Conductivities of CME, CSE and QHE as topological invariants

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What is non – dissipative transport?

(CME,CSE,CVE,QHE,AQHE, …)

Appearance of current (electric, axial, energy) that flows without dissipation.

The conductivities of all known non – dissipative transport phenomena are given by topological invariants.

Plan

1. Our tools: Wigner – Weyl calculus in field theory

- Continuum theory

- Lattice theory ("approximate" version)

- Lettice theory ("precise" version)

2. Applications to quantum Hall effect.

- Topological expression for the QHE conductivity through Green function

- In the presence of inhomogeneities

- non – renormalization by interactions (perturbatively)

3. Applications to Chiral Magnetic Effect (CME)

- No CME in equilibrium (even at finite T and for non – homogeneous systems)

- CME is back out of equilibrium: chiral chemical potential depending on time

- CME contribution to magnetoconductivity: renormalization of the coefficient

Plan

4. Chiral Separation Effect (CSE).

- *Topological expression for chiral separation effect (CSE)*
- *Non – renormalization of the CSE by interactions*
- *Proposal for the experimental observation in magnetic Weyl semimetals*

5. Precise Wigner – Weyl calculus.

- *Infinite rectangular lattice*
- *Finite rectangular lattice*
- *Honeycomb lattice*

Wigner – Weyl calculus in continuum theory
Equilibrium, T=0

model with fermions

$$Z = \int D\bar{\psi} D\psi \; e^{S[\psi,\bar{\psi}]}$$

typical action

$$S[\bar{\psi},\psi] = \int d^4x \bar{\psi}(x) \hat{Q}(\partial_x) \psi(x)$$

$$\hat{Q}(\partial_x) = i\gamma^\mu \partial_\mu - M$$

Green function

$$(i\gamma_\mu \partial_x^\mu - m) G(x-y) = \delta(x-y)$$

Wigner – Weyl calculus in continuum theory

Weyl symbol of operator

$$A_W(x,p) \equiv \int_{-\infty}^{\infty} dy e^{-ipy} \langle x + \frac{y}{2} | \hat{A} | x - \frac{y}{2} \rangle = \int_{-\infty}^{\infty} dq e^{iqx} \langle p + \frac{q}{2} | \hat{A} | p - \frac{q}{2} \rangle$$

Wigner – Weyl calculus in continuum theory

Moyal product

$$A_W(x,p) \star B_W(x,p) = A_W(x,p) e^{\overleftrightarrow{\Delta}} B_W(x,p)$$

$$\overleftrightarrow{\Delta} \equiv \frac{i}{2} \left(\overleftarrow{\partial}_x \overrightarrow{\partial}_p - \overleftarrow{\partial}_p \overrightarrow{\partial}_x \right)$$

Weyl symbol of the product of two operators

$$\boxed{(AB)_W(x,p) \equiv A_W(x,p) \star B_W(x,p)}$$

Wigner – Weyl calculus in continuum theory

model with fermions

$$Z = \int D\bar{\psi} D\psi \ e^{S[\psi,\bar{\psi}]}$$

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Green function

$$(i\gamma_\mu \partial_x^\mu - m)G(x-y) = \delta(x-y)$$

Groenewold equation

$$\boxed{(\hat{Q}\hat{G})_W = Q_W \star G_W = 1}$$

Mathematical tools

Lattice models

Example of Wilson fermions

In the presence of gauge field

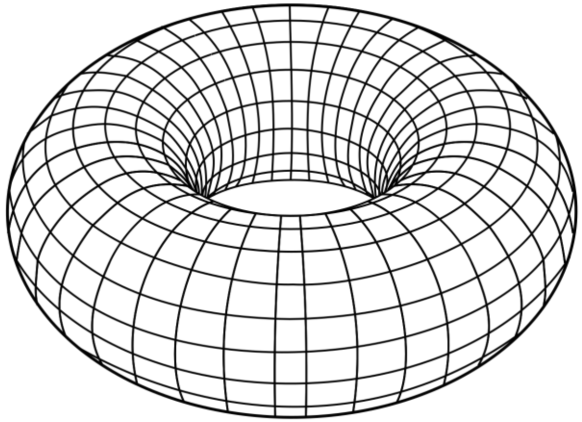
$$S_F^{(W)} = \sum_{\substack{n,m \\ \alpha,\beta}} \hat{\bar{\psi}}_\alpha(n) D^{(W)}_{\alpha\beta}(n,m) \hat{\psi}_\beta(n)$$

$$D_{\mathbf{x},\mathbf{y}} = -\frac{1}{2}\sum_i \left[(1+\gamma^i)\delta_{\mathbf{x}+\mathbf{e}_i,\mathbf{y}} + (1-\gamma^i)\delta_{\mathbf{x}-\mathbf{e}_i,\mathbf{y}}\right] U_{\mathbf{x},\mathbf{y}} + (m^{(0)}+4)\delta_{\mathbf{x},\mathbf{y}}$$

$$U_{x,y} = P e^{i\int_x^y d\boldsymbol{\xi} A(\boldsymbol{\xi})}$$

<u>Approximate</u> Wigner – Weyl calculus for the lattice models

Mathematical tools



Weyl symbol of operator (momentum space)

$$[\hat{A}]_W(x_n,p) = \int_{\mathcal{M}} dq e^{iqx_n} \langle p + \frac{q}{2}|\, \hat{A}\, |p - \frac{q}{2}\rangle$$

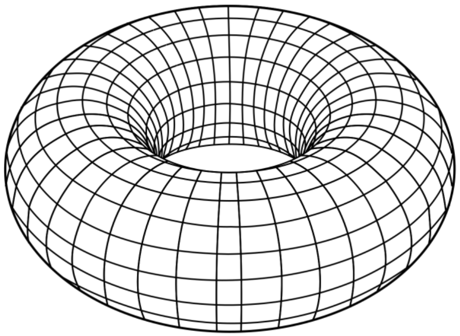
Approximate Wigner – Weyl calculus for the lattice models

Mathematical tools

Weyl symbol of operator (momentum space)

$$[\hat{A}]_W(x_n,p) = \int_{\mathcal{M}} dq e^{iqx_n} \langle p+\frac{q}{2}|\,\hat{A}\,|p-\frac{q}{2}\rangle$$

Weyl symbol of the product of two operators



This identity is approximate. It is valid for the near diagonal operators

$$(AB)_W(x_n,p) \equiv A_W(x_n,p) \star B_W(x_n,p)$$

Mathematical tools

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$$(AB)_W(x_n,p) \equiv A_W(x_n,p) \star B_W(x_n,p)$$

It is valid for the near diagonal operators

partition function

$$Z = \int D\bar{\psi} D\psi \; e^{S[\psi,\bar{\psi}]}$$

Action

$$S[\psi,\bar{\psi}] = \int_{\mathcal{M}} \frac{d^D p}{|\mathcal{M}|} \bar{\psi}(p) \hat{Q}(i\partial_p, p) \psi(p)$$

Lattice model for the description of electrons in crystals:

The typical Lattice Dirac operator ***Q*** *is almost diagonal if the external magnetic field strength is much smaller than 10 000 Tesla while wavelength of external electromagnetic field is much larger than 1 nanometer*

Mathematical tools

This identity is approximate.

$$(AB)_W(x_n,p) \equiv A_W(x_n,p) \star B_W(x_n,p)$$

It is valid for the near diagonal operators

partition function

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Action

$$S[\psi,\bar{\psi}] = \int_{\mathcal{M}} \frac{d^D p}{|\mathcal{M}|} \bar{\psi}(p) \hat{Q}(i\partial_p, p)\psi(p)$$

Lattice model for the regularization of continuum quantum field theory:

The typical Lattice Dirac operator ***Q*** *is almost diagonal when we approach continuum limit of the lattice model.*

Mathematical tools

We can use the approximate Wigner – Weyl calculus dealing with ***any lattice regularized continuum quantum field theory*** *and dealing with the lattice models of solid state physics* ***if the external magnetic field strength is much smaller than***
10 000 Tesla *while wavelength of external electromagnetic field is much larger than*
1 nanometer

Mathematical tools

partition function

$$Z = \int D\bar{\psi} D\psi \; e^{S[\psi,\bar{\psi}]}$$

Action

$$S[\psi,\bar{\psi}] = \int_{\mathcal{M}} \frac{d^D p}{|\mathcal{M}|} \bar{\psi}(p) \hat{Q}(i\partial_p, p) \psi(p)$$

Green function

$$G(p_1,p_2) = \langle p_1 | \, G \, | p_2 \rangle = \frac{1}{Z} \int D\bar{\psi} D\psi \bar{\psi}(p_2) \psi(p_1) \exp\left(\int \frac{d^D p}{|\mathcal{M}|} \bar{\psi}(p) \hat{Q}(i\partial_p, p) \psi(p) \right)$$

Groenewold equation

$$Q_W(p,x) \star G_W(p,x) = 1$$

Moyal product

$$\star_{xp} \equiv e^{\frac{i}{2}\left(\overleftarrow{\partial_x} \overrightarrow{\partial_p} - \overleftarrow{\partial_p} \overrightarrow{\partial_x} \right)}$$

Electric current

$$j_i(x) = \frac{\delta \log Z}{\delta A_k(x)} = -\int_{\mathcal{M}} \frac{d^D p}{|\mathcal{M}|} \mathrm{tr}\left[G_W(x,p) \partial_{p_i} Q_W(x,p) \right]$$

Precise Wigner – Weyl calculus for the lattice models (the details at the end of the talk, if time remains)

Finite rectangular lattice:

M.A. Zubkov (2023)
Journal of Physics A: Mathematical and Theoretical 56 (39), 395201

Infinite rectangular lattice:

I.V. Fialkovsky, M.A. Zubkov (2020)
Nuclear Physics B 954, 114999

Infinite honeycomb lattice:

R. Chobanyan, M.A. Zubkov
arXiv preprint arXiv:2302.00723

*We can use the precise Wigner – Weyl calculus dealing with **any lattice regularized continuum quantum field theory** and dealing with the lattice models of solid state physics **if the external magnetic field strength is of the order of 10 000 Tesla (unphysical!)** while wavelength of external electromagnetic field is of the order of **1 nanometer***

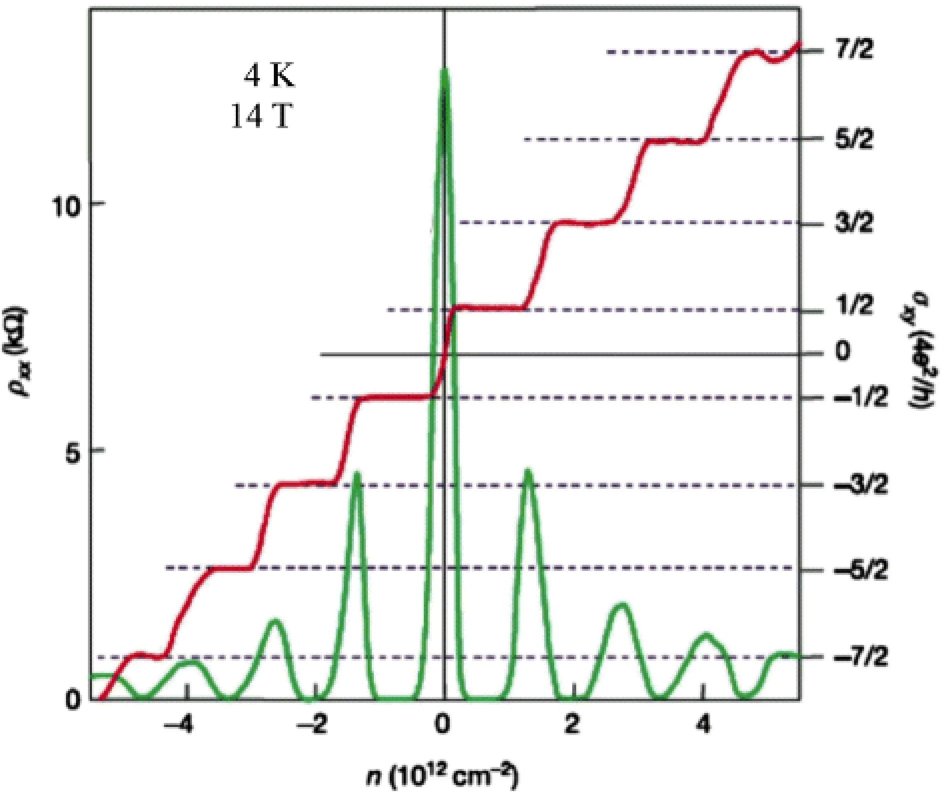
Which is more important, we can use this formalism for artificial lattices, when magnetic flux through the EFFECTIVE lattice cell is compared to 1

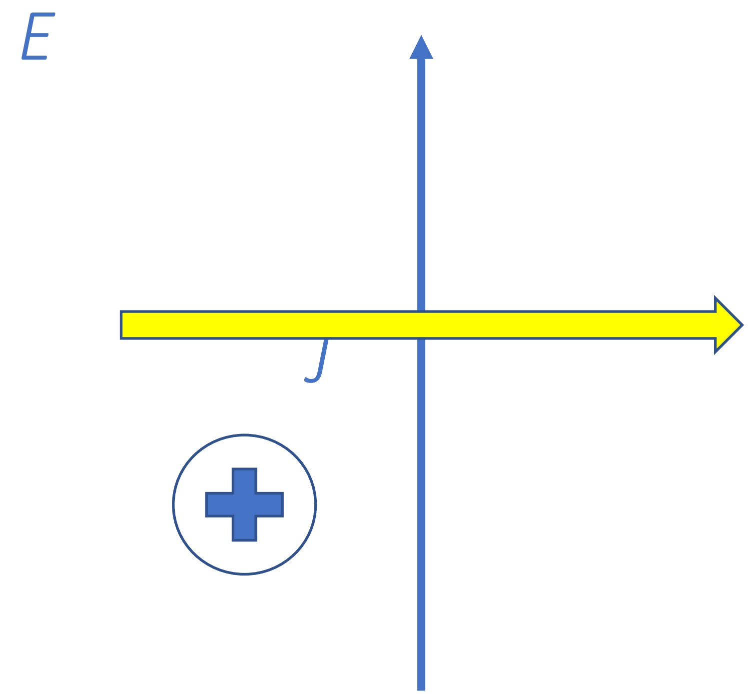
And also for the precise treatment of lattice regularized QFT

Applications to Quantum Hall Effect

QHE

Electric current orthogonal to electric field in the presence of magnetic field



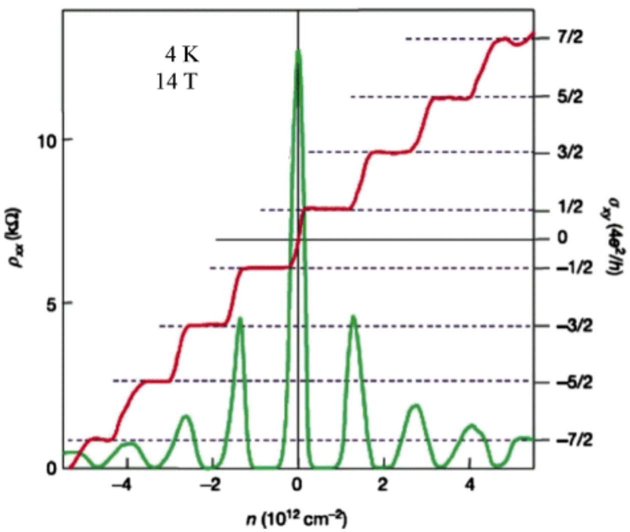


Geim, Novoselov, et all, Nature 438(7065):197-200 *graphene*

Quantum Hall Effect

QHE

constant magnetic field, no interactions, no disorder
k is Bloch vector,
|u(k)> is the eigenvector of Hamiltonian



$$\sigma_H = \frac{\mathcal{N}}{2\pi}$$

$$\sigma_{xy} = \frac{e^2}{h}\frac{1}{2\pi}\int d^2k[\boldsymbol{\nabla}\times\boldsymbol{A}(\boldsymbol{k})]$$

$$\boldsymbol{A}(\boldsymbol{k}) = -\mathrm{i}\,\langle u(\boldsymbol{k})|\boldsymbol{\nabla}|u(\boldsymbol{k})\rangle\,.$$

TKNN invariant

D. J. Thouless, M. Kohmoto, M. P. Nightingale, and M. den Nijs
Phys. Rev. Lett. 49, 405 (1982)

Intrinsic Anomalous Quantum Hall Effect

QHE

homogeneous system

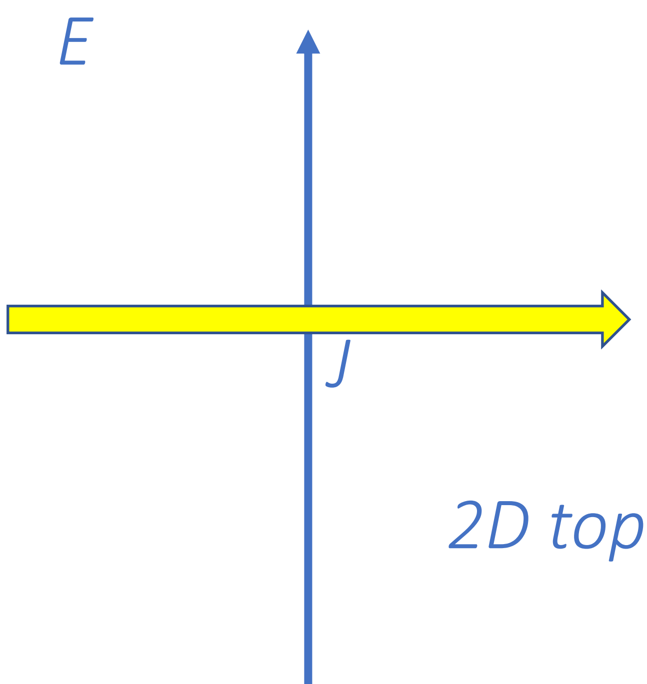
no magnetic field

no interactions

no disorder

T. Matsuyama, Quantization of Conductivity Induced by Topological Structure of Energy Momentum Space in Generalized QED in Three-dimensions, Prog. Theor. Phys 77, 711 (1987)

$$\mathcal{N} = \frac{\epsilon_{ijk}}{3!\,4\pi^2} \int d^3p\, \mathrm{Tr}\left[G(p)\frac{\partial G^{-1}(p)}{\partial p_i}\frac{\partial G(p)}{\partial p_j}\frac{\partial G^{-1}(p)}{\partial p_k}\right]$$



$$\sigma_H = \frac{\mathcal{N}}{2\pi}$$

2D topological insulator

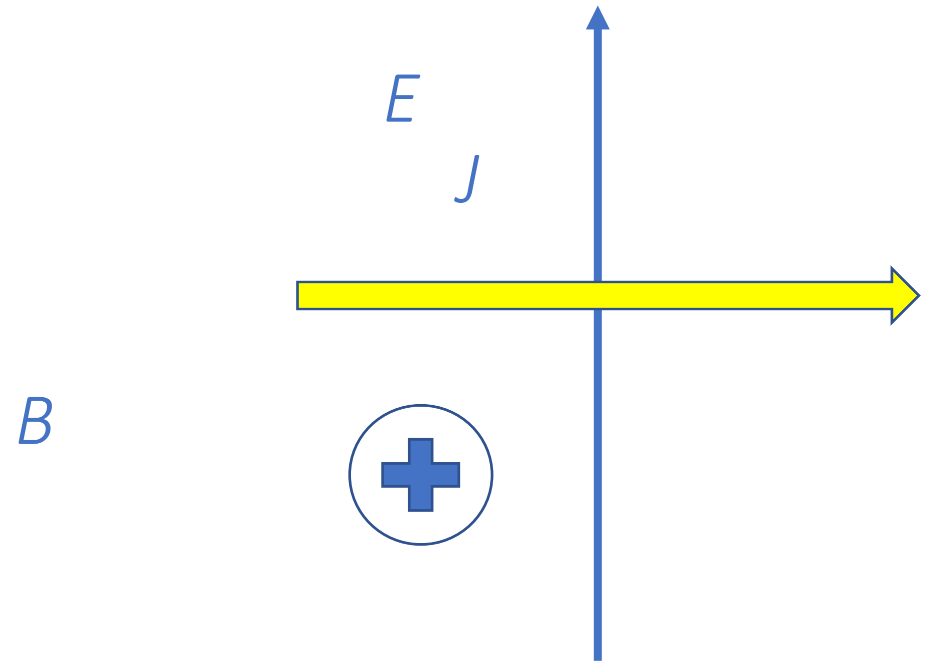
Applications to Quantum Hall Effect

QHE

Equilibrium, T=0

non-homogeneous system

Average electric current



2+1 D:

$$\langle j^k \rangle = -\frac{1}{2\pi} \mathcal{N} \epsilon^{3kj} E_j,$$

$$\mathcal{N} = \frac{T\epsilon_{ijk}}{\mathcal{S}\, 3!\, 4\pi^2} \int d^3p d^3x \, \mathrm{Tr} \left[G_W(p,x) * \frac{\partial Q_W(p,x)}{\partial p_i} * \frac{\partial G_W(p,x)}{\partial p_j} * \frac{\partial Q_W(p,x)}{\partial p_k} \right]$$

M.A. Zubkov [*,1], Xi Wu

Annals of Physics 418 (2020) 168179

Applications to Quantum Hall Effect

QHE

Equilibrium, T=0

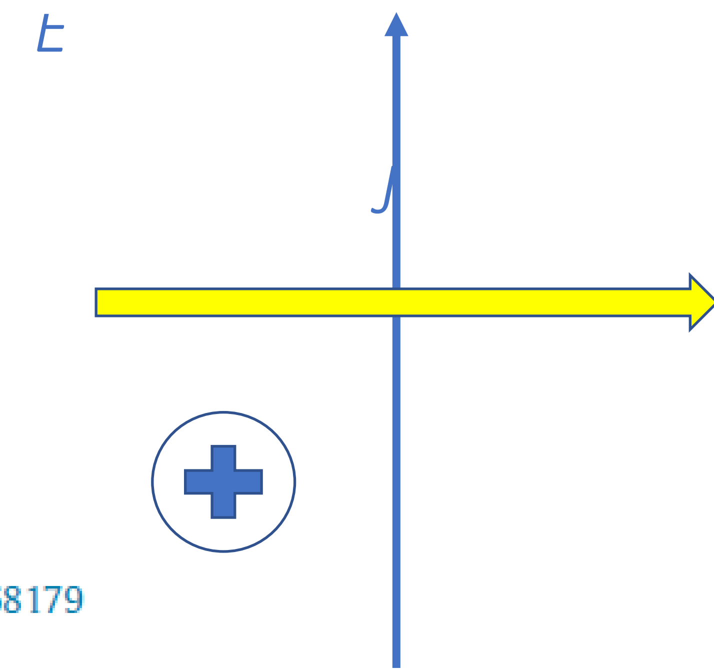
<u>non-homogeneous system</u>

Average electric current

3 + 1 D:

$$\langle j^k \rangle = -\frac{1}{2\pi^2}\epsilon^{kjl4}\mathcal{N}_l E_j$$

$$\mathcal{N}_l = -\frac{T\epsilon_{ijkl}}{\mathcal{V}\,3!\,8\pi^2}\int d^4x d^4p \,\mathrm{Tr}\left[G_W(p,x) * \frac{\partial Q_W(p,x)}{\partial p_i} * \frac{\partial G_W(p,x)}{\partial p_j} * \frac{\partial Q_W(p,x)}{\partial p_k}\right]$$



B

M.A. Zubkov [*,1], Xi Wu

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Quantum Hall Effect ***Equilibrium, T=0*** **QHE**

non-homogeneous system

Average electric current

2+1 D:

$$\langle j^k \rangle = -\frac{1}{2\pi}\mathcal{N}\epsilon^{3kj}E_j,$$

$$\mathcal{N} = \frac{T\epsilon_{ijk}}{\mathcal{S}\,3!\,4\pi^2}\int d^3p d^3x \,\mathrm{Tr}\left[G_W(p,x) * \frac{\partial Q_W(p,x)}{\partial p_i} * \frac{\partial G_W(p,x)}{\partial p_j} * \frac{\partial Q_W(p,x)}{\partial p_k}\right]$$

smooth deformation of the system

↓

the system without disorder, elastic deformations etc, with constant magnetic field

N is not changed!

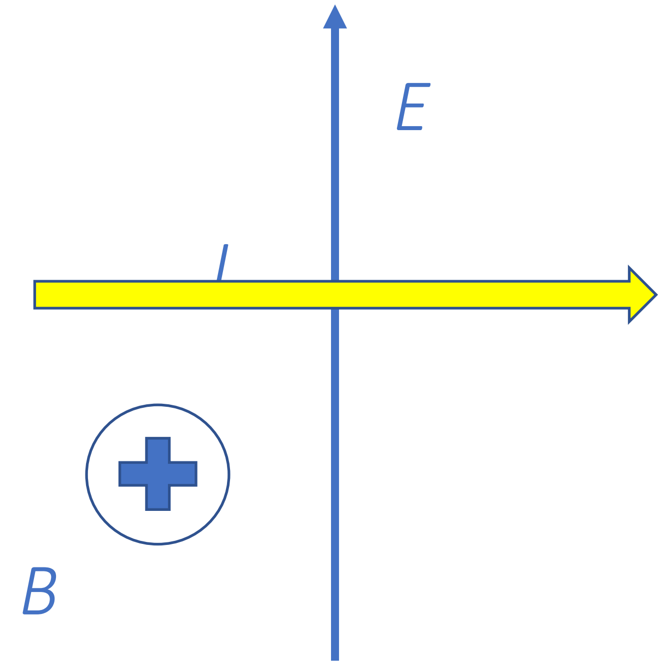
If N is known for less complicated system, we know it also for the more complicated one

QHE

The absence of (perturbative) interaction corrections to Quantum Hall Effect

equilibrium, T=0

Electric current orthogonal to electric field in the presence of magnetic field



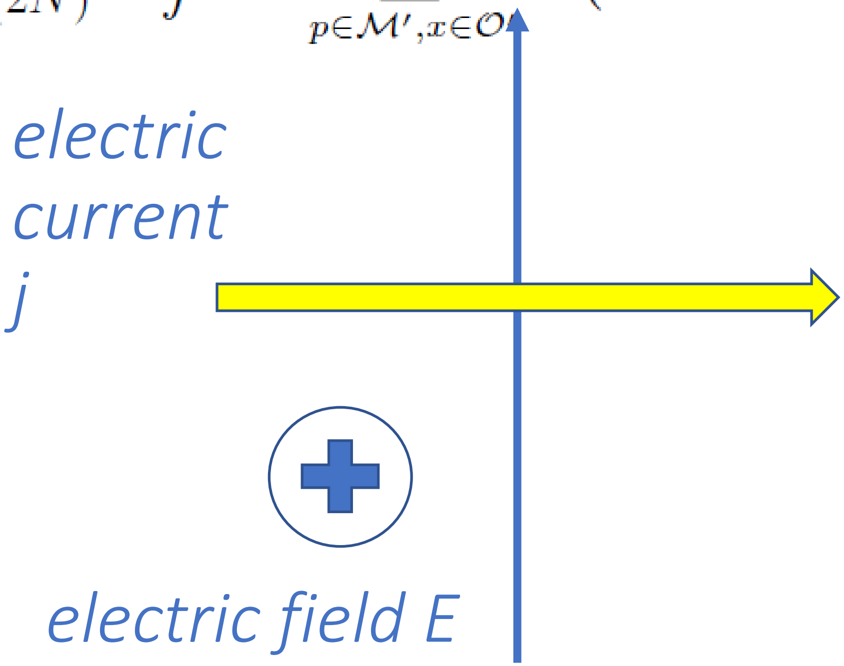
C.X. Zhang, M.A. Zubkov
Annals of Physics 444, 169016

Precise Wigner – Weyl calculus (finite rectangular lattice

QHE

$$\bar{\sigma}^{ij} = \frac{\mathcal{N}}{2\pi}\epsilon^{ij}$$

$$\mathcal{N} = \frac{1}{3!}\epsilon^{\mu\nu\rho}\frac{1}{(2N)^{2D}}\int d\Pi^3 \sum_{p\in\mathcal{M}',x\in\mathcal{O}} \mathrm{tr}\left(\partial_{\Pi^\mu}\hat{Q}_W^{\mathrm{M}} \star \hat{G}_W^{\mathrm{M}} \star \partial_{\Pi^\nu}\hat{Q}_W^{\mathrm{M}} \star \hat{G}_W^{\mathrm{M}} \star \partial_{\Pi^\rho}\hat{Q}_W^{\mathrm{M}} \star \hat{G}_W^{\mathrm{M}}\right)$$



M.A. Zubkov (2023)

Journal of Physics A: Mathematical and Theoretical 56 (39), 395201

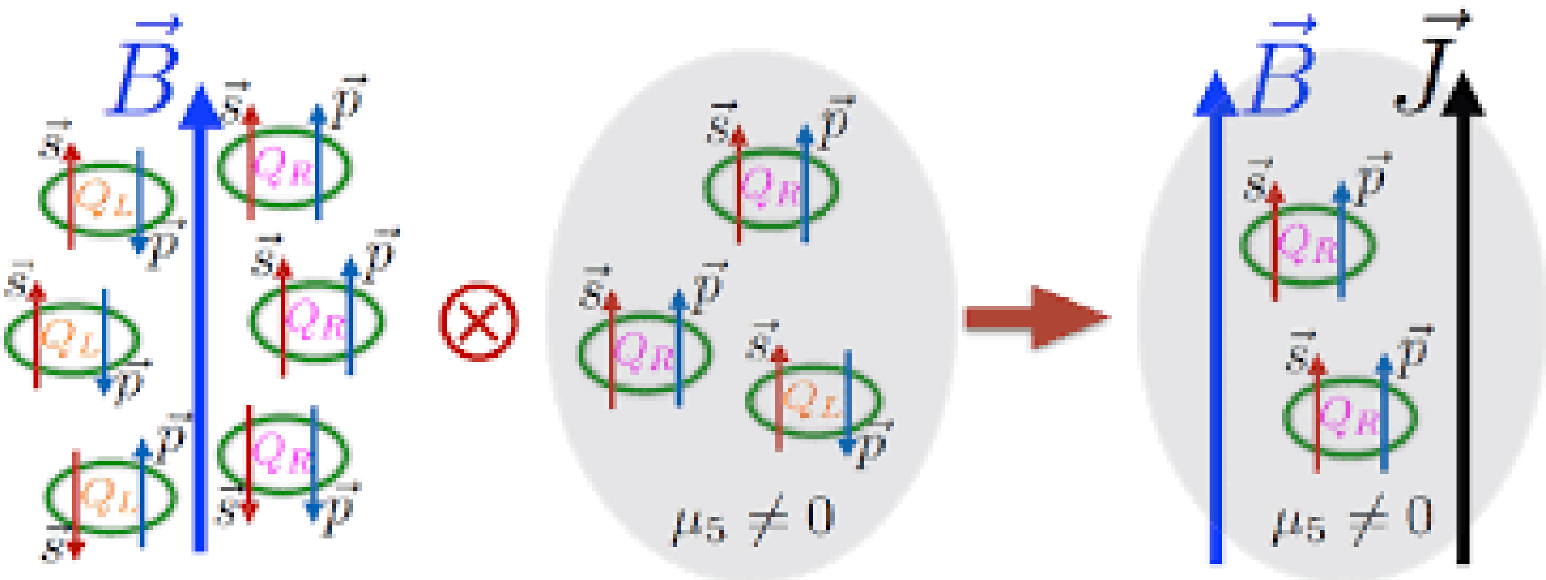
Applications to Chiral Magnetic Effect

CME

non-homogeneous system, equilibrium, T=0

Average electric current

3 + 1 D:



D.E. Kharzeev, J. Liao, S.A. Voloshin, G. Wang,
Progress in Particle and Nuclear Physics, Volume 88, 2016, Pages 1-28,

Applications to Chiral Magnetic Effect **CME**

non-homogeneous system, equilibrium, T=0

Average electric current

3 + 1 D:

$$\bar{j}^k = \frac{1}{4\pi^2}\epsilon^{ijkl}\mathcal{M}_l F_{ij}$$

topological invariant:

$$\mathcal{M}_l = \frac{-iT\epsilon_{ijkl}}{3!V\,8\pi^2}\int_{\mathcal{M}} d^D x \int d^D p \,\mathrm{Tr}\left[G_W^{(0)} \star \partial_{p_i} Q_W^{(0)}(p,x) \star G_W^{(0)} \star \partial_{p_j} Q_W^{(0)}(p,x) \star G_W^{(0)} \star \partial_{p_k} Q_W^{(0)}\right]$$

external magnetic field: $F_{ij} = \epsilon_{ijk} B_k$

C. Banerjee, M. Lewkowicz, M.A. Zubkov
Physics Letters B, 136457 (2021)

Homogeneous systems: M.A.Zubkov, Physical Review D 93 (10), 105036 (2016)

Chiral magnetic effect ***Equilibrium, T=0*** CME

non-homogeneous system

Average electric current

$$\bar{J}^k = \frac{1}{4\pi^2}\epsilon^{ijkl}\mathcal{M}_l F_{ij}$$

$$\mathcal{M}_l = \frac{-iT\epsilon_{ijkl}}{3!V8\pi^2}\int d^D x \int_{\mathcal{M}} d^D p \mathrm{Tr}\left[\mathrm{G}_\mathrm{W}^{(0)} \star \partial_{\mathrm{p_i}}\mathrm{Q}_\mathrm{W}^{(0)}(\mathrm{p},\mathrm{x}) \star \mathrm{G}_\mathrm{W}^{(0)} \star \partial_{\mathrm{p_j}}\mathrm{Q}_\mathrm{W}^{(0)}(\mathrm{p},\mathrm{x}) \star \mathrm{G}_\mathrm{W}^{(0)} \star \partial_{\mathrm{p_k}}\mathrm{Q}_\mathrm{W}^{(0)}\right]$$

smooth deformation of the system

↓

the system without any inhomogeneity

M is not changed!

We know that in homogeneous systems M = 0

Absence of equilibrium chiral magnetic effect, M.A. Zubkov
Physical Review D 93 (10), 105036

↓

No CME in non – uniform systems at T=0

Applications to Chiral Magnetic Effect

CME

non-homogeneous system, equilibrium, T>0

Average electric current

$$\bar{J}^k = \frac{1}{4\pi^2}\epsilon^{ijk4}\mathcal{M}_4 F_{ij}$$

topological invariant:

$$\mathcal{M}_4 = 2\pi T \sum_{\omega} \mathcal{N}_4(\omega) \qquad \omega = 2\pi T(n+1/2),\ n \in Z,\ 0 \le n < N, \text{ where } N = 1/T$$

$$\mathcal{N}_4(\omega) = \frac{-i\epsilon_{ijk4}}{3!V8\pi^2}\int d^{D-1}x \int_{\mathcal{B}} d^{D-1}p\,\mathrm{Tr}\left[G_W^{(0)} \star \partial_{p_i} Q_W^{(0)}(p,x) \star G_W^{(0)} \star \partial_{p_j} Q_W^{(0)}(p,x) \star G_W^{(0)} \star \partial_{p_k} Q_W^{(0)}\right]$$

Response of N to chiral chemical potential is zero

⇓

No CME at T>0

C. Banerjee, M. Lewkowicz, M.A. Zubkov
Physics Letters B, 136457 (2021)

The absence of CME at T>0 **for homogeneous** systems has been reported earlier in
C.G. Beneventano, M. Nieto, E.M. Santangelo J. Phys. A, 53 (46) (2020), Article 465401,

Chiral Magnetic Effect ***non-equilibrium systems*** **CME**

Keldysh technique

Green functions (lower sign for fermions)

$$\left\{\hat{G}^R\right\}_{(\alpha_1;\alpha_2)}(x_1;x_2) \equiv -i\theta(t_1-t_2)\left\langle\left[\Psi_{\alpha_1}(x_1),\Psi^\dagger_{\alpha_2}(x_2)\right]_{+}\right\rangle$$

$$\left\{\hat{G}^A\right\}_{(\alpha_1;\alpha_2)}(x_1;x_2) \equiv \ \ i\theta(t_2-t_1)\left\langle\left[\Psi_{\alpha_1}(x_1),\Psi^\dagger_{\alpha_2}(x_2)\right]_{+}\right\rangle$$

$$\left\{\hat{G}^K\right\}_{(\alpha_1;\alpha_2)}(x_1;x_2) \equiv -i\left\langle\left[\Psi_{\alpha_1}(x_1),\Psi^\dagger_{\alpha_2}(x_2)\right]_{-}\right\rangle,$$

$$\left\{\hat{G}^<\right\}_{(\alpha_1;\alpha_2)}(x_1;x_2) \equiv \mp i\left\langle\Psi^\dagger_{\alpha_2}(x_2)\Psi_{\alpha_1}(x_1)\right\rangle$$

Keldysh Green function

$$\hat{G}(t,x|t',x') = -i\begin{pmatrix} \langle T\Phi(t,x)\Phi^+(t',x')\rangle & -\langle\Phi^+(t',x')\Phi(t,x)\rangle \\ \langle\Phi(t,x)\Phi^+(t',x')\rangle & \langle\tilde{T}\Phi(t,x)\Phi^+(t',x')\rangle \end{pmatrix}$$

$$\begin{pmatrix} G^{--} & G^{-+} \\ G^{+-} & G^{++} \end{pmatrix}$$

$$G^{\mathrm{A}} = G^{--} - G^{+-} = G^{-+} - G^{++}$$

$$G^{\mathrm{R}} = G^{--} - G^{-+} = G^{+-} - G^{++}$$

$$G^< \qquad G^{-+}$$

CME

Keldysh technique and Wigner – Weyl calculus.

Keldysh Green function

$$\hat{G}(t,x|t',x') = -i \begin{pmatrix} \langle T\Phi(t,x)\Phi^+(t',x')\rangle & -\langle \Phi^+(t',x')\Phi(t,x)\rangle \\ \langle \Phi(t,x)\Phi^+(t',x')\rangle & \langle \tilde{T}\Phi(t,x)\Phi^+(t',x')\rangle \end{pmatrix}$$

$$= \begin{pmatrix} G^{--} & G^{-+} \\ G^{+-} & G^{++} \end{pmatrix}$$

$$G^{\mathrm{A}} = G^{--} - G^{+-} = G^{-+} - G^{++}$$

$$G^{\mathrm{R}} = G^{--} - G^{-+} = G^{+-} - G^{++}$$

$$G^{<} = G^{-+}$$

Wigner transformation

$$\hat{G}(X_1, X_2) = \langle X_1|\hat{\mathbf{G}}|X_2\rangle \qquad A(X_1, X_2) = \langle X_1|\hat{A}|X_2\rangle$$

$$A_W(X|P) = \int d^{D+1}Y\, e^{\mathrm{i}Y^\mu P_\mu} A(X+Y/2, X-Y/2)$$

Moyal product

$$(A \star B)(X|P) = A(X|P)\, e^{-\mathrm{i}(\overleftarrow{\partial}_{X^\mu}\overrightarrow{\partial}_{P_\mu} - \overleftarrow{\partial}_{P_\mu}\overrightarrow{\partial}_{X^\mu})/2} B(X|P)$$

Lesser representation

$$\hat{\mathbf{G}}^{(<)} = U\hat{\mathbf{G}}V,$$

CME

U= $\quad$ *V* $\frac{1}{\sqrt{2}}\begin{pmatrix} 2 & 0 \\ 1 & -1 \end{pmatrix}$ $\quad$ $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 0 \\ -1 & 2 \end{pmatrix}$

$$\hat{\mathbf{G}}^{(<)} = \begin{pmatrix} G^{\mathrm{R}} & 2G^{<} \\ 0 & G^{\mathrm{A}} \end{pmatrix}$$

$$G^{\mathrm{A}} = G^{--} - G^{+-} = G^{-+} - G^{++}$$

$$G^{\mathrm{R}} = G^{--} - G^{-+} = G^{+-} - G^{++}$$

$G^{<}$ *=* G^{-+}

The inverse Q of Green function

$$\hat{\mathbf{Q}}\hat{\mathbf{G}} = 1$$

After Wigner transformation

$$\hat{Q} * \hat{G} = 1$$

CME

Response of electric current to external field strength

$$J^i = -\frac{1}{4}\int \frac{d^{D+1}\pi}{(2\pi)^{D+1}}\,\mathrm{tr}\Big(\hat{G}\star\partial_{\pi^\mu}\hat{Q}\star\hat{G}\star\partial_{\pi^\nu}\hat{Q}\star\hat{G}\partial_{\pi_i}\hat{Q}\Big)^{<}\mathcal{F}^{\mu\nu}$$
$$-\frac{1}{4}\int \frac{d^{D+1}\pi}{(2\pi)^{D+1}}\,\mathrm{tr}\Big(\partial_{\pi_i}\hat{Q}\hat{G}\star\partial_{\pi^\mu}\hat{Q}\star\hat{G}\star\partial_{\pi^\nu}\hat{Q}\star\hat{G}\Big)^{<}\mathcal{F}^{\mu\nu}.$$

Electric con

systems

$$J^i = \sigma^{ij}\mathcal{F}_{0j}$$

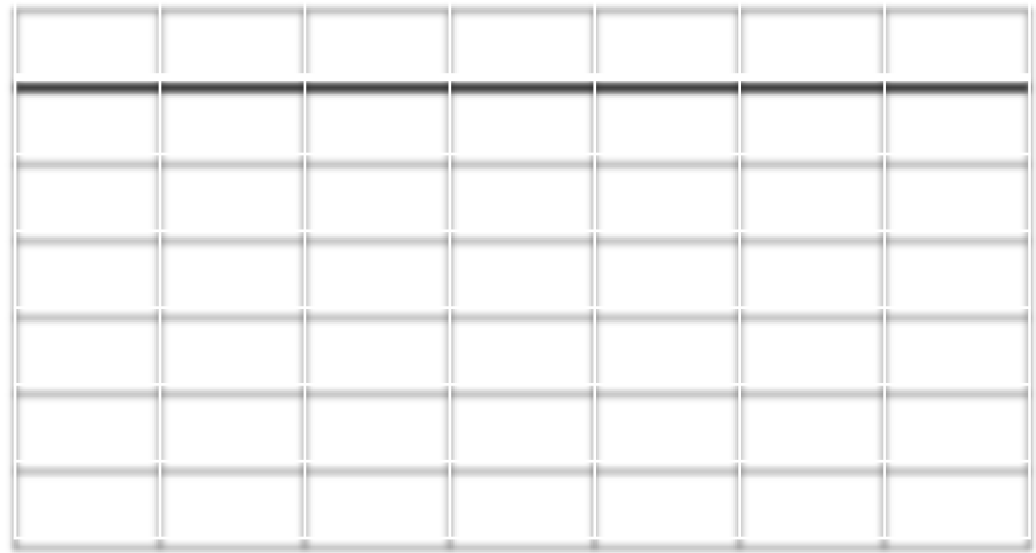
$$\sigma^{ij} = \frac{1}{4}\int \frac{d^{D+1}\pi}{(2\pi)^{D+1}}\,\mathrm{tr}\left(\partial_{\pi_i}\hat{Q}\left[\hat{G}\star\partial_{\pi_{[0}}\hat{Q}\star\partial_{\pi_{j]}}\hat{G}\right]\right)^{<} + \mathrm{c.c.}$$

C Banerjee, IV Fialkovsky, M Lewkowicz, CX Zhang, MA Zubkov
Journal of Computational Electronics 20, 2255-2283 (2021)

CME

Lattice model with Wilson fermions

Out of equilibrium



Thermal equilibrium (in Euclidean space - time)

$$Q_W^M(\pi) = \sum_{\mu=1}^{3} \gamma^\mu g_\mu(\pi) - im(\pi) + \gamma^4 g_4(\pi_4)$$

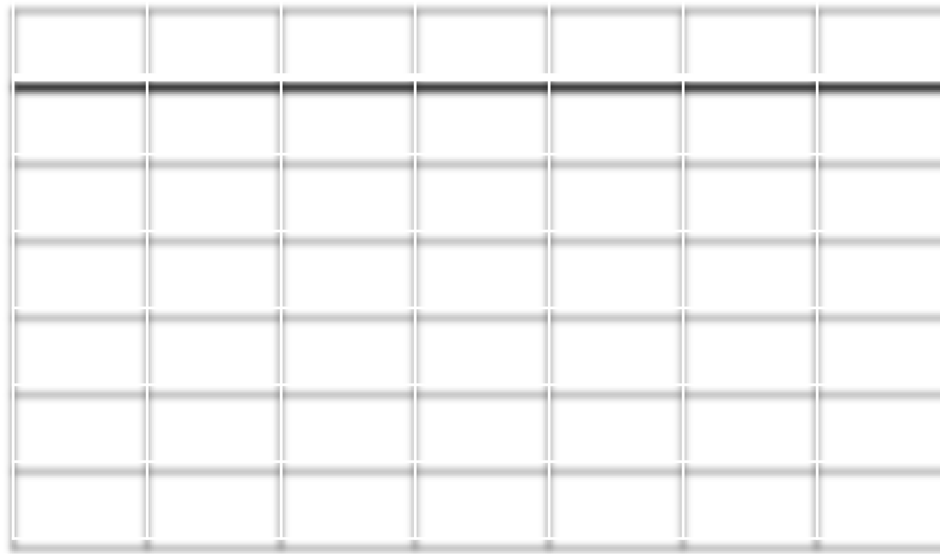
$$g_i = \sin(\pi_i)$$

$$m(\pi) = m^{(0)} + \sum_{i=1}^{4} (1 - \cos(\pi_i))$$

CME

Lattice model with Wilson fermions

Out of equilibrium



Real time dynamics (in Minkowski space - time)

$$Q_W^M(\pi)|_{\pi_4=-i\pi_0} = \sum_{\mu=1}^{3}\gamma^\mu g_\mu(\pi) \quad ($$

$$-i\left(\sum_{i=1}^{3}(1-\cos(\pi_i)) + (1-\mathrm{ch}(\pi_0))\right) - i\gamma^4 \mathrm{sh}(\pi_0)$$

CME

Lattice model with Wilson fermions

Out of equilibrium

Keldysh Green function

$$\hat{Q} = \left(\begin{array}{cc} Q_{--} & Q_{-+} \\ Q_{+-} & Q_{++} \end{array} \right)$$

$$Q_{++} = -\mathcal{Q}(\pi_0, \vec{\pi}) + \mathrm{i}\epsilon\partial_{\pi_0}\mathcal{Q}(\pi_0, \vec{\pi})\frac{1-\rho(\pi_0)}{1+\rho(\pi_0)}$$

$$Q_{--} = \mathcal{Q}(\pi_0, \vec{\pi}) + +\mathrm{i}\epsilon\partial_{\pi_0}\mathcal{Q}(\pi_0, \vec{\pi})\frac{1-\rho(\pi_0)}{1+\rho(\pi_0)}$$

$$Q_{+-} = -2\mathrm{i}\epsilon\partial_{\pi_0}\mathcal{Q}(\pi_0, \vec{\pi})\frac{1}{1+\rho(\pi_0)},$$

$$Q_{-+} = 2\mathrm{i}\epsilon\partial_{\pi_0}\mathcal{Q}(\pi_0, \vec{\pi})\frac{\rho(\pi_0)}{1+\rho(\pi_0)}.$$

$$\pi = P - A(X)$$

initial one – particle distribution

$$f(\pi_0) = \rho(\pi_0)(1 + \rho(\pi_0))^{-1}$$

CME

$$\hat{Q} = \begin{pmatrix} Q_{--} & Q_{-+} \\ Q_{+-} & Q_{++} \end{pmatrix}$$

$$\begin{aligned}
Q_{++} &= -\Big(\sum_{\mu=1}^{3}\gamma^\mu g_\mu(\pi) - im(\vec{\pi}, -i\pi_0 - i\mu_5(t)\gamma^5) \\
&\quad +\gamma^4 g_4(-i\pi_0 - i\mu_5(t)\gamma^5) - \gamma^4 \epsilon\, e^{-\pi_0\gamma^4}\frac{1-\rho(\pi_0)}{1+\rho(\pi_0)}\Big), \\
Q_{--} &= \sum_{\mu=1}^{3}\gamma^\mu g_\mu(\pi) - im(\vec{\pi}, -i\pi_0 - i\mu_5(t)\gamma^5) \\
&\quad +\gamma^4 g_4(-i\pi_0 - i\mu_5(t)\gamma^5) + \gamma^4 \epsilon\, e^{-\pi_0\gamma^4}\frac{1-\rho(\pi_0)}{1+\rho(\pi_0)}, \\
Q_{+-} &= -2\gamma^4 \epsilon\, e^{-\pi_0\gamma^4}\frac{1}{1+\rho(\pi_0)}, \\
Q_{-+} &= 2\gamma^4 \epsilon\, e^{-\pi_0\gamma^4}\frac{\rho(\pi_0)}{1+\rho(\pi_0)}. \qquad (32)
\end{aligned}$$

time depending chiral chemical potential

$$\delta\mu_5(t) = \delta\mu_5^{(0)}\cos\omega_0 t$$

Response of electric current both to magnetic field and to chiral chemical potential

CME

$$J^i = \Sigma_{CME} B^i$$

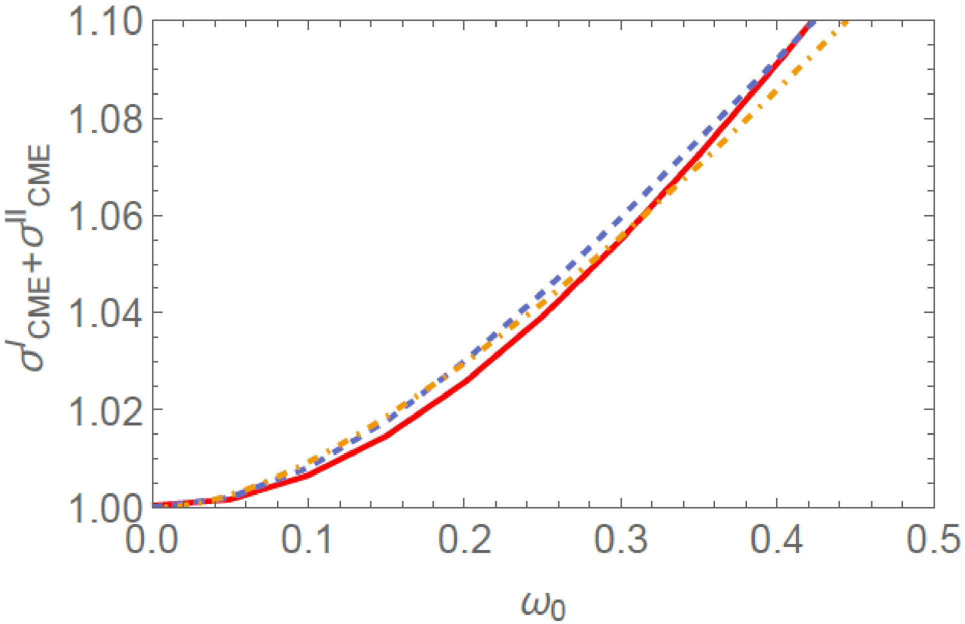
response to chiral chemical potential

$$\delta\mu_5(t) = \delta\mu_5^{(0)} \cos\omega_0 t$$

$$\Delta\Sigma_{CME} = \frac{1}{4\pi^2}\sigma_{CME}(\omega_0)\delta\mu_5^{(0)} e^{i\omega_0 t} + (c.c.)$$

two parts of conductivity

$$\sigma_{CME}(\omega_0) = \sigma_{CME}^{(I)}(\omega_0) + \sigma_{CME}^{(II)}(\omega_0)$$



$T = \frac{1}{10a}$ (solid line), $\frac{1}{20a}$ (dashed line), $\frac{1}{50a}$ (dashed - dotted line)

Response of electric current both to magnetic field and to chiral chemical potential

CME

$$J^i = \Sigma_{CME} B^i$$

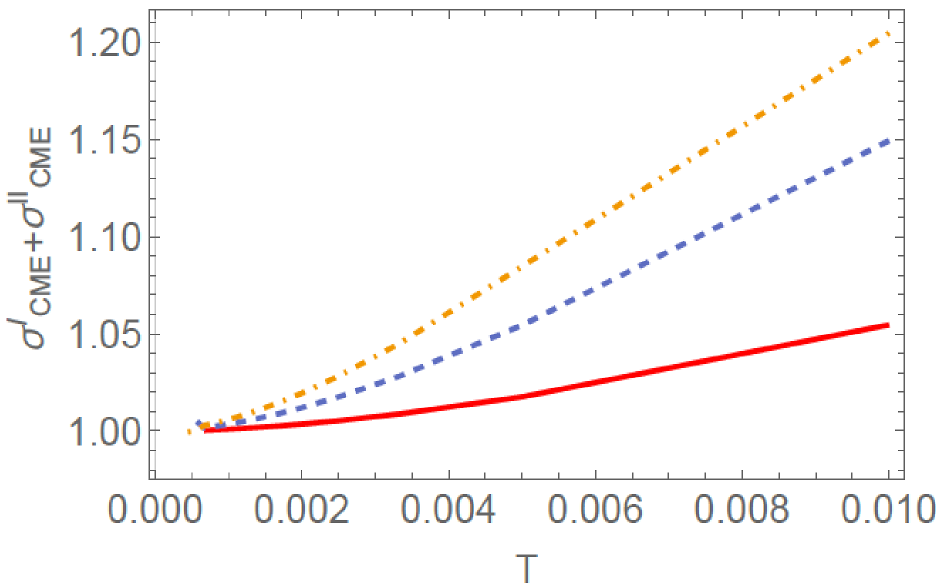
response to chiral chemical potential

$$\delta\mu_5(t) = \delta\mu_5^{(0)} \cos\omega_0 t$$

$$\Delta\Sigma_{CME} = \frac{1}{4\pi^2}\sigma_{CME}(\omega_0)\delta\mu_5^{(0)} e^{i\omega_0 t} + (c.c.)$$

two parts of conductivity

$$\sigma_{CME}(\omega_0) = \sigma^{(I)}_{CME}(\omega_0) + \sigma^{(II)}_{CME}(\omega_0)$$



C. Banerjee, M. Lewkowicz, M.A. Zubkov
Physical Review D 106 (7), 074508 (2022)

$x = \omega_0/T = 30$ (solid line), $x = 60$ (dashed line), $x = 80$ (dashed dotted line)

Out of equilibrium the CME is back!!!

When chiral chemical potential is time dependent, the CME conductivity depends on frequency w. In the continuum limit the conventional value of CME conductivity is reproduced for any ratio w/T.

CME contribution to magneto – conductivity (the today talk by R.Abramchuk at the present conference)

CME

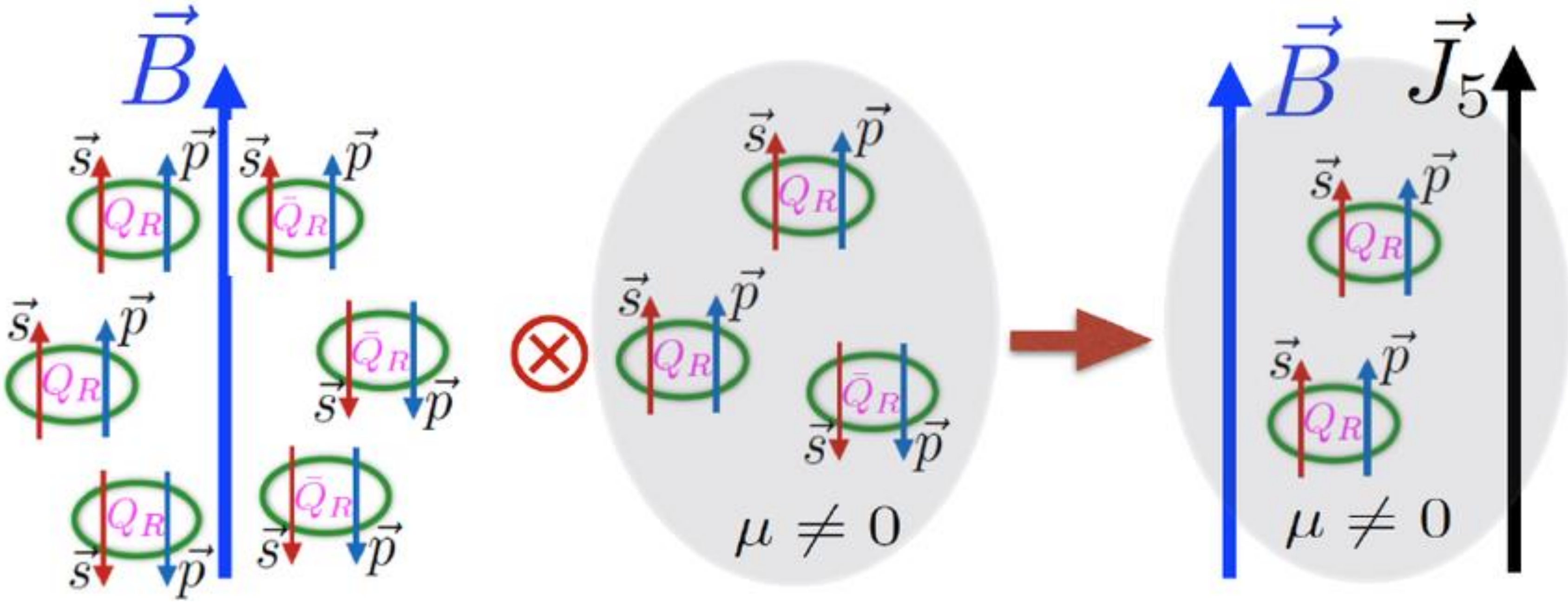
Chiral Kinetic Theory & CME	our NDT calculation
$\rho_5 = \frac{E^j H_j}{4\pi^2}\ \tau_5$	$\rho_5 \approx \frac{E^j H_j}{4\pi^2}\ \frac{1}{2\epsilon} \ln\left(4\frac{\mu^2}{m^2}\right)$
$\sigma_{ij}^{CME} = \frac{3}{2}\ \frac{H_i H_j}{4\pi^2}\ \frac{v_F^3}{\pi^2 T^2 + \mu^2}\ \tau_5$	$\sigma_{jk}^{(2)} \approx \frac{3}{2}\ \frac{H_j H_k}{4\pi^2}\ \frac{v_F^3}{\epsilon^2}\ \frac{1}{2\epsilon} \ln\left(\frac{4\mu^2}{m^2}\right)$
	where $\epsilon \approx \frac{1}{2\pi} \frac{w^2}{\rho_0} \frac{\mu^2 T}{u^2}$,
	$m^2 \ll \mu^2$ and $\frac{u\mu}{T} \ll 1$

$$\tau_5 \sim \frac{1}{2\epsilon} \ln\left(\frac{4\mu^2}{m^2}\right) \sim \frac{\pi \rho_0 u^2}{w^2 \mu^2 T} \ln\left(\frac{4\mu^2}{m^2}\right) \qquad (32)$$

CHIRAL SEPARATION EFFECT

CSE

Axial current along magnetic field in the presence of chemical potential



D.E. Kharzeev, J. Liao, S.A. Voloshin, G. Wang,
Progress in Particle and Nuclear Physics, Volume 88, 2016, Pages 1-28,

$$J_5^k = -\frac{1}{4\pi^2}\epsilon^{ijk0}\mu F_{ij}$$

A. Metlitski and Ariel R. Zhitnitsky,Phys. Rev. D 72 (2005), 045011

Lattice Dirac operator Q

CSE

Is 4 x 4 matrix expressed through the Gamma matrices

$$j_k^5(x) = -\int_{\mathcal{M}} \frac{d^D p}{|\mathcal{M}|} \mathrm{tr}\, [\gamma^5 G_W(x,p)\partial_{p_k} Q_W(x,p)]$$

The system with Fermi surface of arbitrary complicated form

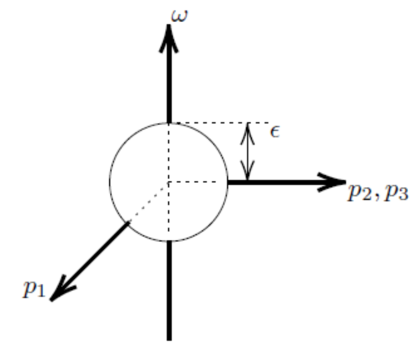
$$\bar{J}_5^k = -\frac{\mathcal{N}}{4\pi^2}\epsilon^{ijk0}\mu F_{ij} \qquad \mathcal{N} = -\frac{1}{48\pi^2 \mathbf{V}} \int_{\Sigma_3} \int d^3x\, \mathrm{tr} \left[\gamma^5 G_W \star dQ_W \star G_W \wedge \star dQ_W \star G_W \star \wedge dQ_W\right]$$

Surface Σ_3 surrounds the singularities

of $\left[\gamma^5 G_W^{(0)} \star dQ_W^{(0)} \star G_W^{(0)} \wedge \star dQ_W^{(0)} \star G_W^{(0)} \star \wedge dQ_W^{(0)}\right]$

γ^5 commutes/anticommutes with Q

in small vicinity of Σ_3

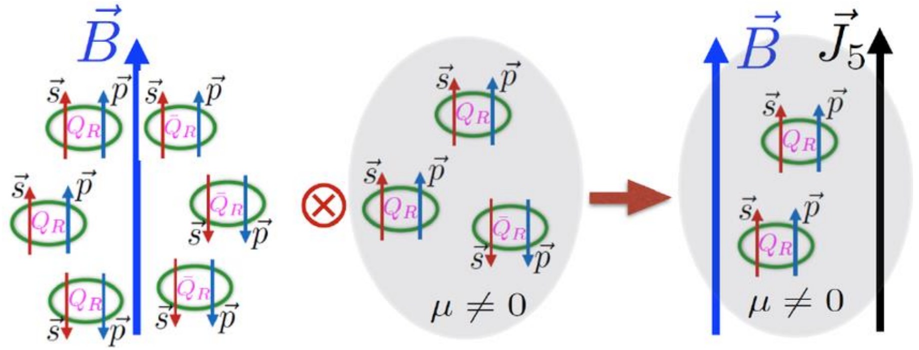


M.Suleymanov, M.Zubkov, Physical Review D 102 (7), 076019 (2020)

Lattice Dirac operator Q

CSE

is 4 x 4 matrix expressed through the Gamma matrices



The system with Fermi surface of arbitrary complicated form

$$\bar{J}_5^k = -\frac{\mathcal{N}}{4\pi^2}\epsilon^{ijk0}\mu F_{ij}$$

Irrespective of the form of the Fermi surface the value of

$\mathcal{N}$ **is equal to the number of chiral**

4 – component Dirac fermions

M.Suleymanov, M.Zubkov, Physical Review D 102 (7), 076019 (2020)

Non – renormalization of CSE

by interactions in QCD

CSE

$$\bar{J}^k_5 = -\frac{\mathcal{N}}{4\pi^2}\epsilon^{ijk0}\mu F_{ij}$$

Chemical potential is counted from the level, where

the CSE disa

$$\mathcal{N} = -\frac{1}{48\pi^2 \mathbf{V}}\int_{\Sigma_3}\int d^3x\, \mathrm{tr}\left[\gamma^5 G_W \star dQ_W \star G_W \wedge \star dQ_W \star G_W \star \wedge dQ_W\right]$$

Σ_3

Surf $\left[\gamma^5 G^{(0)}_W \star dQ^{(0)}_W \star G^{(0)}_W \wedge \star dQ^{(0)}_W \star G^{(0)}_W \star \wedge dQ^{(0)}_W\right]$ larities

of

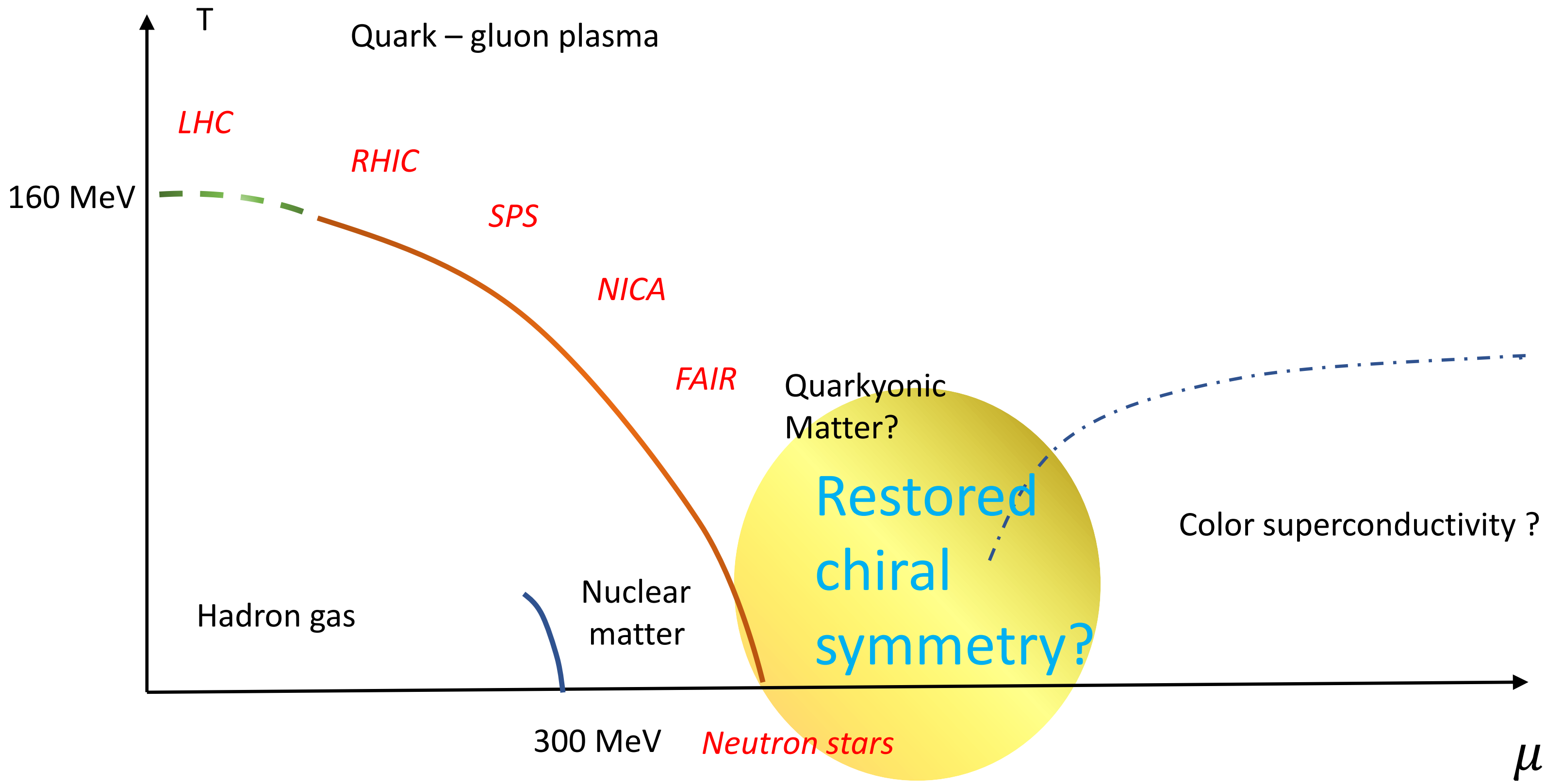
The Green function entering this expression is the complete one with interactions taken into account

M.Zubkov, R.Abramchuk Physical Review D 107 (9), 094021 (2023)

$$\bar{J}_5^k = -\frac{\mathcal{N}}{4\pi^2}\epsilon^{ijk0}\mu F_{ij}$$

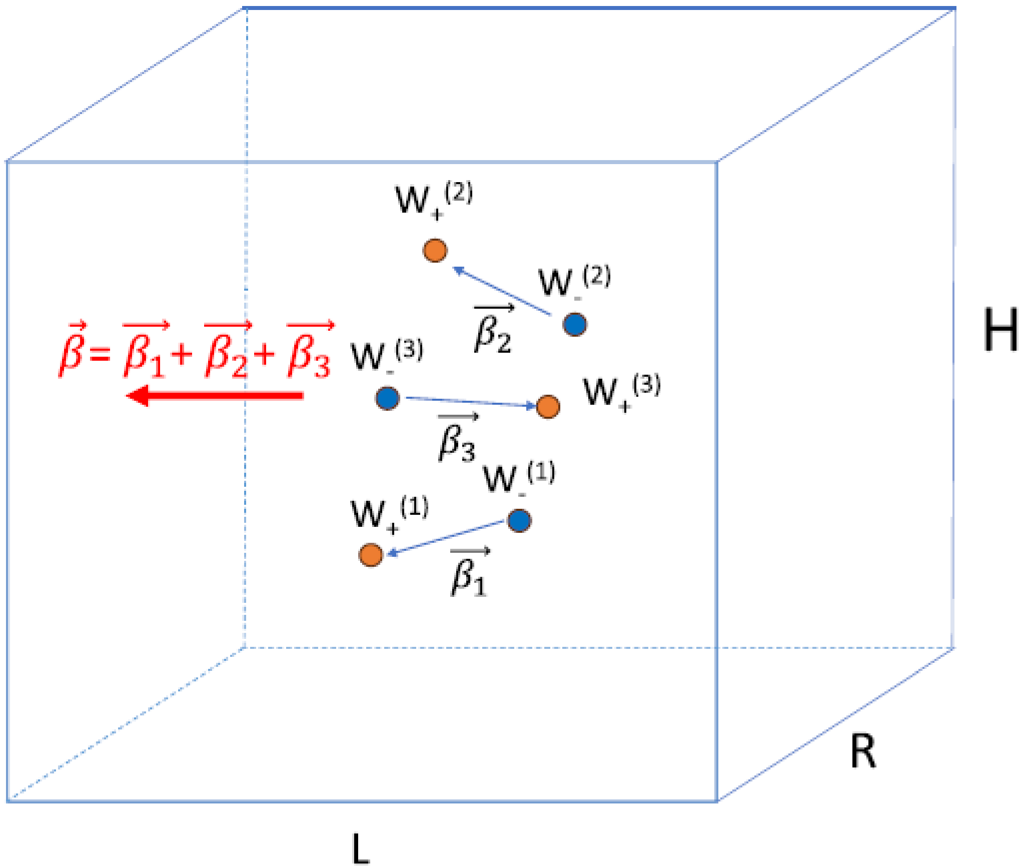
CSE

Chemical potential is counted from the level, where the CSE disappears (the position of the phase transition)



Non – renormalization of CSE by interactions in magnetic Weyl semimetals

CSE



Weyl fermions near Weyl points in momentum space

Non – renormalization of CSE by interactions in magnetic Weyl semimetals

CSE

$$J^5_k(x) = \sigma_{\rm CSE} B_k \delta\mu \qquad \sigma_{\rm CSE} = \frac{\mathcal{N}}{2\pi^2}$$

Chemical potential is counted from the level of Weyl point

$$\mathcal{N} = -\frac{1}{48\pi^2 \mathbf{V}} \int_{\Sigma_3} \int d^3x \, {\rm tr} \left[\gamma^5 G_W \star dQ_W \star G_W \wedge \star dQ_W \star G_W \star \wedge dQ_W \right]$$

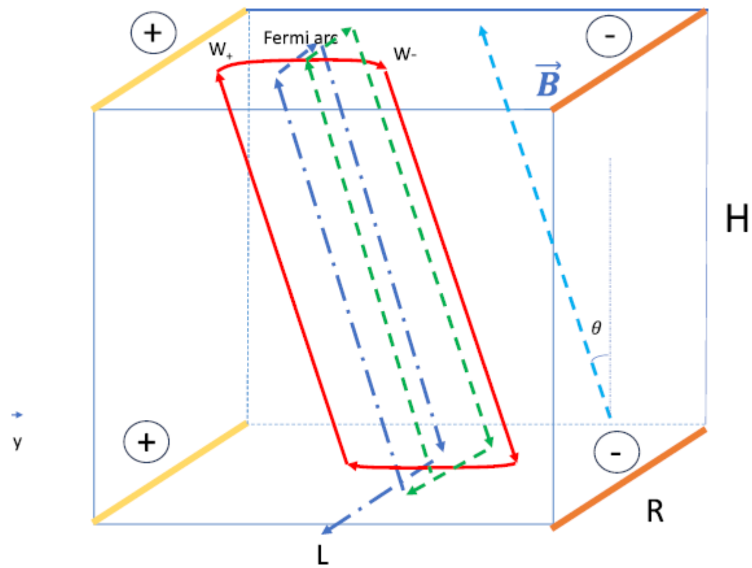
Surface Σ_3 surrounds the positions of Weyl points

The Green function entering this expression is the complete one with interactions taken into account

M A Zubkov 2024 *J. Phys.: Condens. Matter* 36 415501

Proposal for experimental detection of CSE in magnetic Weyl semimetals

CSE



Contribution to QHE conductivity due to the CSE

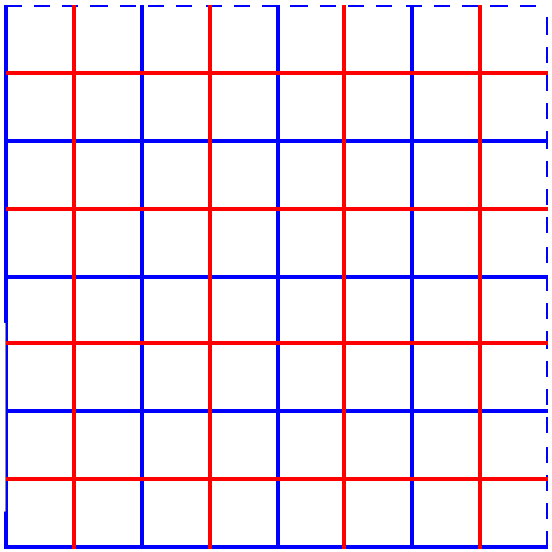
$$\Sigma_{xy}^{\text{Weyl}} = 2\frac{e\left(\mu-\mu_0\right)\beta}{4\pi^2\hbar^2}\frac{1}{B_\perp v_F^{(s)}}L.$$

Precise Wigner – Weyl calculus. Finite rectangular lattice

$$\mathcal{O} = \{(m_1, ..., m_D) | m_i \in \{0, 1, 2, ..., N-1\}\}$$

$$\mathcal{O}' = \{(m_1, ..., m_D) | m_i \in \{0, 1/2, 1, ..., N-1/2\}\}$$

$$\mathcal{M} = \{(m_1 \frac{2\pi}{N}, ..., m_D \frac{2\pi}{N}) | m_i \in \{0, 1, 2, ..., N-1\}\}$$



refined lattice $\mathcal{O}'$

$$\mathcal{M}' = \{(2\pi m_1/N, ..., 2\pi m_D/N) | m_i \in \{0, 1/2, 1, ..., N-1/2\}\}$$

Weyl symbol of operator

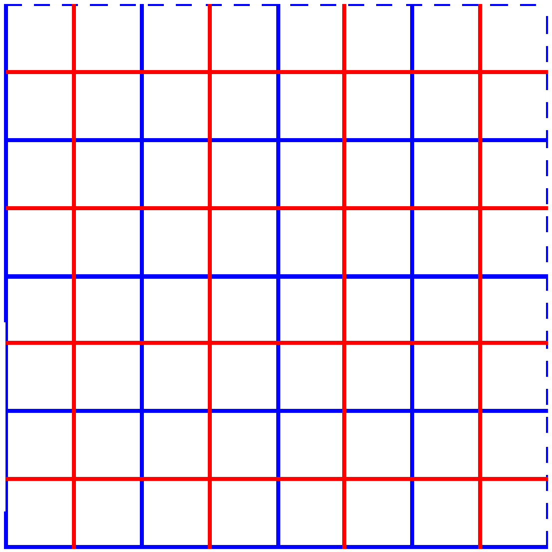
$$A_W(p,q) = \sum_{n_i=0,1; v \in \mathcal{O}'} e^{2ipv} \langle q - v + n/2 | \hat{A} | q + v + n/2 \rangle$$
$$\Pi_i \frac{1 + e^{2iv_i\pi/(N)}}{2} \frac{1 + e^{2\pi i (q_i - v_i + n_i/2)}}{2}.$$

Precise Wigner – Weyl calculus. Finite rectangular lattice

$$\mathcal{O} = \{(m_1, ..., m_D) | m_i \in \{0, 1, 2, ..., N-1\}\}$$

$$\mathcal{O}' = \{(m_1, ..., m_D) | m_i \in \{0, 1/2, 1, ..., N-1/2\}\}$$

$$\mathcal{M} = \{(m_1 \frac{2\pi}{N}, ..., m_D \frac{2\pi}{N}) | m_i \in \{0, 1, 2, ..., N-1\}\}$$



refined lattice $\mathcal{O}'$

$$\mathcal{M}' = \{(2\pi m_1/N, ..., 2\pi m_D/N) | m_i \in \{0, 1/2, 1, ..., N-1/2\}\}$$

Weyl symbol of operator for continuous arguments

$$A_W(p, q) = \sum_{p_1 \in \mathcal{M}'; q_1 \in \mathcal{O}'; p_2 \in \mathcal{M}'; q_2 \in \mathcal{O}'} \frac{1}{(4N^2)^D} e^{2i((p_2-p)(q_1-q)+(q_2-q)(p-p_1))} A_W(p_1, q_1)$$

Properties of Weyl symbol

$$\mathrm{Tr}\,\hat{A} = \frac{1}{(4N)^D} \sum_{p\in\mathcal{M}',q\in\mathcal{O}'} A_W(p,q)$$

$$\mathrm{Tr}\hat{A}\hat{B} = \frac{1}{(4N)^D} \sum_{p\in\mathcal{M}',q\in\mathcal{O}'} A_W(p,q)B_W(p,q)$$

$$(\hat{A}\hat{B})_W(p,q)\Big|_{p\in\mathcal{M}',q\in\mathcal{O}'} = A_W(p,q)e^{\frac{i}{2}(\overleftarrow{\partial}_q\overrightarrow{\partial}_p - \overleftarrow{\partial}_p\overrightarrow{\partial}_q)}B_W(p,q) = A_W(p,q)\star B_W(p,q)$$

$$1_W(p,q)\Big|_{p\in\mathcal{M}',q\in\mathcal{O}'} = 1$$

translation to one lattice spacing

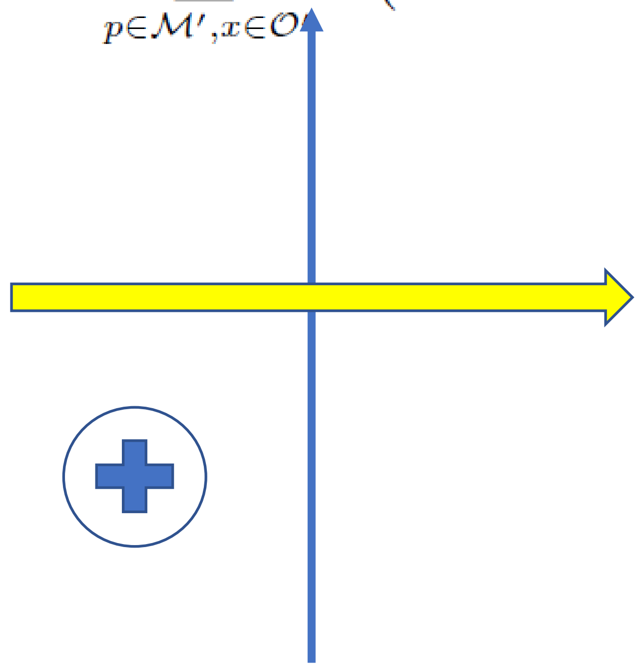
$$T_j(p,q) = e^{ip_j}\Big(\frac{1+e^{2i\pi/N}}{2} + e^{iNp_j}\frac{1-e^{2i\pi/N}}{2}\Big)$$

Applications: QHE

$$\bar{\sigma}^{ij} = \frac{\mathcal{N}}{2\pi}\epsilon^{ij}$$

$$\mathcal{N} = \frac{1}{3!}\epsilon^{\mu\nu\rho}\frac{1}{(2N)^{2D}}\int d\Pi^3 \sum_{p\in\mathcal{M}',x\in\mathcal{O}} \mathrm{tr}\left(\partial_{\Pi^\mu}\hat{Q}^{\mathrm{M}}_W \star \hat{G}^{\mathrm{M}}_W \star \partial_{\Pi^\nu}\hat{Q}^{\mathrm{M}}_W \star \hat{G}^{\mathrm{M}}_W \star \partial_{\Pi^\rho}\hat{Q}^{\mathrm{M}}_W \star \hat{G}^{\mathrm{M}}_W\right)$$

electric current j



electric field E

M.A. Zubkov (2023)

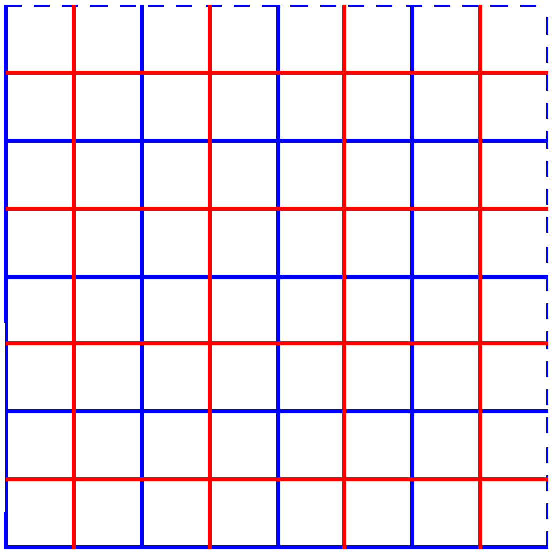
Journal of Physics A: Mathematical and Theoretical 56 (39), 395201

Precise Wigner – Weyl calculus. INFINITE rectangular lattice N → infinity

$$\mathcal{O} = \{(m_1, ..., m_D) | m_i \in \{0, 1, 2, ..., N-1\}\}$$

$$\mathcal{O}' = \{(m_1, ..., m_D) | m_i \in \{0, 1/2, 1, ..., N-1/2\}\}$$

$$\mathcal{M} = \{(m_1 \frac{2\pi}{N}, ..., m_D \frac{2\pi}{N}) | m_i \in \{0, 1, 2, ..., N-1\}\}$$



refined lattice $\mathcal{O}'$

$$\mathcal{M}' = \{(2\pi m_1/N, ..., 2\pi m_D/N) | m_i \in \{0, 1/2, 1, ..., N-1/2\}\}$$

Weyl symbol of operator (momentum space becomes continuous)

$$A_W(p,q) = \int_{\mathcal{M}} d^D p_- \, \langle\langle p - p_- | \hat{A} | p + p_- \rangle\rangle \, e^{2ip_- q} \Pi_i (1 + e^{ip^i_-})$$

$$\langle\langle p_1 | p_2 \rangle\rangle = \delta(p_1 - p_2).$$

Properties of Weyl symbol N → infinity

$$\mathrm{Tr}\,\hat{A} = \frac{1}{(4N)^D} \sum_{p\in\mathcal{M}',q\in\mathcal{O}'} A_W(p,q)$$

$$\mathrm{Tr}\hat{A}\hat{B} = \frac{1}{(4N)^D} \sum_{p\in\mathcal{M}',q\in\mathcal{O}'} A_W(p,q) B_W(p,q)$$

$$(\hat{A}\hat{B})_W(p,q)\Big|_{p\in\mathcal{M}',q\in\mathcal{O}'} = A_W(p,q) e^{\frac{i}{2}(\overleftarrow{\partial}_q\overrightarrow{\partial}_p - \overleftarrow{\partial}_p\overrightarrow{\partial}_q)} B_W(p,q) = A_W(p,q) \star B_W(p,q)$$

$$1_W(p,q)\Big|_{p\in\mathcal{M}',q\in\mathcal{O}'} = 1$$

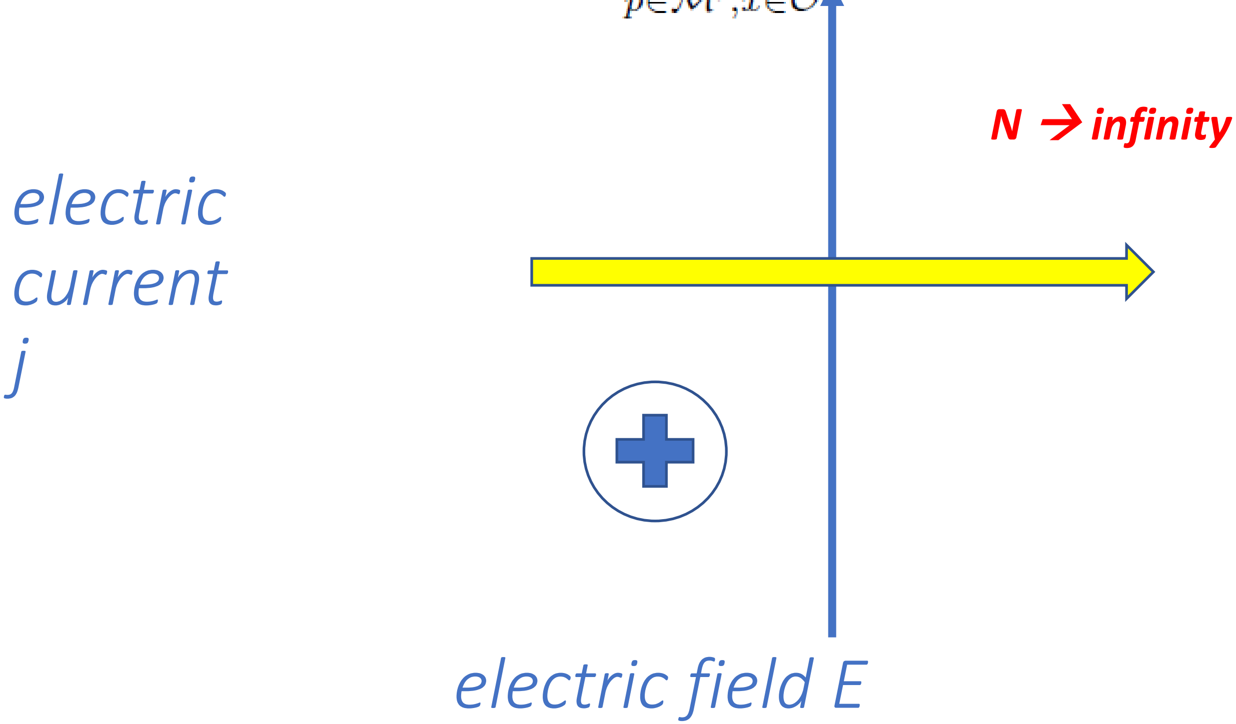
translation to one lattice spacing

$$T_j(p,q) = e^{ip_j}\Big(\frac{1+e^{2i\pi/N}}{2} + e^{iNp_j}\frac{1-e^{2i\pi/N}}{2}\Big) \qquad T_j(p,q) \to e^{ip_j}$$

Applications: QHE

$$\bar{\sigma}^{ij} = \frac{\mathcal{N}}{2\pi}\epsilon^{ij}$$

$$\mathcal{N} = \frac{1}{3!}\epsilon^{\mu\nu\rho}\frac{1}{(2N)^{2D}}\int d\Pi^3 \sum_{p\in\mathcal{M}',x\in\mathcal{O}} \mathrm{tr}\left(\partial_{\Pi^\mu}\hat{Q}^{\mathrm{M}}_W \star \hat{G}^{\mathrm{M}}_W \star \partial_{\Pi^\nu}\hat{Q}^{\mathrm{M}}_W \star \hat{G}^{\mathrm{M}}_W \star \partial_{\Pi^\rho}\hat{Q}^{\mathrm{M}}_W \star \hat{G}^{\mathrm{M}}_W\right)$$



I.V. Fialkovsky, M.A. Zubkov (2020)
Nuclear Physics B 954, 114999

Precise Wigner – Weyl calculus. INFINITE HONEYCOMB lattice

N → infinity

coordinate space

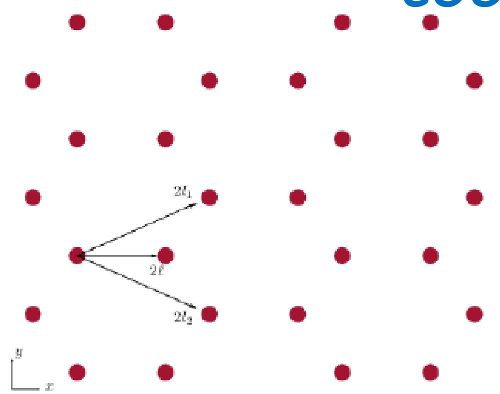


FIG. 2. An illustration of the physical lattice $\mathscr{O}$.

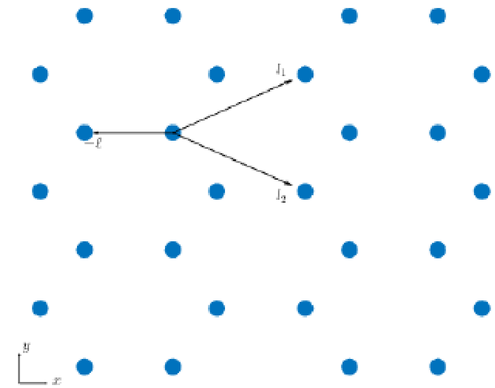


FIG. 5. An illustration of the extended lattice $\mathfrak{D}$.

Momentum space

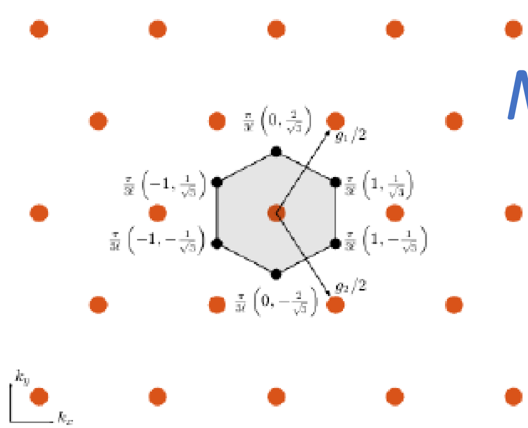


FIG. 3. The first Brillouin zone and the reciprocal lattice of $\mathscr{O}$.

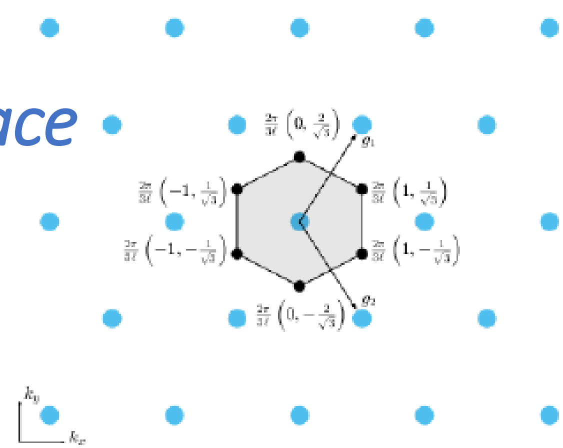


FIG. 6. The first Brillouin zone and the reciprocal lattice of $\mathfrak{D}$.

Precise Wigner – Weyl calculus. INFINITE HONEYCOMB lattice N → infinity

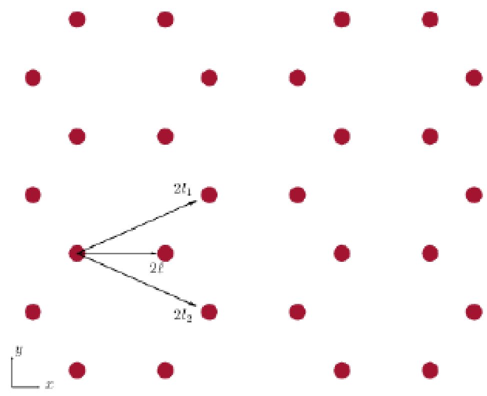


FIG. 2. An illustration of the physical lattice $\mathscr{O}$.

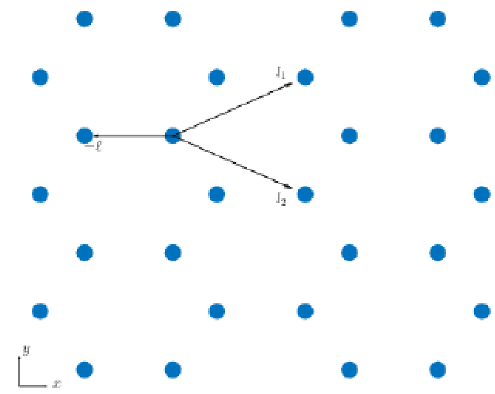


FIG. 5. An illustration of the extended lattice $\mathfrak{D}$.

Weyl symbol of operator

$$A_W(x,p) \equiv \int_{\mathscr{M}} d^2q e^{2ixq} \langle p+q|\hat{A}|p-q\rangle \times \left(1+e^{-2il_1q}+e^{-2il_2q}+e^{-2i(l_1+l_2)q}\right)$$

Properties of Weyl symbol N → infinity

$$\mathrm{Tr}\,\hat{A} = \sum_{x \in \mathfrak{D}} \int_{\mathscr{M}} \frac{d^2 p}{|\mathfrak{M}|} A_W(x,p)$$

$$\mathrm{Tr}\hat{A}\hat{B} = \sum_{x \in \mathfrak{D}} \int_{\mathscr{M}} \frac{d^2 p}{|\mathfrak{M}|} A_W(x,p) B_W(x,p)$$

$$(\hat{A}\hat{B})_W(x,p)\Big|_{p \in \mathcal{M}, x \in \mathfrak{D}} = A_W(p,q) e^{\frac{i}{2}(\overleftarrow{\partial_q}\overrightarrow{\partial_p} - \overleftarrow{\partial_p}\overrightarrow{\partial_q})} B_W(p,q)$$

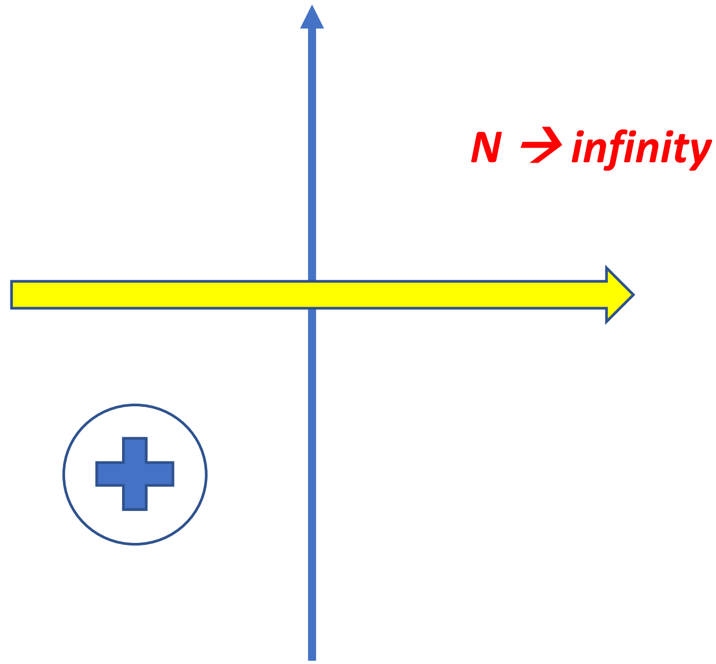
$$1_W(x,p)\Big|_{p \in \mathcal{M}, x \in \mathfrak{D}} = 1$$

Applications: QHE

$$\bar{\sigma}^{ij} = \frac{\mathcal{N}}{2\pi}\epsilon^{ij}$$

$$\mathcal{N} = \frac{1}{24\,\pi^2}\epsilon^{\mu\nu\rho}\frac{1}{|\mathfrak{D}|}\int dP^0 \int_{\mathscr{M}} d^2\vec{P} \sum_{x\in\mathfrak{D}} \mathrm{tr}\left(\partial_{\Pi^\mu}\hat{Q}^{\mathrm{M}}_{W} \star \hat{G}^{\mathrm{M}}_{W} \star \partial_{\Pi^\nu}\hat{Q}^{\mathrm{M}}_{W} \star \hat{G}^{\mathrm{M}}_{W} \star \partial_{\Pi^\rho}\hat{Q}^{\mathrm{M}}_{W}\big|\star \hat{G}^{\mathrm{M}}_{W}\right)$$

electric current j



electric field E

R. Chobanyan, M.A. Zubkov
arXiv preprint arXiv:2302.00723

We can use the precise Wigner – Weyl calculus dealing with ***any lattice regularized continuum quantum field theory*** *and dealing with the lattice models of solid state physics* ***if the external magnetic field strength is of the order of 10 000 Tesla (unphysical!)*** *while wavelength of external electromagnetic field is of the order of* ***1 nanometer***

Which is more important, we can use this formalism for artificial lattices, when magnetic flux through the EFFECTIVE lattice cell is compared to 1

Conclusions

- Wigner – Weyl calculus allows to represent in compact form the conductivities of non – dissipative transport phenomena **<u>in non – uniform systems</u>**.
- Equilibrium systems at zero temperature: QHE conductivity is given by topological invariant composed of the Wigner transformed two-point Green functions. This expression is not renormalized by interactions (perturbatively)

Conclusions

- Equilibrium systems at finite temperatures: CME response of electric current to magnetic field is the topological invariant in phase space → the equilibrium CME is absent
- Out of equilibrium the CME is back if chiral chemical potential depends on time and if the corresponding frequency tends to zero (i.e. the system is approaching to equilibrium).
- The CME contribution to magnetoresistance as calculated using Keldysh technique is given by the standard expression (though with the renormalized expression for the relaxation time).

Conclusions

- CSE conductivity is given by the topological invariant. It is not renormalized by interactions both in QCD and in Weyl semimetals.
- The way to observe experimentally the CSE in magnetic Weyl semimetals is proposed based on the observation of the QHE conductivity.
- Precise Wigner – Weyl calculus is built for the lattice models, which allows to investigate the systems with artificial lattices (when magnetic flux through the lattice cell becomes large) – these are the systems that possess Hofstadter butterfly.

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