi^2}\left\{\left[i\partial_0+\left(\frac{\sigma_3}{2}+\hat{L}_z\right)\omega\right]^2-\frac{1}{4}a^2\right\}+\partial_3^2+\frac{1}{\phi}a\partial_3+\hat{\gamma}^0\hat{\gamma}^3\frac{a}{\phi^2}i\left[i\partial_0+\left(\frac{\sigma_3}{2}+\hat{L}_z\right)\right]+\partial_1^2+\partial_2^2$$

Acceleration case

$$\hat{A} \equiv \frac{\gamma^{\hat{0}}}{\phi}\left(i\partial_0 + \frac{i}{2}a\gamma^{\hat{0}}\gamma^{\hat{3}}\right) + i\gamma^{\hat{\imath}}\partial_i$$

$$A^2 = \frac{1}{\phi^2}\left\{[i\partial_0]^2 - \frac{1}{4}a^2\right\} + \partial_3^2 + \frac{1}{\phi}a\partial_3 + \hat{\gamma}^0\hat{\gamma}^3\frac{a}{\phi^2}i[i\partial_0] + \partial_1^2 + \partial_2^2$$

Projection operators: $\hat{P}^{\pm} = \frac{1}{2}\left(1 \pm \gamma^{\hat{0}}\gamma^{\hat{3}}\right)$

The differential equation can be decoupled into two terms:

$$\hat{A}^2 - s^2 = \left(\hat{Q} + \hat{R}\right)\hat{P}^+ + \left(\hat{Q} - \hat{R}\right)\hat{P}^-$$

$$\hat{Q} = \frac{1}{\phi^2}\left\{[i\partial_0]^2 - \frac{1}{4}a^2\right\} + \partial_3^2 + \frac{1}{\phi}a\partial_3 + \partial_1^2 + \partial_2^2 - s^2$$

$$\hat{R} = \frac{a}{\phi^2}i[i\partial_0]$$

There is no gamma matrices in $\hat{Q}$ and $\hat{R}$

Eigenfunction

$$(\hat{Q} \pm \hat{R}) = \left\{ \frac{1}{\phi^2}\left[E^2 - \frac{1}{4}a^2\right] - p_{l,k}^2 - s^2 + \partial_3^2 + \frac{1}{\phi} a \partial_3 \pm \frac{a}{\phi^2} iE \right\}$$

$$\left\{ a^2 \frac{d^2}{d\phi^2} + \frac{a^2}{\phi}\frac{d}{d\phi} - \left(p_{l,k}^2 + s^2 - \frac{E^2 - \frac{1}{4}a^2}{\phi^2} \mp \frac{a}{\phi^2} i\varepsilon \right) \right\} f^{\pm}(\phi) = 0$$

$$a^2\alpha^2 \left\{ \frac{d^2}{d\alpha^2\phi^2} + \frac{1}{\alpha\phi}\frac{d}{d\alpha\phi} - \left(1 + \frac{\left(-\frac{iE}{a} \pm \frac{1}{2}\right)^2}{\alpha^2\phi^2} \right) \right\} f^{\pm}(\alpha\phi) = 0$$

$$, where\ \phi = 1 + az\ , \alpha = \left[\frac{p_{l,k}^2 + s^2}{a^2}\right]^{\frac{1}{2}}$$

The solution:

$$f_{\Omega}^{\pm}(\alpha\phi) = \frac{\sqrt{(\mp 2i\Omega/a - 1)\cosh(\pi\Omega/a)}}{\pi} K_{\frac{i\Omega}{a} \pm \frac{1}{2}}(\alpha\phi)$$

K_μ is the modified Bessel function

The orthonormal condition :

$$\int_0^\infty \frac{d\xi}{\xi} \Psi_{\Omega}^{\pm}(\xi) \Psi_{\Omega'}^{\pm}(\xi) = \int_0^\infty \frac{d\xi}{\xi} \frac{(\mp 2i\Omega - 1)\cosh \pi\Omega}{\pi^2} K_{i\Omega \pm 1/2}(\alpha\xi) K_{i\Omega' \pm 1/2}(\alpha\xi) = \delta(\Omega, \Omega')$$

Green's functions

$$G(x_1, x_2; s) = \int \frac{dp_0}{(2\pi)} \frac{dp_t^2}{(2\pi)^2} e^{-ip_0(t_1-t_2)+i\boldsymbol{p}_t\cdot\boldsymbol{x}_t} \mathcal{G}(z_1, z_2, p_t; s)$$

G is divided into two part: $G = P^+G^+ + P^-G^-$

With the eigenfunctions and the orthonormal relation, $\mathcal{G}^{\pm}$ are obtained:

$$\mathcal{G}^{\pm}(z_1, z_2, p_t; s) = \int d\Omega - \frac{1}{\left(-i\varepsilon \pm \frac{1}{2}a\right)^2 - \left(i\Omega \pm \frac{1}{2}a\right)^2} \frac{(\pm 2i\Omega/a - 1)\cosh(\pi\Omega/a)}{\pi^2} K_{\frac{i\Omega}{a}\pm\frac{1}{2}}(\alpha\phi_1) K_{\frac{i\Omega}{a}\pm\frac{1}{2}}(\alpha\phi_2)$$

Gap equation

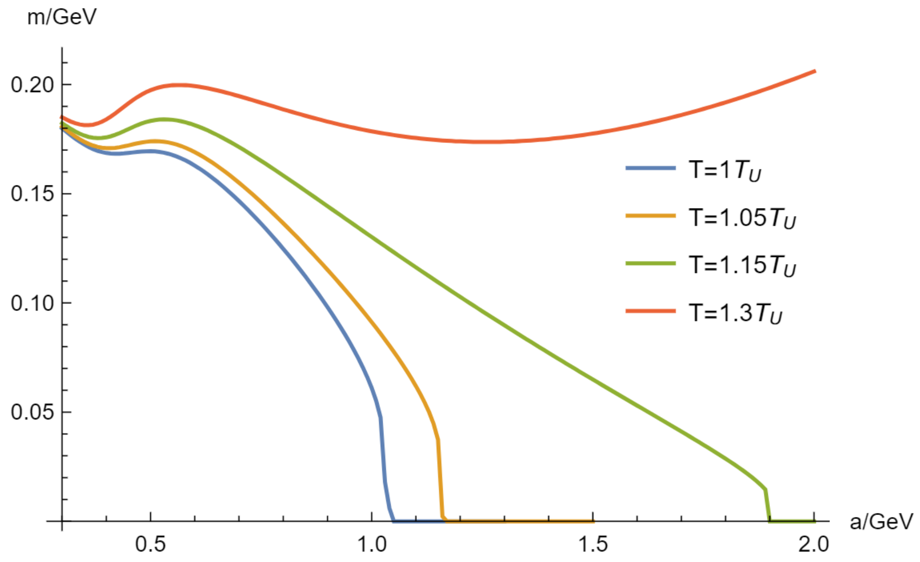
After performed the Matsubara summation, the gap equation become:

T is temperature in acceleration frame, in the past study always been set to $T = T_U$

$$\frac{m}{G} = m \sum_{s_1=\pm} \int_0^\infty d\Omega \int \frac{d^2 p_t}{(2\pi)^2} \frac{-is_1}{a} \frac{\cosh(\pi\Omega/a)}{\pi^2} \left\{ \tanh\left(\frac{\Omega - s_1 ia}{2T}\right) + \tanh\left(\frac{\Omega}{2T}\right) \right\} K^2_{\frac{i\Omega}{a}+s_1\frac{1}{2}}(\alpha\phi).$$

We apply the condition $\phi = 1$, where is the position of a uniform acceleration observer.

- **The acceleration restore the chiral symmetry when T is near T_U.**
- **When the acceleration is small, there is a small fluctuation, possibly due to numerical computation reasons.**
- **As the temperature increases, i.e. $T = 1.3T_U$, the chiral condensate no longer decreases uniformly with changes in acceleration.**



It seems we have some problem if we set $T \neq T_U$

Acceleration and rotation case

$$A^2 = \frac{1}{\phi^2}\left\{\left[i\partial_0 + \left(\frac{\sigma_3}{2} + \hat{L}_z\right)\omega\right]^2 - \frac{1}{4}a^2\right\} + \partial_3^2 + \frac{1}{\phi}a\partial_3 + \hat{\gamma}^0\hat{\gamma}^3\frac{a}{\phi^2}i\left[i\partial_0 + \left(\frac{\sigma_3}{2} + \hat{L}_z\right)\omega\right] + \partial_1^2 + \partial_2^2$$

$$\hat{Q} = \frac{1}{\phi^2}\left\{\left[i\partial_0 + \left(\frac{\sigma_3}{2} + \hat{L}_z\right)\omega\right]^2 - \frac{1}{4}a^2\right\} + \partial_3^2 + \frac{1}{\phi}a\partial_3 + \partial_1^2 + \partial_2^2 - s^2$$

$$\hat{R} = \frac{a}{\phi^2}i\left[i\partial_0 + \left(\frac{\sigma_3}{2} + \hat{L}_z\right)\omega\right]$$

$$\hat{A}^2 - s^2 = \left(\hat{Q} + \hat{R}\right)\hat{P}^+ + \left(\hat{Q} - \hat{R}\right)\hat{P}^-$$

$$\left(\hat{Q} \pm \hat{R}\right) = \left\{\frac{1}{\phi^2}\left[(E + j\omega)^2 - \frac{1}{4}a^2\right] - p_{l,k}^2 - s^2 + \partial_3^2 + \frac{1}{\phi}a\partial_3 \pm \frac{a}{\phi^2}i[E + j\omega]\right\}$$

The form is the same just replace E with ε, $where\ \varepsilon = E + \omega j$.

Green’s functions

$$G(x_1, x_2, s) = \sum_{l,k} \int \frac{dp_0}{2\pi} e^{-ip_0(t_1-t_2)} \frac{1}{2\pi} \frac{2}{J_{l+1}^2(p_{l,k}R)R^2} \mathcal{M}(l, k, x_1, x_2) \mathcal{G}(z_1, z_2, l, k, s)$$

$$\mathcal{M}(l, k, x_1, x_2) = diag\left(\eta_l \eta_l', \xi_l \xi_l', \eta_l \eta_l', \xi_l \xi_l'\right) \; and \; \eta_l = e^{il\theta} J_l\left(p_{l,k} r\right), \xi_l = e^{i(l+1)\theta} J_{l+1}\left(p_{l,k} r\right)$$

Projection operator for rotation part:

$$\widehat{P_1}^{\pm} = \frac{1}{2}\left(1 \pm i\gamma^{\hat{1}}\gamma^{\hat{2}}\right), \mathcal{M} = \widehat{P_1}^{+} \eta\eta' + \widehat{P_1}^{-} \xi_l \xi_l'$$

$$G = \sum_{s_1, s_2 = \pm 1} \frac{1}{2}(1 + s_1 \gamma^{\hat{0}} \gamma^{\hat{3}}) \frac{1}{2}(1 + s_2 i \gamma^{\hat{1}} \gamma^{\hat{2}}) \mathcal{G}^{s_1, s_2}$$

$$\mathcal{G}^{s_1, s_2} = \sum_{l,k} \int \frac{dp_0}{2\pi} - \frac{1}{\left(-i\varepsilon \pm \frac{1}{2}a\right)^2 - \left(i\Omega \pm \frac{1}{2}a\right)^2} \frac{(\mp 2i\Omega/a - 1)\cosh(\pi\Omega/a)}{\pi^2} K_{\frac{i\Omega}{a} \pm \frac{1}{2}}(\alpha\phi_1) K_{\frac{i\Omega}{a} \pm \frac{1}{2}}(\alpha\phi_2)$$

$$\times \frac{1}{2\pi} \frac{1}{N_{l,k}} e^{i(j - s_2/2)(\theta_1 - \theta_2)} J_{j - s_2/2}(p_{l,k} r_1) J_{j - s_2/2}(p_{l,k} r_2)$$

Gap equation

When $m_0 = 0$, we can set π condensate $\pi = 0$

$$\frac{m}{G} = i\,\mathrm{tr}(S)$$

When $T = \frac{a}{2\pi}$, , $m_0 = 0$, the gap equation become

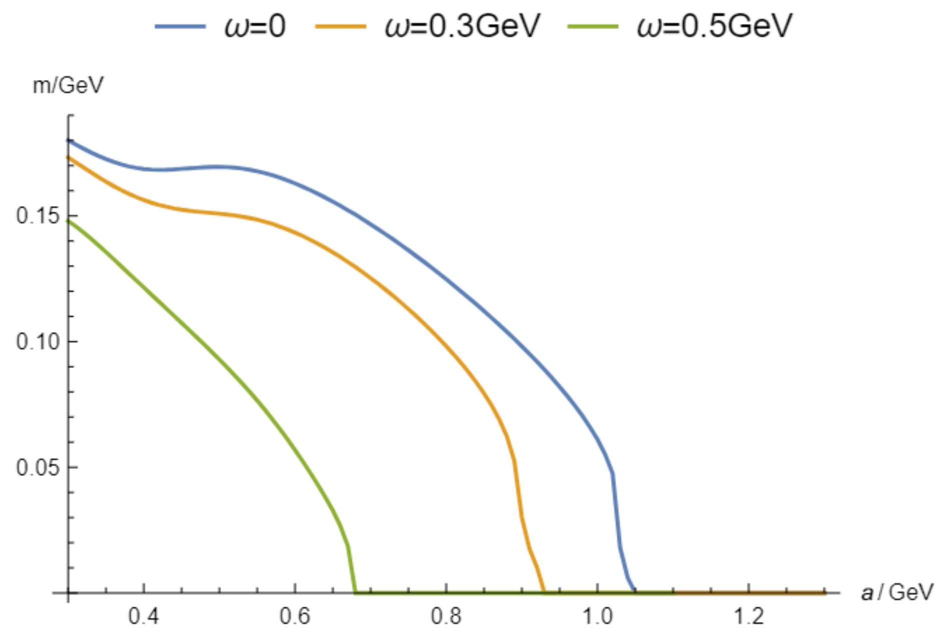
$$\frac{1}{G} = \sum_{l,k,s_1} \int d\,\Omega \frac{1}{2\pi} \frac{1}{N_{l,k}} \frac{-is_1}{2a} \frac{cosh(\pi\,\Omega/a)}{\pi^2} \left\{ \tan\mathrm{h}\left(\frac{\Omega-\omega j}{2T}\right) + \tan\mathrm{h}\left(\frac{\Omega+\omega j}{2T}\right) \right\} \times K^2_{\frac{i\Omega}{a}+s_1\frac{1}{2}}(\alpha\phi)\left[J_l^2(p_{l,k}r) + J_{l+1}^2(p_{l,k}r)\right]$$

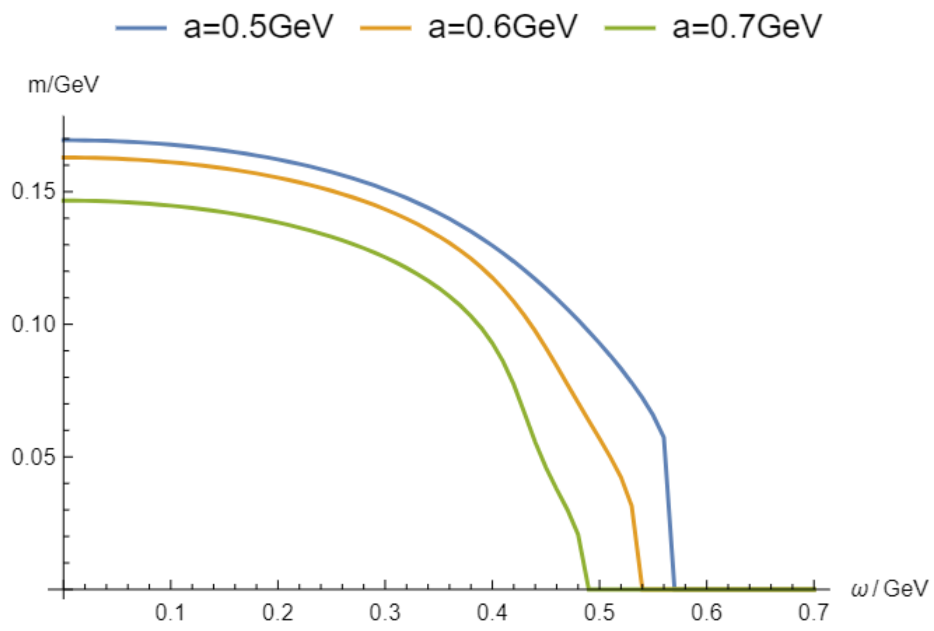
If we ignore the boundary and take the non rotation limit, we have

$$1 = G \int d\Omega \int \frac{d^2 p_t}{(2\pi)^2} \frac{-i}{a} \frac{sinh(\pi\Omega/a)}{\pi^2} \left\{ K^2_{\frac{i\Omega}{a}+\frac{1}{2}}(\alpha) - K^2_{\frac{i\Omega}{a}-\frac{1}{2}}(\alpha) \right\}$$

which is consistent with previous work.

Result





- $T = T_U$
- **both acceleration and rotation restore the chiral symmetry**

Critical acceleration

$$\frac{1}{G} = \sum_{l,k,s_1} \int d\,\Omega \frac{1}{2\pi} \frac{1}{N_{l,k}} \frac{-is_1}{2a} \frac{cosh(\pi\,\Omega/a)}{\pi^2} \left\{ \tanh\left(\frac{\Omega - \omega j}{2T}\right) + \tanh\left(\frac{\Omega + \omega j}{2T}\right) \right\}$$
$$\times K^2_{\frac{i\Omega}{a}+s_1\frac{1}{2}}(\alpha\phi)\left[J_l^2(p_{l,k}r) + J_{l+1}^2(p_{l,k}r)\right]$$

Where, $\alpha = \left[\frac{p_{l,k}^2 + m^2}{a^2}\right]^{\frac{1}{2}}$

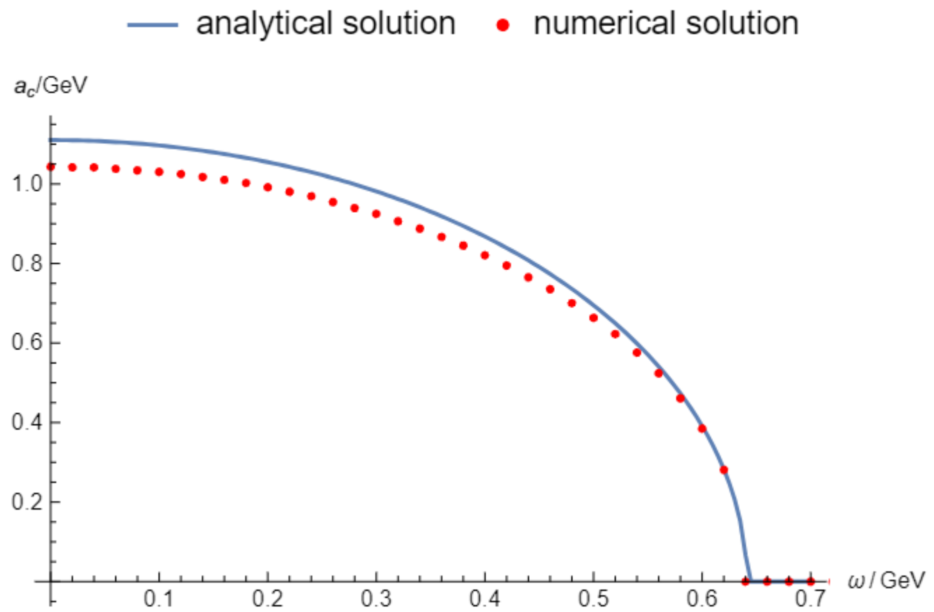
We define critical acceleration a_c, when $m = 0$

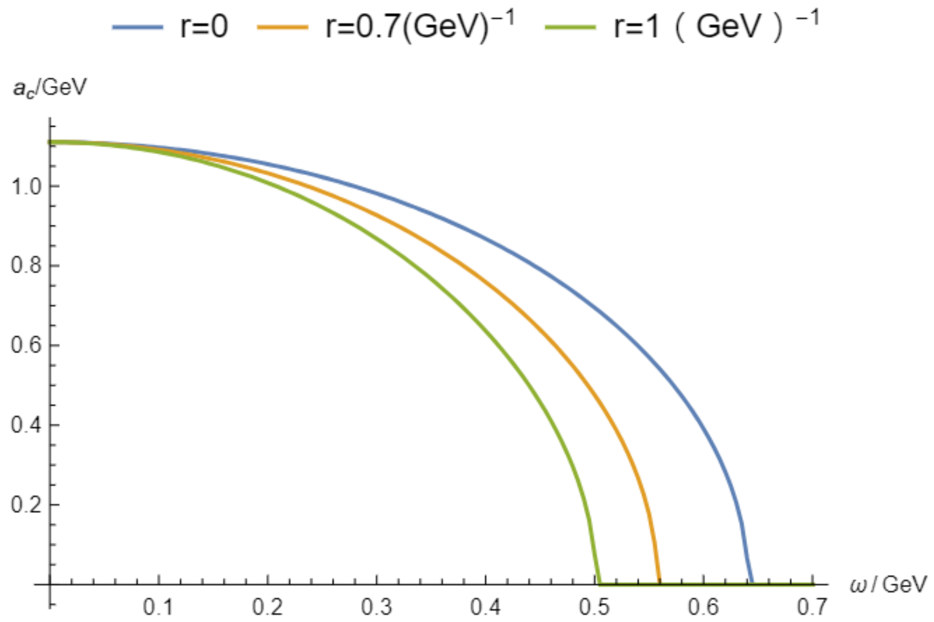
We can work out the relationship between a_c and ω analytically

$$G\left(\frac{\Lambda^2}{2\pi^2} - \frac{a^2(r^4\omega^4 + 1) - r^2\omega^4 + 3\omega^2}{24\pi^2(r^2\omega^2 - 1)^2}\right) = 1$$

Critical acceleration

$$G\left(\frac{\Lambda^2}{2\pi^2}-\frac{a^2(r^4\omega^4+1)-r^2\omega^4+3\omega^2}{24\pi^2(r^2\omega^2-1)^2}\right)=1$$





The critical acceleration (a_c) decreases as the angular velocity increases.
The effects of rotation become increasingly significant with increasing radius.
The rotation at $r=0$ still has an effect because fermions have a spin of 1/2.

Gap equation

The case $m_0 \neq 0$

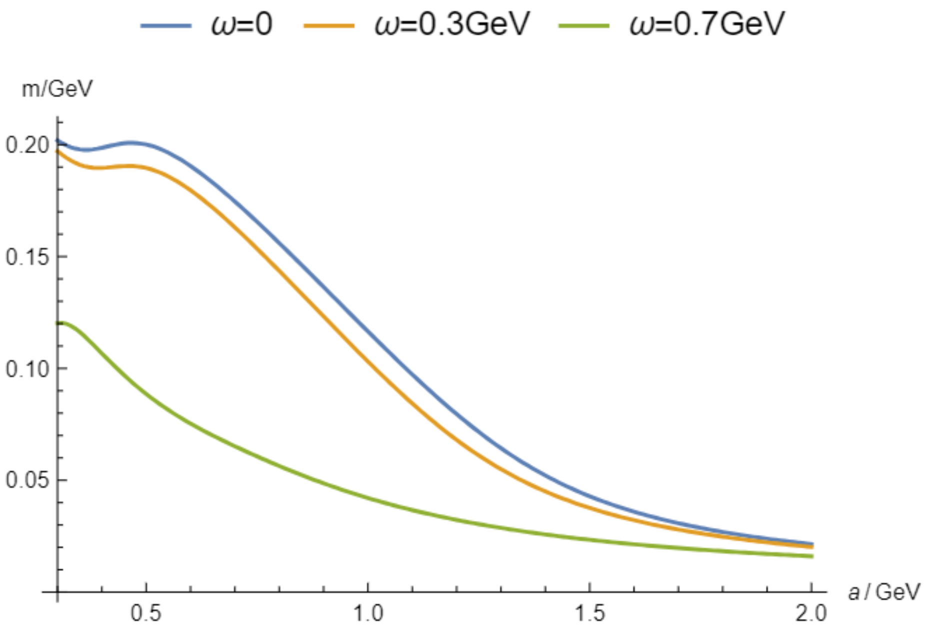
$$\frac{m-m_0}{G} = m\sum_{l,k,s_1}\int d\Omega \frac{1}{2\pi}\frac{1}{N_{l,k}}\frac{-is_1}{2a}\frac{cosh(\pi\Omega/a)}{\pi^2}\left\{\tanh\left(\frac{\Omega-\omega j-s_1 ia}{2T}\right)+\tanh\left(\frac{\Omega+\omega j}{2T}\right)\right\}$$
$$\times K^2_{\frac{i\Omega}{a}+s_1\frac{1}{2}}(\alpha\phi)\left[J_l^2(p_{l,k}r)+J_l^2(p_{l+1,k}r)\right] - i\pi\sum_{l,k,s_1}\int d\Omega\frac{1}{2\pi}\frac{1}{N_{l,k}}\frac{-i}{2a}\frac{cosh(\pi\Omega/a)}{\pi^2}$$
$$\times\left\{\tanh\left(\frac{\Omega-\omega j-s_1 ia}{2T}\right)+\tanh\left(\frac{\Omega+\omega j}{2T}\right)\right\}K^2_{\frac{i\Omega}{a}+s_1\frac{1}{2}}(\alpha\phi)\left[J_l^2(p_{l,k}r)-J_{l+1}^2(p_{l,k}r)\right]$$

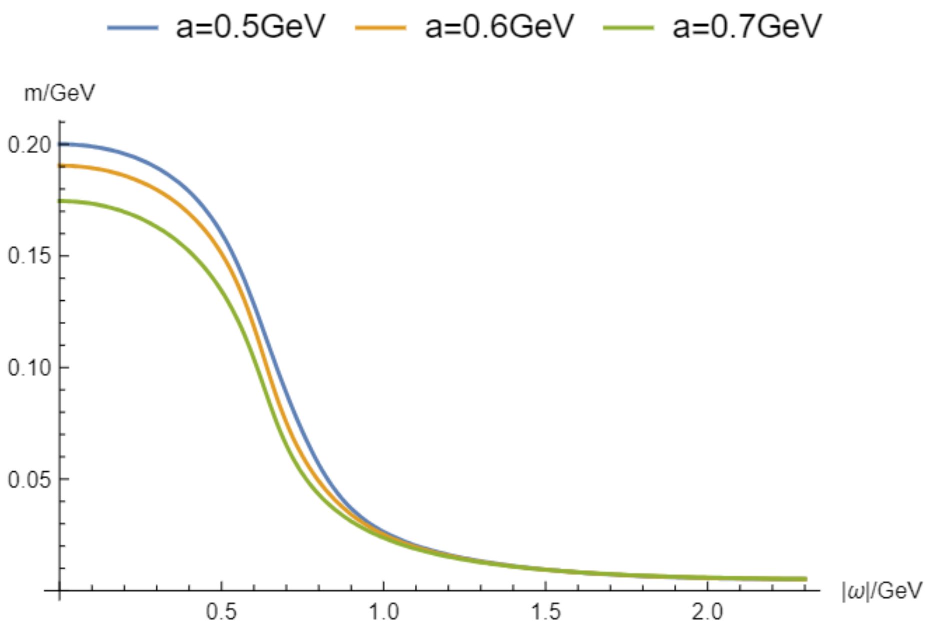
$$\frac{\pi}{G} = \pi\sum_{l,k,s_1}\int d\Omega \frac{1}{2\pi}\frac{1}{N_{l,k}}\frac{-is_1}{2a}\frac{cosh(\pi\Omega/a)}{\pi^2}\left\{\tanh\left(\frac{\Omega-\omega j-s_1 ia}{2T}\right)+\tanh\left(\frac{\Omega+\omega j}{2T}\right)\right\}$$
$$\times K^2_{\frac{i\Omega}{a}+s_1\frac{1}{2}}(\alpha\phi)\left[J_l^2(p_{l,k}r)+J_l^2(p_{l+1,k}r)\right] + im\sum_{l,k,s_1}\int d\Omega\frac{1}{2\pi}\frac{1}{N_{l,k}}\frac{-i}{2a}\frac{cosh(\pi\Omega/a)}{\pi^2}$$
$$\times\left\{\tanh\left(\frac{\Omega-\omega j-s_1 ia}{2T}\right)+\tanh\left(\frac{\Omega+\omega j}{2T}\right)\right\}K^2_{\frac{i\Omega}{a}+s_1\frac{1}{2}}(\alpha\phi)\left[J_l^2(p_{l,k}r)-J_{l+1}^2(p_{l,k}r)\right]$$

$$\frac{m-m_0}{G} = mI_1(\rho) - \pi^0 I_2(\rho)$$
$$\frac{\pi^0}{G} = \pi^0 I_1(\rho) + mI_2(\rho)$$

Gap equation

The case $m_0 \neq 0$





$$\frac{m - m_0}{G} = mI_1(\rho) - \pi^0 I_2(\rho)$$

$$\frac{\pi^0}{G} = \pi^0 I_1(\rho) + mI_2(\rho)$$

- **When $T=a/2\pi$, the second term vanish such that no pion condensate.**
- **The chiral symmetry would not restore with non-vanish current quark mass.**

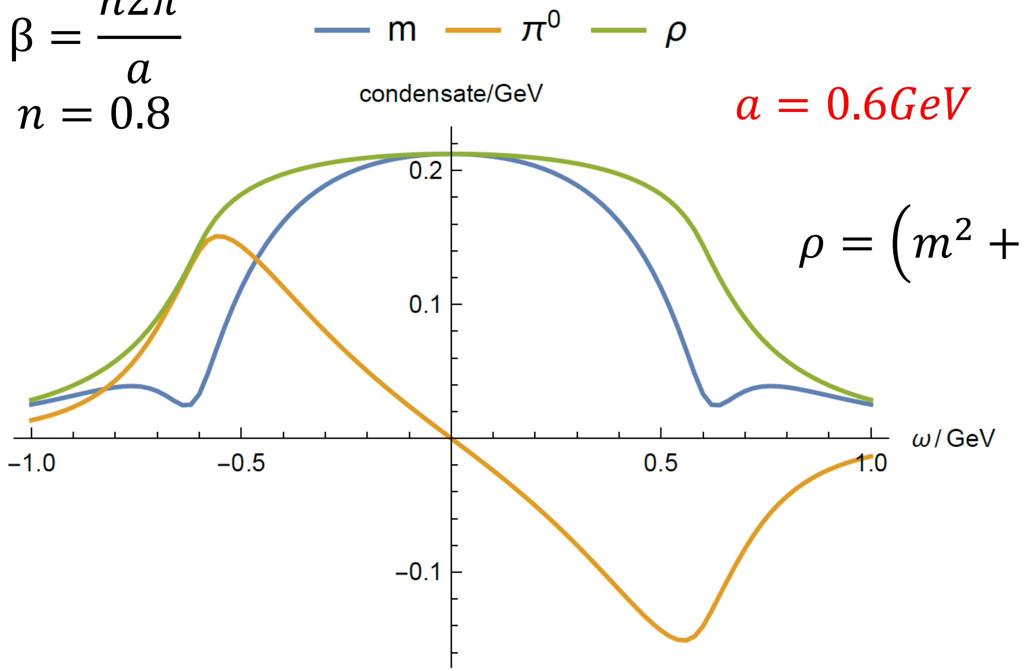
Result in $T \neq T_U$

$\beta = \frac{n2\pi}{a}$

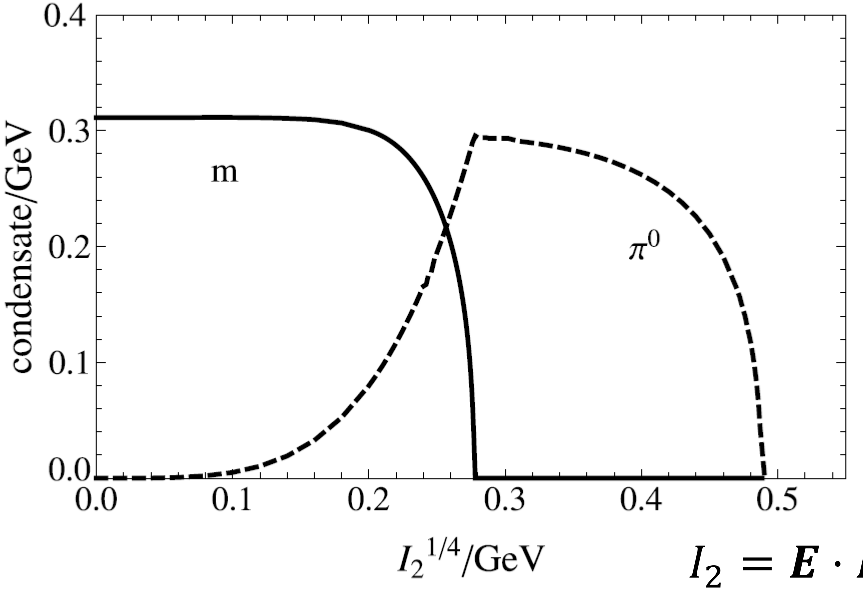
$n = 0.8$

$a = 0.6GeV$

$\rho = \left(m^2 + {\pi^0}^2\right)^{1/2}$



Condensate as functions of rotation $\boldsymbol{\omega}$



$I_2 = \boldsymbol{E} \cdot \boldsymbol{B}$

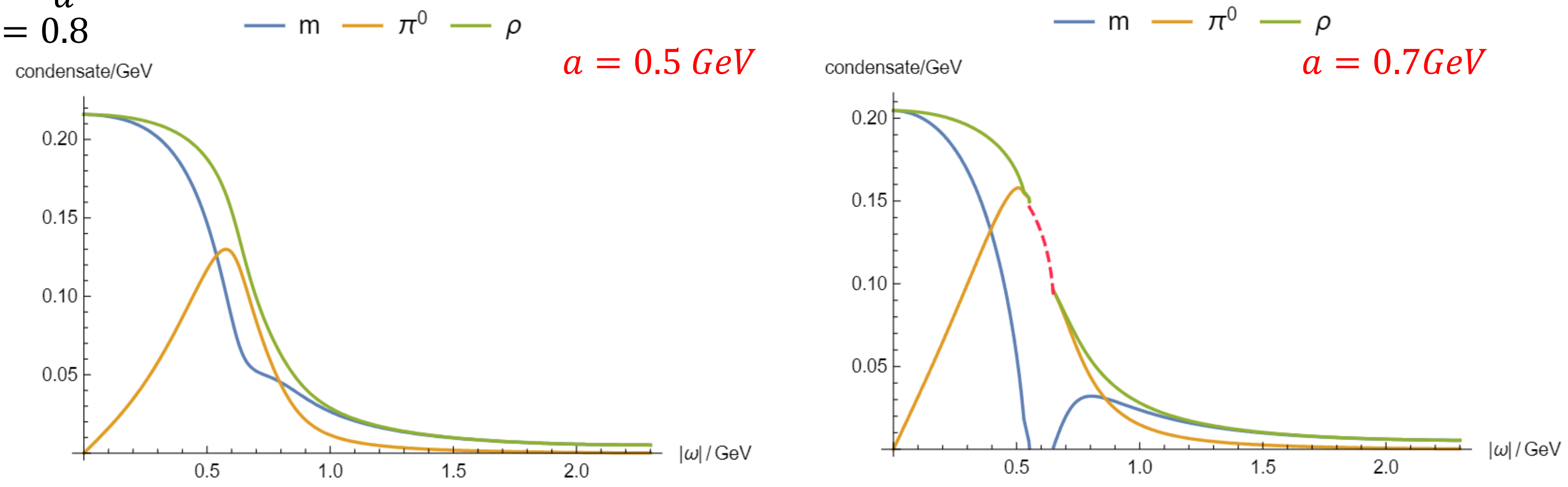
The constitute quark mass m and $\boldsymbol{\pi^0}$ condensate as functions of $\boldsymbol{I^{\frac{1}{4}}}$
(Cao, G., & Huang, X. G. (2016). Physics Letters B, 757, 1-5.
arXiv:1509.06222)

- Acceleration and rotation induced the pion ($\boldsymbol{\pi^0}$) condensate when $\boldsymbol{T \neq T_U}$
- The pion ($\boldsymbol{\pi^0}$) condensate is a odd function of $\boldsymbol{\omega}$ and constitute quark mass m is even
- The presence of $\boldsymbol{a \cdot \omega}$ tends to diminish $\boldsymbol{\sigma}$ condensate while drive $\boldsymbol{\pi^0}$ condensation which is similar to the presence of $\boldsymbol{E \cdot B}$

Result from gap equation

$$\beta = \frac{n2\pi}{a}$$

$n = 0.8$



condensate as a function of rotation with different acceleration

- The minimum of m is depend on a
- With a larger a, the constitute quark mass will decrease to zero while the rotation increasing.
- There exist a region(red dash line) we cannot find solutions that satisfy both gap equations simultaneously.
- In the parallel electromagnetic study, they call this "chiral instability"

Summary

- The acceleration and rotation restore the chiral symmetry
- The presence of $a \cdot \omega$ induce the pion condensate when $T \neq a/2\pi$
- Exist a chiral rotation from σ to π^0

Outlook

- The relationship between temperature in acceleration frame and inertial frame remains unclear.
- The phase structure is complex and seems inexplicable in $T - a$ plane.

Thanks!