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Chiral transport phenomena in core-collapse supernovae

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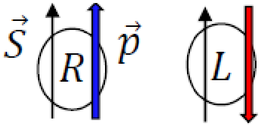
Exotic transport in chiral matter

- Chiral fermions (massless fermions) : $J^{\mu} = J^{\mu}_{\mathrm{R}} + J^{\mu}_{\mathrm{L}}, \quad J^{\mu}_{5} = J^{\mu}_{\mathrm{R}} - J^{\mu}_{\mathrm{L}}.$

chirality=helicity ($\vec{S} \cdot \hat{p}$)



classically : $\partial_{\mu} J^{\mu}_{R/L} = 0$

- Chiral anomaly : $\partial_{\mu} J^{\mu}_{R/L} = \pm \dfrac{\mathbf{E} \cdot \mathbf{B}}{4\pi^{2}} \Longrightarrow \partial_{\mu} J^{\mu}_{5} = \dfrac{\mathbf{E} \cdot \mathbf{B}}{2\pi^{2}}$ S. Adler, J. Bell, R. Jackiw, 69

- Chiral magnetic effect (CME) : $J^{\mu} = \boxed{\xi_{B}} B^{\mu}, \quad \xi_{B} = \dfrac{\mu_{5}}{2\pi^{2}}.$

parity odd

axial chemical potential

$(\mu_{5} = (\mu_{\mathrm{R}} - \mu_{\mathrm{L}})/2)$

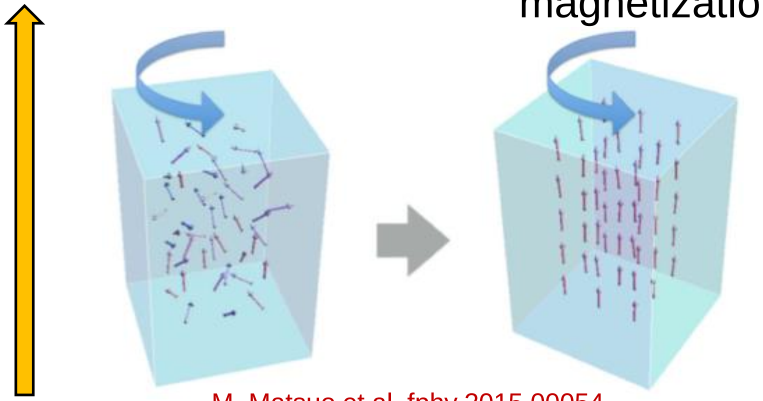
parity violation

(in a quasi-equilibrium cond.)

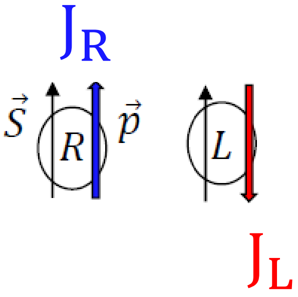
A. Vilenkin, PRD 22, 3080 (1980)

K. Fukushima, D. Kharzeev, H. Warringa, PRD78, 074033 (2008)

magnetic field: B



M. Matsuo et al, fphy.2015.00054



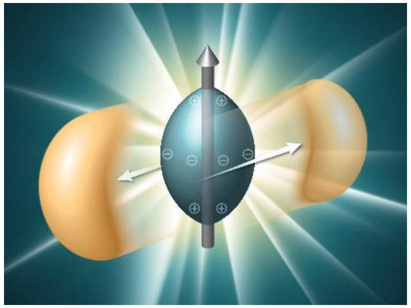
$J = J_R + J_L \neq 0$

when $n_R - n_L = n_5 \neq 0$

Where to find chiral transport in the real world?

- Nuclear physics : relativistic heavy ion collisions

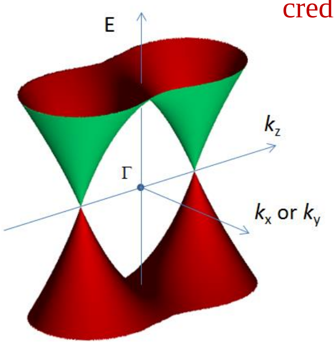
Review : D. Kharzeev, et.al, Prog. Part. Nucl. Phys. 88, 1 (2016)



credit : BNL

- Condensed matter : Weyl semimetals

Textbook : E. V. Gorbar, et.al,
“Electric Properties of Dirac and Weyl Semimetals”
(World Scientific, 2021)



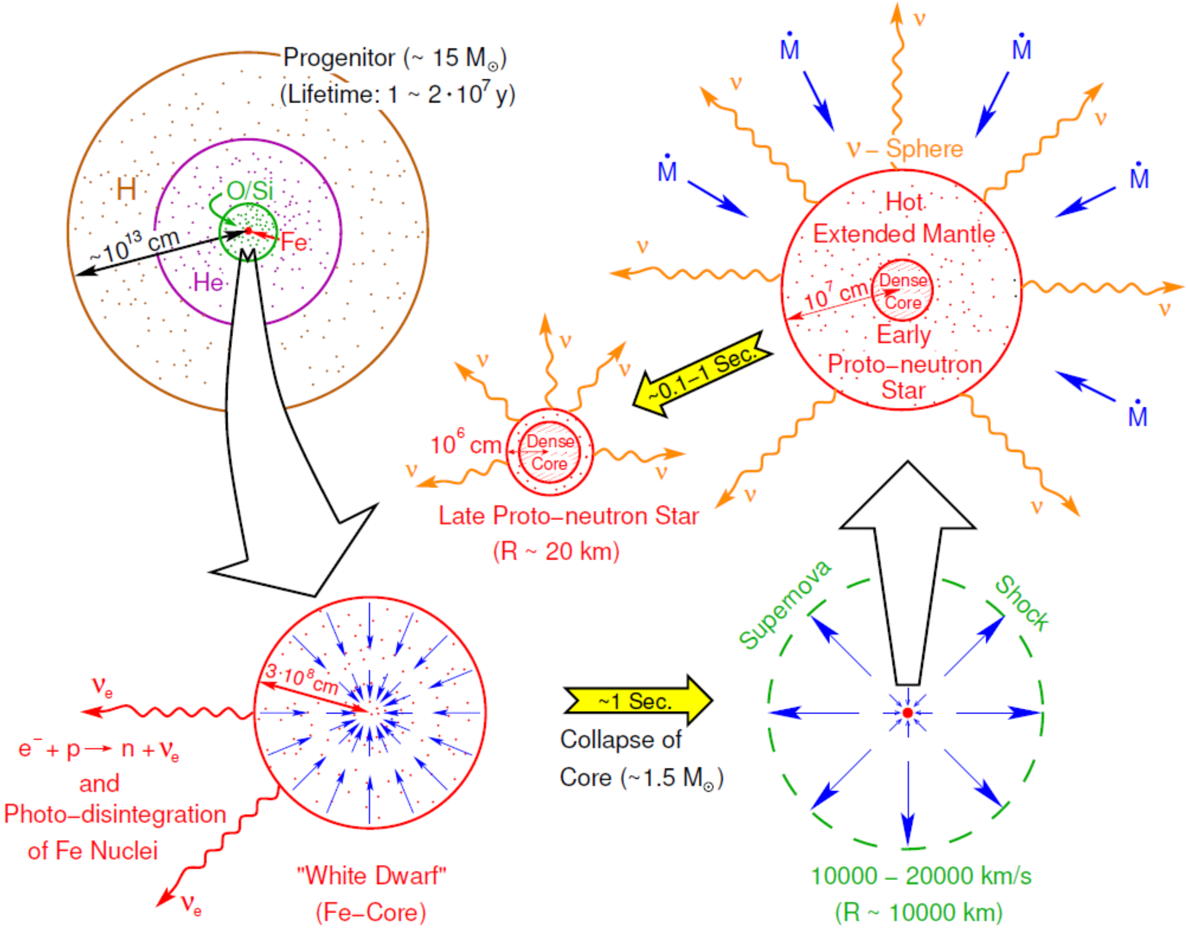
Borisenko et al., Phys. Rev. Lett. 113, 027603 (2014)

- Astrophysics : core-collapse supernovae

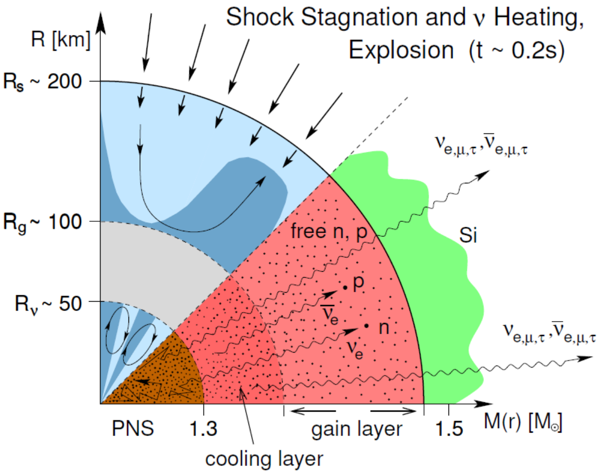
Review : K. Kamada, N. Yamamoto, DY,
Prog. Part. Nucl. Phys. 129 (2023) 104016

credit : RIKEN

Evolution of core-collapse supernovae (CCSN)

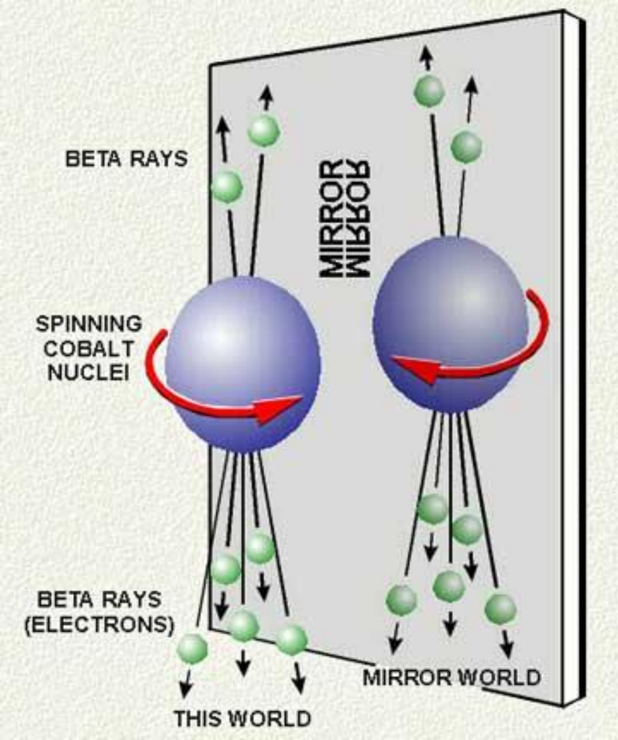


H.-Th. Janka, arXiv:1702.08713



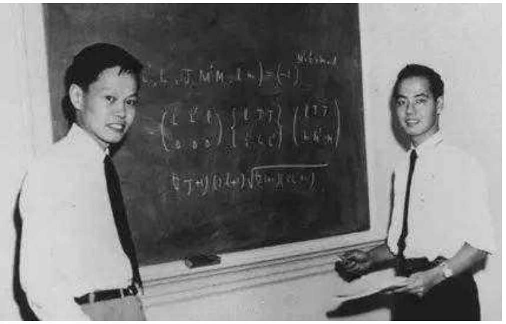
H.-Th. Janka et al., astro-ph/0612072

Parity violation & weak interaction



http://physics.nist.gov/GenInt/Parity/cover.html]

$$n \to p + e_{\mathrm{L}} + \bar{\nu}_{\mathrm{R}}$$



Lee & Yang (th) 1956

Wu et al., (exp) 1957

- Neutrinos play an important role for CCSN and interact with matter via the weak int.
- Intrinsic parity violation for weak interaction : essential for chiral effects
- What will be the transport properties for “chiral” leptons under parity (chirality) violation?

ultrarelativistic (nearly massless) neutrinos & electrons considered ; no BSM

Chiral effects on dense stars?

- Three long-standing puzzles :

○ Pulsar kicks : the origin of momentum asymmetry for neutron stars? $v \sim 10^2$ km/s
A. Lyne, D. Lorimer, Nature 369, 127 (1994).
V. Kaspi, et al., Nature 381, 584 (1996).

○ Magnetars : the origin of strong & stable magnetic fields? $B \sim 10^{15}$ G
review: A. K. Harding, D. Lai, Rept. Prog. Phys. 69, 2631 (2006)

○ Explosions of CCSN with the observed energy?

❖ Chiral effects as a "microscopic" mechanism may provide possible explanations (qualitatively) in a self consistent framework. (disclaimer : not the only solution)

- Radiation hydrodynamics : e.g. S. W. Bruenn, Astrophys. Jl. Suppl. 58 (1985) 771.

magneto-hydrodynamics (MHD) + relativistic kinetic theory (Boltzmann eq.)

matter (e, n, p) in equilibrium radiation (ν) out of equilibrium

- Constructing chiral radiation hydrodynamics :

chiral magneto-hydrodynamics (ChMHD) + chiral kinetic theory (CKT)

review : K. Kamada, N. Yamamoto, DY, PPNP 129 (2023) 104016

Chiral radiation transport equation

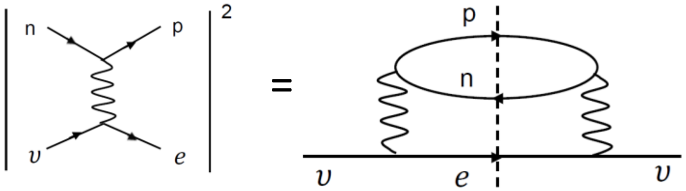
- CKT for neutrinos : $\underbrace{q \cdot D}_{\text{Boltzmann eq. in the inertial frame}} f_q^{(\nu)} = \underbrace{(1 - f_q^{(\nu)})\Gamma_q^{<} - f_q^{(\nu)}\Gamma_q^{>}}_{\text{collision term with quantum corrections}},$

N. Yamamoto & DY, APJ 895 (2020), 1

- Neutrino absorption : $\bar{\Gamma}_q^{\lessgtr} \approx \bar{\Gamma}_q^{(0)\lessgtr} + \bar{\Gamma}_q^{(\omega)\lessgtr}(q \cdot \omega) + \bar{\Gamma}_q^{(B)\lessgtr}(q \cdot B),$

$\nu_{\mathrm{L}}^{\mathrm{e}}(q) + \mathrm{n}(k) \rightleftharpoons \mathrm{e}_{\mathrm{L}}(q') + \mathrm{p}(k')$

vorticity & magnetic field corrections :
breaking spherical symmetry & axisymmetry



analytic expressions : $$\bar{\Gamma}_q^{(0)>} \approx \frac{(g_{\mathrm{V}}^2 + 3g_{\mathrm{A}}^2)}{\pi} G_{\mathrm{F}}^2 (q \cdot u)^3 (1 - f_{0,q}^{(\mathrm{e})}) \left(1 - \frac{3q \cdot u}{Mc^2}\right) \frac{n_{\mathrm{n}} - n_{\mathrm{p}}}{1 - \mathrm{e}^{\beta(\mu_{\mathrm{p}} - \mu_{\mathrm{n}})}},$$

$$\bar{\Gamma}_q^{(\omega)>} \approx \frac{(g_{\mathrm{V}}^2 + 3g_{\mathrm{A}}^2)}{2\pi M} G_{\mathrm{F}}^2 (q \cdot u)^2 (1 - f_{0,q}^{(\mathrm{e})}) \left(\frac{2}{E_{\mathrm{i}}} + \beta f_{0,q}^{(\mathrm{e})}\right) \frac{n_{\mathrm{n}} - n_{\mathrm{p}}}{1 - \mathrm{e}^{\beta(\mu_{\mathrm{p}} - \mu_{\mathrm{n}})}},$$

$$\bar{\Gamma}_q^{(B)>} \approx \frac{(g_{\mathrm{V}}^2 + 3g_{\mathrm{A}}^2)}{2\pi M} G_{\mathrm{F}}^2 (q \cdot u) (1 - f_{0,q}^{(\mathrm{e})}) \left(1 - \frac{8q \cdot u}{3Mc^2}\right) \frac{n_{\mathrm{n}} - n_{\mathrm{p}}}{1 - \mathrm{e}^{\beta(\mu_{\mathrm{p}} - \mu_{\mathrm{n}})}}.$$

$\bar{\Gamma}_q^{(0)>}$: S. Reddy, M. Prakash, J. M. Lattimer, PRD58:013009,1998

Neutrino flux driven by magnetic fields

- Energy-momentum tensor & neutrino current :

$$T^{\mu\nu}_{(\nu)} = \int_q 4\pi\delta(q^2)\Big(q^\mu q^\nu f^{(\nu)}_q - q^{\{\mu}S^{\nu\}\rho}_q \mathcal{D}_\rho f^{(\nu)}_q\Big)\,,$$

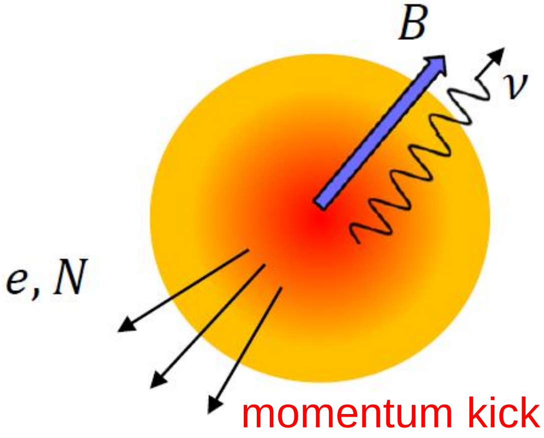
$$J^\mu_{(\nu)} = \int_q 4\pi\delta(q^2)\Big(q^\mu f^{(\nu)}_q - S^{\mu\rho}_q \mathcal{D}_\rho f^{(\nu)}_q\Big)\,, \qquad \mathcal{D}_\mu f^{(\nu)}_q \equiv D_\mu f^{(\nu)}_q - \mathcal{C}_\mu[f^{(\nu)}_q]$$

- The momentum kick from neutrinos near equilibrium :

N. Yamamoto & DY,
PRD 104, 123019 (2021)

$$\Delta T^{i0}_\nu = -\kappa(\boldsymbol{\nabla}\cdot\boldsymbol{v})\mu_\nu B^i, \quad \kappa = \frac{1}{72\pi M G_{\rm F}^2(g_{\rm V}^2+3g_{\rm A}^2)}\,\frac{{\rm e}^{2\beta(\mu_{\rm n}-\mu_{\rm p})}}{n_{\rm n}-n_{\rm p}}$$

$$\text{for } \mu_{\rm n}-\mu_{\rm p} \gg T = \beta^{-1}.$$



❖ Pulsar kicks : $\Delta T^{i0}_{\rm e} = -\Delta T^{i0}_\nu$

(momentum con.)

❖ Effective CME (w/o μ_5) : $\Delta T^{i0}_{\rm e} = \mu_{\rm e}\Delta J^i_{\rm e}$

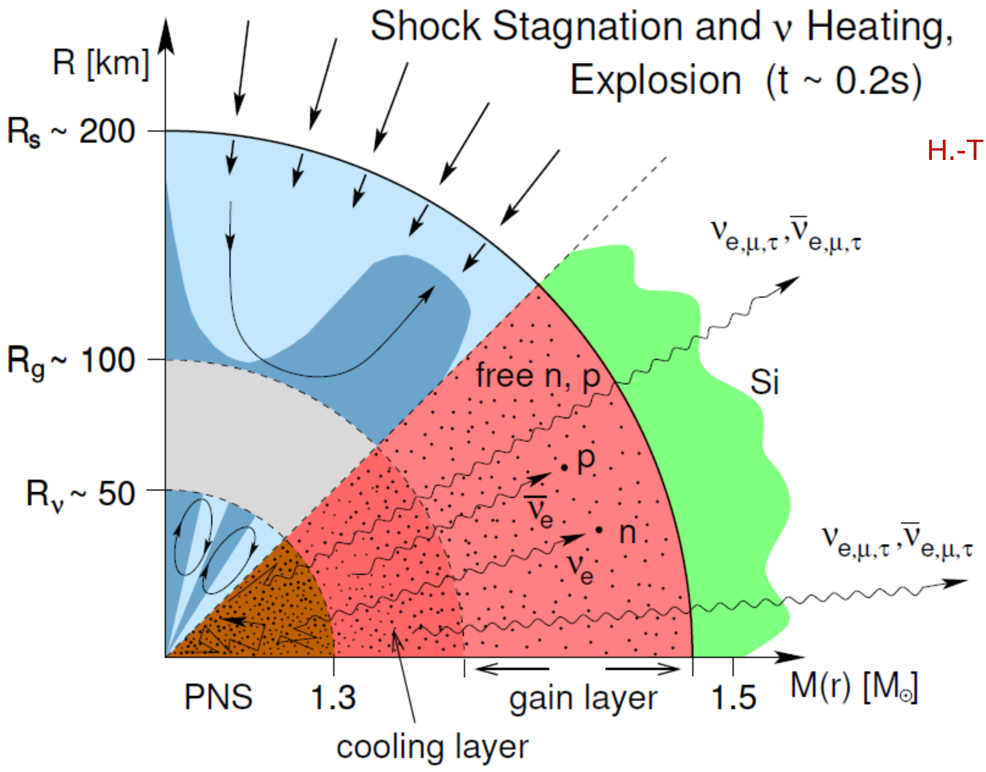
⇨ $\Delta J^i_{\rm e} = \xi_B B^i,$ (effective CME out of equilibrium)

$$\xi_B = -\kappa(\boldsymbol{\nabla}\cdot\boldsymbol{v})\frac{\mu_\nu}{\mu_{\rm e}}\,. \quad \text{(effective } \mu_5\text{)}$$

see also
A. Vilenkin, Astrophys. J. 451, 700 (1995)
M. Kaminski et al., PLB 760, 170 (2016)
K. Fukushima, C. Yu, arXiv: 2401.04568

Shock revival by neutrino heating

- Heating by non-equilibrium neutrinos :



H.-Th. Janka et al., astro-ph/0612072

❖ How to obtain chiral effects from non-equilibrium neutrinos in the gain region?

We can solve the chiral kinetic equation for left-handed electrons near equilibrium with neutrino radiation.

Effective CME from neutrinos out of equilibrium

- Effective CME from neutrino radiation (far from equilibrium) :

N. Yamamoto, DY,
PRL 131, 012701 (2023)

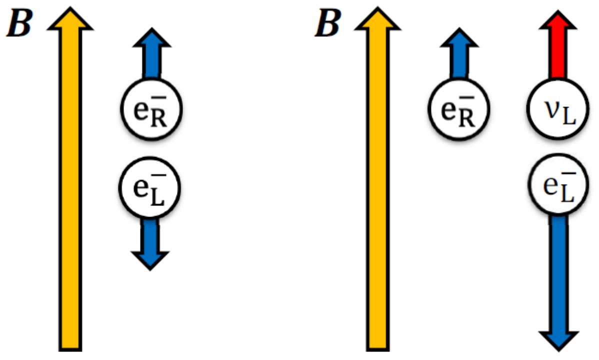
$$j^\mu_B(x) \approx e^2 \int \frac{d^4q}{(2\pi)^3}\frac{\delta(q^2)}{q_0}\left(B^\mu q\cdot\partial_q - q\cdot B\partial^\mu_{\bar q}\right)\delta f^{(\mathrm{e})}_{\mathrm{W}} = \xi_B B^\mu,$$

$$\delta f^{(\mathrm{e})}_{\mathrm{W}}(q,x) = -\frac{1}{q_0}\int_0^{\Delta t} \mathrm{d}t'_0 F_{\mathrm{W}}(q,x')|_{\mathrm{c}}\,,$$

$$F_{\mathrm{W}} \approx \frac{(q\cdot u)^3}{\pi}(g_{\mathrm{V}}^2+3g_{\mathrm{A}}^2)G_{\mathrm{F}}^2(n_{\mathrm{p}}-n_{\mathrm{n}})\left[\frac{\bar f^{(\mathrm{e})}(1-f^{(\nu)})}{1-\mathrm{e}^{\beta(\mu_{\mathrm{n}}-\mu_{\mathrm{p}})}} + \frac{(1-\bar f^{(\mathrm{e})})f^{(\nu)}}{1-\mathrm{e}^{\beta(\mu_{\mathrm{p}}-\mu_{\mathrm{n}})}}\right].$$

$\Delta t \nearrow \;\Rightarrow\; \xi_B \nearrow$

(neutrino emission time)



(a) $j_B = 0$ (b) $j_B \neq 0$

- Approximate upper bounds (in the gain region) :

$\boxed{\xi_B^{\mathrm{tot}} \approx -0.5\,\mathrm{MeV}}$ for $\Delta t = 0.1$ s

- ☐ Kick velocity : $v_{\mathrm{kick}} \sim \frac{|T^{i0}_{B,\mathrm{tot}}|}{\rho_{\mathrm{core}}} \approx \left(\frac{eB}{10^{13\text{-}14}\,\mathrm{G}}\right)\mathrm{km/s}\,.$

$B \sim 10^{15-16}$ G

for

$v_{\mathrm{kick}} \sim 10^2$ km/s

see also A. Vilenkin, Astrophys. J. 451, 700 (1995).

Chiral plasma instability

- Chiral plasma instability (CPI) :

M. Joyce, M. E. Shaposhnikov, PRL **79**, 1193 (1997)

Y. Akamatsu, N. Yamamoto, PRL 111, 052002 (2013)

- Anomalous Maxwell's eq. :

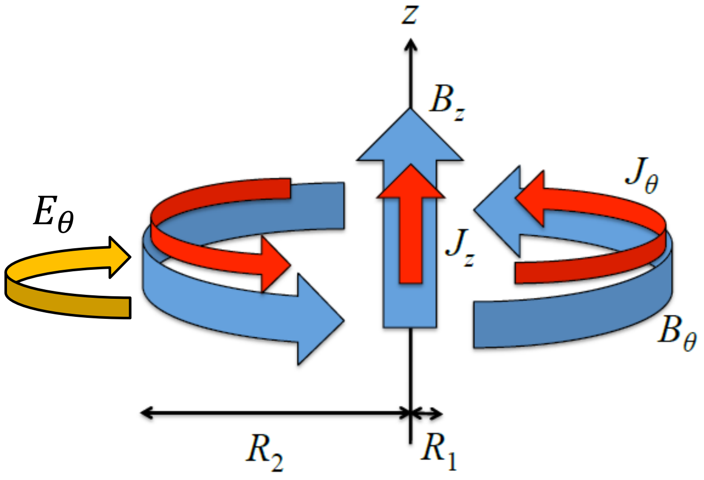
$$\partial_t \boldsymbol{B} = -\boldsymbol{\nabla}\times\boldsymbol{E}, \qquad \boldsymbol{\nabla}\times B = \eta^{-1}\boldsymbol{E} + \boxed{\xi_B \boldsymbol{B}}.$$

CME

$$\Rightarrow \quad \frac{\partial \boldsymbol{B}}{\partial t} = \boxed{\eta\boldsymbol{\nabla}^2\boldsymbol{B}} + \boxed{\eta\,\boldsymbol{\nabla}\times(\xi_B\boldsymbol{B})}$$

diffusion

CME (instability)



Y. Akamatsu, N. Yamamoto, PRD 90, 125031 (2014)

- Unstable modes at long wavelength : $\delta\boldsymbol{B} \propto \mathrm{e}^{\sigma t + \mathrm{i}\boldsymbol{k}\cdot\boldsymbol{x}}$

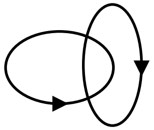
$$\sigma = \eta k(\xi_B - k)$$ (σ > 0 for small k, long wavelength)

- Helicity conservation :

$$\frac{dH_{\text{tot}}}{dt} = 0,$$

$$H_{\text{tot}} \equiv N_5 + \frac{H_{\text{mag}}}{4\pi^2},$$

$$N_5 \equiv \int d^3x\, n_5, \quad H_{\text{mag}} \equiv \int d^3x\, \boldsymbol{A}\cdot\boldsymbol{B}.$$



(linking of magnetic fluxes)

exchange btw the (effective) axial charge & magnetic helicity

Y. Hirono, D. Kharzeev, Y. Yin, PRD 92, 125031 (2015)

Local ChMHD simulations

- ChMHD : energy-momentum & charge conservation + helicity conservation

J. Matsumoto, N. Yamamoto, DY, PRD 105, 123029 (2022)

see also Y. Masada et al., PRD 98, 083018 (2018)

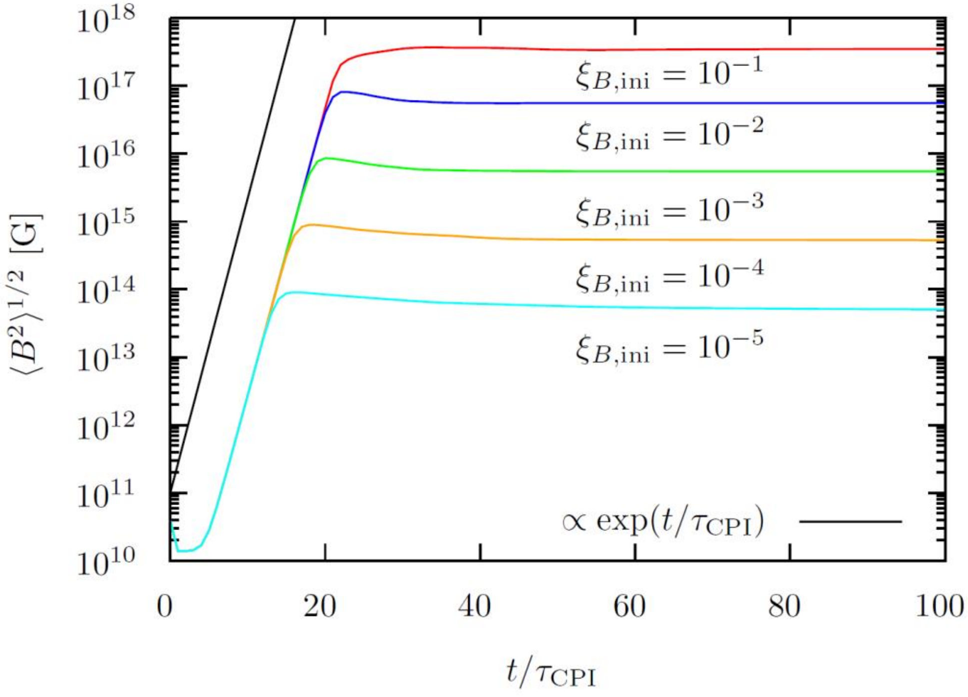
- Local simulations :

resistivity : $\eta = 1$ viscosity : $\nu = 0.01$

in the units of $100\,\mathrm{MeV} = 1$

TABLE I. Summary of the simulation runs.

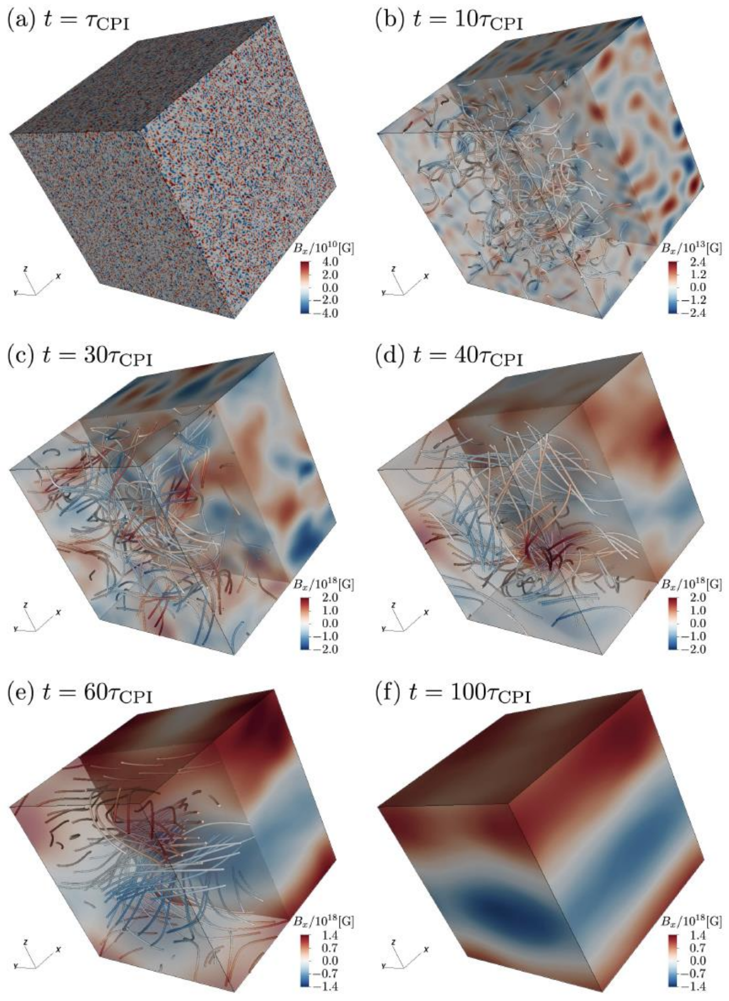
Name	L	$\xi_{B,\mathrm{ini}}$	τ_{CPI}
Model 1	8×10^2	10^{-1}	4×10^2
Model 2	8×10^3	10^{-2}	4×10^4
Model 3	8×10^4	10^{-3}	4×10^6
Model 4	8×10^5	10^{-4}	4×10^8
Model 5	8×10^6	10^{-5}	4×10^{10}



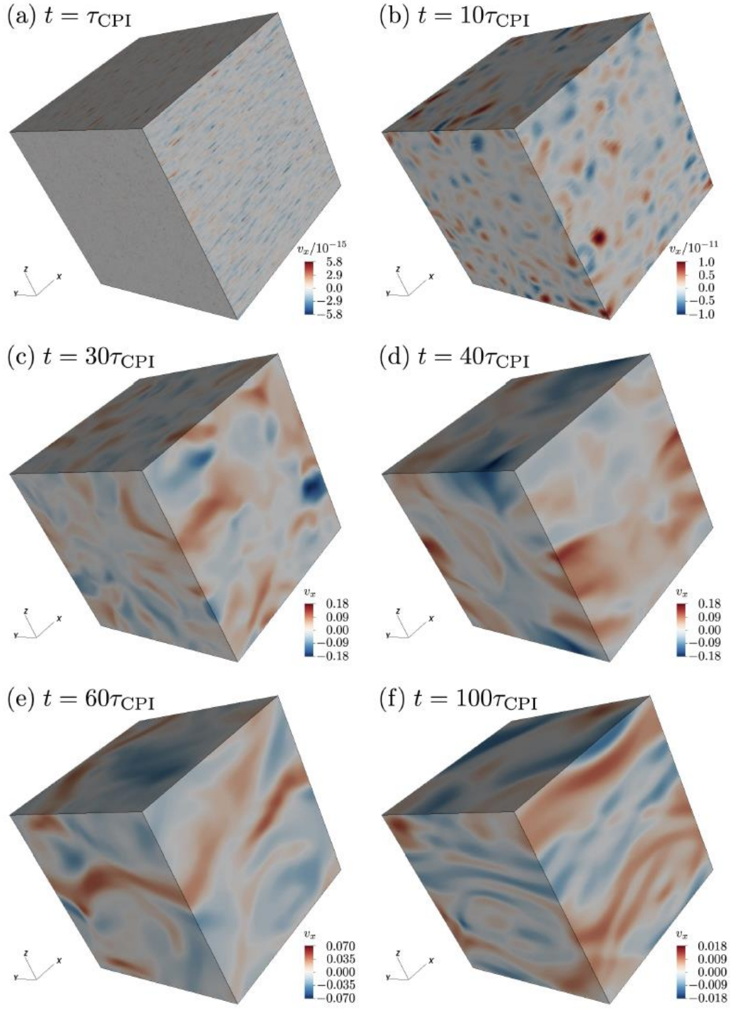
$$B_{\mathrm{CPI}} \sim \mu_{5,\mathrm{eff}}^2$$

Inverse cascade

- Inverse cascade : opposite to the direct energy cascade for turbulence in 3D



magnetic field



fluid velocity

Neutrino spin Hall effect

- Wigner functions up to $\mathcal{O}(\hbar)$, $\mathcal{O}(\bar{\Sigma}_\chi)$, and $\mathcal{O}(\Sigma_\chi^{\lessgtr})$: N. Yamamoto, DY, PRD 109, 056010 (2024)

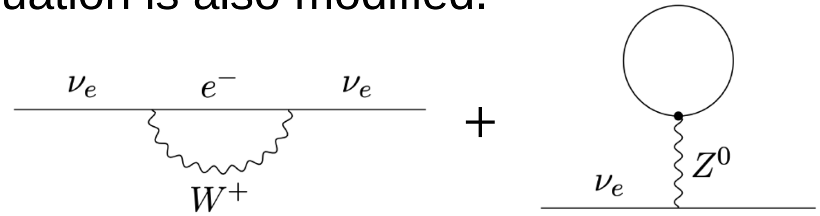
$$\mathcal{W}_\chi^{<\mu} = 2\pi \left[\delta(\tilde{q}^2)\left(\tilde{q}^\mu + \chi\hbar S_{\tilde{q}}^{\mu\nu}\tilde{\mathcal{D}}_\nu\right) + \frac{\chi\hbar}{2}\delta'(\tilde{q}^2)\epsilon^{\mu\nu\rho\sigma}\tilde{q}_\nu \boxed{\left(F_{\rho\sigma} + \Delta_{[\rho}\bar{\Sigma}_{\chi\sigma]}\right)} \right] f_\chi ,$$

$$\tilde{\mathcal{D}}_\rho = \mathcal{D}_\rho + (\Delta_\nu \bar{\Sigma}_{\chi\rho})\partial_q^\nu - (\partial_{q\nu}\bar{\Sigma}_{\chi\rho})\partial^\nu , \quad \tilde{q}_\mu = q_\mu - \bar{\Sigma}_{\chi\mu} .$$

e.g., gradient of thermal mass :
effective EM fields

⇨ The chiral kinetic equation is also modified.

- Neutrino self-energy :



$\sim O(G_F)$

($O(G_F^2)$ from collisions)

$$\bar{\Sigma}_{\rm L}^\mu = V n^\mu + \mathcal{O}(\hbar), \quad V = \frac{G_{\rm F}}{\sqrt{2}} \left[\left(1 + 4\sin^2\theta_{\rm W}\right) N_{\rm e} - N_{\rm n} + \left(1 - 4\sin^2\theta_{\rm W}\right) N_{\rm p} \right] .$$

D. Notzold and G. Raffelt, Nucl. Phys. B 307 (1988) 924

⇨ neutrino spin Hall effect : $J^\mu_{\rm SHE} = \hbar\epsilon^{\mu\nu\rho\sigma}\ell_\nu n_\sigma(\partial_\rho V)$,

anisotropic neutrino distribution: $\ell_\nu = \int \frac{d^4q}{(2\pi)^3}\frac{\delta(q_0 - |\mathbf{q}|)}{2q_0}\partial_{q\nu}f^{(\nu)}(q)$.

❖ Spin polarization from vorticity & shear corrections for massive quarks of $O(g^2)$

S. Fang, S. Pu, DY, PRD 109, 034034 (2024)

Summary & outlook

❖ Summary

- ✓ The chiral effect stemming from “parity violation” should be included for leptonic transport with the weak interaction.
- ✓ Chiral effects with magnetic fields : “effective CME” induced by neutrino radiation, chiral plasma instability, helicity conservation, inverse cascade.
- ✓ Relevant to pulsar kicks, magnetars, and explosion dynamics of CCSN.

❖ Outlook

- ☐ Ultimate goal : global simulations for chiral radiation hydrodynamics with consistent inclusion of the effective CME from neutrino radiation.
- ☐ Chiral effects from vorticity (chiral vortical effect), temperature/chemical-potential gradients (spin-Hall effect), etc.
- ☐ Other applications of chiral effects to dense astrophysical systems?

Thank you!

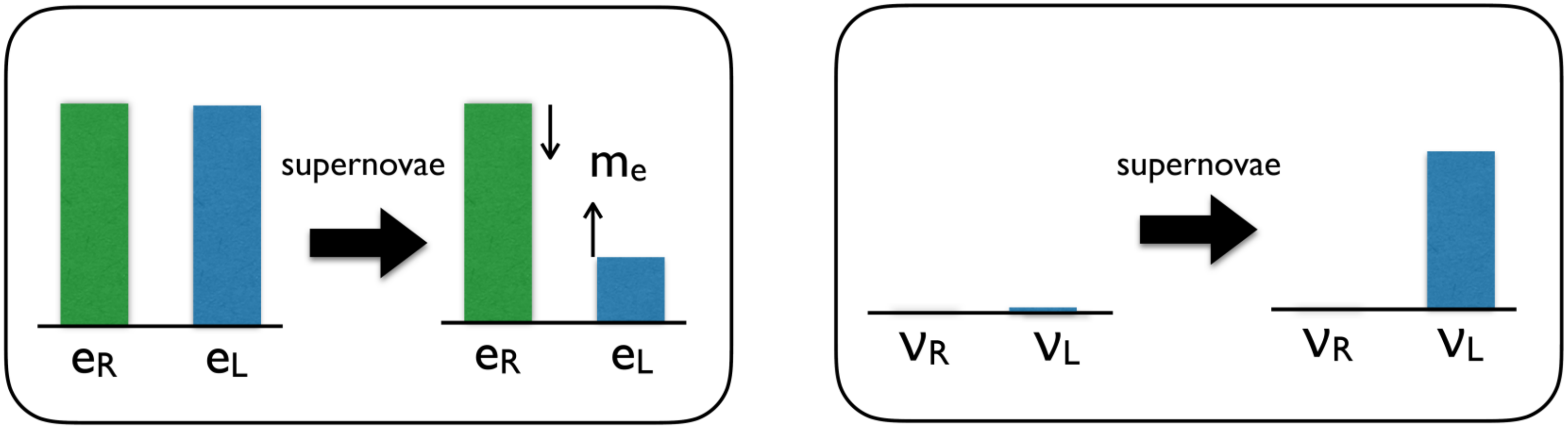
Generation of chirality imbalance

- Electron capture process in supernovae :

A. Ohnishi, N. Yamamoto, 2014, arXiv:1402.4760

an innate lefthander

$$p + \underline{e^{\mathrm{L}}} \to n + \boxed{\nu_e^{\mathrm{L}}}$$



D. Grabowska, D.B. Kaplan, S. Reddy, PRD 91 (8) (2015) 085035

- Back-reaction from non-equilibrium neutrinos.

$$\nu_{\mathrm{L}}^{\mathrm{e}}(q) + \mathrm{n}(k) \rightleftharpoons \mathrm{e}_{\mathrm{L}}(q') + \mathrm{p}(k')$$

(relativistic electrons/neutrinos will be treated as massless pts.)

Chiral kinetic theory from QFT

- Wigner functions : $\grave{S}^<_{\rm L}(q,x) \equiv \int_y e^{-\frac{iq\cdot y}{\hbar}} \langle \psi^\dagger_{\rm L}(x,y/2)\psi_{\rm L}(x,-y/2)\rangle \equiv \sigma^\mu \mathcal{L}^<_\mu(q,x)$

see e.g. Y. Hidaka, S. Pu, DY, PRD 95, 091901 (2017)

review : Y. Hidaka S. Pu, Q, Wang, DY, PPNP 127 (2022) 103989

- Perturbative solution up to $\mathcal{O}(\hbar)$: (~∂/q : gradient expansion)

$$\mathcal{L}^{<\mu} = 2\pi\Big[\delta(q^2)\big(q^\mu - \hbar S^{\mu\nu}_{(n)}\mathcal{D}_\nu\big) - \hbar\tilde{F}^{\mu\nu}q_\nu\delta'(q^2)\Big]f_{\rm L}\,,$$

$$\mathcal{D}_\mu\mathcal{L}^<_\nu \equiv \big(\nabla_\mu - \Gamma^\lambda_{\mu\rho}q^\rho\partial_{q\lambda} + F_{\rho\mu}\partial^\rho_q\big)\mathcal{L}^<_\nu - \Sigma^<_\mu\mathcal{L}^>_\nu + \Sigma^>_\mu\mathcal{L}^<_\nu,\quad S^{\mu\nu}_{(n)} = \frac{\epsilon^{\mu\nu\alpha\beta}q_\alpha n_\beta}{2q\cdot n}.$$

- Chiral kinetic equation :

$$0 = \delta\big(q^2 - \hbar F_{\alpha\beta}S^{\alpha\beta}_{(n)}\big)\Bigg\{\Bigg[q\cdot\mathcal{D} - \hbar\left(\frac{S^{\mu\nu}_{(n)}F_{\mu\rho}n^\rho}{q\cdot n} + \big(D_\mu S^{\mu\nu}_{(n)}\big)\right)\mathcal{D}_\nu - \hbar S^{\mu\nu}_{(n)}\big(\nabla_\mu F^\lambda{}_\nu - q^\rho R^\lambda_{\rho\mu\nu}\big)\partial_{q\lambda}\Bigg]f_{\rm L}$$
$$-\hbar\frac{\epsilon^{\mu\nu\alpha\beta}n_\nu}{2q\cdot n}\big((1-f_{\rm L})\Delta^>_\alpha\Sigma^<_\beta - f_{\rm L}\Delta^<_\alpha\Sigma^>_\beta\big)\Bigg\}.$$

- Current and EM tensor : $J^\mu = 2\int_q \mathcal{L}^{<\mu},\quad T^{\mu\nu} = \int_q \big(\mathcal{L}^{<\mu}q^\nu + \mathcal{L}^{<\nu}q^\mu\big).$

Chiral kinetic equation for electrons

- Chiral kinetic equation for left-handed electrons near equilibrium:

N. Yamamoto, DY, PRL 131, 012701 (2023)

$$f_{\mathrm{L}}^{(\mathrm{e})} = \bar{f}_{\mathrm{L}}^{(\mathrm{e})} + \delta f_{\mathrm{L}}^{(\mathrm{e})}$$

⇨ $$\Box_q f_{\mathrm{L}}^{(\mathrm{e})} \approx -q\cdot n\hat{\tau}_{\mathrm{EM}}^{-1}\delta f_{\mathrm{L}}^{(\mathrm{e})} - F_{\mathrm{W}},$$

$$\Box_q f_{\chi}^{(\mathrm{e})} = \left(q^{\mu} + \chi\hbar\frac{S_q^{\mu\nu}eF_{\mu\rho}n^{\rho}}{q\cdot n}\right)\Delta_{\mu} f_{\chi}^{(\mathrm{e})},$$

$$\Delta_{\mu} = D_{\mu} + eF_{\lambda\mu}\partial_q^{\lambda}$$

χ = ±1 for R/L.

❖ collision term with neutrinos : $F_{\mathrm{W}} = \bar{f}_{\mathrm{L}}^{(\mathrm{e})}\Gamma_{\mathrm{W}}^{>} - (1-\bar{f}_{\mathrm{L}}^{(\mathrm{e})})\Gamma_{\mathrm{W}}^{<}$

- Modified relaxation-time approx. : $f_{\mathrm{L}}^{(\mathrm{e})} = \bar{f}_{\mathrm{L}}^{(\mathrm{e})} + \delta f_{\mathrm{LEM}}^{(\mathrm{e})} + \delta f_{\mathrm{LW}}^{(\mathrm{e})},$

$$\mathcal{O}(\delta f_{\mathrm{LEM}}^{(\mathrm{e})}) \approx \mathcal{O}(\tau_{\mathrm{EM}}/L)$$

$$\mathcal{O}(\delta f_{\mathrm{LW}}^{(\mathrm{e})}) \approx \mathcal{O}(\tilde{\epsilon}^4 G_F^2)$$

Effective CME from neutrino radiation

- Kinetic equation breaks into : $\Box_q \bar{f}^{(\mathrm{e})}_{\mathrm{L}} \approx -q\cdot n\hat{\tau}^{-1}_{\mathrm{EM}}\delta f^{(\mathrm{e})}_{\mathrm{LEM}},$

$$\Box_q \delta f^{(\mathrm{e})}_{\mathrm{W}} \approx (1-\bar{f}^{(\mathrm{e})}_{\mathrm{L}})\Gamma^{<}_{\mathrm{W}} - \bar{f}^{(\mathrm{e})}_{\mathrm{L}}\Gamma^{>}_{\mathrm{W}} = -F_{\mathrm{W}}.$$

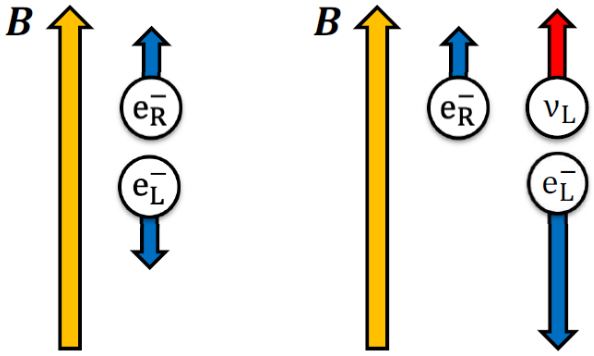
- Neutrino absorption on nucleons :

$$F_{\mathrm{W}} \approx \frac{(q\cdot u)^3}{\pi}(g_{\mathrm{V}}^2+3g_{\mathrm{A}}^2)G_{\mathrm{F}}^2(n_{\mathrm{p}}-n_{\mathrm{n}})\left[\frac{\bar{f}^{(\mathrm{e})}_q(1-f^{(\nu)}_q)}{1-\mathrm{e}^{\beta(\mu_{\mathrm{n}}-\mu_{\mathrm{p}})}}+\frac{(1-\bar{f}^{(\mathrm{e})}_q)f^{(\nu)}_q}{1-\mathrm{e}^{\beta(\mu_{\mathrm{p}}-\mu_{\mathrm{n}})}}\right]$$

- Ignoring electric fields : $q\cdot\partial\delta f^{(\mathrm{e})}_{\mathrm{W}} \approx -F_{\mathrm{W}}$

$$\Rightarrow \quad \delta f^{(\mathrm{e})}_{\mathrm{W}}(q,x) = -\frac{1}{q_0}\int_0^{x_0}\mathrm{d}x'_0 F_{\mathrm{W}}(q,x')|_{\mathrm{c}}\,,$$

$$|_{\mathrm{c}} = \{x'^{\mu}_{\perp} = x^{\mu}_{\perp}, x'^{\mu}_{\parallel} = x^{\mu}_{\parallel} - \bar{q}^{\mu}(x_0-x'_0)/q_0\}.$$



(a) $\boldsymbol{j}_B = \boldsymbol{0}$ (b) $\boldsymbol{j}_B \neq \boldsymbol{0}$

❖ Effective CME :

$$\boxed{j^{\mu}_B \approx \hbar e^2\int\frac{\mathrm{d}^4q}{(2\pi)^3}\frac{\delta(q^2)}{q_0}\left(B^{\mu}q\cdot\partial_q - q\cdot B\partial^{\mu}_{\bar{q}}\right)\delta f^{(\mathrm{e})}_{\mathrm{W}}}$$

Chiral magnetohydrodynamics

- Chiral magnetohydrodynamics (ChMHD) equations:

$$\partial_\mu J^\mu = 0$$

$$\partial_\mu T^{\mu\nu} = F^{\nu\lambda} J_\lambda$$

energy-momentum & charge conservation

$$\left\{\begin{aligned}
&\frac{\partial \rho}{\partial t} + \boldsymbol{\nabla} \cdot (\rho \boldsymbol{v}) = 0\,, && (17)\\
&\frac{\partial}{\partial t}(\rho \boldsymbol{v}) + \boldsymbol{\nabla} \cdot \left[\rho \boldsymbol{v}\boldsymbol{v} - \boldsymbol{B}\boldsymbol{B} + \left(P + \frac{B^2}{2}\right)\mathbf{I}\right] = \boldsymbol{S}\,, && (18)\\
&\frac{\partial}{\partial t}\left(\frac{1}{2}\rho v^2 + \frac{1}{\Gamma - 1}P + \frac{B^2}{2}\right) + \boldsymbol{\nabla} \cdot \left[\left(\frac{1}{2}\rho v^2 + \frac{\Gamma}{\Gamma - 1}P\right)\boldsymbol{v} \right. \\
&\left. + \boldsymbol{E} \times \boldsymbol{B}\right] = -\boldsymbol{S} \cdot \boldsymbol{v} - \Delta \boldsymbol{J} \cdot \boldsymbol{E}\,, && (19)
\end{aligned}\right.$$

$$\partial_\nu F^{\nu\mu} = J^\mu$$

Maxwell + CME

$$\partial_\mu J_5^\mu = \frac{\mathbf{E} \cdot \mathbf{B}}{2\pi^2}$$

helicity conservation

$$\left\{\boxed{\begin{aligned}
&\frac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{\nabla} \times (\boldsymbol{v} \times \boldsymbol{B}) + \eta \boldsymbol{\nabla}^2 \boldsymbol{B} + \eta\, \boldsymbol{\nabla} \times (\xi_B \boldsymbol{B})\,, && (20)\\
&\frac{\partial n_{5,\mathrm{eff}}}{\partial t} = \frac{1}{2\pi^2}\boldsymbol{E} \cdot \boldsymbol{B}\,, && (21)
\end{aligned}}\right.$$

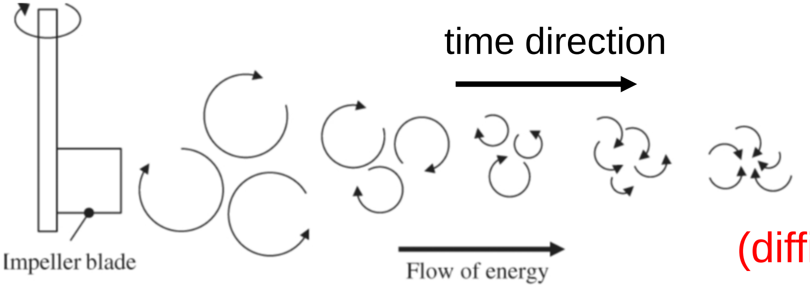
$$\boldsymbol{S} = \rho\nu \boldsymbol{\nabla}^2 \boldsymbol{v} + \frac{1}{3}\rho\nu \boldsymbol{\nabla}(\boldsymbol{\nabla} \cdot \boldsymbol{v}) \quad \text{(viscous correction)}$$

J. Matsumoto, N. Yamamoto, DY, PRD 105, 123029 (2022)

see also Y. Masada et al., PRD 98, 083018 (2018)
A. Brandenburg et al., AJL 845, L21 (2017)

Direct & inverse energy cascades

- 3D supernova simulations are more difficult to achieve explosion than 2D
- Turbulence in 3D : direct energy cascade



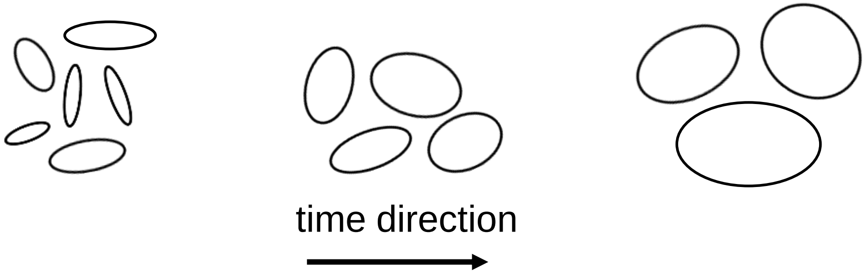
time direction

https://doi.org/10.1515/htmp-2016-0043

Kolmogorov 1941

(difficult to explode)

- Turbulence in 2D : inverse energy cascade



time direction

Kraichnan, Leith, Batchelor
1967-1969

(easier to explode)

- Chiral effects in 3D : inverse cascade