QCD PD in the $T - eB$ plane for pion masses beyond its physical value

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Based on arXiv: 2407:14449

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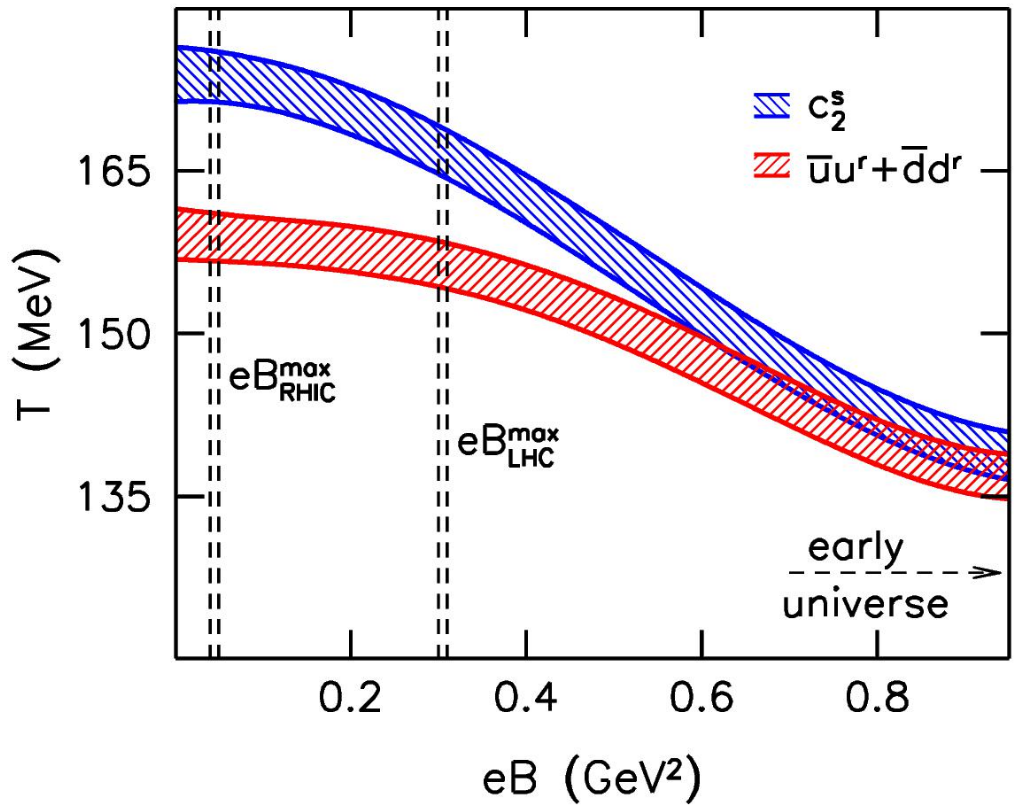
Outline:

- QCD phase diagram (PD) in the presence of a magnetic field (eB)
- What is the magnetic catalysis (MC) and inverse magnetic catalysis (IMC)?
- What is the role played by the pion mass?
- The effects in effective models, such as Nambu-Jona-Lasinio model
- Conclusion

Phase diagram

- QCD PD in the T-eB plane

[G. S. Bali et al., JHEP 02 (2012) 044]



- What happens if we increase the magnetic field further?

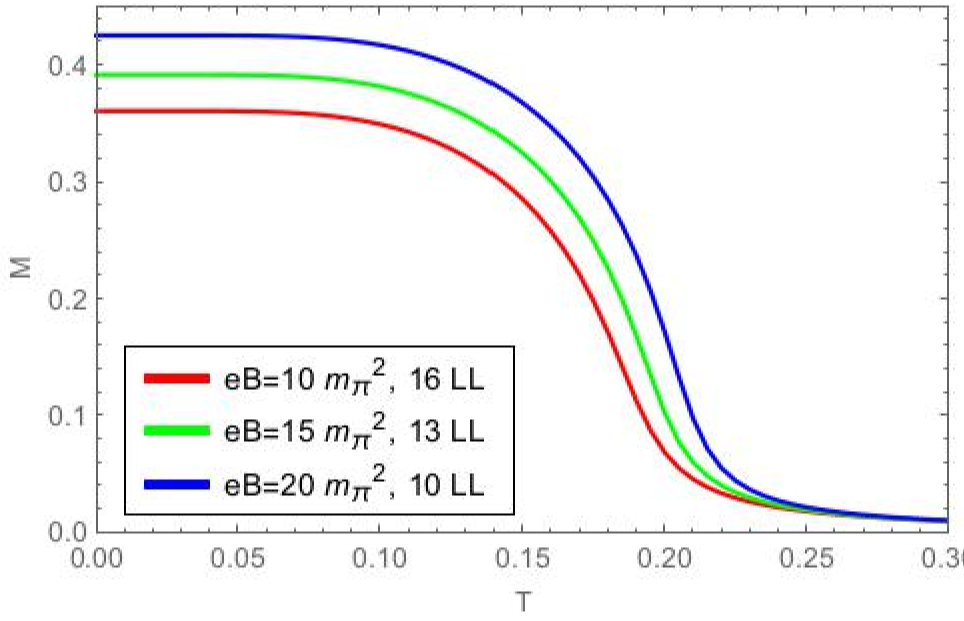
➡ There is possiblity of finiding a CEP at high enough eB [Endrődi 2015 and D'Elia et al., 2021]

Magnetic catalysis

- Did we always know about such a PD?

- Different effective model results and as well as lattice QCD calculations agreed on an increasing T_{CO} as a function of eB [V. P. Gusynin et al., NPB462, 249 (1996), M. D'Elia et al, PRD 82, 051501 (2010)]

For more such references: [G. S. Bali et al., PRD 86, 071502]



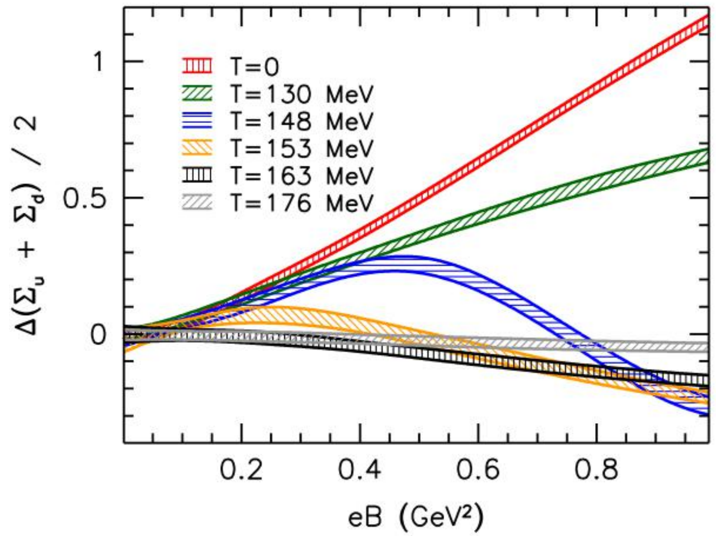
- Two things to observe:

I) Increasing T_{CO} and

II) Increasing condensate (MC) with eB

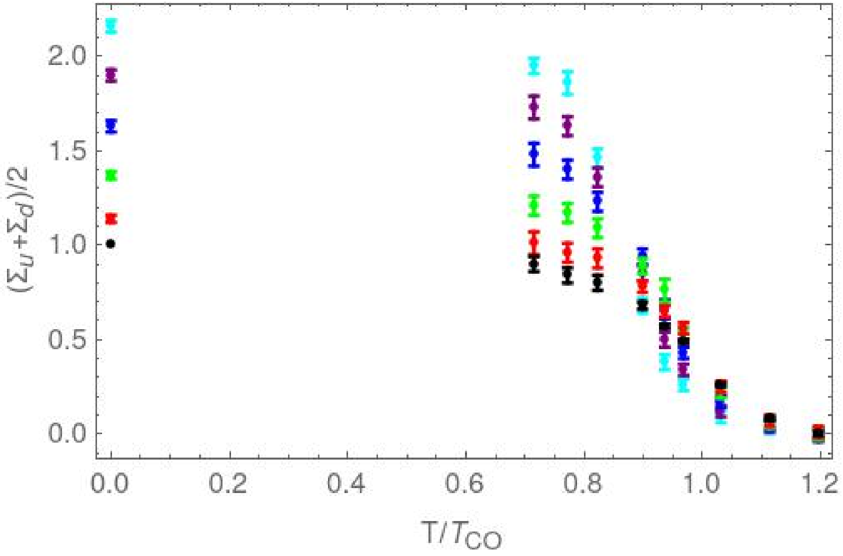
[CAI et al., PRD 99, 094028]

Inverse Magnetic catalysis



- Updated lattice QCD calculation and there is a change in our understanding!
  [G. S. Bali et al., PRD 86, 071502]

- Now we observe IMC effect around the transition temperature.

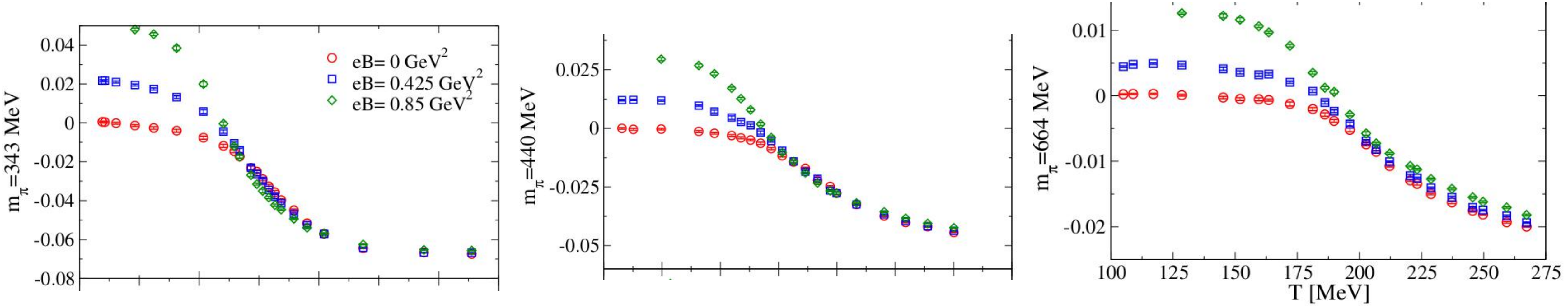


- IMC effect and T_{CO} accompany each other.
  A word of caution!

- Why such properties could not be observed in the previous lattice studies?
  ⟹ What do we learn from this?

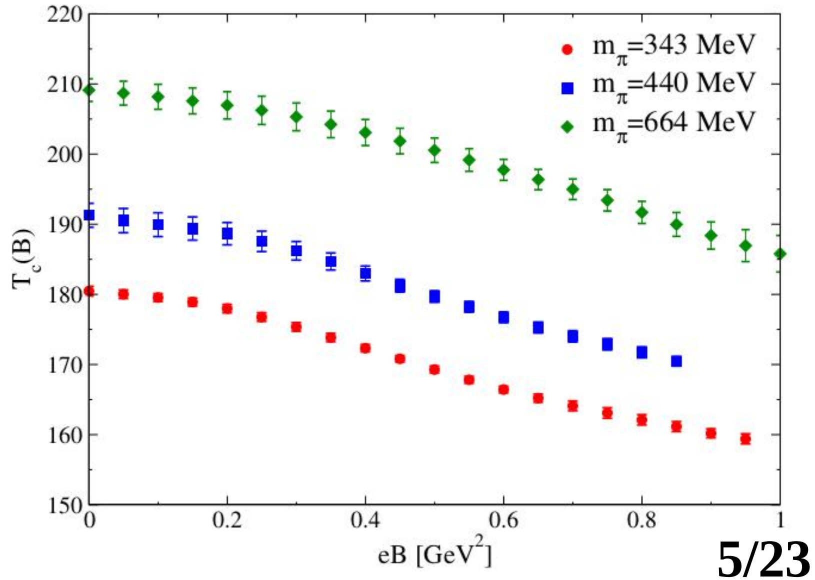
The role of pion mass

- The condensates with increasing pion mass [M. D'Elia et al., PRD 98, 054509, G. Endroidi et al., JHEP07(2019)007]



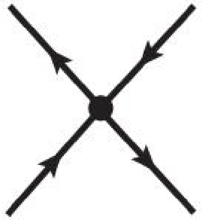
- The PD with increasing pion mass

- Conclusion: No IMC but decreasing T_{CO}



An effective model perspective

- The standard NJL Lagrangian



Local Four point interaction

$$\mathcal{L}_{\rm NJL} = \bar{\psi}(i\gamma_\mu\partial^\mu - m_0)\psi + \frac{G_S}{2}[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2]$$

- Cannot capture the IMC effect on its own. Needs an eB or eB and T dependent coupling constant [R. L. S. Farias et al., PRC 90, 025203; M. Ferreira et al., PRD 89 116011]

- Such examples:

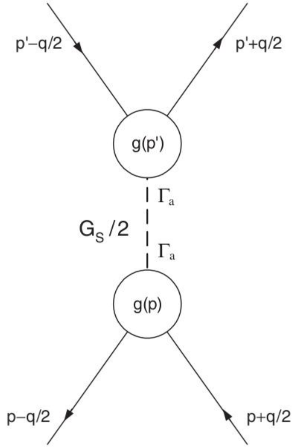
a) $$G_S(eB,T) = c(eB)\left(1 - \frac{1}{1+e^{\beta(eB)(T_a(eB)-T)}}\right) + \gamma(eB)$$ [R. Farias et al., EPJA(2017) 53: 101

b) $$G_S(\xi) = G_S^0\frac{1+a\xi^2+b\xi^3}{1+c\xi^2+d\xi^4}$$ with $\xi = eB/\Lambda_{QCD}^2$ [M. Ferreira et al., PRD 89 116011]

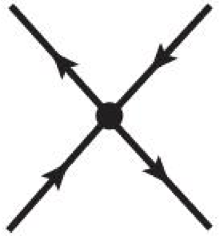
Non-local model

- On the other hand a non-local version can capture the IMC effect without tweaking the model further

- The non-local interaction:

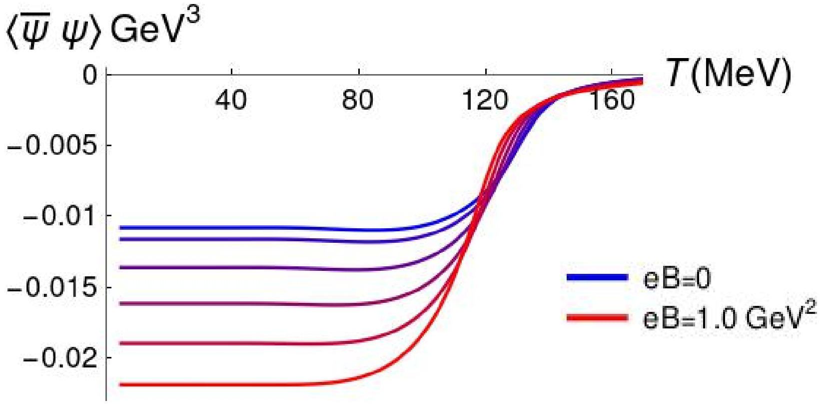


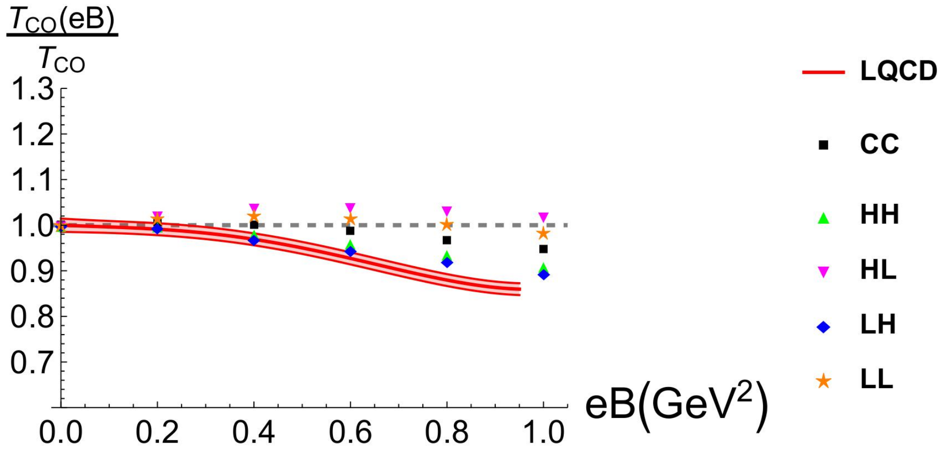
Difference



$$j_a(x) = \int d^4 z\, g(z) \bar{\psi}\left(x + \frac{z}{2}\right) \Gamma_a \psi\left(x - \frac{z}{2}\right)$$

$g(z)$ is the non-local form factors [Birse et al., for example: NPA 582 655, Schmidt et al., PRC 50 435]





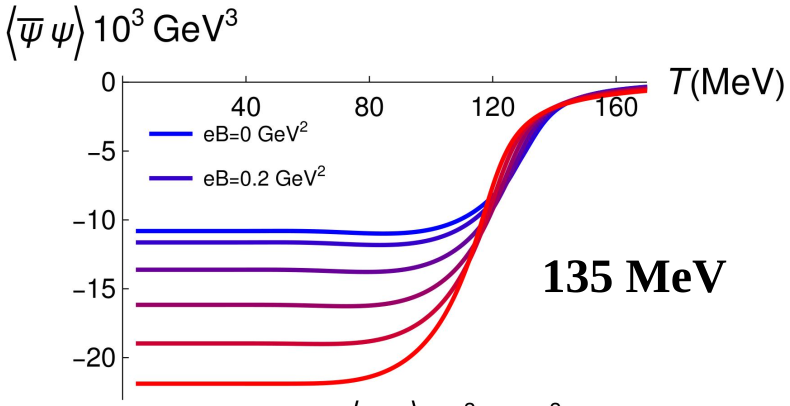
[Pagura et al., PRD 95, 034013; CAI et al., PRD 104, 114026]

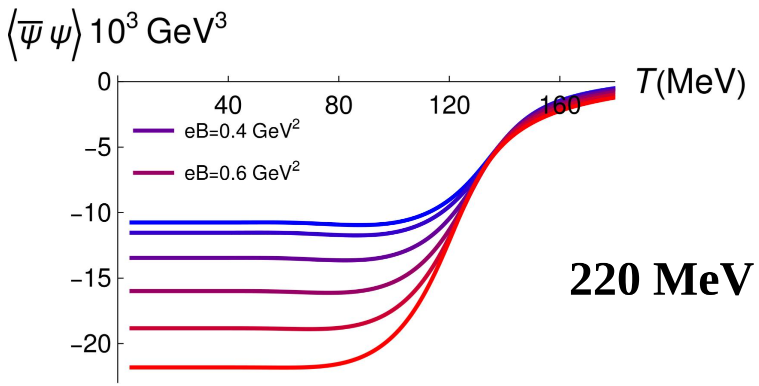
To go beyond physical pion mass

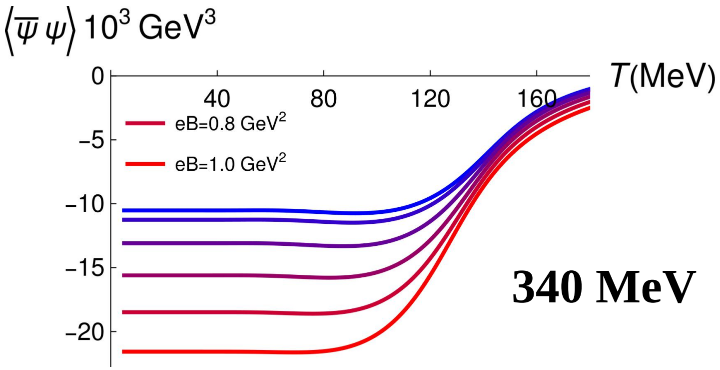
m_π (MeV)	m (MeV)
135	6.94
220	18.28
340	42.89
440	70.78

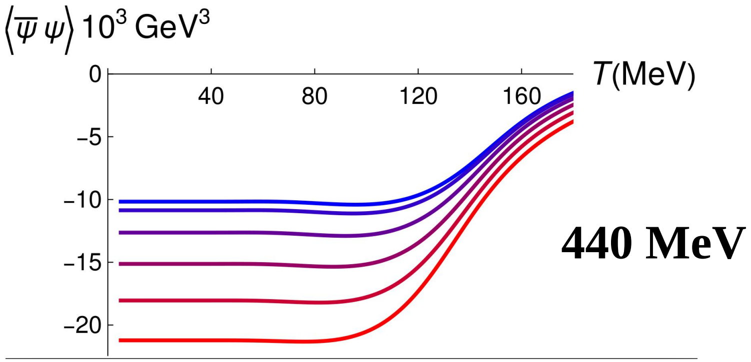
[CAI et al., arXiv: 2407:14449]

TABLE II. Parameter m for different pion masses, with Λ and G_0 kept fixed to their values at physical point ($\Lambda = 605.05$ MeV and $G_0 = 29.38/\Lambda^2$).







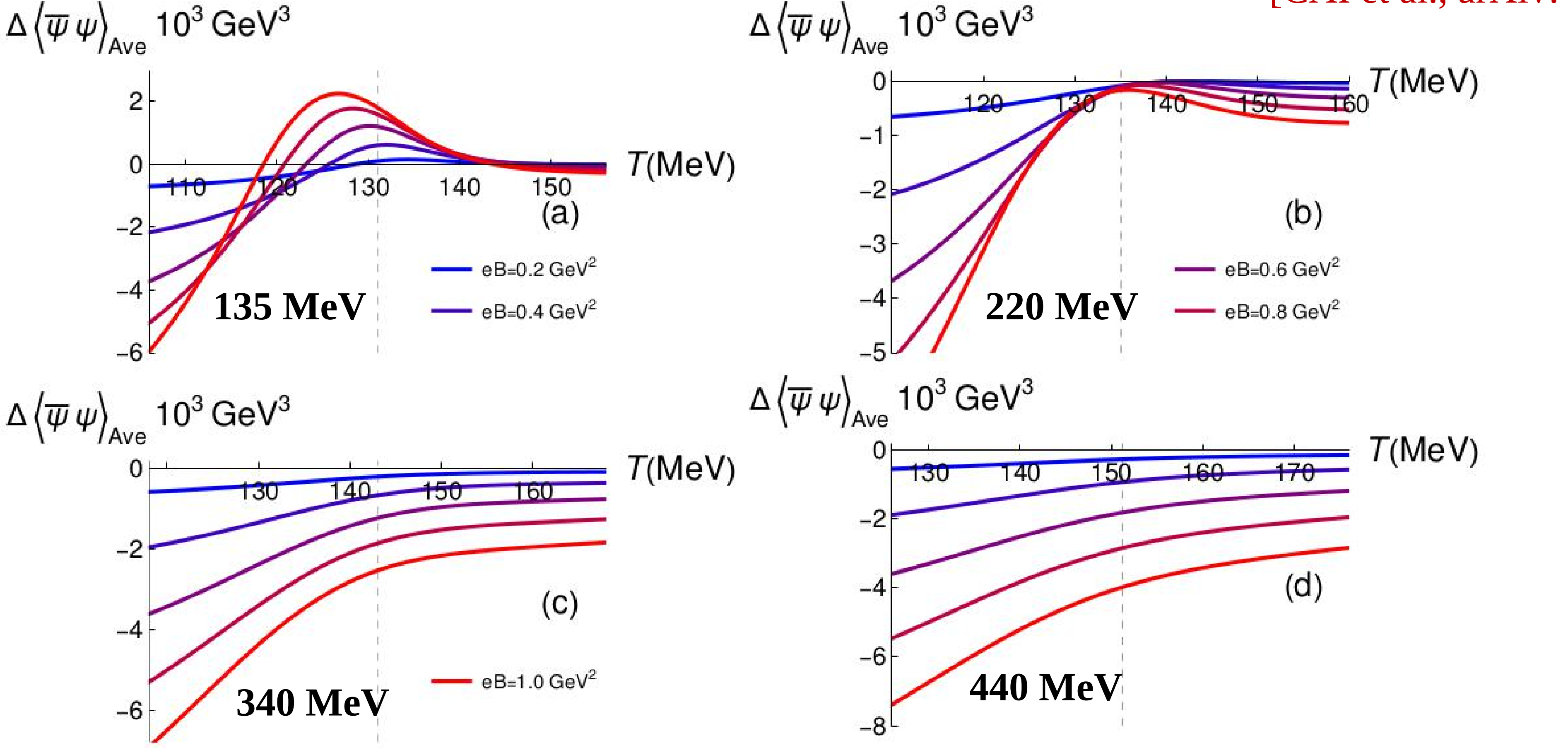


Condensate-average differences

- The condensate-average difference is defined as the average of the u and d quark condensates from which the value of the condensate average for $eB = 0$ is subtracted.

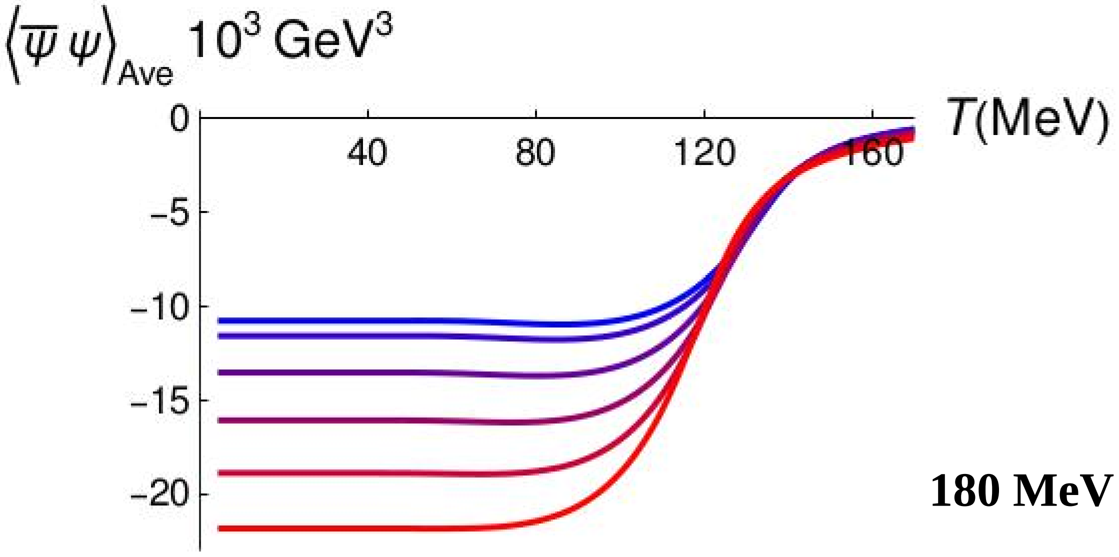
$$\Delta\langle\bar{\psi}\psi\rangle_{\mathrm{Ave}} = \langle\bar{\psi}\psi\rangle_{\mathrm{Ave}}(eB \neq 0) - \langle\bar{\psi}\psi\rangle_{\mathrm{Ave}}(eB = 0)$$

[CAI et al., arXiv: 2407:14449]

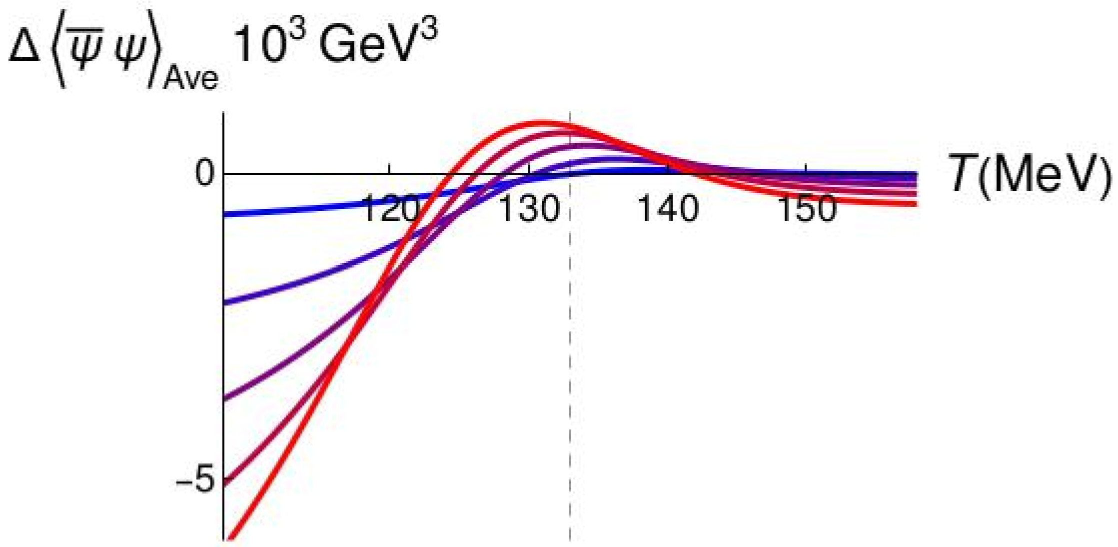


Pion mass value of 180 MeV

- Presence of the IMC effect for a higher pion mass value:

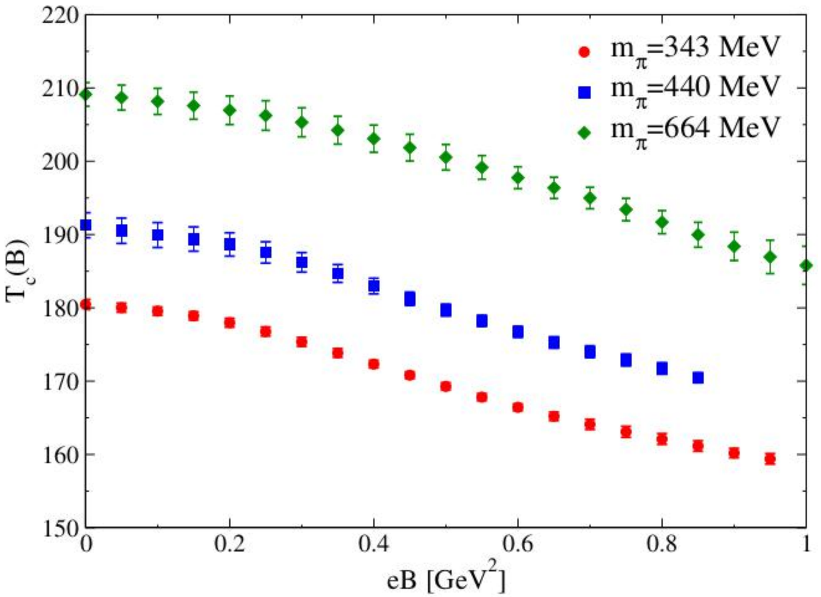


180 MeV



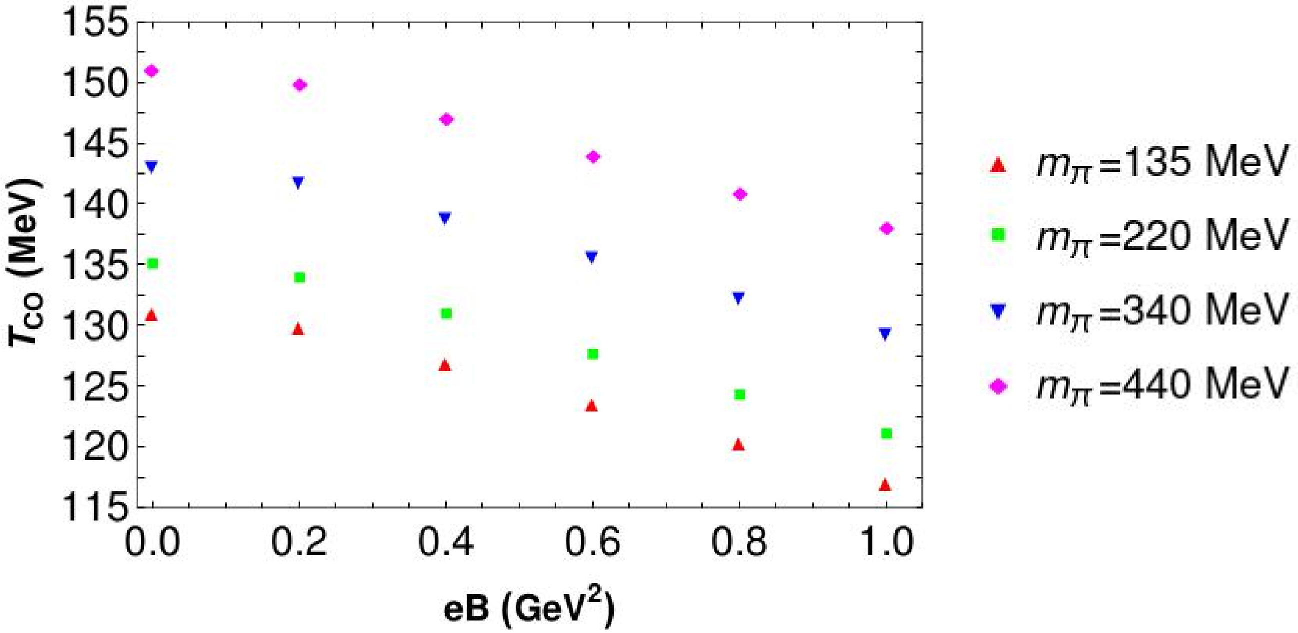
[CAI et al., arXiv: 2407:14449]

PD for beyond physical pion mass



Lattice QCD

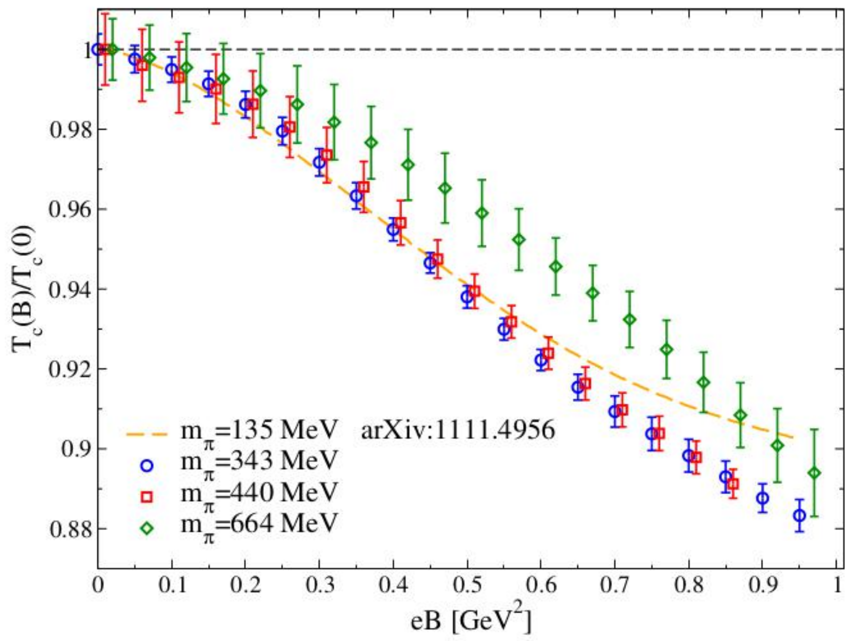
[M. D’Elia et al., PRD 98, 054509]



Nonlocal NJL model

[CAI et al., arXiv: 2407:14449]

PD for beyond physical pion mass



Lattice QCD

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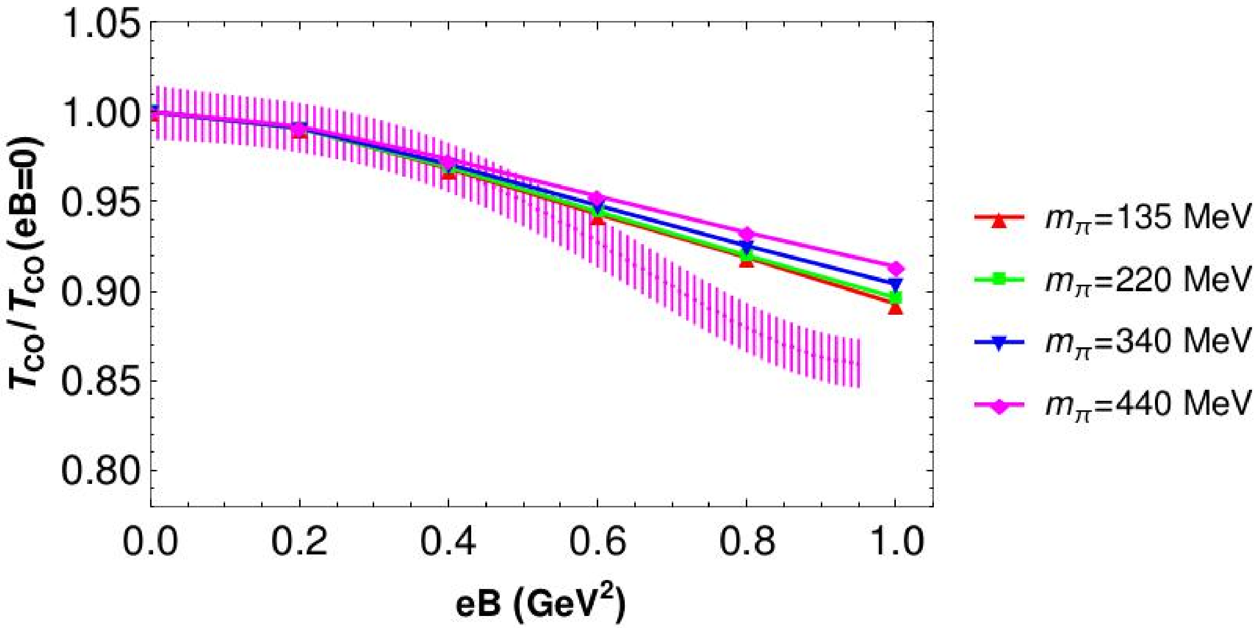


Nonlocal NJL model

[CAI et al., arXiv: 2407:14449]

Upshots

- We observe that with increasing pion mass, the IMC effect disappears.
- The decreasing trend of T_{CO} persists up to the tested pion mass values.
- This is qualitatively consistent with the LQCD results.
- The value of the pion mass at which the IMC effect goes away is lower than that found in the LQCD study.

2-flavour local model

- In the local 2-flavour model:

$$G_S(eB,T) = c(eB)\left(1 - \frac{1}{1+e^{\beta(eB)(T_a(eB)-T)}}\right) + \gamma(eB)$$

[R. Farias et al., EPJA(2017) 53: 101

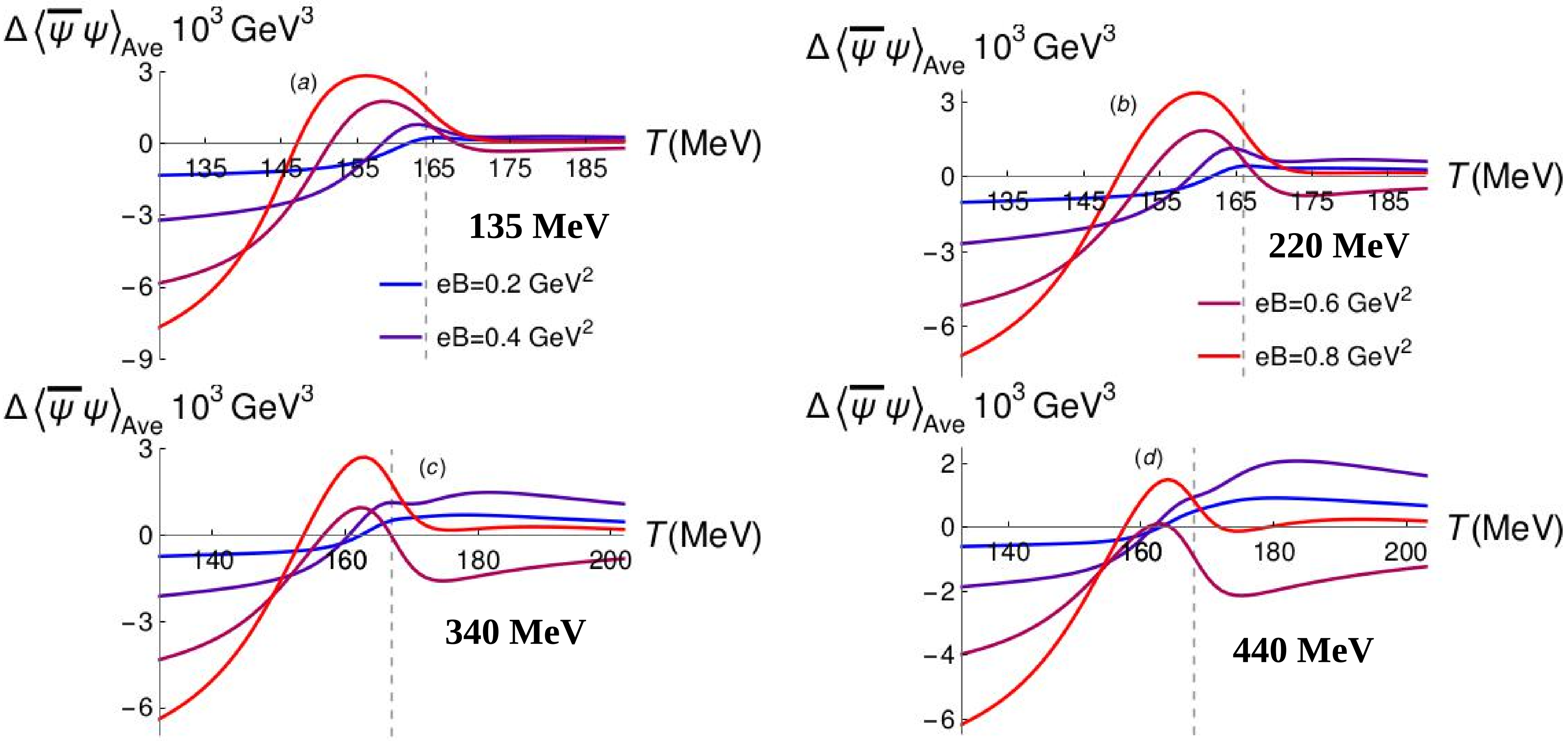
m_π (MeV)	m (MeV)
135	5.5
220	13.2
340	32.2
440	54.2

TABLE III. Parameter m for different pion masses, with Λ and G_0 kept fixed to their values at physical point ($\Lambda = 650$ MeV and $G_0 = 4.5$ GeV^{-2}).

[CAI et al., arXiv: 2407:14449]

2-flavour local model

- The IMC effect for different pion masses:

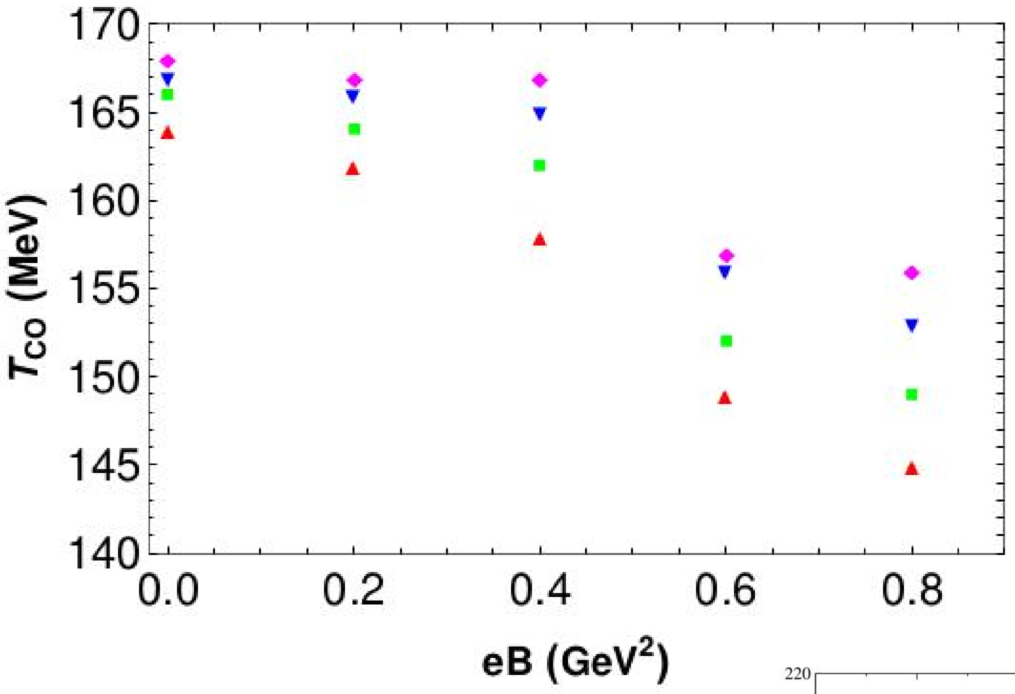


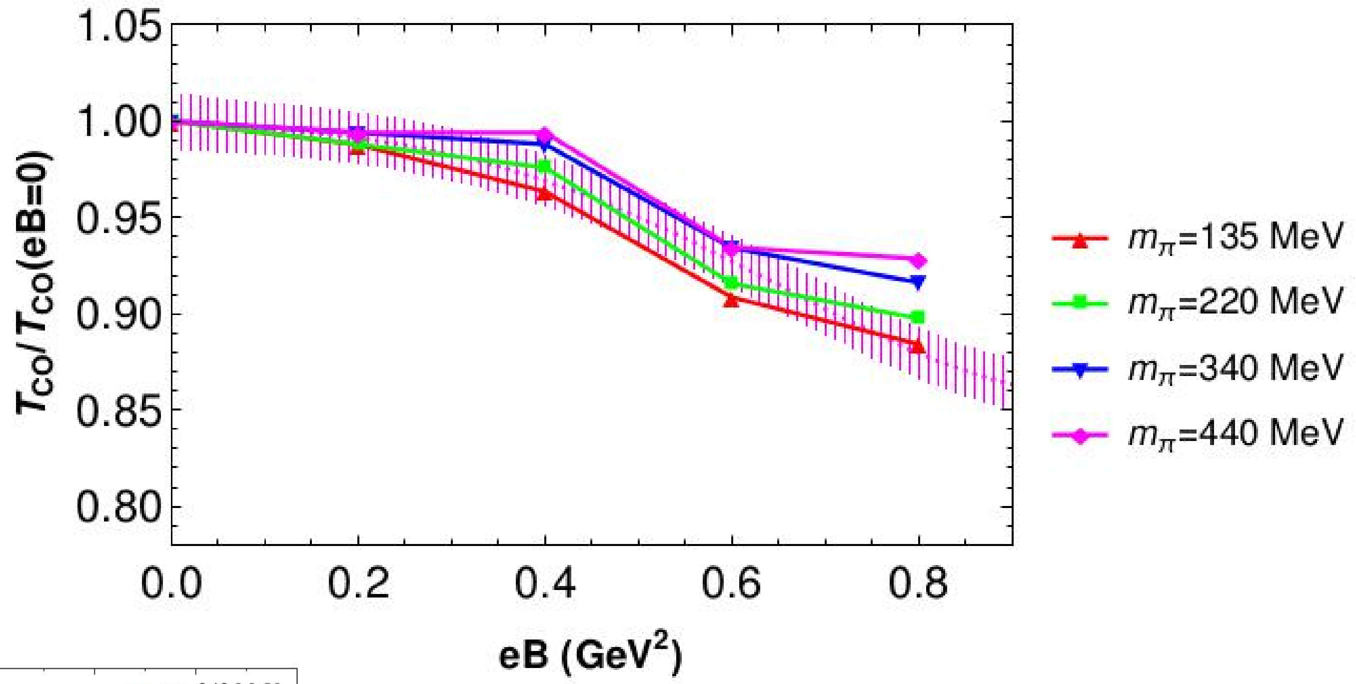
[CAI et al., arXiv: 2407:14449]

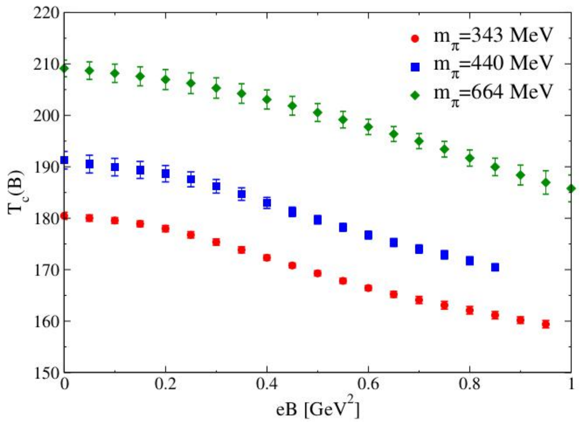
PD for beyond physical pion mass in 2-flavour local model

- The decreasing trend of T_{CO} for different pion masses:

[CAI et al., arXiv: 2407:14449]







[M. D'Elia et al., PRD 98, 054509]

2+1-flavour local model

- In the local 2+1-flavour model:

$$\mathcal{L}_{\rm sym} = G_1 \sum_{a=0}^{8} \left[\left(\bar{\psi}\lambda_a\psi\right)^2 + \left(\bar{\psi}\,i\gamma_5\lambda_a\psi\right)^2 \right]$$

$$\mathcal{L}_{\rm det} = -G_2 \left[\det \bar{\psi}_i(1-\gamma_5)\psi_j + \det \bar{\psi}_i(1+\gamma_5)\psi_j\right]$$

$$G_1(\xi) = G_1^0 \frac{1 + a\xi^2 + b\xi^3}{1 + c\xi^2 + d\xi^4} \quad \text{with} \quad \xi = eB/\Lambda^2_{QCD}$$

[M. Ferreira et al., PRD 89 116011]

m_π (MeV)	m (MeV)
135	5.5
220	14.5
340	34.1
440	56.1

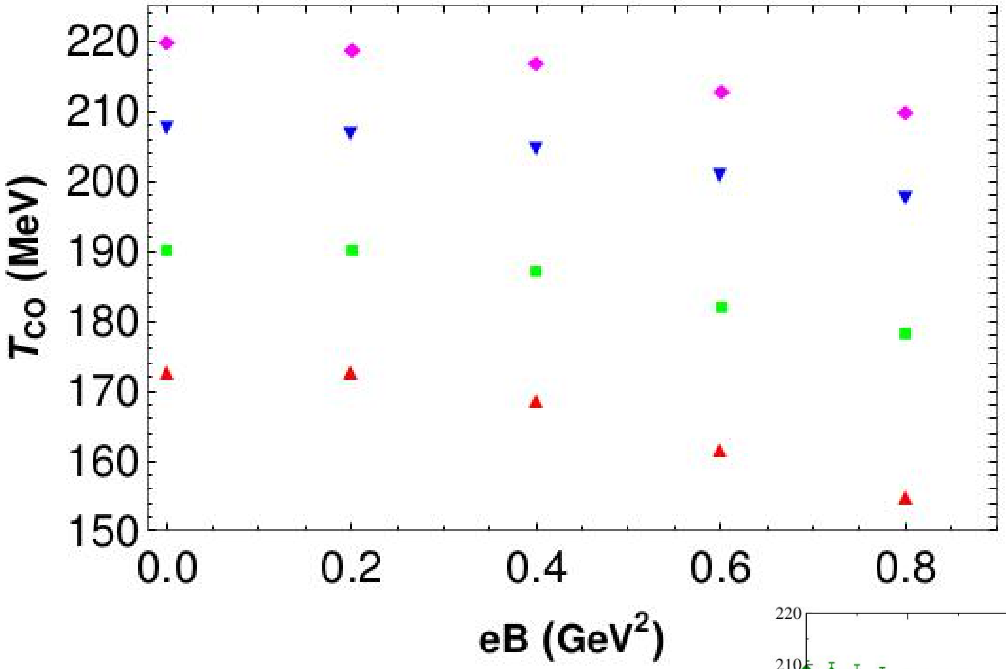
TABLE IV. Parameter m for different pion masses, with Λ, G_1 and G_2 kept fixed to their values at physical point ($\Lambda = 602.3$ MeV and $G_1 = 3.67/\Lambda^2$ and $G_2 = 12.36/\Lambda^5$). The strange-to-light quark mass ratio is kept at its physical value, 25.58.

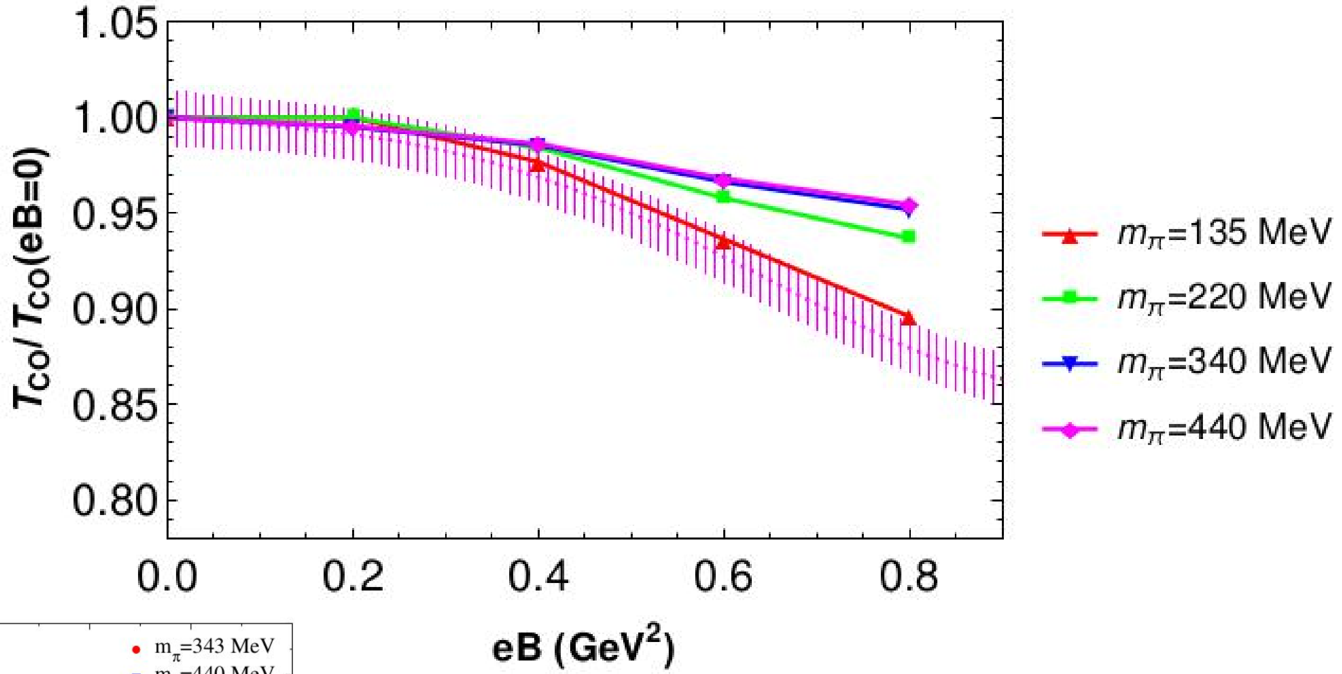
[CAI et al., arXiv: 2407:14449]

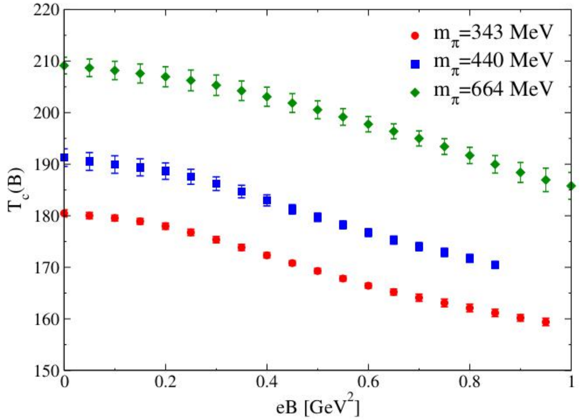
PD for beyond physical pion mass

- The decreasing trend of T_{CO} for different pion masses:

[CAI et al., arXiv: 2407:14449]



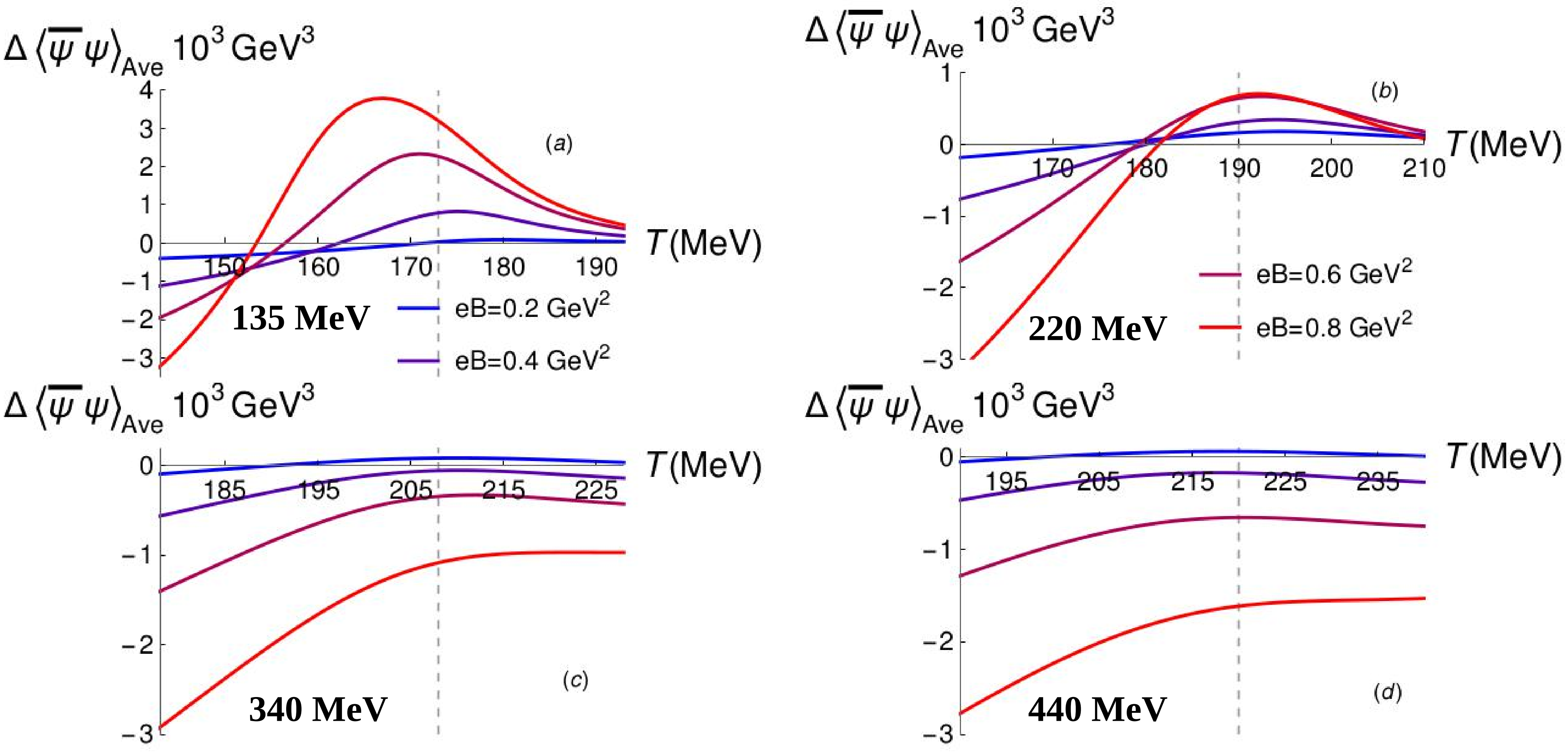




[M. D'Elia et al., PRD 98, 054509]

2+1-flavour local model

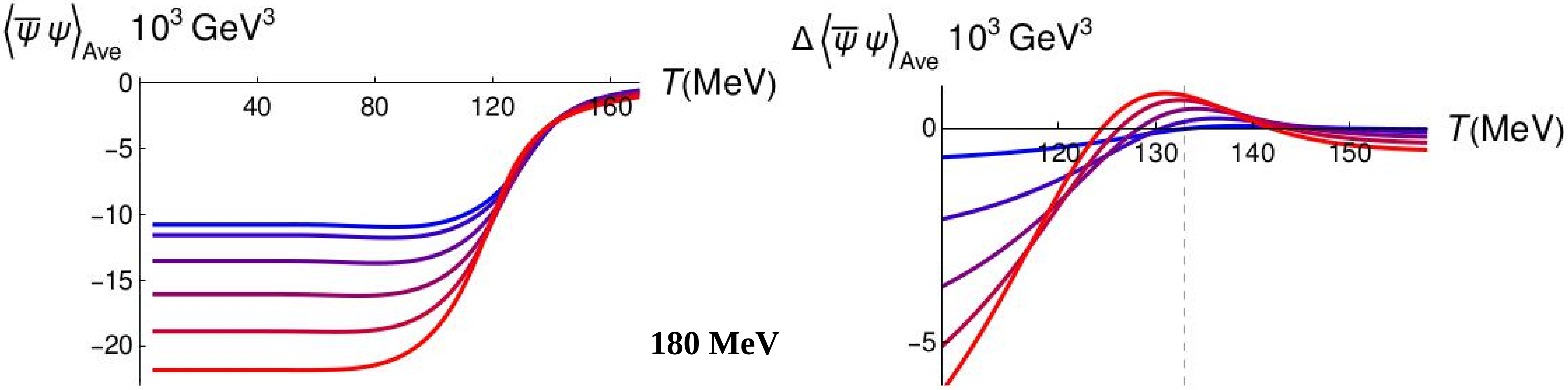
- The IMC effect for different pion masses:



[CAI et al., arXiv: 2407:14449]

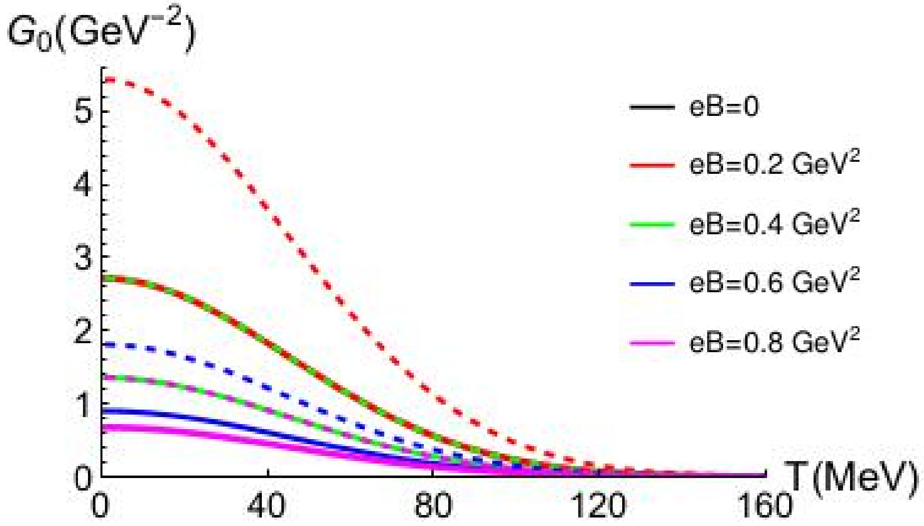
Flashback

- The consistent IMC effect for a higher pion mass value in a nonlocal framework:

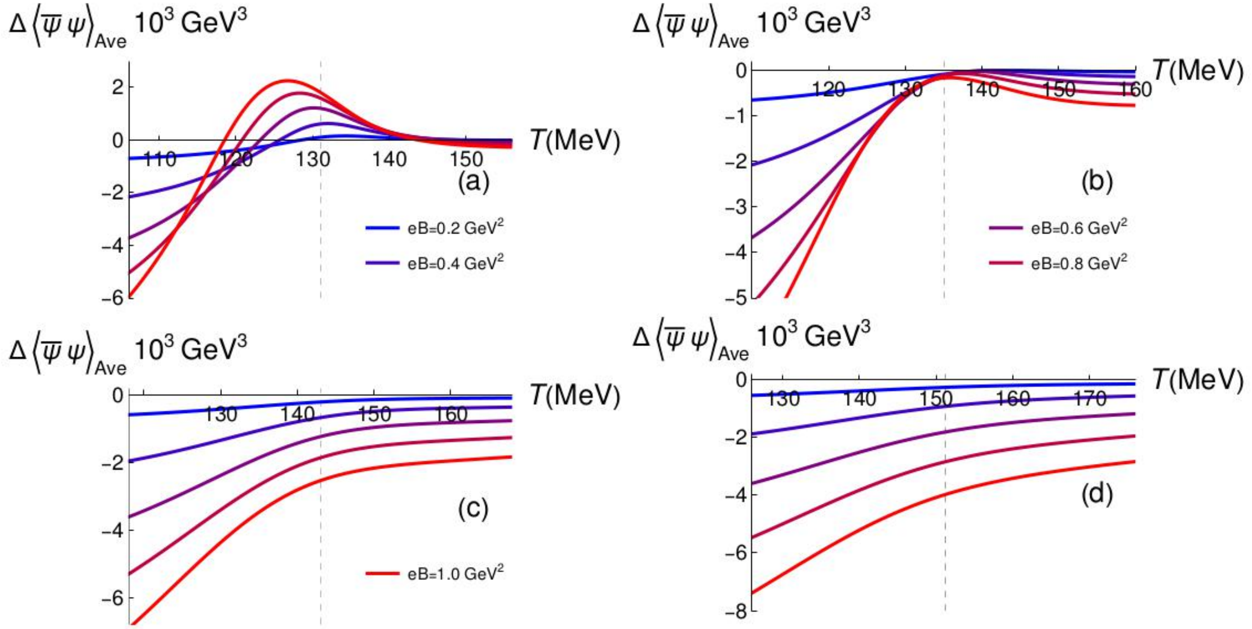


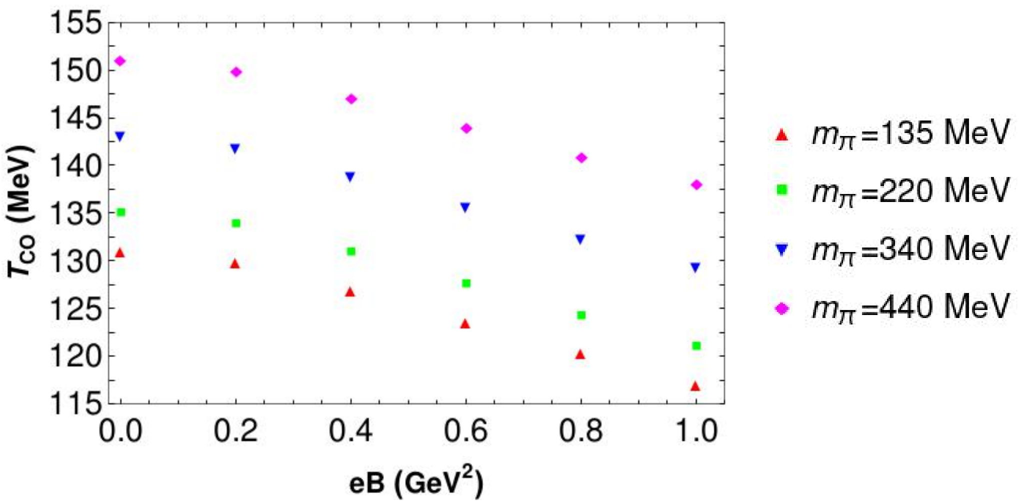
[CAI et al., arXiv: 2407:14449]

Behaviour of the coupling constant



Nonlocal 2-flavour





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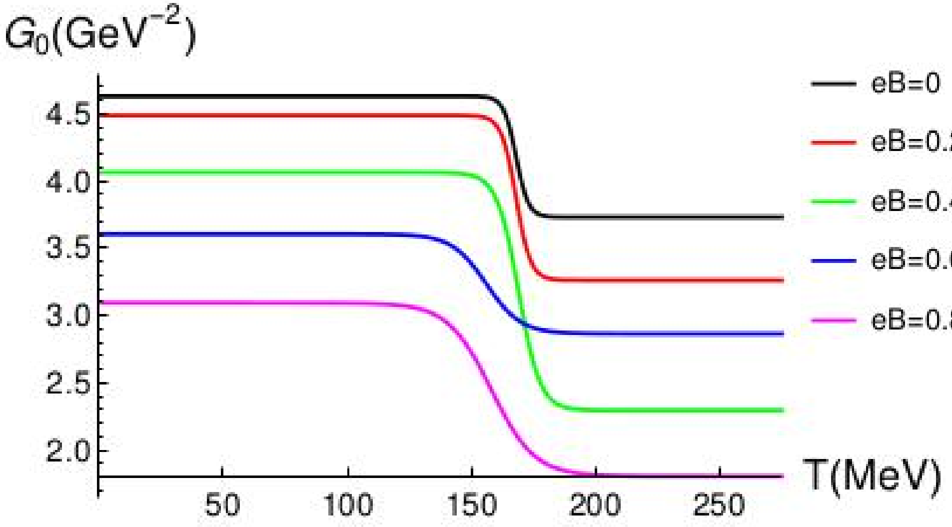
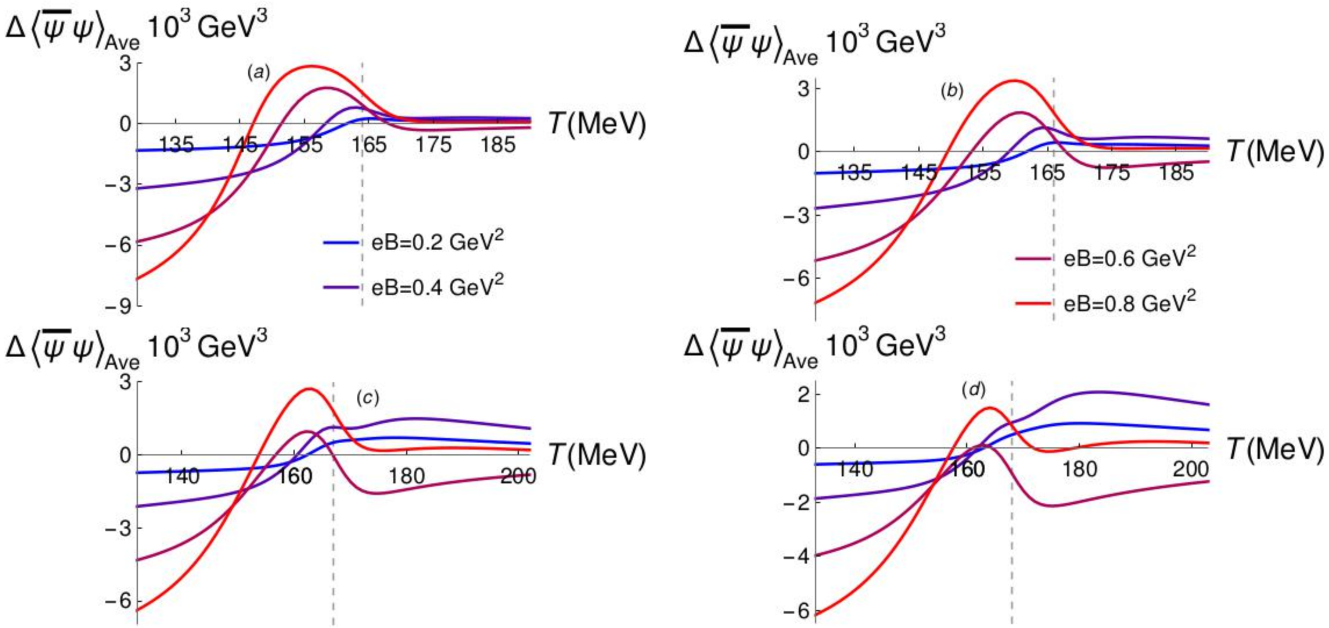
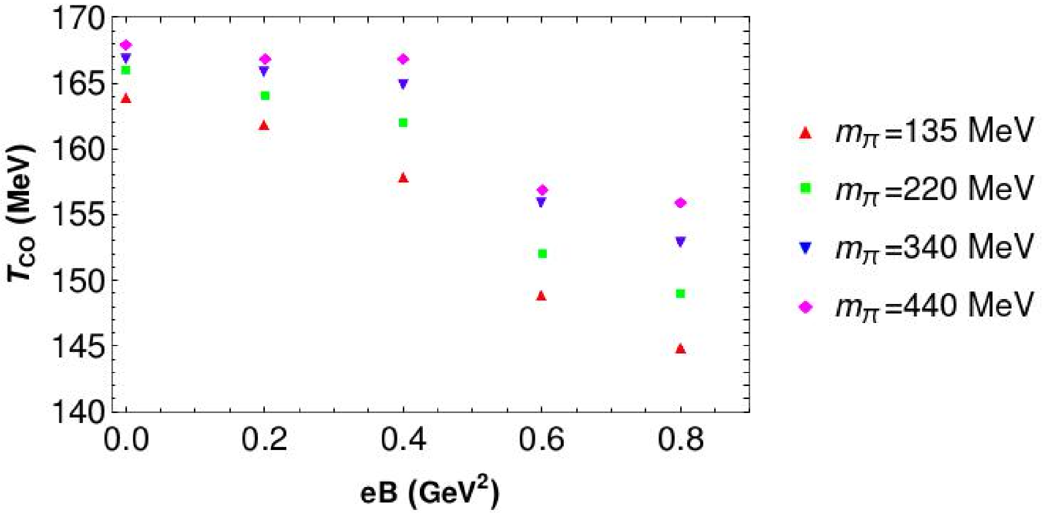


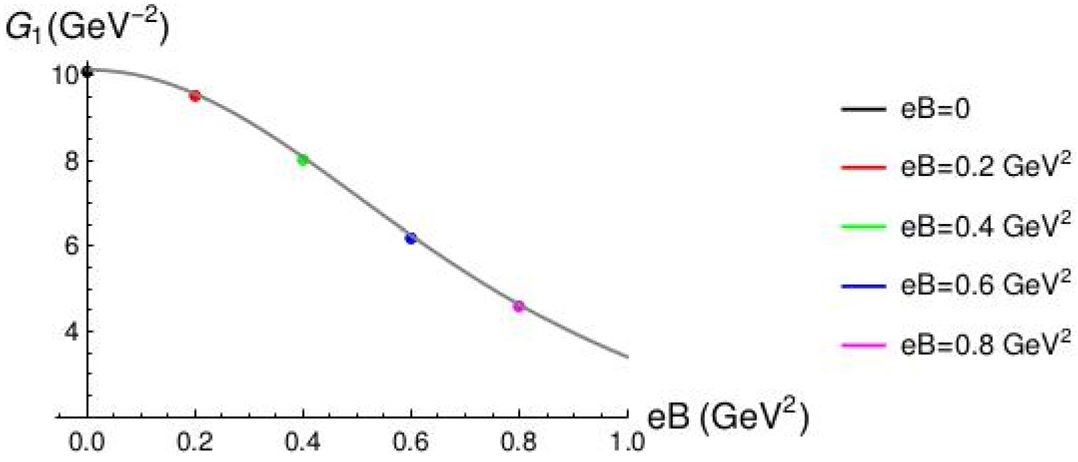
Local 2-flavour



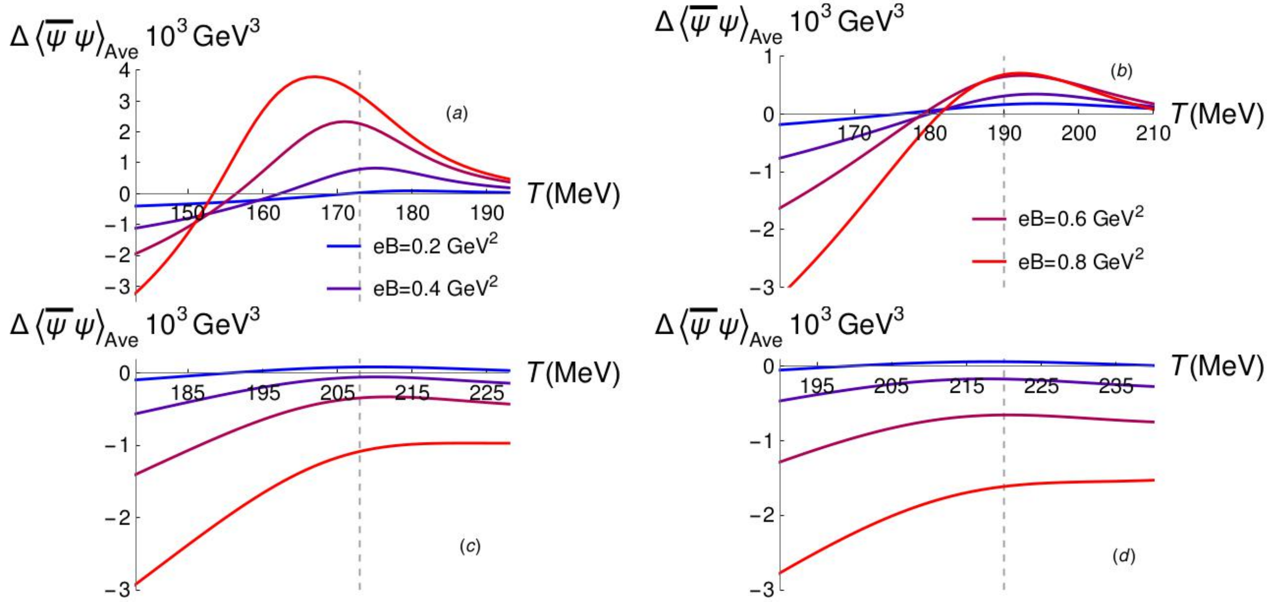


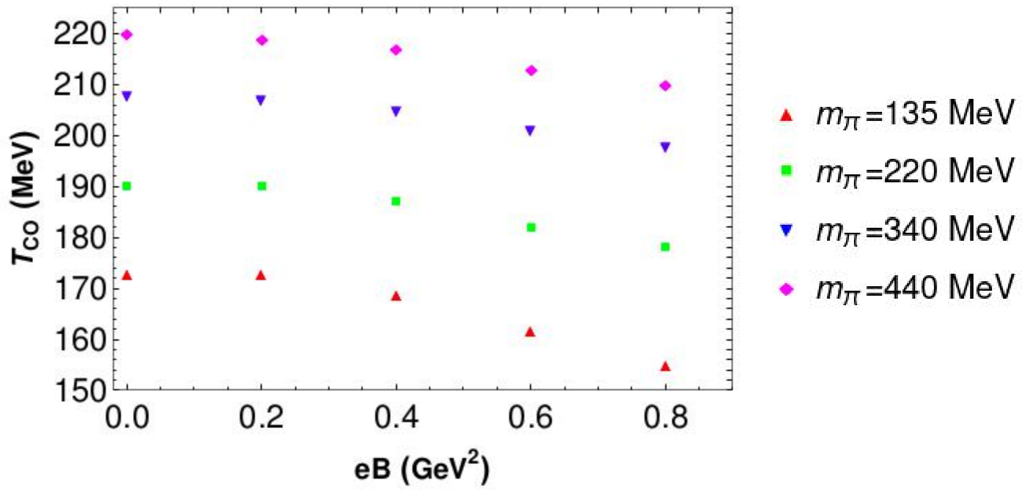
[CAI et al., arXiv: 2407:14449]

Behaviour of the coupling constants



Local (2+1)-flavour





[CAI et al., arXiv: 2407:14449]

Conclusion

- Lattice QCD shows that beyond certain unphysical pion mass the IMC effect disappears but the decreasing trend of the T_{CO} continues.

- We find out that the models are capable of capturing qualitatively the LQCD results for heavier (unphysical) pions.

- In this regard, the key feature in the models is the incorporation of the effect of a reduction in the coupling constant with increasing energy.

- For the local NJL model, this agreement depends on how the parameters of the model are fit at the physical point.

- The nonlocal version, on the other hand, captures the physics more naturally.....

Thank you

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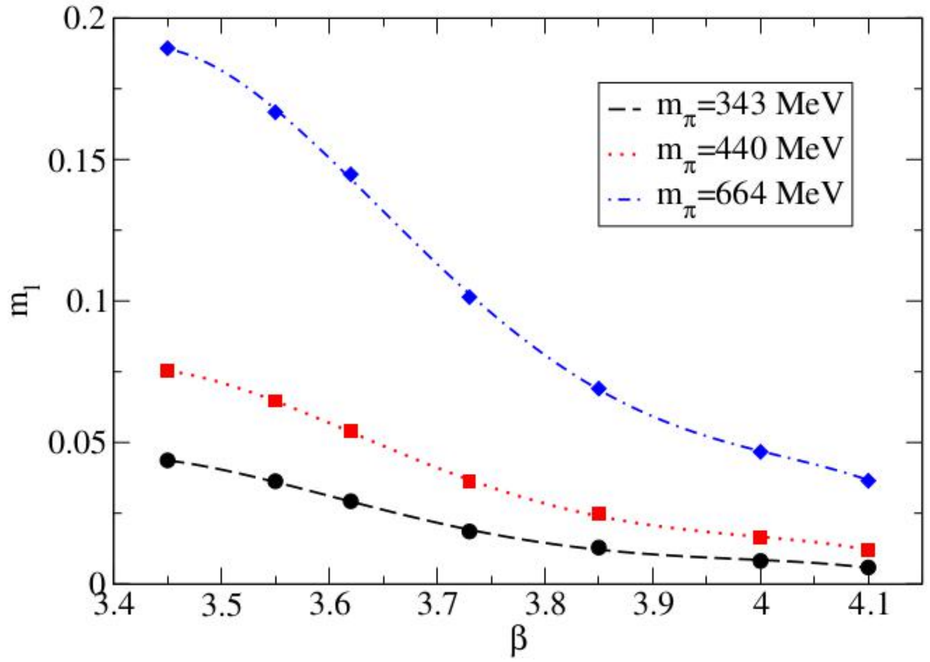


FIG. 1: Lines of constant pion mass in the $m_\ell - \beta$ plane.

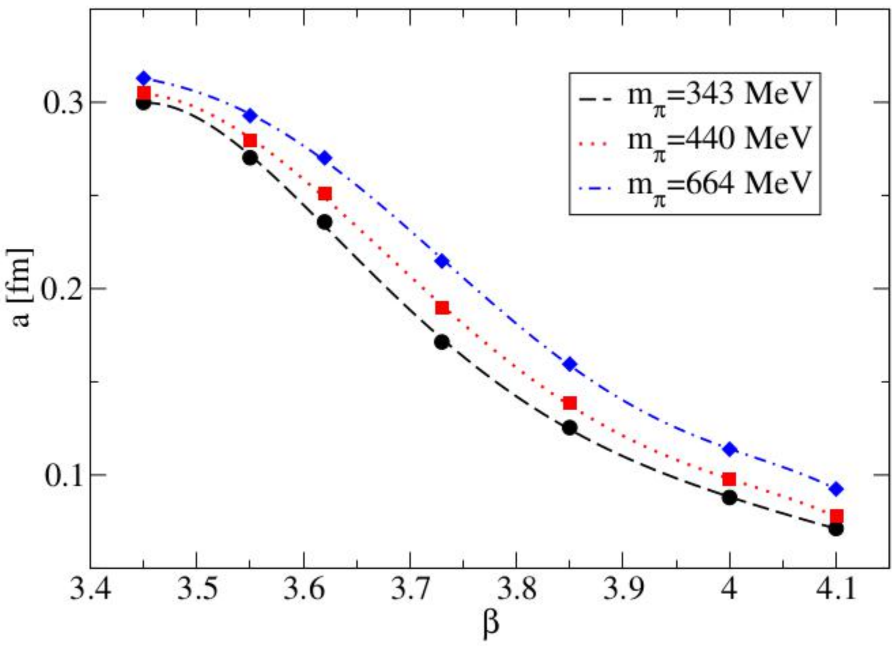


FIG. 2: Dependence of the lattice spacing a on β along the lines of constant pion mass.

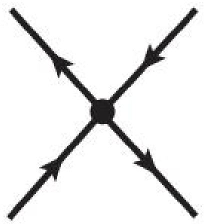
Condensate and f_π beyond physical pion mass

m_π (MeV)	m (MeV)	$\langle\bar{\psi}_i\psi_i\rangle^{1/3}$ (MeV)	f_π (MeV)
135	6.94	221.1	92.9
220	18.28	220.7	95.7
340	42.89	218.6	101.0
440	70.78	214.9	105.6

TABLE II. Parameter m for different pion masses, with Λ and G_0 kept fixed to their values at physical point ($\Lambda = 605.05$ MeV and $G_0 = 29.38/\Lambda^2$). The condensate and the pion decay constant are given for different pion masses.

$$\mathcal{L}_{\rm NJL} = \bar{\psi}(i\gamma_\mu \partial^\mu - m_0)\psi + \frac{G_S}{2}[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2]$$

The standard NJL Lagrangian



Local Four point interaction

Local version, the most popular one: advantages and disadvantages. Why do we need the non-local version?

Let's understand the basic working method of non-local version.

Let's first define a current and the corresponding action.

$$J_a^\mu = \bar{\psi}(x)\gamma^\mu \frac{\lambda_a}{2}\psi(x) \quad \longrightarrow \quad \mathcal{S}_{\rm int} = -\int d^4x \int d^4y J_a^\mu(x)\mathcal{G}_{\mu\nu}^{ab}(x-y)J_b^\nu(y)$$

Equivalent to gluonic field correlator

What should be the form of this correlator? And how to introduce it?

Non-local interactions are introduced in two alternative ways:

a) Instanton liquid picture: (Bowler and Birse, for example: NPA 582 (1995) 655-664 and others)

$$j_a(x) = \int d^4y d^4z r(y-x) r(x-z) \bar{\psi}(y) \Gamma_a \psi(z)$$

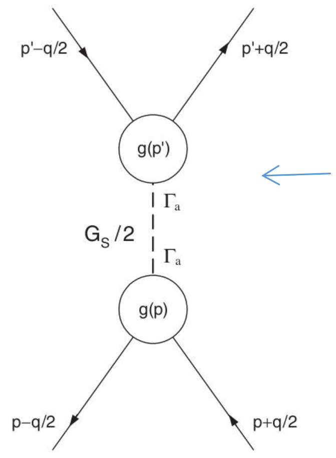
b) One gluon exchange: (Sebastian Schmidt et al, PRC 50 435 (1994))

$$j_a(x) = \int d^4z g(z) \bar{\psi}\left(x + \frac{z}{2}\right) \Gamma_a \psi\left(x - \frac{z}{2}\right)$$

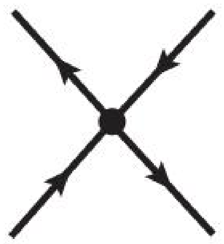
r(*x*-*y*) and *g*(*z*) are the non-local covariant form factors. Please see: N. N. Scoccola et al, PRD 74, 054026 (2006)

The major properties of these two methods are more or less the same. We will mainly focus on OGE method.

With the OGE method the non-local interaction looks like this:



← Difference →



Let's look at the action:

$$\frac{\mathcal{S}_{\mathrm{E}}^{\mathrm{MF}}}{V^{(4)}} = -4N_{\mathrm{c}} \int \frac{d^4 p}{(2\pi)^4} \ln[p^2 + M^2(p)] + \frac{\bar{\sigma}^2}{2G}$$

The constant mass is found from the gap equation:

$$M(p) = m_q + \mathcal{C}(p)\bar{\sigma}$$

with the mean field $\bar{\sigma} = \langle \sigma \rangle$

By the principle of least action we can get the mean field:

$$\bar{\sigma} = 8N_{\mathrm{c}}G \int \frac{d^4 p}{(2\pi)^4} \mathcal{C}(p) \frac{M(p)}{p^2 + M^2(p)}$$

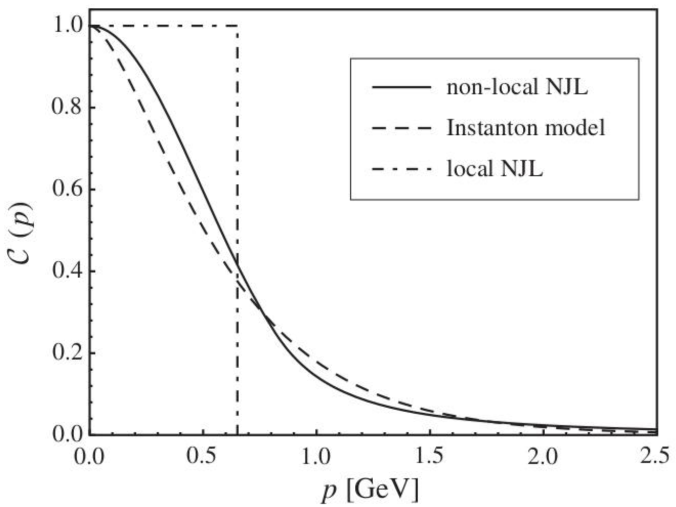
How to get the condensate:

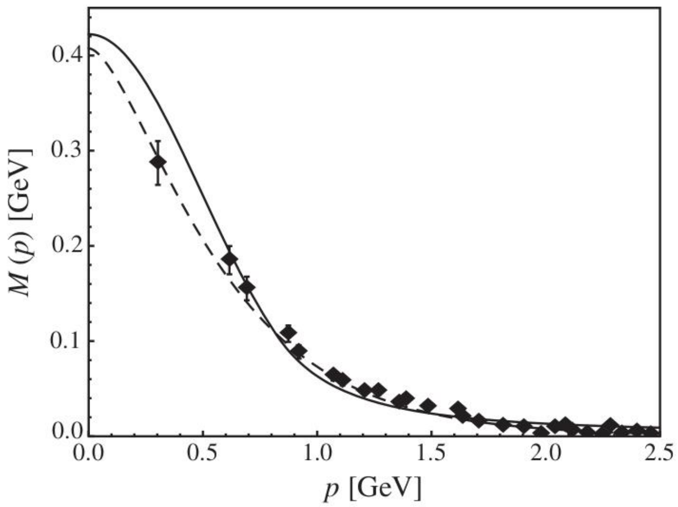
$$\langle \bar{\psi} \psi \rangle = -4N_{\mathrm{f}}N_{\mathrm{c}} \int \frac{d^4 p}{(2\pi)^4} \left[\frac{M(p)}{p^2 + M^2(p)} - \frac{m_q}{p^2 + m_q^2} \right]$$

Differences with the local version?

$$M = -G\langle \bar{\psi} \psi \rangle = \bar{\sigma}$$

A good article to understand the basic
T. Hell et al, PRD 79 014022 (2009)





Advantages and disadvantages

IMC appears: N. N. Scoccola et al PRD 95, 034013 (2017):

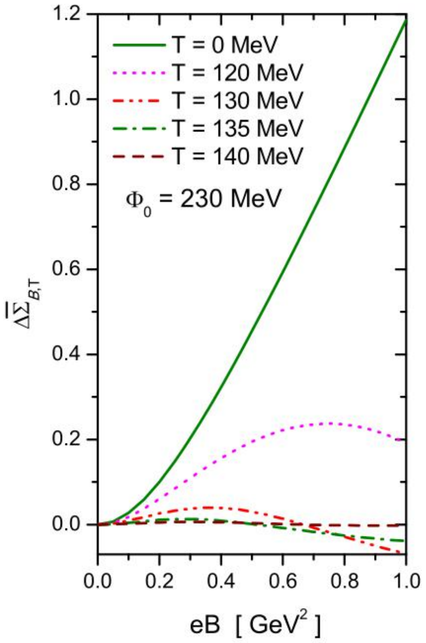
$$j_a(x) = \int d^4z g(z) \bar{\psi}\left(x + \frac{z}{2}\right)\Gamma_a \psi\left(x - \frac{z}{2}\right)$$

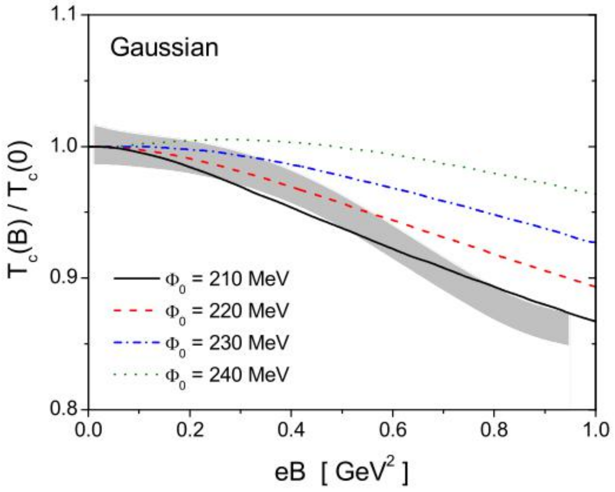
"In the case of the nonlocal model under consideration, the situation is more complicated since the inclusion of gauge interactions implies a change not only in the kinetic terms of the Lagrangian but also in the nonlocal currents."

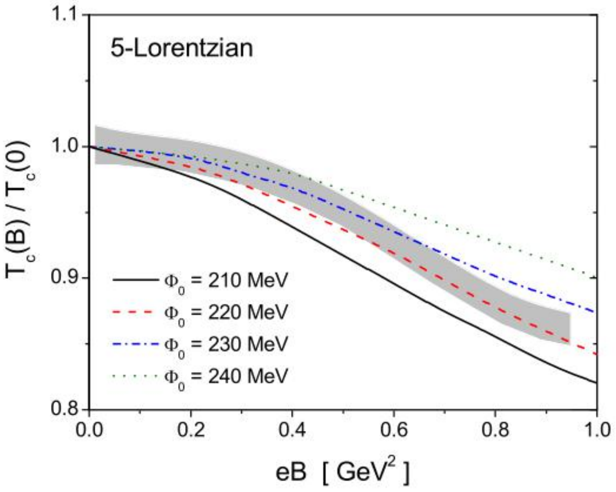
$$\psi(x - z/2) \to W(x, x - z/2)\psi(x - z/2)$$

Similarly for, $\overline{\psi}(x + z/2)$

$$W(s,t) = \mathrm{P}\, \exp\left[-i\hat{Q}\int_s^t dr_\mu \mathcal{A}_\mu(r)\right]$$







$$\langle \bar{\psi}_f(x)\psi_f(x)\rangle = \frac{\partial \Omega}{\partial m} = -\,4N_c \int \frac{d^4q}{(2\pi)^4} \left[\frac{M(q)}{q^2+M^2(q)} - \frac{m}{q^2+m^2}\right]$$

$$\mathcal{G}^{\pm}(p^2) = \frac{1}{G_0} - \,8\,N_c \int \frac{d^4q}{(2\pi)^4}\; h^2(q) \frac{[(q^+\cdot q^-) \mp M(q^+)M(q^-)]}{[(q^+)^2+M^2(q^+)]\,[(q^-)^2+M^2(q^-)]}$$

$$\mathcal{G}^{-}(-m_\pi^2) = 0$$

$$m_\pi^2\; F_\pi = m\; Z_\pi^{1/2}\; J(-m_\pi^2)$$

$$J(p^2) = \,8\,N_c \int \frac{d^4q}{(2\pi)^4}\; h(q) \frac{[(q^+\cdot q^-) + M(q^+)M(q^-)]}{[(q^+)^2+M^2(q^+)]\,[(q^-)^2+M^2(q^-)]}$$