Latest updates on ideal-spin hydrodynamics

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Outline

Motivations:

- Do we understand the surprising success of local-equilibrium (LEQ) assumption (A. Palermo's talk)
- How can we simplify semi-classical spin hydrodynamics (a made-up name for formulation of [Weickgenannt et al. (2022)])

Outline:

- What is semi-classical (ideal)-spin hydrodynamics?
- The poor man's toolbox (I): linearized spin hydro
- The poor man's toolbox (II): conformal Bjorken flow
- Final words

Recap: surprising success of the local-equilibrium (LEQ) assumption

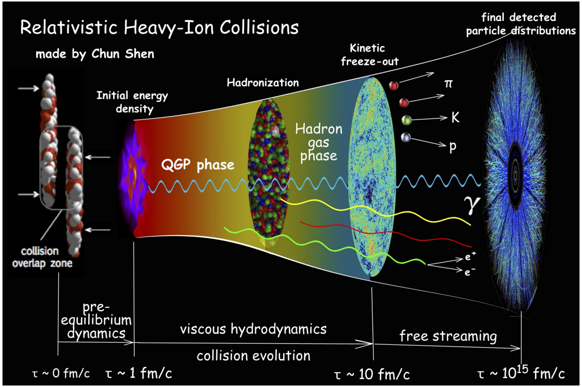
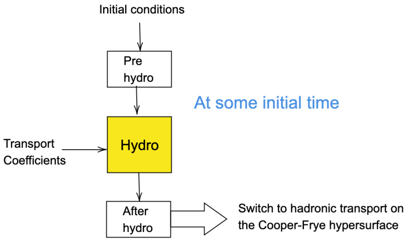
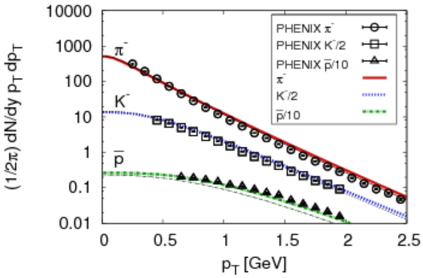


Fig. by Chun Shen (https://u.osu.edu/vishnu/2014/08/06/sketch-of-relativistic-heavy-ion-collisions/)

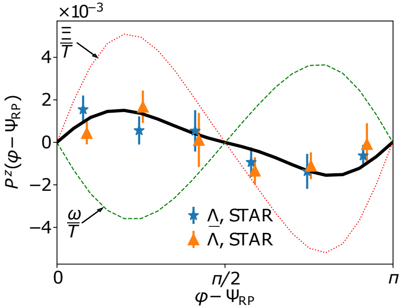
Parent cell's field (in spacetime) $\longrightarrow$ final hadron's properties (in phase space)





[Schenke et al. (2010)]

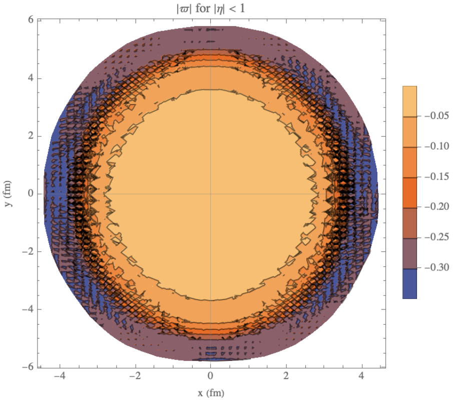
Gradients of the fields of parent fluid cell $\longrightarrow$ polarization of final-state hadrons [Becattini et al. (2013)]

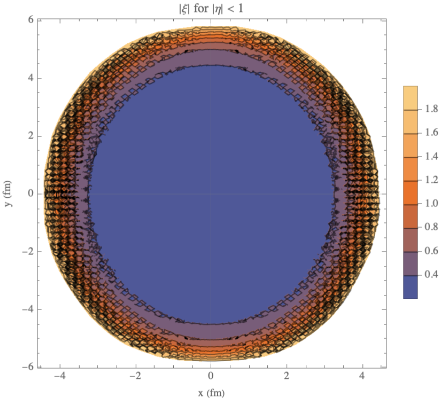


[Becattini et al. (2021)]

The sign puzzle seems to have been solved but ...

- **It is not clear how and when the spin degrees of freedom are equilibrated**





Thermal vorticity and thermal shear are small and of the same order on the FO surface (surface analysis with VHLLE output [Karpenko et al. (2014)])

What is semi-classical spin hydrodynamics?

Standard dissipative hydro

- Conservation of energy-momentum and angular momemtun

$$\partial_\mu T^{\mu\nu} = 0 \qquad \partial_\lambda J^{\lambda\mu\nu} = 0$$

- Perfect fluid ($J^{\lambda\mu\nu}$ is automatically conserved)

$$T^{\mu\nu}_{(0)} = \varepsilon u^\mu u^\nu - P\Delta^{\mu\nu} \qquad J^{\lambda\mu\nu} = L^{\lambda\mu\nu} \equiv 2T^{\lambda[\nu}x^{\mu]}$$

- Dissipative hydrodynamics (Schematically ϵ_∂ is the expansion parameter $\rightarrow$ David Wagner's talk)

$$T^{\mu\nu} = T^{\mu\nu}_{(0)} + \epsilon_\partial T^{\mu\nu}_{(1)} + \epsilon_\partial^2 T^{\mu\nu}_{(2)} + \cdots$$

Notations and conventions:

$$\eta_{\mu\nu} = \mathrm{diag}(1,-1,-1,-1) \qquad A^{[\mu\nu]} \equiv \frac{1}{2}(A^{\mu\nu} - A^{\nu\mu})$$

This dog wants to perform a coordinate transformation

. . . and the definition of $L^{\lambda\mu\nu}$ annoys it!

Standard dissipative hydro

- Question of covariance ($\rightarrow$ A. Chiarini's poster)

$$L^{\lambda\mu\nu} \rightarrow \mathcal{L}^{\lambda\, h} = K^h_\nu T^{\lambda\nu} \qquad h = 1, 2, 3$$

  K^h are generators of rotations $D_\mu K^h_\nu + D_\nu K^h_\mu = 0$
- Then $D_\mu T^{\mu\nu} = 0 \implies D_\mu \mathcal{L}^\mu = 0 \rightarrow Q^h = \int_\Sigma \mathrm{d}\Sigma_\mu\, K^{\mu h}$ being conserved
- Quantum nature of the fluid encoded in transport coefficients

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* D is the covariant derivative
* In Minkowski coordinates h is related to $\mu\nu$ e.g., $K^{\lambda(yx)} = (0, -y, x, 0)$
* Σ is a Cauchy hypersurface

Happy now? Remember this is only in flat spacetime!

Three assumptions of semiclassical spin hydro

- Macroscopic quantum corrections $\rightarrow$ semi-classical expansion in $\hbar$ (for more details $\rightarrow$ David Wagner's talk)
- We truncate all the **equations** up to first order in $\hbar$ [Weickgenannt et al. (2022)]
- **Assumption I**: decomposition of angular momentum

$$J^{\lambda\mu\nu} = L^{\lambda\mu\nu} + \hbar S^{\lambda\mu\nu}$$

or the covariant form: $\mathcal{J}^{\lambda\, h} = T^{\lambda\nu} K_\nu^h + \frac{1}{2}\hbar S^{\lambda\mu\nu} D_{[\mu} K_{\nu]}^h$

- $D_\lambda \mathcal{J}^{\lambda\, h} = 0$ if the spacetime is flat ($\implies D_\lambda D_\mu K_\nu = 0$) and

$$\hbar D_\lambda S^{\lambda\mu\nu} = 2T^{[\nu\mu]}$$

Three assumptions of semiclassical spin hydro

- **Assumption II**: In global equilibrium

$$T^{[\nu\mu]} \overbrace{=}^{\text{geq}} 0 \qquad D_\lambda S^{\lambda\mu\nu} \overbrace{=}^{\text{geq}} 0 \qquad T^{(\mu\nu)} \overbrace{=}^{\text{geq}} T^{\mu\nu}_{(0)}$$

- **Assumption III**:

$$T^{(\mu\nu)} = \underbrace{\overbrace{T^{(\mu\nu)}}^{\mathcal{O}\left(\hbar^0\right)}}_{\text{standard}} + \mathcal{O}\left(\hbar^2\right) \qquad T^{[\mu\nu]} = \mathcal{O}\left(\hbar^2\right)$$

- No back-reaction from spin to fluid $D_\mu T^{(\mu\nu)} = \mathcal{O}\left(\hbar^2\right)$

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* In global equilibrium $\beta^\mu = u^\mu / T$ is a Killing vector

The symmetric part of the energy-momentum tensor

$$T^{(\mu\nu)} = \mathcal{E} u^\mu u^\nu - \mathcal{P}\Delta^{\mu\nu} + \mathcal{Q}^\mu u^\nu + \mathcal{Q}^\nu u^\mu + \mathcal{T}^{\mu\nu}$$

Semiclassical Landau frame

$$\mathcal{E} = u_\mu u_\nu T^{(\mu\nu)} = \varepsilon + \mathcal{O}\left(\hbar^2\right) \qquad \mathcal{P} = -\frac{1}{3}\Delta_{\alpha\beta}T^{\alpha\beta} = p + \Pi + \mathcal{O}\left(\hbar^2\right)$$

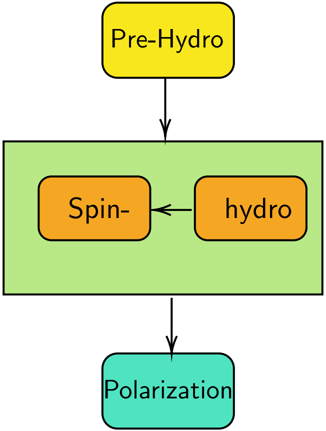
$$\mathcal{Q}^\mu = \Delta^{\mu\alpha}u^\beta T_{\alpha\beta} = \mathcal{O}\left(\hbar^2\right) \qquad \mathcal{T}^{\mu\nu} = \Delta^{\mu\nu}_{\alpha\beta}T^{(\alpha\beta)} = \pi^{\mu\nu} + \mathcal{O}\left(\hbar^2\right)$$

MIS-type EOM for dissipative fluxes

$$\tau_\Pi \dot{\Pi} + \Pi = -\zeta\theta + \cdots \qquad \tau_\pi \dot{\pi}^{\langle\mu\nu\rangle} + \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} + \cdots$$

Notations:

$$\dot{X} \equiv u^\mu D_\mu X \qquad \Delta^{\mu\nu}_{\alpha\beta} := \Delta^{(\mu}_{\alpha}\Delta^{\nu)}_{\beta} - \Delta^{\mu\nu}\Delta_{\alpha\beta}/3$$



- N_f fluid fields $\{\varphi^A\} = \{\varepsilon, u^\mu, \Pi, \pi^{\mu\nu}, \cdots\}$
- By solving standard dissipative hydro (DNMR in our case) we find $\{\varphi^A\}$
- Also N_s spin degrees $\{\psi^A\}$ of freedom (spin potential ...)
- Knowing $\{\varphi^A\}$ we can solve $\hbar D_\lambda S^{\lambda\mu\nu} = 2T^{[\nu\mu]}$ and other required equations to find $\{\psi^A\}$
- Inserting the results into the formula for S^μ on the FO surface we find the polarization

Ideal-spin hydrodynamics

- 24 components of $S^{\lambda\mu\nu}$ and 6 equations $\rightarrow$ further equations are needed (e.g., from the method of moments)
- In ideal-spin approximation d.o.f in $S^{\lambda\mu\nu}$ are only the 6 ones in $\Omega^{\mu\nu}$

$$S^{\lambda\mu\nu} = Au^\lambda\Omega^{\mu\nu} + Bu^\lambda u_\alpha\Omega^{\alpha[\mu}u^{\nu]} + Cu^\lambda\Omega^{\alpha[\mu}\Delta^{\nu]}{}_\alpha + Du_\alpha\Omega^{\alpha[\mu}\Delta^{\nu]\lambda} + E\Delta^\lambda{}_\alpha\Omega^{\alpha[\mu}u^{\nu]}$$

- Constraint from Assumption III: $B - C - D + T\frac{\partial E}{\partial T} = 0$

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* $\{A, B, C, D, E\}$ are functions of ε or, equivalently, T
* In quantum kinetic theory

$$A = \frac{\hbar T^2}{4m^2}\frac{\partial}{\partial T}(\varepsilon - 3P) \qquad B = \frac{\hbar T^2}{4m^2}\frac{\partial \varepsilon}{\partial T} \qquad C = D = E = -\frac{\hbar T^2}{4m^2}\frac{\partial P}{\partial T}$$

Ideal-spin hydrodynamics

- The antisymmetric part of energy-momentum tensor

$$T^{[\mu\nu]} = -\hbar^2\Gamma^{(\kappa)} u^{[\mu} \left(\kappa^{\nu]} + \varpi^{\nu]\alpha} u_\alpha\right) + \frac{1}{2}\hbar^2\Gamma^{(\omega)} \epsilon^{\mu\nu\rho\sigma} u_\rho \left(\omega_\sigma + \beta\Omega_\sigma\right) + \hbar^2\Gamma^{(a)} u^{[\mu} \left(\beta a^{\nu]} + \nabla^{\nu]}\beta\right)$$

- Why "ideal": **spin** contributions to entropy production is of higher order in $\hbar$

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* $\{\Gamma^{(\kappa)}, \Gamma^{(\omega)}, \Gamma^{(a)}\}$ are functions of ε or, equivalently, T
* Notations

$$\varpi_{\mu\nu} = -D_{[\mu}\beta_{\nu]} \qquad \beta = 1/T \qquad \Omega^\mu = -\tfrac{1}{2}\epsilon^{\mu\nu\alpha\beta} u_\nu D_\alpha u_\beta \qquad \nabla^\mu = \Delta^{\mu\nu} D_\nu$$

$$\kappa^\mu = -\Omega^{\mu\nu} u_\nu \qquad \omega^\mu = \tfrac{1}{2}\epsilon^{\mu\nu\alpha\beta} u_\nu \Omega_{\alpha\beta} \qquad a_\mu = u^\alpha D_\alpha u_\mu$$

Please stay with me, I will entertain you in a moment.

The poor man's toolbox

We need a solution of hydrodynamics to feed into the spin equations

The poor man's toolbox

We need a solution of hydrodynamics to feed into the spin equations

In a poor man's toolbox:

- **Damping of spin waves in a hydrostatics background** [D. Wagner, M.S, and D. H. Rischke arXiv:2405.00533] (Similar works: [Ambrus et al. (2022)] and [Singh et al. (2023)])
- **Linear spin hydro** [J. Sammet, M.S., D. Wagner, and D. H. Rischke, in preparation] (Similar works: [Ren et al. (2024)] and [Daher et al. (2024)])
- **Bjorken spin hydro** [A. Chiarini, M.S., D. Wagner, and D. H. Rischke work in progress] [(Kind of) similar work: [Singh et al. (2021)]]
- **Rigidly rotating fluid** [A. Chiarini, M.S., D. Wagner, A. Dash, and D. H. Rischke work in progress] $\to$ A. Chiarini's poster

What can we possibly learn from each one?

Different contributions can be investigated in these three simple setups

$$\tau_\kappa \dot{\kappa}^{\langle\mu\rangle} = \overbrace{F(\theta,\dot{\beta})}^{\text{Bjorken}}\,\kappa^\mu + (\cdots)\left(\overbrace{\omega^{\mu\nu}}^{\text{Rigid rot.}} + \overbrace{\sigma^{\mu\nu}}^{\text{Bjorken}}\right)\kappa_\nu$$

$$+ \overbrace{\epsilon^{\mu\alpha\beta\sigma}\omega_\sigma G_{\alpha\beta}}^{\text{Linear and Rigid rot.}} + (\cdots) \overbrace{u_\mu T^{[\mu\nu]}}^{\text{Linear and Rigid rot.}}$$

Equations for both κ and ω are relaxation-type equations

$$\tau_\kappa = -\frac{A-B-C}{\hbar\Gamma^{(\kappa)}} \qquad \tau_\omega = -\frac{E}{\hbar\Gamma^{(\kappa)}}$$

In quantum kinetic theory

$$\tau_\kappa = \frac{T}{2m^2\Gamma^{(\kappa)}}(\varepsilon+P) \qquad \tau_\omega = \frac{T}{4m^2\Gamma^{(\omega)}}(\varepsilon+P)\left(1-\frac{1}{v_s^2}\right)$$

Linear regime

Linearized ideal-spin hydrodynamics

Main idea: linearized equations of hydrodynamics in a homogeneous equilibrium configuration have linear wave solutions

- Independent degrees of freedom in $T^{(\mu\nu)}$ are $\varphi \in \{\beta = 1/T, u^\mu, \cdots\}$ and in $S^{\lambda\mu\nu}$ are $\psi \in \{\kappa^\mu, \omega^\mu, \cdots\}$
- For each $X \in \{\phi, \psi\}$: $X_0 \to X_0 + \delta X$ (X_0 is constant in a homogeneous equilibrium configuration)
- Fourier transform

$$\delta X(x) = \int \frac{\mathrm{d}^4 k}{(2\pi)^4} e^{ik\cdot x/\hbar} \delta X(k)$$

Linearized ideal-spin hydrodynamics

- Insert δX into the EOM (in Fourier space)

$$ik_\mu \delta T^{(\mu\nu)} = 0 \qquad ik_\lambda \delta S^{\lambda\mu\nu} = \delta T^{[\nu\mu]}$$ and EOM for dissipative fluxes

- EOM to the matrix form $\mathbf{M}\,\delta\vec{\mathbf{X}} = 0$
- This equation has solutions iif $\det(\mathbf{M}) = 0$
- $\implies$ dispersion relations for eigenfrequencies $\omega = \omega(\mathbf{k})$ (if $k^\mu = (\omega, \mathbf{k})$)
- Performing this procedure for the example of ideal-spin hydrodynamics and DNMR (or MIS) theory with shear viscosity alone $\rightarrow$ spin and fluid waves are decoupled!

$$\omega^2 - i\hbar a\omega - v_{\mathfrak{s}}^2\mathbf{k}^2 - \hbar^2 b = 0 \qquad a = \frac{\tau_\kappa + \tau_\omega}{\tau_\kappa\tau_\omega} \quad b = \frac{1}{\tau_\kappa\tau_\omega} \quad v_{\mathfrak{s}}^2 = \frac{\Gamma^{(\kappa)}\tau_\kappa}{4\Gamma^{(\omega)}\tau_\omega}$$

Where did all fluid's contributions go?

- Let's assume a fluid with N_f degrees of freedom: φ^A where $A = 1 \cdots N_f$ (all possible dissipative contributions, multiple charges etc)
- Such that all these equations in the linear order are written as

$$M_{\mathrm{f}}^{AB} \delta\varphi^B = \mathcal{O}\left(\hbar^2\right)$$

- And $S^{\lambda\mu\nu}$ with N_s degrees of freedom: ψ^A where $A = 1 \cdots N_s$ (ideal and dissipative)
- The N_s equations in linear order take the form

$$M_{\mathrm{s}}^{AB} \delta\psi^B + \underbrace{M_{\mathrm{fs}}^{AB} \delta\varphi^B}_{\text{source terms}} = 0$$

Where did all fluid's contributions go?

- All equations can be collectively written as

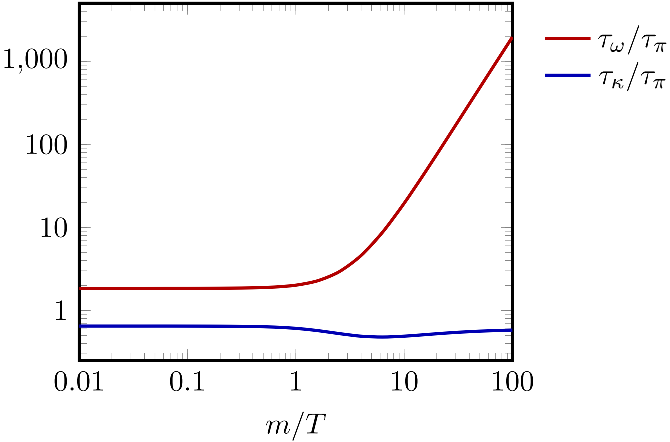
$$\begin{pmatrix} M_{\mathrm{f}} & \mathcal{O}(\hbar^2) \\ M_{\mathrm{fs}} & M_{\mathrm{s}} \end{pmatrix} \begin{pmatrix} \delta\varphi \\ \delta\psi \end{pmatrix} = \mathbf{0} \,.$$

- But

$$\begin{vmatrix} M_{\mathrm{f}} & 0 \\ M_{\mathrm{fs}} & M_{\mathrm{s}} \end{vmatrix} = \begin{vmatrix} M_{\mathrm{f}} & 0 \\ 0 & M_{\mathrm{s}} \end{vmatrix} = \det(M_{\mathrm{f}})\det(M_{\mathrm{s}})$$

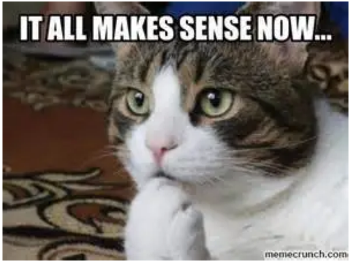
In the absence of back-reaction from the spin to the fluid, the linear characteristic equation that determines the spin modes is decoupled from the fluid modes.

Timescales in spin hydrodynamics



Relaxation times simiar/larger than typical dissipative timescale τ_π for small/large m/T [Wagner et al. (2024)]

Timescales in spin hydrodynamics

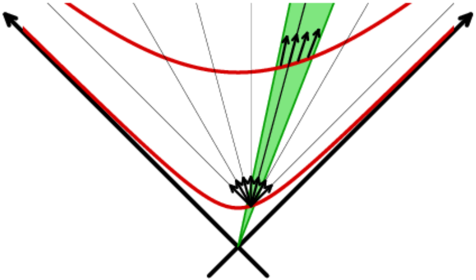


- At late times and small $\mathbf{k}$ the timescale t_d in damping factor $\exp\left(-t/t_d\right)$ is determined by τ_ω
- Spin degrees of freedom relax quite fast in high-energy collisions
- ...while these timescales for low-energy collisions might be even larger than the lifetime of the fireball
- A possible explanation of why the results of [Becattini et al. (2021)] are consistent with data

Bjorken flow

Conformal Bjorken flow

A theorist's favorite solution which unfortunatley does not have thermal vorticity.



Conformal Bjorken flow

- Let's assume conformal symmetry: $\varepsilon = 3P \sim T^4 \quad \tau_\pi = C_{\tau\pi}/T \quad \eta = C_\eta \frac{\varepsilon+P}{T}$
- Reasons to love Bjorken flow (as a theorist):
  1. A coordinate system (Milne) in which $u^\mu = (1, \mathbf{0})$:

$$\mathrm{d}s^2 = \mathrm{d}\tau^2 - \mathrm{d}x^2 - \mathrm{d}y^2 - \tau^2\,\mathrm{d}{\eta_s}^2 \qquad \tau^2 = t^2 - z^2 \qquad \tanh\eta_s = \frac{z}{t}$$

  2. All quantities are functions of τ only $\implies$ EOM become ODEs
  3. Energy-momentum tensor is diagonal $T^\mu_\nu = \mathrm{diag}(\varepsilon, P_\perp, P_\perp, P_\parallel)$
- The pressure anisotropy is due to the shear-stress tensor (which has one degree of freedom)

$$\mathcal{A} \equiv \frac{P_\parallel - P_\perp}{P_{\mathrm{EQ}}}$$

- There is a clever parameterization found by Heller and Spalinski (2015):

$$w = T\tau \qquad f(w) = 1 + \frac{\tau}{T}\frac{\mathrm{d}T}{\mathrm{d}\tau} \qquad \mathcal{A} = 18\left(f(w) - \frac{2}{3}\right)$$

Conformal Bjorken flow

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- There is a clever parameterization found by Heller and Spalinski (2015):

$$w = T\tau \qquad f(w) = 1 + \frac{\tau}{T}\frac{\mathrm{d}T}{\mathrm{d}\tau} \qquad \mathcal{A} = 18\left(f(w) - \frac{2}{3}\right)$$

- Slow-roll approximation captures the MIS **attractor** at early and late times

$$f(w) = \frac{2}{3} - \frac{w}{8C_{\tau\pi}} + \frac{\sqrt{64C_\eta C_{\tau\pi} + 9w^2}}{24C_{\tau\pi}} \qquad C_{\tau\pi} = T\tau_\pi \qquad C_\eta = \frac{\eta}{s}$$

Bjorken ideal-spin conformal hydrodynamics

- The coefficients from conformal symmetry

$$\{A, B, C, D, E\} \to \{A, B, C, D, E\}T^3 \qquad \{\Gamma^{(\kappa)}, \Gamma^{(\omega)}\} \to \{\Gamma^{(\kappa)}, \Gamma^{(\omega)}\}T^4$$

- Constraint becomes algebraic

$$3E + B - C - D = 0$$

- Using rotational symmetry

$$\kappa^\mu = \left(0, \kappa_\perp(\tau), 0, \frac{\kappa_\parallel(\tau)}{\tau}\right) \qquad \omega^\mu = \left(0, \omega_\perp(\tau), 0, \frac{\omega_\parallel(\tau)}{\tau}\right)$$

Spin equations of motion

- The same equations are found for $\mathbf{x} \in \{\omega, \kappa\}$:

$$C_{\tau\mathbf{x}} \left(w \frac{\mathrm{d}}{\mathrm{d}w} + \frac{\mathcal{A}}{6} \right) \mathbf{x}_\perp + (w - \rho_\mathbf{x}) \mathbf{x}_\perp = 0$$
$$C_{\tau\mathbf{x}} \left(w \frac{\mathrm{d}}{\mathrm{d}w} + \frac{\mathcal{A}}{6} \right) \mathbf{x}_\parallel + w \mathbf{x}_\parallel = 0$$

- The timescales are redefined as $C_{\tau\mathbf{x}} = T\tau_\mathbf{x}$
- Couplings to the shear tensor: $T\rho_\kappa = D/(\hbar\Gamma^\kappa)$ and $T\rho_\omega = E/(\hbar\Gamma^\omega)$

A quick solution

- Using the slow-roll approximation we find at late times

$$\mathbf{x}_\perp \propto \exp\left(-\frac{C_{\tau\mathbf{x}}}{C_{\tau\pi}^2} w\right) w^{C_{\tau\mathbf{x}}/\rho_{\mathbf{x}}} \qquad \mathbf{x}_\parallel \propto \exp\left(-\frac{C_{\tau\mathbf{x}}}{C_{\tau\pi}^2} w\right)$$

- Numerical inspection with the assumption of a very small initial value of $\mathbf{x}$ shows that:
  - τ_π has a much more important effect than τ_ω and τ_κ
  - A large value of ρ_κ (ρ_ω) can amplify a very small initial spin potential to a very large one

Final words

- Numerically solving spin EOM on top of an uncharged fluid in global equilibrium with a non-vanishing thermal vorticity
- ...to understand better the equilibration timescale of spin degrees of freedom [A. Chiarini, M.S., D. Wagner, A. Dash, and D. H. Rischke, work in progress]
- Multi-formalism polarization calculator code [N. Saß, M.S, A. Palermo, David Wagner, H. Elfner, and Dirk H. Rischke, work in progress]


Thank you for your attention,
please do not ask
hard question
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Backup

Local Λ polarization

Solving standard dissipative hydrodynamics and feeding the results into polarization formula of [Weickgenannt et al. (2022)] on the freezeout surface: preliminary results signal that the standard shear tensor contributes in the right direction [N. Saß, M.S, A. Palermo, David Wagner, H. Elfner, and Dirk H. Rischke, work in progress]

$$
\begin{aligned}
S^{\mu}(p)_{\mathrm{NS}} = \int d\Sigma \cdot p \, \frac{f_{0p}}{2\mathcal{N}} \Bigg\{ - \frac{\hbar}{2m} \tilde{\Omega}^{\mu\nu} p_{\nu} + \left(g^{\mu}_{\nu} - \frac{u^{\mu} p_{\langle \nu \rangle}}{E_p} \right) \\
\times \left[\mathfrak{e} \chi_{\mathfrak{p}} \left(\tilde{\Omega}^{\nu\rho} - \tilde{\varpi}^{\nu\rho} \right) u_{\rho} - \chi_{\mathfrak{q}} \mathfrak{d} \beta_0 \sigma_{\rho}{}^{\langle \alpha} \epsilon^{\beta \rangle \nu \sigma \rho} u_{\sigma} p_{\langle \alpha} p_{\beta \rangle} \right] \Bigg\}
\end{aligned}
$$

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