Longitudinal spin polarization in a thermal model with dissipative effects

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Based On : PLB 814, 136096 (2021); PRD 103, 014030 (2021); arXiv: 2405.05089

Collaborators : S. Banerjee, W. Florkowski, A. Jaiswal, R. Ryblewski, A. Kumar

(Krakow-NISER Collabortion)

Section Outline :

Introduction :

- Non-central heavy-ion collisions produce large global angular momentum leading to spin polarization of hadrons.

  [STAR Collaboration, *Nature 548, 62 (2017)*]

- Theoretical models assuming equilibration of spin degrees of freedom, explains the global spin polarization.

  [F. Becattini et. al. *PRC 77, 024906 (2008), PRC 88, 034905 (2013), Ann. Phys. 338, 32 (2013)*]

- Until recently, the same models did not explain longitudinal spin polarization.

  [W. Florkowski et. al., *PRC 99, 044910 (2019)*, F. Becattini et. al., *EPJC 79, 741 (2019)*]

Features of Non-central Collisions :

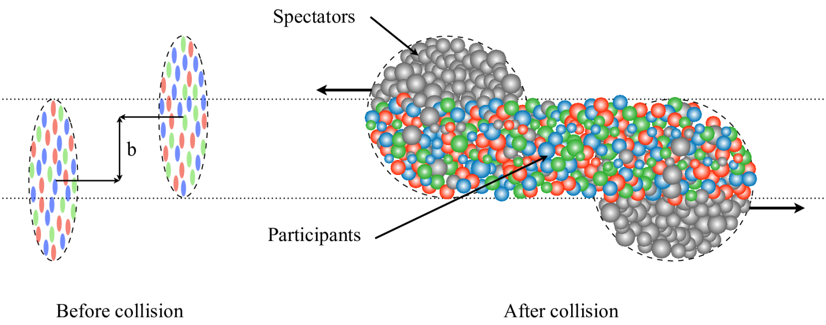


Figure 1: Heavy-ion collision experiments. [LHC Collaboration, JINST 17 (2022) 05, P05009]

- Properties of the matter produced :
  - Behaves like a fluid (Hydrodynamics applicable).
  - The viscosity (η/s) is lowest (Dissipative hydrodynamics required).
  - Highly vortical (for non-central collisions).

Features of Non-central Collisions :

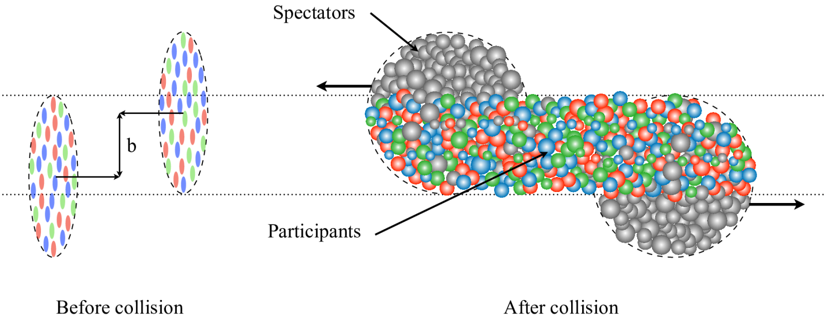


Figure 1: Heavy-ion collision experiments. [LHC Collaboration, JINST 17 (2022) 05, P05009]

- Special feature of Non-Central Collisions :
  - Large Magnetic Field. [A. Bzdak and, V. Skokov, Phys. Lett. B 710 (2012) 171–174]
  - Large Angular Momentum. [F. Becattini et. al. Phys. Rev. C 77 (2008) 204906]
  - Particle polarization at small $\sqrt{S_{NN}}$. [STAR Collaboration, Nature 548 62-65, 2017]

Generation of Angular Momentum :

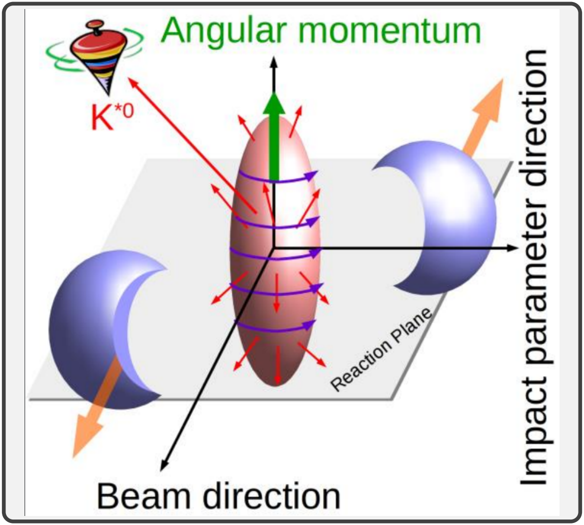


Figure 2: Generation of angular momentum in non-central collisions. [B. Mohanty, ICHEP 2020]

Generation of Angular Momentum :

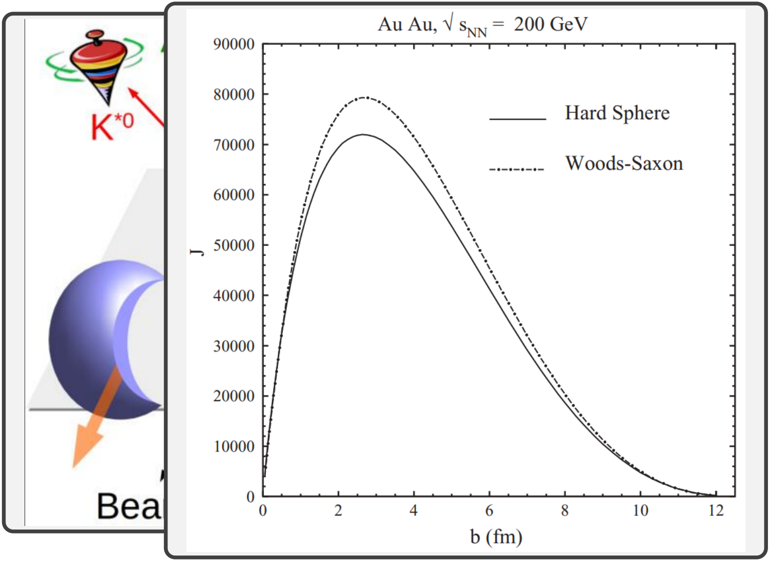


Figure 2: Generation of angular momentum in non-central collisions. [B. Mohanty, ICHEP 2020]

Angular momentum vs impact parameter. [Becattini, Piccinini and, Rizzo, Phys. Rev. C 77 (2008) 024906]

Particle Polarization :

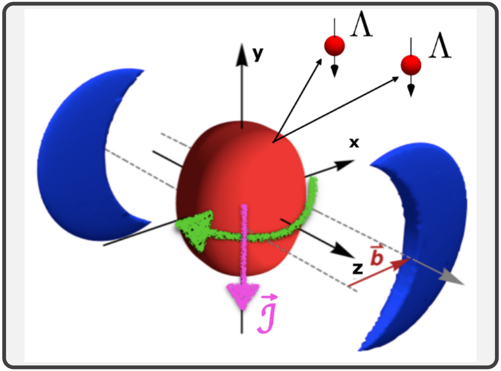
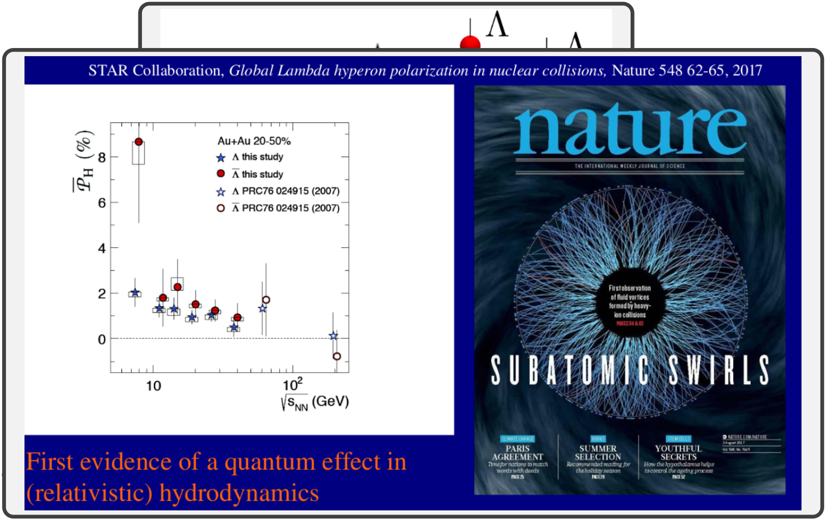


Figure 3: Origin of particle polarization. [W. Florkowski *et al*, PPNP 108 (2019) 103709]

- Large angular momentum → Local vorticities → spin alignment.

  [Z.-T. Liang and X.-N. Wang, Phys. Rev. Lett. 94, 102301 (2005); Phys. Lett. B 629, 20 (2005)]

Particle Polarization :



Experimental evidence, [STAR Collaboration, Nature 548, 62 (2017), Phys. Rev. Lett. 123, 132301 (2019), Phys. Rev. Lett. 126, 162301 (2021)]

Theoretical models assuming equilibration of spin d.o.f. explains the data.

Particle Polarization :

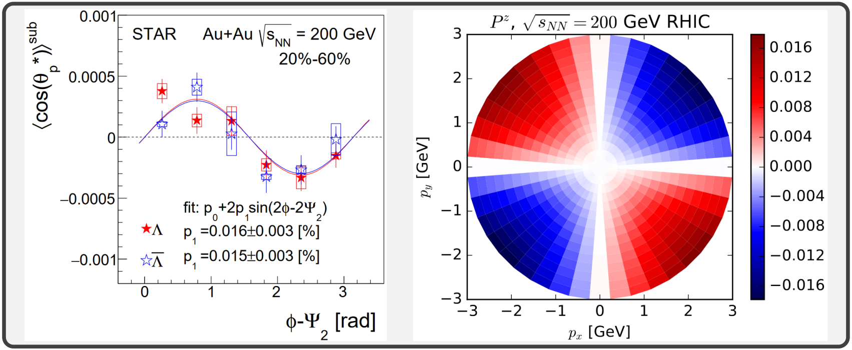


Figure 4: Observation (L) and prediction (R) of longitudinal polarization.

[Left: Phys. Rev. Lett. **123** 132301 (2019); Right: Phys. Rev. Lett. **120** 012302 (2018)]

- Inclusion of shear-induced polarization (SIP) solves the problem with extra constraints.
  [Fu et. al. Phys. Rev. Lett. 127, 142301 (2021); Becattini et. al. Phys. Lett. B 820 136519 (2021)]
- Still the resolution remains ambiguous.
  [Florkowski et. al., Phys. Rev. C 100, 054907 (2019); Phys.Rev.C 105, 064901 (2022)]
- Do dissipative forces play any role and solve the problem?

Summary of the Problem :

The main problem we wish to address is :

- Study the effects of dissipative forces on spin polrization.
  - $\rightarrow$ Formulate Dissipative Spin-hydrodynamics.
  - $\rightarrow$ Explore phenomenological implications.

Section Outline :

- Inspired by the success of Relativistic Hydrodynamics (RH) in explaining the multitude of properties of QGP evolution, development of a framework of RH with spin was started.

[P. Romatschke, IJMPE 19 (2010) 1-53, J. Y. Ollitrault EJP 29 (2008) 275-302, Jaiswal and Roy AHEP 2016 (2016) 9623034]

[F. Becattini *et al*, Annals Phys. 338 (2013) 32-49, Phys. Rev. C 95 (2017) 5, 054902, EPJC 77 (2017) 4, 213]

[W. Florkowski *et al*, Phys. Rev. C 97 (2018) 4, 041901, Phys. Rev. D 97 (2018) 11, 116017]

[D. Montenegro *et al*, Phys. Rev. D 96 (2017) 5, 056012, Phys. Rev. D 96 (2017) 7, 076016]

How to include internal degrees of freedom in a macroscopic theory?

[J. Weyssenhoff, A. Raabe, Acta Phys. Pol. 9 (1947) 7]

Relativistic Spin-hydrodynamics :

- Origin of spin is purely quantum mechanical.

- A theory with spin should be built up from Quantum Field Theory (QFT).

- For a hydrodynamic description of a spin-polarizable fluid starting from QFT, it was proved that a spin-polarization tensor ($\omega^{\mu\nu}$) must be introduced.

  [F. Becattini *et al*, Phys. Lett. B 789 (2019) 419-425]

- It has been argued that, at global equilibrium, the spin-polarization tensor should be same as the thermal vorticity.

  [F. Becattini *et al*, Annals Phys. 338 (2013) 32-49, Phys. Rev. C 95 (2017) 5, 054902, EPJC 77 (2017) 4, 213]

  [N. Weickgenannt *et al*, Phys. Rev. Lett. 127 (2021) 5, 052301]

$$\boxed{\omega^{\mu\nu}|_{\text{geq}} \propto \varpi^{\mu\nu} = \left(\partial^\mu \beta^\nu - \partial^\nu \beta^\mu\right)/2}$$

$\beta^\mu = u^\mu/T$ is the inverse temperature four-vector.

Relativistic Spin-hydrodynamics :

- Theories of ideal spin-hydrodynamics were formulated for fluids in equilibrium.

  [D. Montenegro *et al*, Phys. Rev. D 96, 056012 (2017), Phys. Rev. D 96, 076016 (2017)]

  [W. Florkowski *et al*, Phys. Rev. C 97, 041901 (2018), Phys. Rev. D 97, 116017 (2018)]

- But, we want description of fluid with non-thermalized spin, where the relation, $\omega^{\mu\nu} \propto \varpi^{\mu\nu}$ may not hold.

- Here, we want to understand, how $\omega^{\mu\nu}$ and hence a out-of-equilibrium system of spin-polarizable particles evolves in presence of magnetic field.

Relativistic Spin-hydrodynamics :

- We first note that spin-polarization originates from the rotation of fluid.

- Hence, we will have to deal with three conserved currents :

$$\partial_\mu N^\mu = 0, \qquad \partial_\mu T^{\mu\nu} = 0, \qquad \partial_\lambda J^{\lambda,\mu\nu} = 0$$

where, $J = L + S$. Also, $L^{\lambda,\mu\nu} = x^\mu T^{\lambda\nu} - x^\nu T^{\lambda\mu}$.

- For symmetric $T^{\mu\nu}$ we have, $\boxed{\partial_\lambda S^{\lambda,\mu\nu} = 0}$

$$N^\mu = N^\mu_{\text{eq}} + \delta N^\mu, \qquad T^{\mu\nu} = T^{\mu\nu}_{\text{eq}} + \delta T^{\mu\nu}, \qquad S^{\lambda,\mu\nu} = S^{\lambda,\mu\nu}_{\text{eq}} + \delta S^{\lambda,\mu\nu}$$

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- The dissipative parts require microscopic description → Kinetic Theory.

Kinetic Theory with Spin :

- To import spin in kinetic theory (KT), we start from the Wigner function ($\mathcal{W}_{\alpha\beta}$), that bridges the gap between QFT and KT.

- For spin-1/2 particles we set up kinetic equation of $\mathcal{W}_{\alpha\beta}$ using Dirac equation,

$$\left[\gamma \cdot \left(p + \frac{i}{2}\partial\right) - m\right] \mathcal{W}_{\alpha\beta} = \mathcal{C}\left[\mathcal{W}_{\alpha\beta}\right]$$

[Xin-Li Sheng, *PhD Thesis (2019)*, N. Weickgenannt et al, *PRL 127 (2021) 5, 052301*, *PRD 100, 056018 (2019)*.]

- The Wigner function can be decomposed as,

$$\mathcal{W}_{\alpha\beta} = \frac{1}{4}\left(\mathcal{F} + i\gamma^5\mathcal{P} + \gamma^\mu\mathcal{V}_\mu + \gamma^5\gamma^\mu\mathcal{A}_\mu + \frac{1}{2}\Sigma^{\mu\nu}\mathcal{S}_{\mu\nu}\right)_{\alpha\beta}$$

$\mathcal{F} \rightarrow$ scalar component,

$\mathcal{P} \rightarrow$ pseudoscalar component,

$\mathcal{V}_\mu \rightarrow$ vector component,

$\mathcal{A}_\mu \rightarrow$ axial vector component,

$\mathcal{S}_{\mu\nu} \rightarrow$ tensor component.

where, the γ-matrices are the 4×4 Dirac γ-matrices and, $\Sigma^{\mu\nu} = i\gamma^{[\mu}\gamma^{\nu]}$.

Kinetic Theory with Spin :

- For spin-hydrodynamics it suffices to consider only $\mathcal{F}$ and $\mathcal{A}_\mu$ components.

  [Xin-Li Sheng, *PhD Thesis (2019)*]

	Scalar Component	***Axial Component***
Kin. Eq.	$k^\mu \partial_\mu \mathcal{F}(x,k) = \mathcal{C}_\mathcal{F}$	$k^\mu \partial_\mu \mathcal{A}^\nu(x,k) = \mathcal{C}^\nu_\mathcal{A}$
RTA	$C_\mathcal{F} = \frac{(k\cdot u)}{\tau_{\rm eq}}\Big[\mathcal{F}_{\rm eq}(x,k) - \mathcal{F}(x,k)\Big]$	$C^\nu_\mathcal{A} = \frac{(k\cdot u)}{\tau_{\rm eq}}\Big[\mathcal{A}^\nu_{\rm eq}(x,k) - \mathcal{A}^\nu(x,k)\Big]$
Dist. fn.	$\mathcal{F}^\pm(x,k) = 2m\int_{p,s} f^\pm(x,p,s)\ \delta^{(4)}(k \mp p)$	$\mathcal{A}^\mu_\pm(x,k) = 2m\int_{p,s} s^\mu f^\pm(x,p,s)\ \delta^{(4)}(k \mp p)$

[S.B., W. Florkowski, A. Jaiswal, A. Kumar and, R. Ryblewski, *PLB 814 (2021) 136096*, *PRD 103 (2021) 1, 014030*]

Momentum measure $\rightarrow \quad \int_p(\cdots) \rightarrow \int \mathrm{dP}(\cdots), \quad \int \mathrm{dP} = d^3p/\,(2\pi)^3\, p^0.$

Spin measure $\rightarrow \quad \int_s(\cdots) \rightarrow \int \mathrm{dS}(\cdots), \quad \int \mathrm{dS} = (m/\pi\mathfrak{s}) \int d^4s\,\delta(s\cdot s + \mathfrak{s}^2).$

Relativistic Kinetic Equation :

- We take the equilibrium (extended) phase-space distribution function to be :

$$f^{\pm}_{\text{eq}}(x,p,s) = e^{-\beta(u\cdot p)\pm\xi}\left(1+\frac{1}{2}\omega_{\mu\nu}s^{\mu\nu}\right)+\mathcal{O}(\omega^2)$$

[F. Becatinni et al., *Annals Phys. 338 (2013) 32-49*, W. Florkowski et al., *PRD 97 (2018) 11, 116017*]

- Near local equilibrium $f(x,p,s)$ is expanded using Chapman-Enskog :

$$f^{\pm}(x,p,s) = f^{\pm}_{\text{eq}}(x,p,s) + \delta f^{\pm}(x,p,s).$$

[de Groot, van Leewan, van Weert, *'Relativistic Kinetic Theory - Principle and Applications (1980)'.*]

- δf is the non-equilibrium correction and is obtained from the Boltzmann equation,

$$p^{\mu}\partial_{\mu}f^{\pm}(x,p,s) = -\frac{(u\cdot p)}{\tau_{\text{eq}}}\delta f^{\pm}(x,p,s)$$

[Anderson, Witting, *Physica 74 (3) (1974) 466–488.*]

- Solution:

$$\delta f^{\pm}_{s} = -\frac{\tau_s}{u\cdot p}e^{\pm\xi-\beta\cdot p}\left[\left(\pm p^{\mu}\partial_{\mu}\xi - p^{\lambda}p^{\mu}\partial_{\mu}\,\beta_{\lambda}\right)\left(1+\frac{1}{2}s^{\alpha\beta}\omega_{\alpha\beta}\right)+\frac{1}{2}p^{\mu}s^{\alpha\beta}\left(\partial_{\mu}\omega_{\alpha\beta}\right)\right]$$

- The conserved currents are expressed in kinetic theory as,

$$N^{\mu}=\int_{p,s}p^{\mu}\left(f^{+}-f^{-}\right);\qquad T^{\mu\nu}=\int_{p,s}p^{\mu}p^{\nu}\left(f^{+}+f^{-}\right);\qquad S^{\lambda,\mu\nu}=\int_{p,s}p^{\lambda}s^{\mu\nu}\left(f^{+}+f^{-}\right)$$

Dissipative Currents in Spin-hydrodynamics:

- The dissipative quantities are defined as,

$$n^{\mu} = \Delta^{\mu}_{\alpha} \int dP \int dS \, p^{\alpha} \left(\delta f^{+} - \delta f^{-}\right)$$
$$\Pi = -\frac{\Delta_{\alpha\beta}}{3} \int dP \int dS \, p^{\alpha} p^{\beta} \left(\delta f^{+} + \delta f^{-}\right)$$
$$\pi^{\mu\nu} = \Delta^{\mu\nu}_{\alpha\beta} \int dP \int dS \, p^{\alpha} p^{\beta} \left(\delta f^{+} + \delta f^{-}\right)$$
$$\delta S^{\lambda,\mu\nu} = \int dP \int dS \, p^{\lambda} s^{\mu\nu} \left(\delta f^{+} + \delta f^{-}\right)$$

where, $\Delta^{\mu\nu}_{\alpha\beta} = (1/2)(\Delta^{\mu}_{\alpha}\Delta^{\nu}_{\beta} + \Delta^{\nu}_{\beta}\Delta^{\mu}_{\alpha}) - (1/3)\Delta^{\mu\nu}\Delta_{\alpha\beta}$ is a traceless symmetric projection operator.

Dissipative Currents in Spin-hydrodynamics:

- The non-equilibrium parts give the transport coefficients:

$$\delta N^{\mu} = \tau_{\text{eq}} \beta_n (\nabla^{\mu} \xi),$$
$$\delta T^{\mu\nu} = \tau_{\text{eq}} \left[-\beta_{\Pi}\, \Delta^{\mu\nu}\, \theta + 2\, \beta_{\pi}\, \sigma^{\mu\nu} \right],$$
$$\delta S^{\lambda,\mu\nu} = \tau_{\text{eq}} \Big[B_{\Pi}^{\lambda,\mu\nu} \theta + B_{n}^{\phi\lambda,\mu\nu} (\nabla_{\phi} \xi) + B_{\pi}^{\alpha\beta\lambda,\mu\nu} \sigma_{\alpha\beta} + B_{\Sigma}^{\rho\gamma\phi\lambda,\mu\nu} (\nabla_{\rho} \omega_{\gamma\phi}) \Big]$$

- By choosing the Landau frame and matching conditions we found the following relations:

$$\dot{\xi} = \xi_{\theta}\, \theta, \qquad \dot{\beta} = \beta_{\theta}\, \theta, \qquad \beta \dot{u}_{\mu} = -\nabla_{\mu} \beta + \frac{n_0\, \tanh \xi}{(\mathcal{E} + \mathcal{P})} (\nabla_{\mu} \xi)$$
$$\dot{\omega}^{\mu\nu} = \mathcal{D}_{\Pi}^{\mu\nu} \theta + \mathcal{D}_{\text{n}}^{\mu\nu\alpha} (\nabla_{\alpha} \xi) + \mathcal{D}_{\pi}^{\mu\nu\alpha\beta} \sigma_{\alpha\beta} + \mathcal{D}_{\Sigma}^{\lambda\mu\nu\alpha\beta\gamma} (\nabla_{\alpha} \omega_{\beta\gamma}),$$

[S.B., W. Florkowski, A. Jaiswal, A. Kumar and, R. Ryblewski, *PLB 814 (2021) 136096, PRD 103, 014030 (2021)*]

Section Outline :

Pauli-Lubanski and the Polarization

- Pauli Lubanski Vector : $$E_p \frac{d\Delta\Pi_\alpha(x,p)}{d^3p} = -\frac{1}{2}\epsilon_{\alpha\mu\nu\beta}\, \Delta\Sigma_\lambda E_p \frac{dS^{\lambda,\mu\nu}}{d^3p}\frac{p^\beta}{m}.$$

  [W. Florkowski, A. Kumar and, R. Ryblewski, *Prog. Part. Nucl. Phys. 108 103709 (2019)*]

- Polarization : $$\langle P(\phi_p)\rangle = \frac{\int p_T dp_T E_p \frac{d\Pi^z(p)}{d^3p}}{\int d\phi_p p_T dp_T E_p \frac{dN(p)}{d^3p}}$$

- $$E_p \frac{dN(p)}{d^3p} = \frac{4\cosh\xi}{(2\pi)^3}\int \Delta\Sigma_\lambda p^\lambda e^{-\beta . p}, \qquad \xi = \mu/T, \beta^\mu = u^\mu/T.$$

- Further Contraints : $\omega \longrightarrow \varpi, \qquad \varpi_{0i} = 0$

Thermal Vorticity : $\varpi_{\mu\nu} = -\frac{1}{2}(\partial_\mu\beta_\nu - \partial_\nu\beta_\mu)$

Thermal Model

- Single freeze-out model.
- $\tau_f^2 = t^2 - x^2 - y^2 - z^2$ with $x^2 + y^2 \leq r_{max}^2$.
- Asymmetry of the fireball boundary :
  $x = r_{\text{max}}\sqrt{1-\epsilon}\,\cos\phi \qquad y = r_{\text{max}}\sqrt{1+\epsilon}\,\sin\phi$
- Hydrodynamic flow:
  $u^\mu = \frac{1}{N}(\,t,\, x\sqrt{1+\delta}\,,\, y\sqrt{1-\delta}\,, z\,)$
  $N = \sqrt{\tau^2 - (x^2 - y^2)\delta}$

[W. Broniowski, A. Baran, and W. Florkowski, *AIP Conf. Proc. 660 (2003) 1, 185-195*]

Parametrizing the Flow and Hypersurface

Table 1: Freeze-out Temperature (T_f) = 165 MeV.

c %	ϵ	δ	τ_f [fm]	r_{max} [fm]
0-15	0.055	0.12	7.666	6.540
15-30	0.097	0.26	6.258	5.417
30-60	0.137	0.37	4.266	3.779

[W. Broniowski, A. Baran, and W. Florkowski, *AIP Conf. Proc. 660 (2003) 1, 185-195*]

Section Outline :

Polarization for 30-60% Centrality

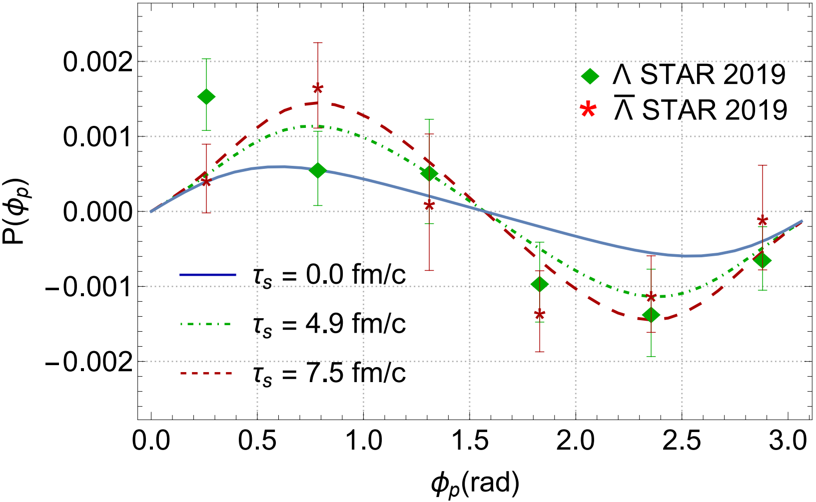

Figure 5: Polarization. $\tau_s = 7.5$ fm for $\overline{\Lambda}$ ($\chi_r^2 = 0.6$) and, $\tau_s = 4.9$ fm for Λ ($\chi_r^2 = 1.5$).

- τ_s is in agreement with [Hidaka et. al., arXiv: 2312.08266, Wagnar et. al., arXiv: 2405.00533]

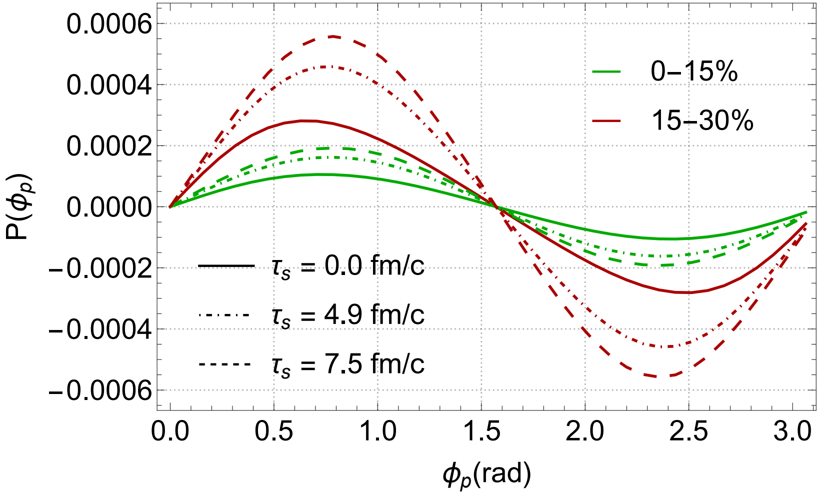

Figure 6: Λ and $\bar{\Lambda}$ for different centralities

.

Next Steps Forward :

- Does magnetic field play any role?

  [Sun, Yan, *arXiv: 2401.07458*]

- Recall:

$$\delta f_s = -\frac{\tau_s}{u \cdot p} e^{-\beta \cdot p} \left[-p^\lambda p^\mu \left(\partial_\mu \beta_\lambda\right) \left(1 + \frac{1}{2} s^{\alpha\beta} \omega_{\alpha\beta}\right) + \frac{1}{2} p^\mu s^{\alpha\beta} \left(\partial_\mu \omega_{\alpha\beta}\right) \right]$$

- The theory of spin magnetohydrodynamics indicates vorticity plays a role in the evolution of the system.

  [S.B., W. Florkowski, A. Jaiswal, A. Kumar and, R. Ryblewski, *PRL 129 (2022), 192301*]

$$X = \tau_{\mathrm{eq}} \Big[\beta_{X\Pi}\, \theta + \beta^{\alpha}_{Xn} \left(\nabla_\alpha \xi\right) + \beta^{\alpha}_{Xa} \dot{u}_\alpha + \beta^{\alpha\beta}_{X\pi} \sigma_{\alpha\beta} + \beta^{\alpha\beta}_{X\Omega} \Omega_{\alpha\beta} + \beta^{\alpha\beta}_{XF} \left(\nabla_\alpha B_\beta\right) + \beta^{\alpha\beta\gamma}_{X\Sigma} \left(\nabla_\alpha \omega_{\beta\gamma}\right) \Big],$$

where, $X \equiv n^\mu,\ \Pi,\ \pi^{\mu\nu},\ \delta S^{\lambda,\mu\nu}$.

- Evolution of spin-polarization tensor (obtained from spin-matching) is given by,

$$\dot{\omega}^{\mu\nu} = \mathcal{D}^{\mu\nu}_{\Pi}\, \theta + \mathcal{D}^{\mu\nu\gamma}_{\mathrm{n}} \left(\nabla_\gamma \xi\right) + \mathcal{D}^{\mu\nu\gamma}_{\mathrm{a}} \dot{u}_\gamma + \mathcal{D}^{\mu\nu\rho\kappa}_{\pi} \sigma_{\rho\kappa} + \mathcal{D}^{\mu\nu\rho\kappa}_{\Omega} \Omega_{\rho\kappa} + \mathcal{D}^{\mu\nu\phi\rho\kappa}_{\Sigma} \left(\nabla_\phi \omega_{\rho\kappa}\right)$$

Section Outline :

Summary and Outlook :

- **Summary :**
  1. Although SIP resolves the “spin sign puzzle", multiple models exist and details differ.
  2. Inclusion of dissipative forces provides another alternative solution.
  3. Using thermal vorticity as proxy for spin polarization tensor should be reconsidered.

- **Outlook :**
  1. May explore the consequences of magnetic field.
  2. Development of a 3+1 D code for spin-hydrodynamic code is essential.

Thank you.