Anomalous Hall instability in chiral magnetohydrodynamics

The 8th International Conference on Chirality, Vorticity, and Magnetic Field in Quantum Matter

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July 26, 2024

Outline

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Quark Gluon Plasma

The relativistic heavy ion collision experiments show that the quark gluon plasma (QGP) is in **strong magnetic fields** and **fluid vortical fields** .

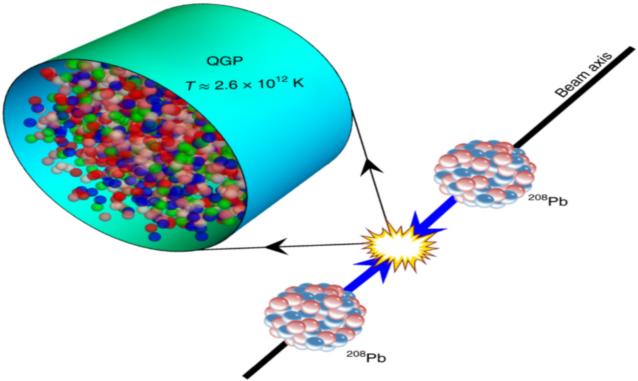

Figure 1: Schematic representation of a ^{208}Pb-^{208}Pb collision at the LHC.

Quark Gluon Plasma

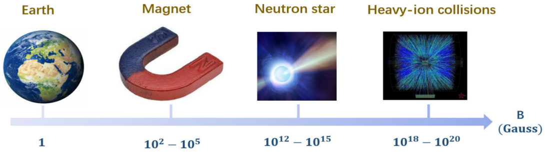

Figure 2: The magnetic field strength for different physical systems

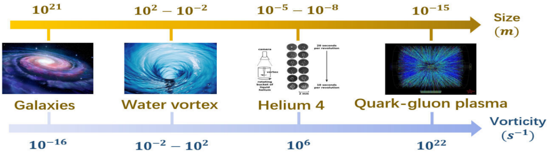

Figure 3: The vorticity in fluid systems at different scales

Magnetohydrodynamics

One can use-**Magnetohydrodynamics (MHD)** equations to describe a conducting fluid

$$\nabla\cdot\boldsymbol{B}=0,\qquad \nabla\times\boldsymbol{E}=-\partial_t\boldsymbol{B},\qquad \nabla\times\boldsymbol{B}=\mu_0(\boldsymbol{j}+\epsilon_0\partial_t\boldsymbol{E}) \tag{1}$$

$$\boldsymbol{j}=\sigma(\boldsymbol{E}+\boldsymbol{v}\times\boldsymbol{B}),\qquad \partial_t\rho+\nabla\cdot(\rho\boldsymbol{v})=0,\qquad P=P(\rho,T) \tag{2}$$

$$\rho[\partial_t\boldsymbol{v}+(\boldsymbol{v}\cdot\nabla)\boldsymbol{v}]=-\nabla P+\eta_s\nabla^2\boldsymbol{v}+(\zeta+\frac{\eta_s}{3})\nabla(\nabla\cdot\boldsymbol{v})+\boldsymbol{f} \tag{3}$$

$$\rho T(\partial_t s+\boldsymbol{v}\cdot\nabla s)=\sigma'_{ik}\frac{\partial v_i}{\partial x_k}+\nabla\cdot(\kappa\nabla T)+\frac{1}{\mu_0^2\sigma}(\nabla\times\boldsymbol{B})^2 \tag{4}$$

where $\boldsymbol{f}=\boldsymbol{j}\times\boldsymbol{B}+n\boldsymbol{E}$, when $\eta_s=\zeta=\kappa=0$,$\sigma=+\infty$, one obtains the **ideal-MHD** .

(η_s,ζ-shear,bulk viscosity, κ,σ-thermal,electrical conductivity, μ_0-vacuum magnetic permeability, ϵ_0-vaccum dielectric constant, ρ-mass density, s-entropy per unit mass, σ'_{ik}-viscous stress tensor, $\boldsymbol{f}$-external force density per unit volume).

Alfvén Wave

Considering ideal MHD, one obtains the known MHD waves - **Alfvén wave** in incompressible fluid by *Alfvén.H.(1942)*

$$\partial_t \boldsymbol{b} = (\boldsymbol{B}_0 \cdot \nabla)\boldsymbol{v}, \qquad \partial_t \boldsymbol{v} = \frac{1}{\mu_0 \rho_0}(\boldsymbol{B}_0 \cdot \nabla)\boldsymbol{b} \tag{5}$$

where $\boldsymbol{B}_0$-background field, $\boldsymbol{b}, \boldsymbol{v}$-perturbations, $\boldsymbol{B}_0 \perp \boldsymbol{b}(\boldsymbol{v})$, ρ_0-mass density, μ_0-vacuum magnetic permeability . For plane waves

$$\boldsymbol{v}, \boldsymbol{b} \sim exp[i(\boldsymbol{k} \cdot \boldsymbol{x} - \omega t)] \tag{6}$$

the dispersion relation is

$$\omega^2 - \frac{(\boldsymbol{B}_0 \cdot \boldsymbol{k})^2}{\mu_0 \rho_0} = 0 \Rightarrow \omega = \frac{\boldsymbol{B}_0 \cdot \boldsymbol{k}}{\sqrt{\mu_0 \rho_0}} \tag{7}$$

Chiral Anomaly

Generally, a symmetry of a classical theory is also a symmetry of the quantum theory. However, if it is not, we call the symmetry "Anomaly". Here we focus on the **Chiral Anomaly**

$$\partial_\mu j_5^\mu = -\frac{e^2}{16\pi^2}\epsilon^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta} \tag{8}$$

the axial current j_5^μ is not conserved for chiral fermions with $m = 0$. *"Adler,Bell and Jackiw(1969) or Fujikawa(1979)"* .
Anomalous phenomena, the chiral separation effect(CSE) and the chiral magnetic effect(CME) *"A.Vilenkin(1980); K.Fukushima et al.(2008), ..."*

$$\boldsymbol{j}_5 = \frac{\mu_V e^2}{2\pi^2}\boldsymbol{B} \qquad \boldsymbol{j}_V = \frac{\mu_5 e^2}{2\pi^2}\boldsymbol{B} \tag{9}$$

also the chiral vortical effect(CVE) are *"A.Vilenkin(1980), ..."*

$$\boldsymbol{j}_V = \frac{\mu_V\mu_5}{\pi^2}\boldsymbol{\omega} \qquad \boldsymbol{j}_5 = (\frac{T^2}{6} + \frac{\mu_V^2+\mu_5^2}{2\pi^2})\boldsymbol{\omega} \tag{10}$$

Chiral Hydrodynamics

The **Chiral Hydrodynamics (CHD)** was pioneered by *"D.T.Son and P.Surowka(2009)"*. Considering the CHD for right-handed chiral fermions in external electromagnetic fields

$$\partial_\mu T^{\mu\nu} = F^{\nu\lambda} j_\lambda \tag{11}$$

$$\partial_\mu j^\mu = C E^\mu B_\mu \tag{12}$$

here $T^{\mu\nu}$ is the energy-momentum tensor, j^μ is right-handed current, $E^\mu = F^{\mu\nu}u_\nu, B^\mu = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}u_\nu F_{\alpha\beta}$ are defined in the fluid rest frame, C is anomaly coefficient. In the Landau-Lifshitz frame *(1959)*

$$T^{\mu\nu} = (\epsilon + P)u^\mu u^\nu - P g^{\mu\nu} + \tau^{\mu\nu} \tag{13}$$

$$j^\mu = n u^\mu + \nu^\mu \tag{14}$$

with dissipative terms $\tau^{\mu\nu}u_\nu = \nu^\mu u_\mu = 0$. We start from relativistic hydrodynamics because the fluid's velocity is significantly large.

Chiral Hydrodynamics

And the entropy current analysis

$$s^\mu = su^\mu + s_1^\mu \tag{15}$$

thus the 2nd law of thermodynamics tells us $\partial_\mu s^\mu \geq 0$

$$\partial_\mu s^\mu = \frac{\tau^{\mu\nu}\partial_\mu u_\nu}{T} - \nu^\mu[\partial_\mu(\frac{\mu}{T}) + \frac{E_\mu}{T}] - \boldsymbol{\frac{\mu}{T} C E^\mu B_\mu} + \partial_\mu[\frac{\mu}{T}\nu^\mu + s_1^\mu] \geq 0 \tag{16}$$

with projection operator-$\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$, so

$$\nu^\mu = -T\sigma\Delta^{\mu\nu}\partial_\nu(\frac{\mu}{T}) + \sigma E^\mu + \boldsymbol{\xi_\omega \omega^\mu} + \boldsymbol{\xi_B B^\mu} \tag{17}$$

two new transport coefficients: ξ_ω-CVE and ξ_B-CME.

Chiral Alfvén wave

The Chiral hydrodynamics eqs. for plasmas of single right-handed chiral fermions in external electromagnetic fields are [1]

$$\partial_\mu T^{\mu\nu} = F^{\nu\lambda} j_\lambda, \qquad \partial_\mu j^\mu = -C E^\mu B_\mu \tag{18}$$

In the Landau-Lifshitz frame [1, 2], taking $|\boldsymbol{v}| \ll 1$, assuming plasmas with homogeneous and static ϵ, P, n, high temperature $T \gg \mu$(and so n)[2]

$$(\epsilon + P)\partial_t \boldsymbol{v} = \xi_\omega \boldsymbol{\omega} \times \boldsymbol{B}, \qquad \boldsymbol{\nabla} \cdot \boldsymbol{v} = 0 \tag{19}$$

here $\xi_\omega \boldsymbol{\omega}$ is the chiral vortical effet(CVE).By setting $\boldsymbol{B} \cdot \boldsymbol{v} = 0$, $\boldsymbol{B} = B\hat{z}$. The plane wave solution of Right-CAW. *N.Yamamoto(2015)*

$$\omega = -\frac{\xi_\omega B}{\epsilon + P} k_z \qquad (\xi_\omega \approx \frac{DT^2}{2}, D = \frac{1}{12}) \tag{20}$$

[1] D.T.Son and P.Surowka,PRL.103,191601(2009)

[2] N.Yamamoto,PRL.115,141601(2015)

Alfvén Wave and CAW

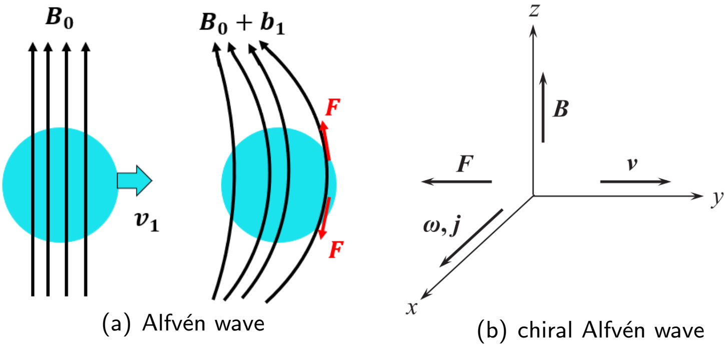

Figure 4: the intuitive pictures

(a):the magnetic tension(restoring force)balances inertia,so disturbances are communicated by Alfvén waves.(b):the CVE induces a Lorentz force, where $\boldsymbol{v} = v(z)\hat{\boldsymbol{y}}$ with $\partial_z v(z) < 0$.

Outline

Chiral MHD

The CHD with the dynamical electromagnetic field

$$\partial_\mu T^{\mu\nu} = F^{\nu\lambda} j_\lambda^V, \quad \partial_\mu j_5^\mu = C E^\mu B_\mu, \quad \partial_\mu j_V^\mu = 0 \tag{21}$$

$$\partial_\mu F^{\mu\nu} = j_V^\nu, \quad \partial_\mu \tilde{F}^{\mu\nu} = 0 \tag{22}$$

where $\tilde{F}^{\mu\nu}$ is the dual of $F^{\mu\nu}$, j_V^μ, j_5^μ are the vector, axial current. Considering the nonrelativistic limit $|\boldsymbol{v}| \ll 1$

$$\begin{aligned}(\epsilon + P)(\partial_t + \boldsymbol{v}\cdot\boldsymbol{\nabla})\boldsymbol{v} &= -\boldsymbol{\nabla} P + n\boldsymbol{E} + \boldsymbol{j}\times\boldsymbol{B} \\ \boldsymbol{j} &= n\boldsymbol{v} + \sigma(\boldsymbol{E} + \boldsymbol{v}\times\boldsymbol{B}) + \xi_B \boldsymbol{B} + \xi_\omega \boldsymbol{\omega} \\ \boldsymbol{\nabla}\cdot\boldsymbol{E} = n, \quad \boldsymbol{\nabla}\times\boldsymbol{B} &= \partial_t\boldsymbol{E} + \boldsymbol{j}, \quad \boldsymbol{\nabla}\times\boldsymbol{E} = -\partial_t\boldsymbol{B}, \quad \boldsymbol{\nabla}\cdot\boldsymbol{B} = 0\end{aligned} \tag{23}$$

In above chiral MHD equations, we ignore the terms $n\boldsymbol{E}, \partial_t\boldsymbol{E}, n\boldsymbol{v}$ and keep our calculations in $\boldsymbol{v} \sim \mathcal{O}(\delta)$ order.

Chiral MHD

perturbation $\boldsymbol{B} = \boldsymbol{B}_0 + \boldsymbol{b} = B_0\hat{\boldsymbol{z}} + \boldsymbol{b}$ ($\boldsymbol{b} \sim \mathcal{O}(\delta)$),by setting $\boldsymbol{B}_0 \cdot \boldsymbol{b} = 0$, also $\boldsymbol{B}_0 \cdot \boldsymbol{v} = 0$

$$(\epsilon + P)\partial_t \boldsymbol{v} = (\boldsymbol{B}_0 \cdot \boldsymbol{\nabla})\boldsymbol{b} - \boldsymbol{\nabla}(\boldsymbol{B}_0 \cdot \boldsymbol{b}) \quad (24)$$

$$\partial_t \boldsymbol{b} = (\boldsymbol{B}_0 \cdot \boldsymbol{\nabla})\boldsymbol{v} + \eta\boldsymbol{\nabla}^2\boldsymbol{b} + \eta\xi_B\boldsymbol{\nabla} \times \boldsymbol{b} - \eta\xi_\omega\boldsymbol{\nabla}^2\boldsymbol{v} \quad (25)$$

here $\eta = 1/\sigma$ is resistivity. For plane-wave

$$\boldsymbol{v}, \boldsymbol{b} \sim e^{i(\boldsymbol{k}\cdot\boldsymbol{x} - \omega t)} \quad (26)$$

then one obtains

$$(\epsilon + P)(-\omega)\boldsymbol{v} = (\boldsymbol{B}_0 \cdot \boldsymbol{k})\boldsymbol{b} \quad (27)$$

$$(\eta\boldsymbol{k}^2 - i\omega)\boldsymbol{b} - i\eta\xi_B\boldsymbol{k} \times \boldsymbol{b} = [i(\boldsymbol{B}_0 \cdot \boldsymbol{k}) + \eta\xi_\omega\boldsymbol{k}^2]\boldsymbol{v} \quad (28)$$

Chiral Plasma Instability

we focus on the CME (setting $\xi_\omega = 0$)

$$\omega^2 + i\eta(k_z^2 - \lambda\xi_B|k_z|)\omega - \frac{B_0^2 k_z^2}{\epsilon + P} = 0 \qquad (\lambda = \pm 1) \tag{29}$$

whose solutions are

$$\omega = \frac{1}{2}[-i\eta k_3(k_3 - \lambda\xi_B) \pm |k_3|\sqrt{-\eta^2(k_3 - \lambda\xi_B)^2 + \frac{4B_0^2}{\epsilon + P}}] \tag{30}$$

The instability appears in a finite intervals $|k_3| \in (0, \xi_B)$- **Chiral Plasma Instability (CPI)** [3].

[3] Y.Akamatsu and N.Yamamoto,PRL.111,052002(2013)

Chiral Magnetovortical Instability

we focus on the CVE (setting $\xi_B = 0$)

$$\omega^2 + i\eta \boldsymbol{k}^2\omega + i\eta\xi_\omega \boldsymbol{k}^2 \frac{\boldsymbol{B}_0 \cdot \boldsymbol{k}}{\epsilon + P} - \frac{(\boldsymbol{B}_0 \cdot \boldsymbol{k})^2}{\epsilon + P} = 0 \tag{31}$$

whose solutions are (small k limit)

$$\omega = \frac{1}{2}[-i\eta \boldsymbol{k}^2 \pm \sqrt{\frac{4(\boldsymbol{B}_0 \cdot \boldsymbol{k})^2}{\epsilon + P} - \eta^2 \boldsymbol{k}^4 - i \cdot \eta\xi_\omega \boldsymbol{k}^2 \frac{4(\boldsymbol{B}_0 \cdot \boldsymbol{k})}{\epsilon + P}}] \tag{32}$$

$$\approx \pm \underbrace{\frac{(\boldsymbol{B}_0 \cdot \boldsymbol{k})}{\sqrt{\epsilon + P}}}_{Alfven\ \ wave} - i\frac{\eta}{2}(1 \pm \frac{\xi_\omega}{\sqrt{\epsilon + P}})\boldsymbol{k}^2 \tag{33}$$

the instability occurs once $\xi_\omega > \sqrt{\epsilon + P}$ and applicable to any $|\boldsymbol{k}|$. Because the CVE and magnetic field are both included, we call this instability **Chiral Magnetovortical Instability (CMVI)**[4].

[4] S.Wang and XG.Huang,PRD.109,L121302(2024)

CPI and CMVI

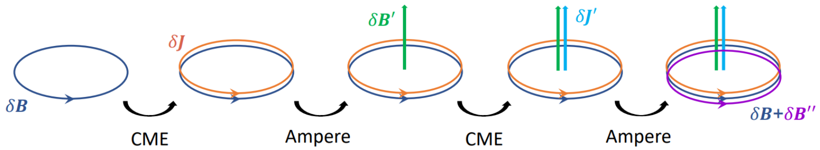

Figure 5: Intuitive picture of the CPI.

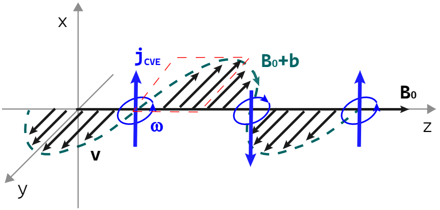

Figure 6: Intuitive picture of the CMVI[4]

Outline

Maxwell-Chern-Simons Electrodynamics

We start from the Maxwell-Chern-Simons(MCS) equations [5].

$$\nabla \cdot \boldsymbol{E} = n - \boldsymbol{\xi} \cdot \boldsymbol{B} \quad (34)$$

$$\nabla \cdot \boldsymbol{B} = 0 \quad (35)$$

$$\nabla \times \boldsymbol{E} = -\partial_t \boldsymbol{B} \quad (36)$$

$$\nabla \times \boldsymbol{B} = \partial_t \boldsymbol{E} + \boldsymbol{j} + \xi_B \boldsymbol{B} + \boldsymbol{\xi} \times \boldsymbol{E} \quad (37)$$

where $n, \boldsymbol{j}$ are source charge and source current without any anomalous effects, ξ_B is the CME coefficient, $\boldsymbol{\xi}$ is the anomalous Hall coefficient. One can introduce the abbreviation $\xi^\mu = (\xi_B, \boldsymbol{\xi})$ and the above equations also describes the axion electrodynamics which have an axion field term $\theta(t, \boldsymbol{x})$

$$\xi_B = \frac{\alpha}{\pi}\mu_5(t, \boldsymbol{x}) = \frac{\alpha}{\pi}\partial_t \theta(t, \boldsymbol{x}) \quad (38)$$

$$\boldsymbol{\xi}(t, \boldsymbol{x}) = \frac{\alpha}{\pi}\nabla\theta(t, \boldsymbol{x}) \quad (39)$$

[5] Z.Qiu,G.Cao and XG.Huang, PRD.95,036002(2017)

Maxwell-Chern-Simons Electrodynamics

Because the CME leads to the CPI, we just need to check the anomalous Hall term. For sourceless MCS equations $n = 0, \boldsymbol{j} = 0$ and one can combine Faraday and Ampere laws to get

$$\nabla \times (\partial_t \boldsymbol{B}) = \partial_t^2 \boldsymbol{E} + \boldsymbol{\xi} \times (\partial_t \boldsymbol{E}) = -\nabla \times (\nabla \times \boldsymbol{E}) \tag{40}$$

where we assume the four-vector ξ^μ to be constant, then the plane wave solutions are

$$\omega^2 = k^2 + \frac{\xi^2}{2} \pm \sqrt{\frac{\xi^4}{4} + (\boldsymbol{\xi} \cdot \boldsymbol{k})^2} \tag{41}$$

here $k = |\boldsymbol{k}|, \xi = |\boldsymbol{\xi}|$. Without loss of generality, we set $\boldsymbol{\xi}$ to be along the $\hat{z}$ direction, so [5]

$$\omega^2 = k_1^2 + k_2^2 + (\sqrt{k_3^2 + \frac{\xi^2}{4}} \pm \frac{\xi}{2})^2 > 0 \tag{42}$$

so there is no instability due to the AHE in a pure chiral electrodynamics.

Anomalous Hall Effect in CMHD

Like the CMVI which occurs when $\boldsymbol{v}$ interact with $\boldsymbol{B}$, we can introduce the hydro parts, thus

$$(\epsilon + P)(\partial_t + \boldsymbol{v} \cdot \nabla)\boldsymbol{v} = -\nabla P + n\boldsymbol{E} + \boldsymbol{j} \times \boldsymbol{B} \tag{43}$$

$$\nabla \cdot \boldsymbol{E} = n - \boldsymbol{\xi} \cdot \boldsymbol{B} \tag{44}$$

$$\nabla \cdot \boldsymbol{B} = 0 \tag{45}$$

$$\nabla \times \boldsymbol{E} = -\partial_t \boldsymbol{B} \tag{46}$$

$$\nabla \times \boldsymbol{B} = \partial_t \boldsymbol{E} + \boldsymbol{j} \tag{47}$$

$$\boldsymbol{j} = n\boldsymbol{v} + \sigma(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}) + \xi_B \boldsymbol{B} + \boldsymbol{\xi} \times \boldsymbol{E} \tag{48}$$

Of course, there is the CME coefficient $\xi_B = \frac{\alpha}{\pi}\mu_5$, however, there is no axial current. In contrast to the chiral MHD, one can name these equations as: Chern-Simons MHD(CSMHD).[6]

[6] M.Kiamari, M.Rahbardar, M.Shokri and N.Sadooghi, PRD.104,076023(2021)

Anomalous Hall Effect in CMHD

One can usually eliminate $\partial_t \boldsymbol{E}$ because the MHD is effective for low frequence waves. Also taking $n = 0$ as charge neutrality assumption. Now we focus on the fluid and the magnetic field, they are governed by

$$(\epsilon + P)(\partial_t + \boldsymbol{v} \cdot \boldsymbol{\nabla})\boldsymbol{v} = -\boldsymbol{\nabla} P + (\boldsymbol{\nabla} \times \boldsymbol{B}) \times \boldsymbol{B} \tag{49}$$

$$\sigma \partial_t \boldsymbol{B} = \sigma \boldsymbol{\nabla} \times (\boldsymbol{v} \times \boldsymbol{B}) + \boldsymbol{\nabla}^2 \boldsymbol{B} + \xi_B \boldsymbol{\nabla} \times \boldsymbol{B} + \boldsymbol{\nabla} \times (\boldsymbol{\xi} \times \boldsymbol{E}) \tag{50}$$

the last term means $\boldsymbol{\nabla} \times (\boldsymbol{\xi} \times \boldsymbol{E}) = \boldsymbol{\xi}(\boldsymbol{\nabla} \cdot \boldsymbol{E}) - (\boldsymbol{\xi} \cdot \boldsymbol{\nabla})\boldsymbol{E}$, one can take curl to replace $\boldsymbol{E}$ with $\boldsymbol{B}$, then

$$(\epsilon + P)\partial_t \boldsymbol{v} = (\boldsymbol{B}_0 \cdot \boldsymbol{\nabla})\boldsymbol{b} \tag{51}$$

$$\begin{aligned} \sigma \partial_t \boldsymbol{\nabla} \times \boldsymbol{b} = {} & \sigma(\boldsymbol{B}_0 \cdot \boldsymbol{\nabla})\boldsymbol{\nabla} \times \boldsymbol{v} + \boldsymbol{\nabla}^2 \boldsymbol{\nabla} \times \boldsymbol{b} \\ & - \xi_B \boldsymbol{\nabla}^2 \boldsymbol{b} - \boldsymbol{\nabla}(\boldsymbol{\xi} \cdot \boldsymbol{b}) \times \boldsymbol{\xi} + (\boldsymbol{\xi} \cdot \boldsymbol{\nabla})\partial_t \boldsymbol{b} \end{aligned} \tag{52}$$

where $\boldsymbol{\nabla} \cdot \boldsymbol{v} = 0$, $\boldsymbol{B} = \boldsymbol{B}_0 + \boldsymbol{b}$, $\boldsymbol{B}_0 \cdot \boldsymbol{v} = \boldsymbol{B}_0 \cdot \boldsymbol{b} = 0$. Without loss of generality, the constant $\boldsymbol{B}_0$ and $\boldsymbol{\xi}$ are taken in z direction $\boldsymbol{B}_0 = (0, 0, B_0), \boldsymbol{\xi} = (0, 0, \xi)$ with $B_0, \xi > 0$, this implies $\boldsymbol{\xi} \cdot \boldsymbol{b} = 0$.

Anomalous Hall Effect in CMHD

The dispersion relations for plane wave

$$[ik\sigma - \lambda(\boldsymbol{\xi}\cdot\boldsymbol{k})]\omega^2 - k^3\omega - ik\sigma\frac{(\boldsymbol{B}_0\cdot\boldsymbol{k})^2}{\epsilon + P} = 0 \qquad (\lambda = \pm 1) \tag{53}$$

If we turn off the anomalous Hall term, eg: $\boldsymbol{\xi} = 0$ or the term $\boldsymbol{\xi}\cdot\boldsymbol{k} = 0$, the dispersion relation will reduce to the famous Alfvén waves which are damped out finally due to the finite electric resistivity $1/\sigma$ in a conventional MHD. We show the solutions first

$$\omega = \frac{k^2 \pm k\sqrt{k^2 - 4\sigma^2(\boldsymbol{B}_0\cdot\hat{\boldsymbol{k}})^2/(\epsilon+P) - i\cdot 4\sigma(\boldsymbol{B}_0\cdot\hat{\boldsymbol{k}})^2\lambda(\boldsymbol{\xi}\cdot\hat{\boldsymbol{k}})/(\epsilon+P)}}{2[i\sigma - \lambda(\boldsymbol{\xi}\cdot\hat{\boldsymbol{k}})]} \tag{54}$$

here $\hat{\boldsymbol{k}} = \boldsymbol{k}/k$. The anomalous Hall effect modifies the classical Alfvén wave's property dramatically.

Anomalous Hall Effect in CMHD

The wave modes are damped out in large k region, however we find a new instability in small k region. Concretely

$$\sqrt{\{...\}} \approx 2\sqrt{\frac{(\boldsymbol{B}_0 \cdot \hat{\boldsymbol{k}})^2}{\epsilon + P} i\sigma[i\sigma - \lambda(\boldsymbol{\xi} \cdot \hat{\boldsymbol{k}})]} \cdot \{1 + \mathcal{O}(k^2)\} \tag{55}$$

$$\approx 2\sqrt{\frac{(\boldsymbol{B}_0 \cdot \hat{\boldsymbol{k}})^2}{\epsilon + P}} \sigma\sqrt{-1} \cdot [1 + \frac{i\lambda(\boldsymbol{\xi} \cdot \hat{\boldsymbol{k}})}{2\sigma} + \mathcal{O}(\frac{1}{\sigma^2})] \tag{56}$$

with $k \ll \sigma B_0/\sqrt{\epsilon + P}, \sqrt{\sigma\xi} B_0/\sqrt{\epsilon + P}, \xi \ll \sigma$. Then

$$\omega_{\pm} \approx \pm u_A k - i\frac{\eta k}{2}[k \pm u_A \lambda(\boldsymbol{\xi} \cdot \hat{\boldsymbol{k}})] \qquad (u_A = \sqrt{\frac{(\boldsymbol{B}_0 \cdot \hat{\boldsymbol{k}})^2}{\epsilon + P}}) \tag{57}$$

taking $\lambda = 1, \boldsymbol{\xi} \cdot \hat{\boldsymbol{k}} > 0$ as an example

$$Im(\omega_-) > 0 \Rightarrow \quad 0 < k < u_A(\boldsymbol{\xi} \cdot \hat{\boldsymbol{k}}) \tag{58}$$

so the anomalous Hall effect causes a **AHI** in chiral MHD.

Outline

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   - Quark Gluon Plasma
   - Magnetohydrodynamics
   - Chiral Hydrodynamics
2. Instabilities
   - Chiral MHD
   - Chiral Plasma Instability
   - Chiral Magnetovortical Instability
3. Anomalous Hall Instability
   - Maxwell-Chern-Simons
   - Anomalous Hall Effect in CMHD
4. Summary and Outlook

Summary and Outlook

1.CPI and CMVI

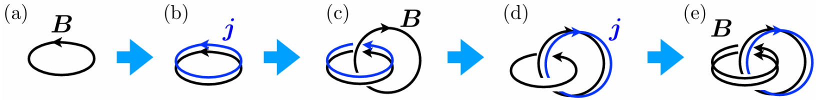

Figure 7: CPI [7]

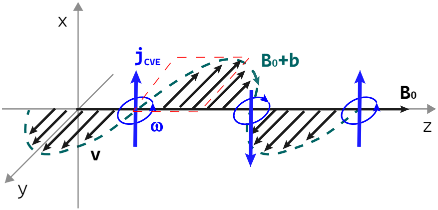

Figure 8: CMVI [4]

[7] N.Yamamoto and R.Yokokura,JHEP07(2023)045

Summary and Outlook

2. One can compare the CMVI with the AHI

$$\omega = \pm\frac{(\boldsymbol{B}_0\cdot\boldsymbol{k})}{\sqrt{\epsilon+P}} - i\frac{\eta}{2}(1\pm\frac{\xi_\omega}{\sqrt{\epsilon+P}})\boldsymbol{k}^2 \tag{59}$$

$$\omega_\pm \approx \pm u_A k - i\frac{\eta k}{2}[k \pm u_A\lambda(\boldsymbol{\xi}\cdot\hat{\boldsymbol{k}})] \qquad (u_A = \sqrt{\frac{(\boldsymbol{B}_0\cdot\hat{\boldsymbol{k}})^2}{\epsilon+P}}) \tag{60}$$

both of them modify the classical Alfvén wave and the instability arises from the **mutual evolution** of the magnetic and fluid fields by anomalous currents. They are "**Alfvénic type's instability**"

3. Some articles have relevant results of the anomalous Hall instability. eg: M.Kiamari,etc.PRD.104,076023(2021)-$\partial_t \boldsymbol{E}$,CSMHD; N.Yamamoto,etc.JHEP07(2023)045-no fluid.

Summary and Outlook

4. We will try to explain this AHI by a simple physical picture like the CPI/CMVI.
5.The instability-AHI can not last forever, we will use axion field $\theta(t, \boldsymbol{x})$-axion electrodynamics to analyze the fate of it. Obviously, $\mu_5 = \partial_t \theta, \boldsymbol{\xi} = \frac{\alpha}{\pi} \boldsymbol{\nabla} \theta$ will decrease when the AHI happens. Like the CPI

$$\partial_t n_5 = C \boldsymbol{E} \cdot \boldsymbol{B} \Rightarrow \chi \partial_t \mu_5 = \frac{C}{V} \int d^3x \boldsymbol{E} \cdot \boldsymbol{B} = -\frac{C}{2} \partial_t \mathcal{H}(t) \qquad (61)$$

where $\mathcal{H}$ is the magnetic helicity. The CPI once happens, however, the fermionic helicity will deplete.

Thank you for your attention!