Chiral kinetic theory in curved space reinterpreted and radiative corrections

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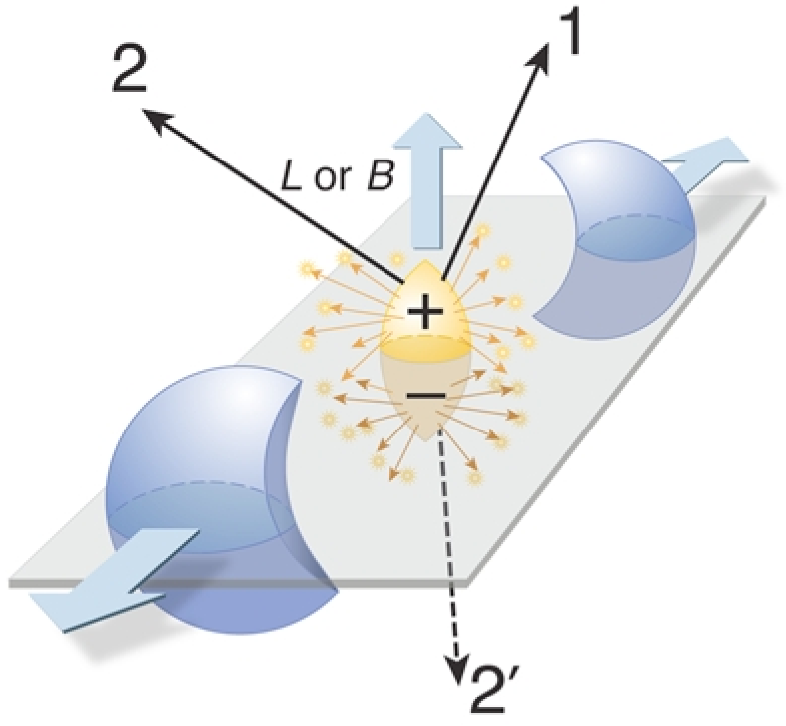
Outline

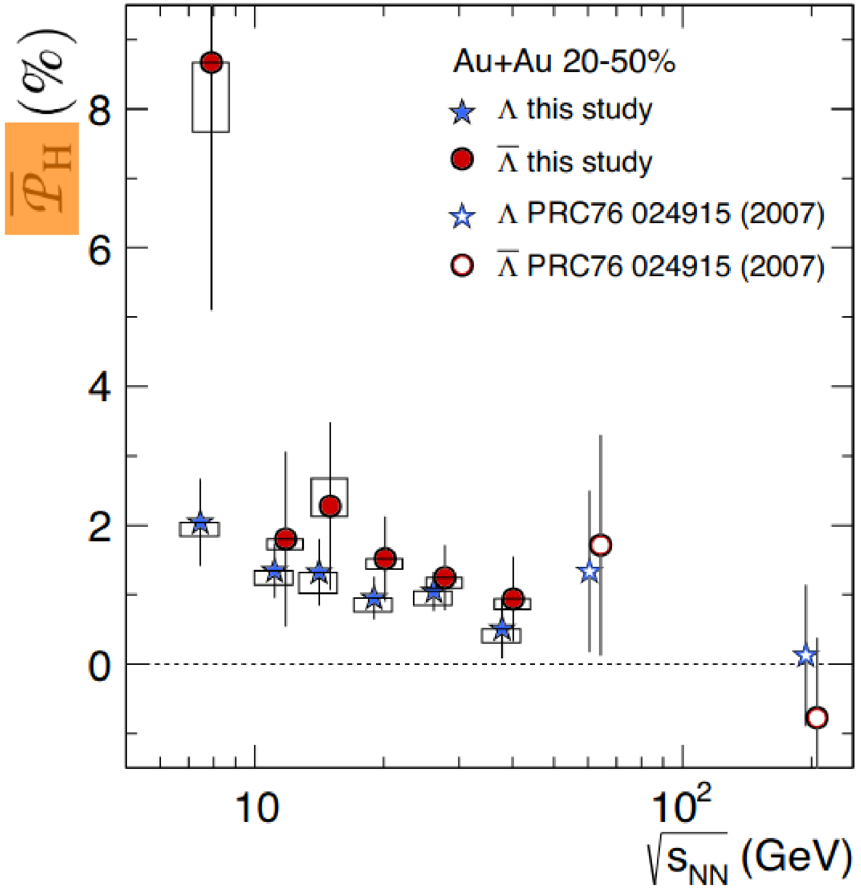
- Spin polarization in heavy ion collisions
- Lessons from quantum(chiral) kinetic theory and limitations
- Radiative corrections to spin coupling to EM fields
- Subtlety in mimicking off-equilbrium state by metric perturbion on equilibrium state
- Radiative corrections to spin coupling to hydro-gradients.
- Conclusion and outlook

global spin polarization in heavy ion collisions



$$L_{ini} \sim 10^5\hbar \rightarrow S_{final}$$

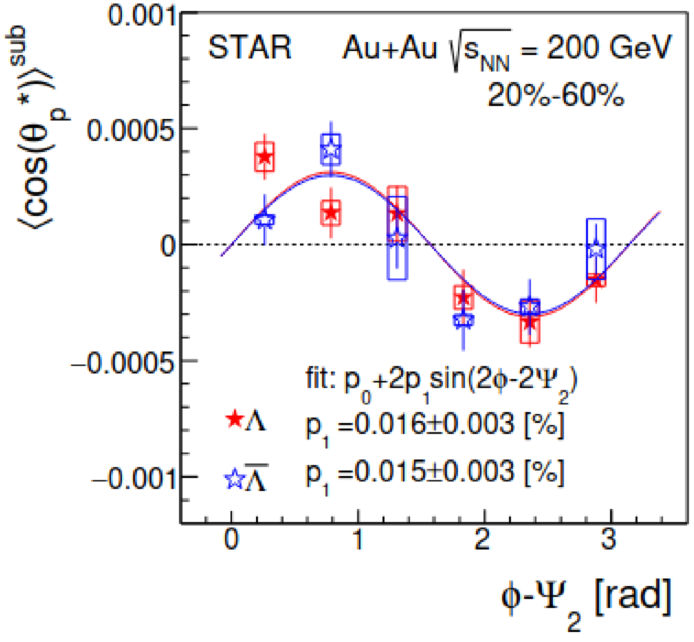
Liang, Wang, PRL 2005, PLB 2005



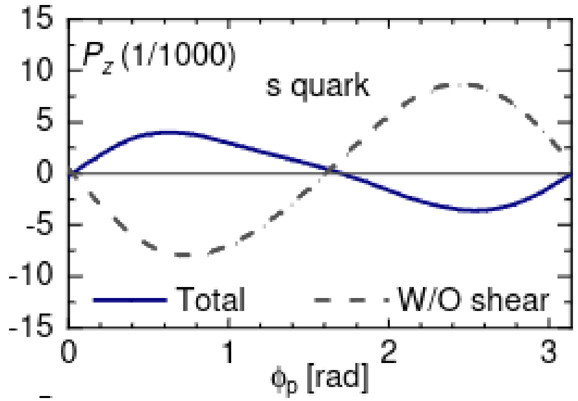
Talks by Voloshin&Palermo, Mon

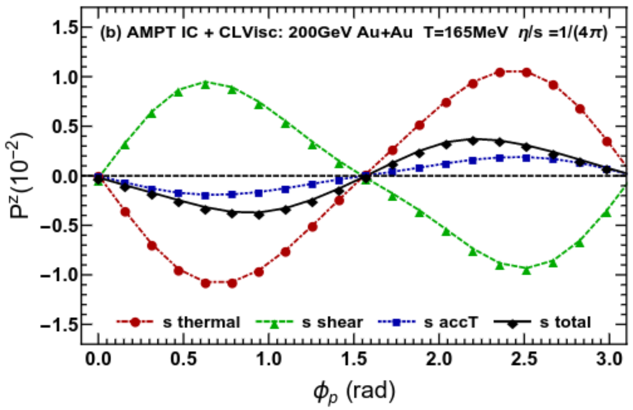
STAR collaboration, Nature 2017 $e^{-\beta(H_0 - \mathbf{S}\cdot\omega)}$

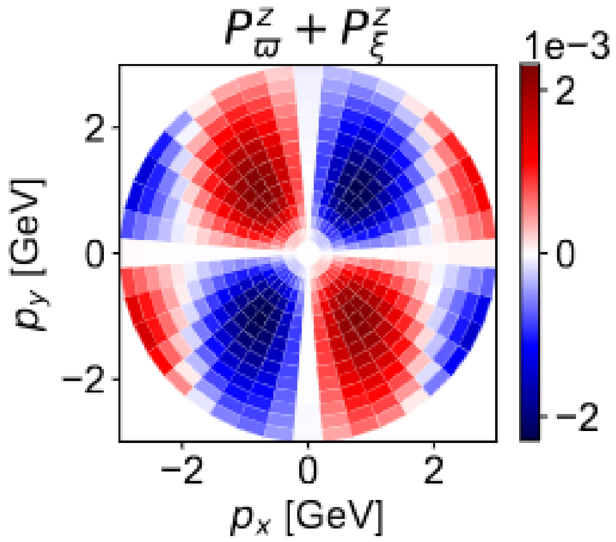
local spin polarization in heavy ion collisions



STAR collaboration, PRL 2019







Fu, Liu, Pang, Song, Yin, PRL 2021
Becattini, et al, PRL 2021
Yi, Pu, Yang, PRC 2021

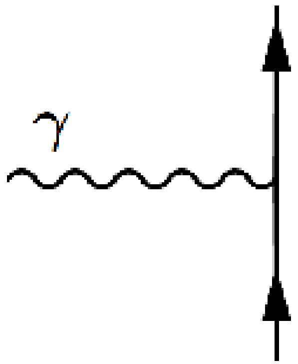
$\mathcal{P}^i \sim \omega^i \quad \mathcal{P}^i \sim \epsilon^{ijk}\hat{p}_j\hat{p}_l\sigma_{kl}$ vorticity + shear

Spin polarization in heavy ion collisions

for $S = \frac{1}{2}$ particle

$S_i \sim B_i$

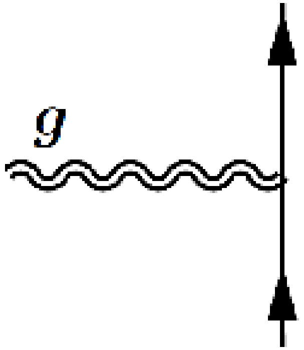
$S_i \sim \epsilon^{ijk} \hat{p}_j E_k$



external EM fields

$S_i \sim \omega_i$

$S_i \sim \epsilon^{ijk} \hat{p}_j \hat{p}_l \sigma_{kl}$



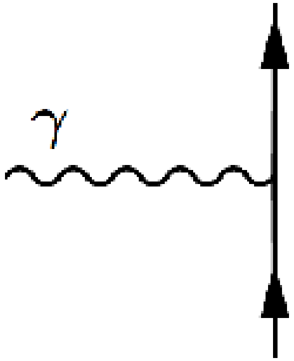
off-equilibrium state: hydro gradient (mimicked by metric)

Spin polarization from correlation functions

Wigner function

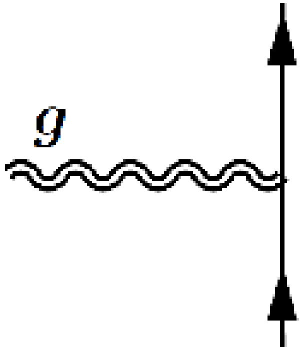
$$S^<_{\alpha\beta}(X = \frac{x+y}{2}, P) = \int d^4(x-y) e^{iP\cdot(x-y)/\hbar} \left(-\langle \bar{\psi}_\beta(y)\psi_\alpha(x)\rangle\right)$$

- Spin polarization in EM fields



$$\langle S^<(X,P)\rangle_{\text{eq},A_\mu}$$

- Spin polarization in off-equilibrium state: hydro gradient



$$\langle S^<(X,P)\rangle_{\text{off-eq}} = \langle S^<(X,P)\rangle_{\text{eq},h_{\mu\nu}}$$

A_μ, $h_{\mu\nu}$ slow-varying $\partial_X \ll P$

Quantum (chiral) kinetic theory

$$\frac{i}{2}\partial\!\!\!/ S^< + P\!\!\!/ S^< = \frac{i}{2}\left(\Sigma^> S^< - \Sigma^< S^>\right)$$

diag part → spin-averaged Boltzmann equation

off-diag part → spin evolution equation

Hidaka, Pu, Wang, Yang, PPNP 2022

Wagner's talk, Fri

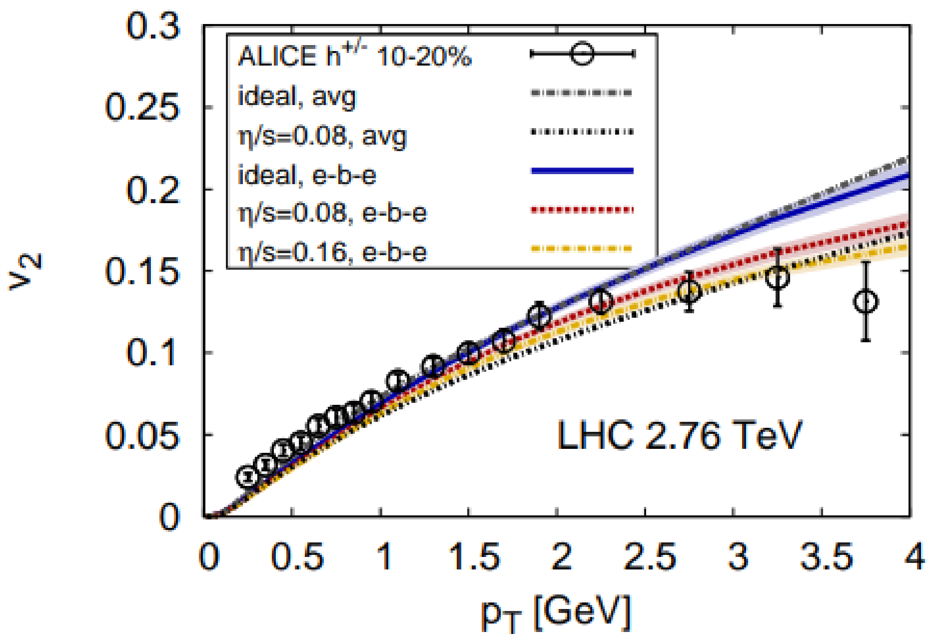
$$S^< = \frac{1}{4}\left(\gamma^\mu V_\mu + \gamma^5\gamma^\mu A_\mu\right) \propto \delta(P^2)$$ up to $O(\partial)$

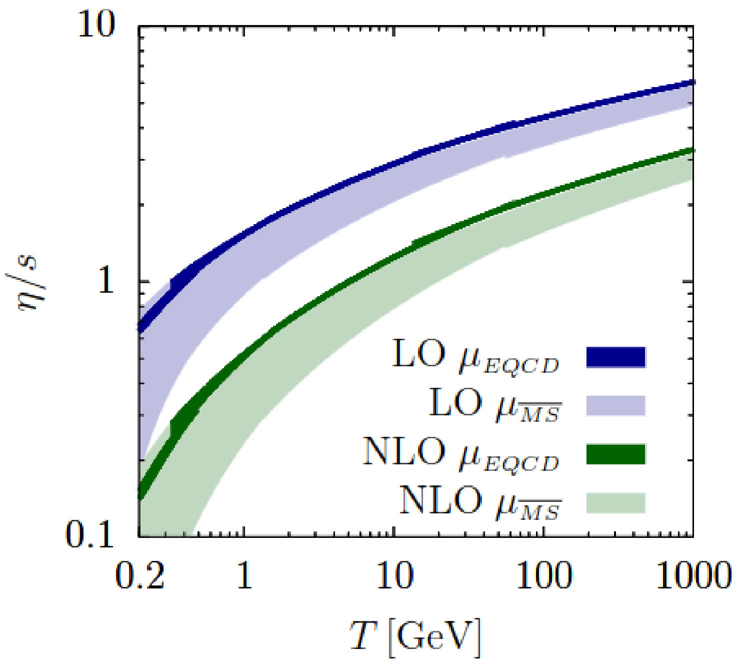
spin polarization

$$S^< = \frac{1}{4}\left[\left(1+\gamma^5\right)\gamma^\mu R_\mu + \left(1-\gamma^5\right)\gamma^\mu L_\mu\right]$$

Limitation of CKT

Phenomenology implementation based on free theory,
but correction in coupling can be significant and crucial

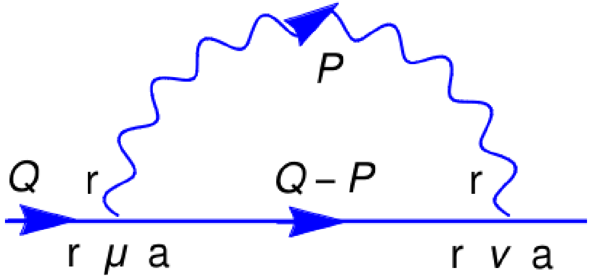




phenomenology $\frac{\eta}{s} \simeq 0.08$

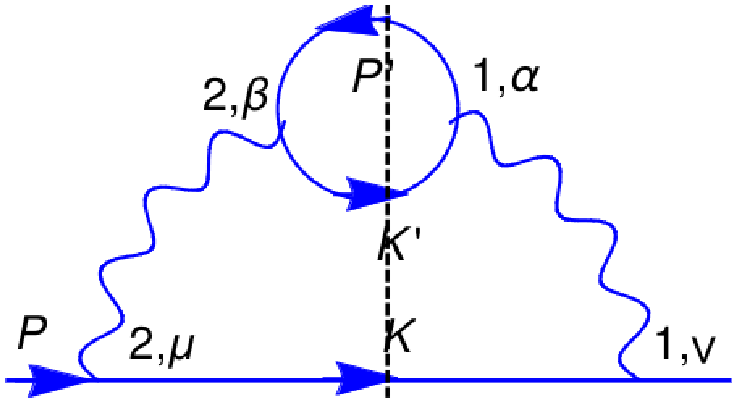
kinetic theory Arnold, Moore, Yaffe 2003
Ghiglieri, Moore, Teaney 2018

How to include coupling corrections?



correction to spectral density, usually ignored

this talk, beyond free particle spectral density



collision term in steady state

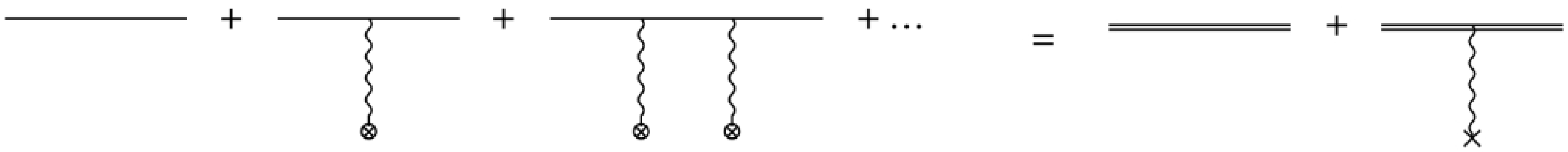
$$\delta f \sim O\left(\frac{\partial}{g^4}\right)$$

$$g^4 \times \delta f \sim O(\partial)$$

diagramatic resummation Gagnon, Jeon, 2006

collisional contribution to spin-shear coupling:
SL, Wang, 2022, 2024

Equivalence of CKT to tree diagrams: EM fields



gauge link

motion modification by EM fields

for right-handed particle

$$\delta S^{<0} = 2\pi\, \mathbf{p}\cdot\mathbf{B}\, \delta'(P^2) f(p_0)$$

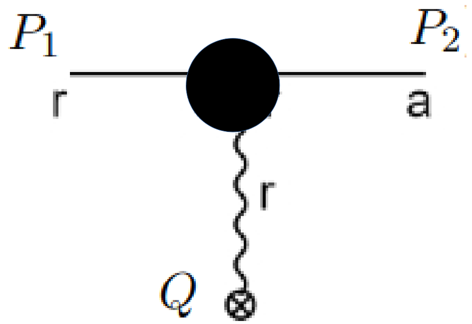
$$\delta S^{<i} = 2\pi \left[\epsilon^{ijk} E_j p_k + p_0 B_i\right] \delta'(P^2) f(p_0)$$

equilibrium distribution unchanged

SL, Tian, 2023

Structure of radiative correction in medium

$$\Gamma^\mu = F_0 u^\mu + F_1 \hat{p}^\mu + F_2 \frac{i\epsilon^{\mu\nu\rho\sigma} u_\nu P_\rho Q_\sigma}{2(P\cdot u)^2}$$



u^μ medium frame vector

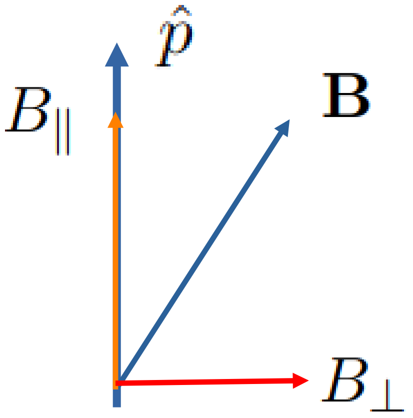
$$S^{<0} = 2\pi F_2 p B_\parallel \delta'(P^2) f(p_0)$$

$$S^{<i} = 2\pi \left[F_0 \epsilon^{ijk} E_j p_k + F_1 p_0 B_\perp^i + F_2 B_\parallel p^i \right] \delta'(P^2) f(p_0)$$

spin Hall effect

spin-perpendicular magnetic coupling

spin-parallel magnetic coupling

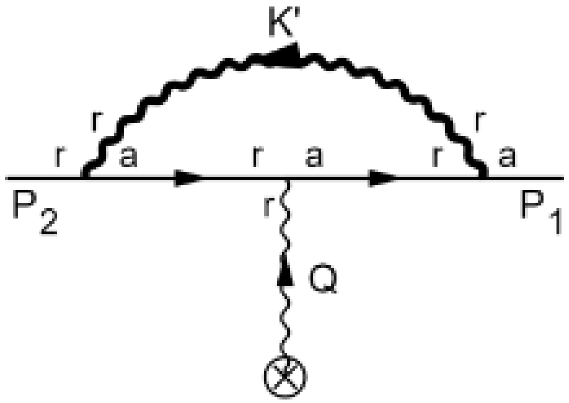


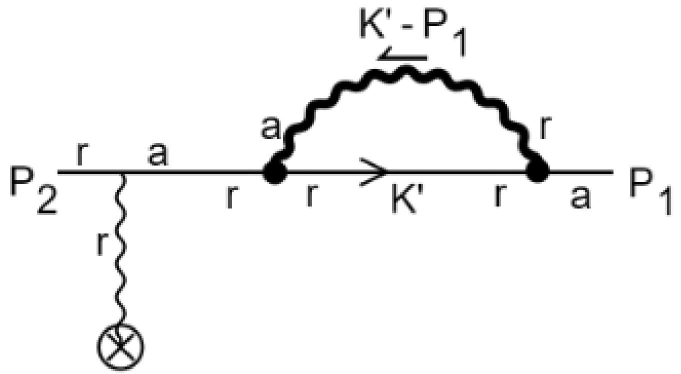
In vacuum $F_0 = F_1 = F_2 = 1$

In medium: lift of degeneracy possible

SL, Tian, 2023

One-loop correction to electromagnetic FF





Types of corrections:
vertex correction +
self-energy correction

$$\delta F_0 = \frac{2m_f^2}{p^2} X + \frac{m_f^2}{p^2}\left(1 - ln\frac{2p^2}{m_f^2}\right),$$

spin Hall effect

$$\delta F_1 = \frac{2m_f^2}{p^2} (X-1) + \frac{m_f^2}{p^2}\left(1 - ln\frac{2p^2}{m_f^2}\right),$$

spin-perpendicular
magnetic coupling

$$\delta F_2 = \frac{2m_f^2}{p^2} X + \frac{m_f^2}{p^2}\left(1 - ln\frac{2p^2}{m_f^2}\right),$$

spin-parallel
magnetic coupling

$$X(p, T)$$

SL, Tian, 2023

FF as renormalized couplings

CKT solution for off-equilibrium state

$$S^{<} = \frac{1}{4}\left[\left(1+\gamma^5\right)\gamma^\mu R_\mu + \left(1-\gamma^5\right)\gamma^\mu L_\mu \right]$$

$$R^\mu = -2\pi\delta(P^2)\left(P^\mu f_n + \frac{\epsilon^{\mu\nu\rho\sigma}P_\rho n_\sigma}{2P\cdot n}\partial_\nu f_n\right)$$

Hidaka, Pu, Yang 2016

n^μ arbitrary frame vector

$n^\mu \to u^\mu$

$$f\left(\frac{P\cdot u(X)}{T(X)}\right) \quad \Longrightarrow \quad S^i \sim \left(\beta\omega^i + \epsilon^{ijk}\hat{p}_l\hat{p}_k\beta\sigma_{jl} + \partial_i\beta\right)$$

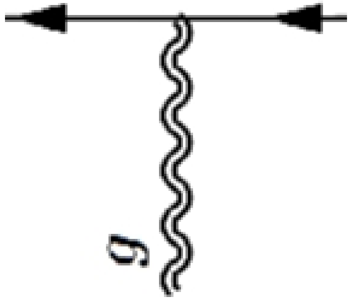
Hidaka, Pu, Yang 2017

Yi, Pu, Yang 2019

degenerate couplings to vorticity, shear, T-grad

Equilibrium state perturbed by metric: vertex+gauge link

perturbation I: motion modified by metric



$$S^<(X,P) = \int d^4y \sqrt{-g(X)} e^{-iP\cdot y} \langle \bar{\psi}_\beta(X+\frac{y}{2})\psi_\alpha(X-\frac{y}{2})\rangle$$

Gao, Huang, Mameda, Liu 2018

$$\bar{\psi}(X+\frac{y}{2}) = \bar{\psi}(X)\exp(\frac{y}{2}\cdot\overleftarrow{D}) \quad \psi(X-\frac{y}{2}) = \exp(-\frac{y}{2}\cdot D)\psi(X)$$

gravitational gauge link

$$D_\mu = \partial_\mu^X + \frac{1}{4}\omega_{\mu,ab}\gamma^{ab} - \Gamma^\lambda_{\mu\nu} y^\nu \partial_\lambda^y$$

perturbation II: rotation of spinor by spin connection

Perturbations can't change equilibrium distribution!

SL, Tian, to appear

Equilibrium state perturbed by metric: CKT

$$\gamma^\mu = e^\mu_a \gamma^a \qquad \bar{\psi} = \psi^\dagger \gamma^{\hat{0}}$$

μ curved index

a flat index

Gao, Huang, Mameda, Liu 2018

$$S^< = \frac{1}{4}\left[\,(1+\gamma^5)\,\gamma^a R_a + (1-\gamma^5)\,\gamma^a L_a\right]$$

Clifford algebra in flat basis

$$R^a = -2\pi\delta(P^2)\left(P^a f_n + \frac{\epsilon^{abcd} n_d}{2P\cdot n} e^\mu_b D_\mu (P_c f_n)\right)$$

f_n : equilibrium distribution

- agree with vertex+gravitational gauge link
- Christoffel term can realize vorticity and T-grad, but not shear!

$$D_\mu = \partial^X_\mu + \Gamma^\lambda_{\mu\nu}\frac{\partial}{\partial P_\nu}P_\lambda$$

$$\langle S^<(X,P)\rangle_{\text{off-eq}} \overset{?}{=} \langle S^<(X,P)\rangle_{\text{eq},h_{\mu\nu}}$$

SL, Tian, to appear

Inequivalence?

What we have

$$R^a = -2\pi\delta(P^2)\left(P^a f_n + \frac{\epsilon^{abcd} n_d}{2P\cdot n} e_b^\mu D_\mu (P_c f_n)\right)$$

f_n : equilibrium distribution

$$D_\mu = \partial_\mu^X + \Gamma^\lambda_{\mu\nu}\frac{\partial}{\partial P_\nu} P_\lambda$$

Radiative correction expect to affect the Christoffel term only

What we want

$$R^a = -2\pi\delta(P^2)\left(P^a f_n + \frac{\epsilon^{abcd} P_c n_d}{2P\cdot n} \partial_b f_n(X)\right)$$

f_n : off-equilibrium distribution

SL, Tian, to appear

A metric induced off-equilibrium state

$$R^a = -2\pi\delta(P^2)\left(P^a f_n + \frac{\epsilon^{abcd}n_d}{2P\cdot n}e^\mu_b D_\mu(P_c f_n)\right)$$

$$D_\mu = \partial^X_\mu + \Gamma^\lambda_{\mu\nu}\frac{\partial}{\partial P_\nu}P_\lambda$$

$$f\left(\frac{p_a e^a_\mu u^\mu}{T}\right)$$

$$u^\mu = (g_{00}^{-1/2}, 0, 0, 0)$$

equilibrium state in curved space

$$u^a = u^\mu e^a_\mu = -g_{00}^{-1/2} v^i$$

off-equilibrium state in flat space

Proper choice of vielbein $e^{\hat{i}}_0 = -v^i$

SL, Tian, to appear

Equivalence from hydrodynamics

$$G^R_{\pi_i\pi_j} = \left(\delta_{ij} - \hat{k}_i\hat{k}_j\right)\frac{\eta k^2}{i\omega - \gamma_\eta k^2} + \hat{k}_i\hat{k}_j\frac{(\epsilon+p)(k^2c_s^2 - i\omega\gamma_s k^2)}{\omega^2 - k^2c_s^2 + i\omega\gamma_s k^2}$$

$$G^R_{\epsilon\epsilon} = \frac{(\epsilon+p)k^2}{\omega^2 - k^2c_s^2 + i\omega\gamma_s k^2}$$

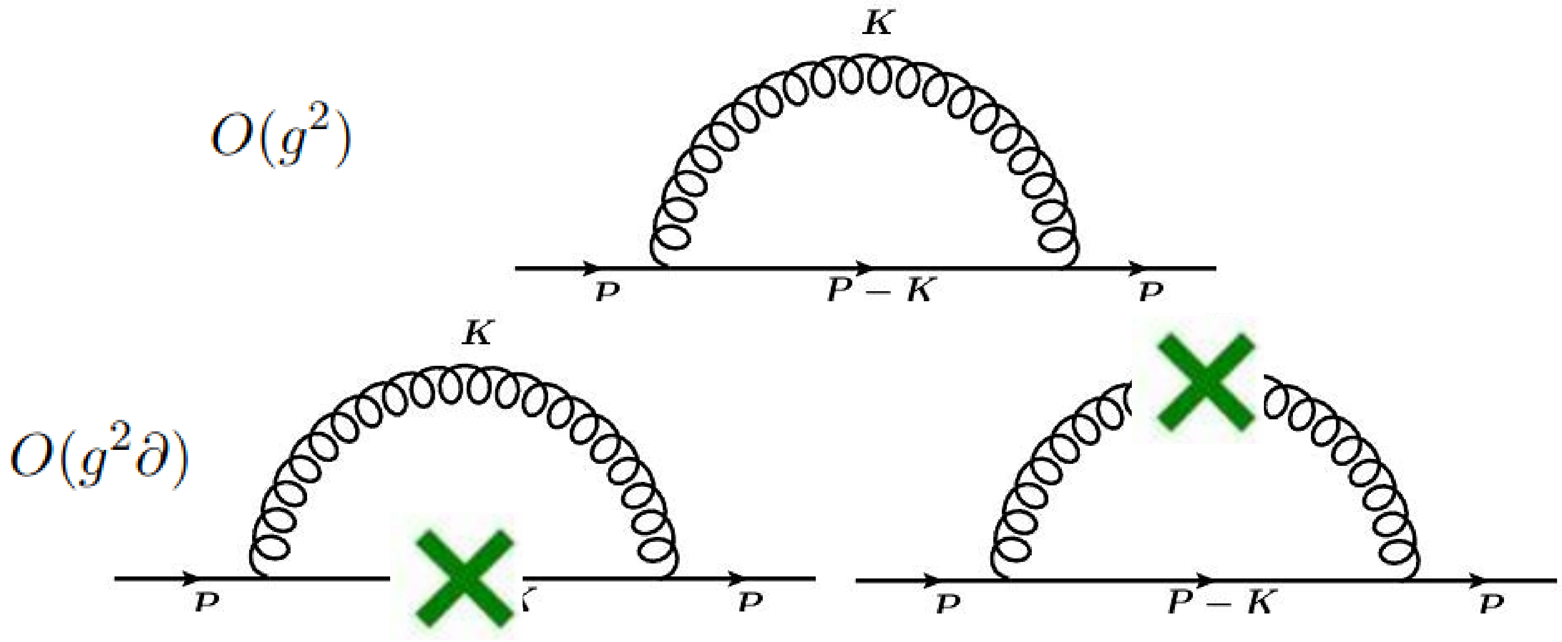
$\omega \to 0$ $\quad G^R_{\pi_i\pi_j} \to -\delta_{ij}(\epsilon+p) \quad G^R_{\epsilon\epsilon} \to -\frac{\epsilon+p}{c_s^2}$

$$\delta\pi_i = h_{0j}G^R_{\pi_i\pi_j} = -(\epsilon+p)\delta_{ij}h_{0j} \qquad \delta\epsilon = \frac{1}{2}h_{00}G^R_{\epsilon\epsilon} = -\frac{(\epsilon+p)}{2c_s^2}h_{00}$$

→ $h_{0i} = -v^i \quad h_{00} = -2\frac{\delta T}{T}$

- Equivalence on time scale >> equilibration of hydro modes
- $\langle S^<(X,P)\rangle_{\text{off-eq}} = \langle S^<(X,P)\rangle_{\text{eq},h_{\mu\nu}}$ holds in collisional theory in principle, may still work in collisionless theory in practice

Radiative corrections to spectral density



off-equilibrium propagators for quark/gluon from collisionless CKT

Hidaka, Pu, Yang 2017

Huang, Mitkin, Sadofyev, Speranza 2020

Hattori, Hidaka, Yamamoto, Yang 2020

SL 2020

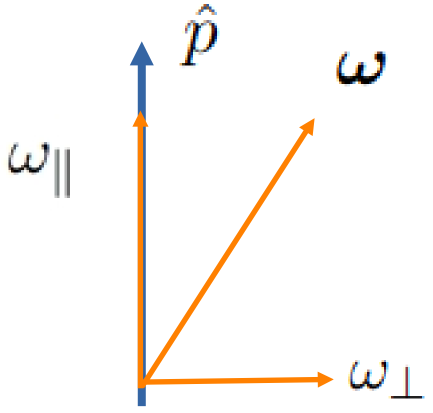
- Equilbrium self-energy doesn’t lead to polarization
- Off-equilibrium correction to spectral density contributes to polarization in addition to CKT results

SL, Tian, to appear

Polarization & damping rates

$$i\frac{\Sigma_{ar}(P)}{g^2} = \gamma^5\gamma_0\omega\cdot\hat{p}A_0(p,T) + \gamma^5\gamma_k\omega_{\|}^k A_{\|}(p,T) + \gamma^5\gamma_k\omega_{\perp}^k A_{\perp}(p,T)$$

$$+\,\epsilon^{ijk}\gamma^5\gamma_k\hat{p}_i\hat{p}_l\sigma_{jl}B(p,T) + \epsilon^{ijk}\gamma^5\gamma_k\hat{p}_i\partial_j\beta C(p,T)$$



A, B, C: complex functions

$$S_{ra}^R = \frac{i(P_\mu - iA_\mu)\bar{\sigma}^\mu}{(P-iA)^2} \qquad S_{ra}^L = \frac{i(P_\mu + iA_\mu)\sigma^\mu}{(P+iA)^2}$$

- lift of degeneracy in coupling to vorticity, shear, T-grad
- splitting of damping rates for R&L-handed particles

magnetic analog:

Dong, SL, 2024

SL, Tian, to appear

Conclusion

- Equilibrium state perturbed by metric can’t describe complete off-equilbrium effect
- An equilbrium state in curved space describes off-equilbrium state in flat space
- Off-equilibrium correction to spectral density contributes to polarization
- Lift of degeneracy of spin coupling to vorticity, shear and T-grad
- Splitting of damping rate of R&L-handed particles

Outlook

- Quantum kinetic theory with spectral density correction
- Accleration? Off-equilibrium effect beyond static limit

Thank you!

$$\left(2k^{\mu} - \frac{\partial \mathrm{Re}\Sigma}{\partial k_{\mu}}\right) \frac{\partial \rho}{\partial X^{\mu}} + \frac{\partial \mathrm{Re}\Sigma}{\partial X_{\mu}} \frac{\partial \rho}{\partial k^{\mu}} = -\left\{\Gamma, \mathrm{Re}\, G_R\right\}_{P.B.}$$

$$\rho(k, X) = \frac{\Gamma(k, X)}{\left(k^2 - m^2 - \Sigma^{\delta}(X) - \mathrm{Re}\,\Sigma_R(k, X)\right)^2 + \left(\Gamma(k, X)/2\right)^2}$$