Relaxation terms for hydrodynamic transport in Weyl semimetals

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Chirality, Vorticity, and Magnetic Field in Quantum Matter

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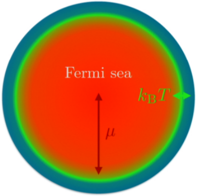
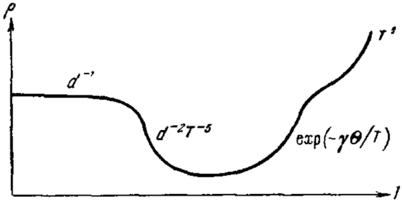
Context. . .

Fermi liquid: long-lived quasiparticles
$\tau_{ee} \gg \tau_{\text{imp}}, \tau_{e\gamma} \Rightarrow$ Wiedemann–Franz law

$$\frac{\kappa}{\sigma T} = L_0 = \frac{\pi^2}{3}$$



Clean, strongly-coupled materials
$\Rightarrow \quad \tau_{ee} \ll \tau_{\text{imp}}, \tau_{e\gamma}$ (no quasiparticles)
conserved momentum $\Rightarrow$ **emergent hydrodynamic transport**
[review, Narozhny (2022)].



Features of transport:

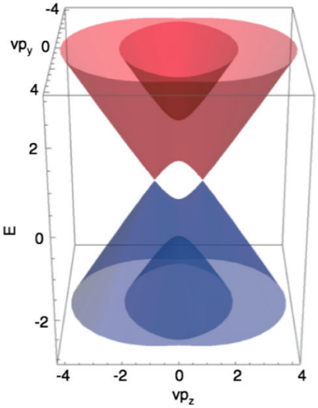
- Gurzhi effect (minimum of resistivity),
- Breakdown of Wiedemann–Franz law,
- Non-local transport.

Examples

Graphene, ultra-pure 2D heterostructures, Dirac/Weyl semimetals, cuprates.

. . . and motivation

Typical band structure of Weyl semimetals [Armitage et al. (2018)].
Examples: NbP, TaAs, TaP, NbAs, WP_2.



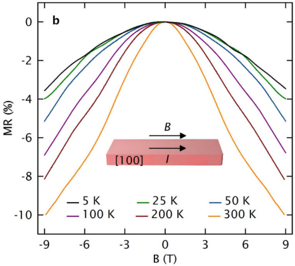


Figure: Negative longitudinal $(B \parallel E)$ magneto-resistance [Nielsen, Ninomiya (1981)]. $\sigma \propto B^2$ in NbP [Niemann et al. (2017)].

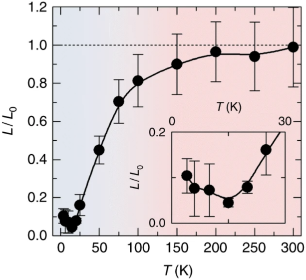


Figure: Breakdown of the Wiedemann-Franz law in WP_2 [Gooth et al. (2018)].

Setup

Conserved charges [Landsteiner et al., Lucas et al., Gorbar et al., Chernodub et al., . . .]

$$\partial_\mu T^{\mu\nu} = F^{\nu\lambda} J_\lambda \qquad \partial_\mu J^\mu = 0 \qquad \partial_\mu J_5^\mu = cE\cdot B$$

Constitutive relations:

- symmetries,
- derivative expansion,
- second law of thermodynamics $\partial_\mu S^\mu \geq 0$.

Relativistic hydrodynamics with $U(1)_V \times U(1)_A$ anomaly [Son, Surówka (2009)] and $B \sim \mathcal{O}(1)$

$$\begin{aligned} T^{\mu\nu} &= \varepsilon u^\mu u^\nu + P\Delta^{\mu\nu} + \xi^\varepsilon \left(u^\mu B^\nu + u^\nu B^\mu\right) + \mathcal{O}(\partial) \\ J^\mu &= n u^\mu + \xi B^\mu + \mathcal{O}(\partial) \\ J_5^\mu &= n_5 u^\mu + \xi_5 B^\mu + \mathcal{O}(\partial) \end{aligned}$$

with $\xi = c\mu_5$, $\xi_5 = c\mu$, and $\xi^\varepsilon = c\mu\mu_5$. Dissipative and hydrostatic terms are $\mathcal{O}(\partial)$.

Linear response and transport

Linear response theory [Martin, Kadanoff (1963)][1]:

$$\begin{pmatrix} \delta \mathbf{J} \\ \delta \mathbf{Q} \end{pmatrix} = \begin{pmatrix} \sigma(\omega) & \alpha(\omega) \\ T\bar{\alpha}(\omega) & \bar{\kappa}(\omega) \end{pmatrix} \begin{pmatrix} \delta \mathbf{E} \\ -\boldsymbol{\nabla} \delta T \end{pmatrix}$$

Compute *longitudinal DC transport* $\mathbf{E} \parallel \mathbf{B} \Rightarrow$ conductivities diverge as $\omega \to 0$.

Indeed, $n\delta\mathbf{E}$ adds momentum, $\mathbf{J} \cdot \delta\mathbf{E} \propto \mathbf{B} \cdot \delta\mathbf{E}$ adds energy, $\delta\mathbf{E} \cdot \mathbf{B}$ adds axial charge $\Rightarrow$ need **energy, momentum and axial charge relaxations**.

[1] Heat current $Q^i = T^{0i} - \mu J^i - \mu_5 J_5^i$ is not anomalous.

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We look for relaxations such that:

- conductivities are finite in DC,
- transport coefficients are Onsager-reciprocal $\alpha = \bar{\alpha}$,
- electric charge is conserved,
- (relaxations are independent).

[1] Heat current $Q^i = T^{0i} - \mu J^i - \mu_5 J_5^i$ is not anomalous.

Diagonal relaxations

Natural choice [Landsteiner et al. (2014), Abbasi et al. (2016), ...]

$$\partial_\mu \delta T^{\mu 0} = \delta(F^{0\lambda} J_\lambda) - \frac{\delta T^{00}}{\tau_{\varepsilon\varepsilon}}$$

$$\partial_\mu \delta T^{\mu i} = \delta(F^{i\lambda} J_\lambda) - \frac{\delta T^{0i}}{\tau_m}$$

$$\partial_\mu \delta J^\mu = 0$$

$$\partial_\mu \delta J_5^\mu = c\delta E \cdot B - \frac{\delta J_5^0}{\tau_{n_5 n_5}}$$

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$$\partial_\mu \delta J^{\mu} = -\frac{\delta J^0}{\tau_{nn}}$$

$$\partial_\mu \delta J_5^{\mu} = c\delta E \cdot B - \frac{\delta J_5^0}{\tau_{n_5 n_5}}$$

Onsager relations require $\tau_{n_5 n_5} = \tau_{nn} = \tau_m = \tau_{\varepsilon\varepsilon}$.

$\Rightarrow$ Cannot have finite DC conductivities, Onsager reciprocal transport and conservation of electric charge.

Generalized relaxation

$$\left.\begin{aligned}
\partial_t \delta\varepsilon + \ldots &= -\frac{1}{\tau_{\varepsilon\varepsilon}}\delta\varepsilon - \frac{1}{\tau_{\varepsilon n}}\delta n - \frac{1}{\tau_{\varepsilon n_5}}\delta n_5 \\
\partial_t \delta n + \ldots &= -\frac{1}{\tau_{n\varepsilon}}\delta\varepsilon - \frac{1}{\tau_{nn}}\delta n - \frac{1}{\tau_{nn_5}}\delta n_5 \\
\partial_t \delta n_5 + \ldots &= -\frac{1}{\tau_{n_5\varepsilon}}\delta\varepsilon - \frac{1}{\tau_{n_5 n}}\delta n - \frac{1}{\tau_{n_5 n_5}}\delta n_5
\end{aligned}\right\} = \hat{\tau}\cdot(\delta\varepsilon, \delta n, \delta n_5)$$

$$\partial_t \delta P^i + \cdots = -\frac{\delta v^i}{\tau_m}$$

Onsager relations imply $\hat{\chi}\cdot\hat{\tau} = \hat{\tau}^T\cdot\hat{\chi}^T$, explicitly

$$0 = \frac{\chi_{nn_5}}{\tau_{\varepsilon n_5}} + \frac{\chi_{nn}}{\tau_{\varepsilon n}} - \frac{\chi_{\varepsilon n_5}}{\tau_{nn_5}} + \frac{\chi_{\varepsilon n}}{\tau_{\varepsilon\varepsilon}} - \frac{\chi_{\varepsilon n}}{\tau_{nn}} - \frac{\chi_{\varepsilon\varepsilon}}{\tau_{n\varepsilon}} \qquad + \text{2 more}$$

Finite DC conductivities, Onsager relations and electric charge conservation $\Rightarrow$
However:

- only σ_{DC} is anomalous (NMR) $\sigma_{\text{DC}} = \sigma_{\text{Drude}} + \alpha B^2$
- entropy production not positive definite.

Kinetic theory

Boltzmann equation (BE) for $f_{\mathbf{p}} = f(t, \mathbf{x}, \mathbf{p})$

$$\partial_t f_{\mathbf{p}} + \mathbf{p} \cdot \boldsymbol{\nabla} f_{\mathbf{p}} = I_{\text{coll}}[f_{\mathbf{p}}]$$

If $I_{\text{coll}} = I_{ee}$, then $I_{ee} = 0$ gives Detailed Balance $\Rightarrow$ Local Thermodynamic Equilibrium

$$f_{\mathbf{p}} = \frac{1}{1 + e^{(\varepsilon_{\mathbf{p}} - \mathbf{u}\cdot\mathbf{p} - \mu)/T}}$$

Integrate BE in momentum space against $\varepsilon_{\mathbf{p}}$, $\mathbf{p}$ and $1 \Rightarrow$ hydrodynamics

$$\int \frac{\mathrm{d}^3\mathbf{p}}{(2\pi)^3} A \; I_{ee} = 0 \qquad \text{for} \qquad A = \{\varepsilon_{\mathbf{p}}, \mathbf{p}, 1\}$$

Charges are conserved in kinetic theory if $I_{\text{coll}} = I_{ee}$.

Relaxation Time Approximation

Momentum relaxation: linearize [Gorbar et al. (2018)]

$$f_{\mathbf{p}} \approx f^{(0)} + (\mathbf{p} \cdot \mathbf{u}) \frac{\partial f^{(0)}}{\partial \varepsilon_{\mathbf{p}}} \qquad \text{with} \qquad f^{(0)} = \frac{1}{1 + e^{(\varepsilon_{\mathbf{p}} - \mu)/T}}$$

Considering $I_{\text{coll}} = I_{ee} + I_{\text{imp}}$ we have

$$I_{\text{imp}} \approx -\frac{f_{\mathbf{p}} - f^{(0)}}{\tau_m} \qquad \Rightarrow \qquad \partial_t P^i + \cdots = -\frac{P^i}{\tau_m}$$

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Energy and charge relaxations: $I_{\mathsf{coll}} = I_{ee} + I_{\mathsf{imp}} + I_{e\gamma}$

$$I_{e\gamma} \approx -\frac{f_{\mathbf{p}} - \bar{f}^{(0)}}{\tau_n} \qquad \Rightarrow \qquad \begin{cases} \partial_t \varepsilon + \cdots = -\frac{\varepsilon - \bar{\varepsilon}}{\tau_n} \\ \partial_t n + \cdots + \cdots = -\frac{n-\bar{n}}{\tau_n} \end{cases}$$

Generalized relaxations from kinetic theory

Consider $\tau_n = \tau_n(\varepsilon_{\mathbf{p}})$ and expand

$$I_{e\gamma} = \sum_{j\geq -2} \varepsilon_{\mathbf{p}}^{j} \frac{f^{(0)} - \bar{f}^{(0)}}{\tau_{j+2}} = \frac{1}{\varepsilon_{\mathbf{p}}^{2}} \frac{f^{(0)} - \bar{f}^{(0)}}{\tau_0} + \frac{1}{\varepsilon_{\mathbf{p}}} \frac{f^{(0)} - \bar{f}^{(0)}}{\tau_1} + \frac{f^{(0)} - \bar{f}^{(0)}}{\tau_2} + \dots$$

Integrate

$$\partial_t \varepsilon + \dots = -\frac{M_1 - \bar{M}_1}{\tau_0} - \frac{n - \bar{n}}{\tau_1} - \frac{\varepsilon - \bar{\varepsilon}}{\tau_2} - \frac{M_3 - \bar{M}_3}{\tau_3} + \dots$$

$$\partial_t n + \dots = -\frac{M_0 - \bar{M}_0}{\tau_0} - \frac{M_1 - \bar{M}_1}{\tau_1} - \frac{n - \bar{n}}{\tau_2} - \frac{\varepsilon - \bar{\varepsilon}}{\tau_3} + \dots$$

Linearize and identify

$$\frac{1}{\tau_{nn}} = \frac{\partial M_0}{\partial n}\frac{1}{\tau_0} + \frac{\partial M_1}{\partial n}\frac{1}{\tau_1} + \frac{1}{\tau_2} + \dots \qquad \frac{1}{\tau_{n\varepsilon}} = \dots$$

Mixed relaxations from kinetic theory **identically** satisfy Onsager $\hat{\chi} \cdot \hat{\tau} = \hat{\tau}^T \cdot \hat{\chi}^T$.

Summary and Outlook

- Hydrodynamic regime of Weyl semimetals $\Rightarrow$ anomalous relativistic two-components fluid.
- Longitudinal magneto-conductivities are divergent in DC $\Rightarrow$ need energy, momentum and axial charge relaxations.
- **Generalized relaxations** are necessary to satisfy fundamental considerations:
  - finite DC conductivities
  - Onsager relations
  - conservation of electric charge
- They can be justified from kinetic theory using energy-dependent RTA.
- **Entropy production** not positive definite.
- **Thermoelectric transport** not anomalous.
- BKG-like model to preserve charge conservation.

For the future:

- Explicit examples from microscopic physics?
- Other relaxations mechanisms?

Thank you for the attention!

Backup slides

Longitudinal magnetoresistance

Linear Response $\delta\mathbf{J} = \sigma\delta\mathbf{E}$. If $J^\mu \sim \mathcal{O}(\partial)$, numerator of $\sigma \sim \mathcal{O}(\partial)$ must be truncated at order one.

$\Longrightarrow$ If $B \sim \mathcal{O}(\partial)$, σ cannot depend on $B^2 \sim \mathcal{O}(\partial^2)$.

Standard order-one anomalous hydrodynamics fails to predict negative magnetoresistance — cfr. [Landsteiner et al. (2014), Lucas et al. (2016)]

Solution

Consider $B \sim \mathcal{O}(1) \Rightarrow$ now $B^2 \sim \mathcal{O}(1)$ and appears in the conductivity.

Anomalous ideal fluid, $\xi^\varepsilon = \frac{1}{2}c\mu^2$ and $\xi = c\mu$ [Ammon et al. (2021)].

Magneto-resistance is now *well-defined* and *physical*.

DC transport I

Longitudinal DC conductivities are infinite $\Rightarrow$ add momentum, energy and charge relaxations [Landsteiner et al. (2014), Abbasi et al. (2016)]

$$\partial_\mu \delta T^{\mu 0} = \delta(F^{0\lambda} J_\lambda) - \frac{\delta T^{00}}{\tau_{\varepsilon\varepsilon}}$$

$$\partial_\mu \delta T^{\mu i} = \delta(F^{i\lambda} J_\lambda) - \frac{\delta T^{0i}}{\tau_m}$$

$$\partial_\mu \delta J^{\mu} = -\frac{\delta J^0}{\tau_{nn}}$$

$$\partial_\mu \delta J_5^{\mu} = c\delta E \cdot B - \frac{\delta J_5^0}{\tau_{n_5 n_5}}$$

Onsager relations $\tau_{\varepsilon\varepsilon} = \tau_{nn} = \tau_{n_5 n_5} = \tau_m \Rightarrow$ unphysical solution.

DC transport II

First suggestion:
anomalous flow is *superfluid*-like [Sadofyev, Yin (2016), Stephanov, Yee (2016)] $\Rightarrow$ relax normal component only, e.g. $\delta J^0 = \delta n + c\mu_5 \mathbf{B} \cdot \delta \mathbf{v} \to \delta n$

$$\partial_\mu \delta T^{\mu 0} = \delta(F^{0\lambda} J_\lambda) - \frac{\delta \varepsilon}{\tau_{\varepsilon\varepsilon}}$$

$$\partial_\mu \delta T^{\mu i} = \delta(F^{i\lambda} J_\lambda) - \frac{\delta P^i}{\tau_m}$$

$$\partial_\mu \delta J^{\mu} = -\frac{\delta n}{\tau_{nn}}$$

$$\partial_\mu \delta J_5^{\mu} = c\delta E \cdot B - \frac{\delta n_5}{\tau_{n_5 n_5}}$$

Onsager relations $\tau_{\varepsilon\varepsilon} = \tau_{nn} = \tau_{n_5 n_5}$, while $\tau_m \geq 0$ is free $\Rightarrow$ still bad.

DC transport III: generalized relaxations

Second suggestion: generalized-mixed relaxations

$$\left.\begin{array}{ll} \text{energy:} & \frac{1}{\tau_{\varepsilon\varepsilon}}\delta\varepsilon + \frac{1}{\tau_{\varepsilon n}}\delta n + \frac{1}{\tau_{\varepsilon n_5}}\delta n_5 \\ \text{charge:} & \frac{1}{\tau_{n\varepsilon}}\delta\varepsilon + \frac{1}{\tau_{nn}}\delta n + \frac{1}{\tau_{n n_5}}\delta n_5 \\ \text{axial charge:} & \frac{1}{\tau_{n_5\varepsilon}}\delta\varepsilon + \frac{1}{\tau_{n_5 n}}\delta n + \frac{1}{\tau_{n_5 n_5}}\delta n_5 \end{array}\right\} = \hat{\tau}\cdot\varphi$$

Onsager relations imply $\hat{\chi}\cdot\hat{\tau} - \hat{\tau}^T\cdot\hat{\chi}^T = 0$, explicitly

$$0 = \frac{\chi_{nn_5}}{\tau_{\varepsilon n_5}} + \frac{\chi_{nn}}{\tau_{\varepsilon n}} - \frac{\chi_{\varepsilon n_5}}{\tau_{nn_5}} + \frac{\chi_{\varepsilon n}}{\tau_{\varepsilon\varepsilon}} - \frac{\chi_{\varepsilon n}}{\tau_{nn}} - \frac{\chi_{\varepsilon\varepsilon}}{\tau_{n\varepsilon}} \qquad + 2\text{ more}$$

- Only σ has NMR in DC, while α and κ have standard Drude form.
- Entropy is not conserved

$$\frac{1}{\tau_{\varepsilon\varepsilon}} - \frac{\mu}{\tau_{n\varepsilon}} - \frac{\mu_5}{\tau_{n_5\varepsilon}} \neq 0 \qquad + 2\text{ more}$$

Collision integrals

We take $I_{\mathsf{coll}} = I_{ee} + I_{\mathsf{imp}} + I_{e\gamma}$ such that

$$I_{\mathsf{imp}} = \int \mathrm{d}^3\mathbf{p}' W_{\mathbf{p}\to\mathbf{p}'}\left[f_{\mathbf{p}} - f_{\mathbf{p}'}\right]\delta(\varepsilon_{\mathbf{p}} - \varepsilon_{\mathbf{p}'}) \qquad \Rightarrow \qquad I_{\mathsf{imp}} \approx -\frac{f_{\mathbf{p}} - f^{(0)}}{\tau_m}$$

and

$$I_{e\gamma} = \int \mathrm{d}^3\mathbf{q} W_{\mathbf{p'},\mathbf{q}\to\mathbf{p}}\left[f_{\mathbf{p}'}(1 - f_{\mathbf{p}})n_{\mathbf{q}} - f_{\mathbf{p}}(1 - f_{\mathbf{p}'})(1 + n_{\mathbf{q}})\right]\delta(\varepsilon_{\mathbf{p}} - \varepsilon_{\mathbf{p}'} - \omega_{\mathbf{q}}) +$$
$$+ \int \mathrm{d}^3\mathbf{q} W_{\mathbf{p}'\to\mathbf{p},\mathbf{q}}\left[f_{\mathbf{p}'}(1 - f_{\mathbf{p}})(1 + n_{\mathbf{q}}) - f_{\mathbf{p}}(1 - f_{\mathbf{p}'})n_{\mathbf{q}}\right]\delta(\varepsilon_{\mathbf{p}} + \omega_{\mathbf{k}} - \varepsilon_{\mathbf{p}'})$$

if phonons in thermal equilibrium

$$I_{e\gamma} \approx -\frac{f_{\mathbf{p}} - \bar{f}^{(0)}}{\tau_n} \qquad \text{with} \qquad \bar{f}^{(0)} = \frac{1}{1 + e^{(\varepsilon_p - \bar{\mu})/\bar{T}}}$$

Mapping relaxations

Consider $\tau_n = \tau_n(\varepsilon_p)$ and expand

$$I_{\mathsf{coll}} = \sum_{j\geq -2} \varepsilon_p^j \frac{f^{(0)} - \bar{f}^{(0)}}{\tau_{j+2}} = \frac{1}{\varepsilon_p^2}\frac{f^{(0)} - \bar{f}^{(0)}}{\tau_0} + \frac{1}{\varepsilon_p}\frac{f^{(0)} - \bar{f}^{(0)}}{\tau_1} + \frac{f^{(0)} - \bar{f}^{(0)}}{\tau_2} + \dots$$

Integrate

$$\partial_t \varepsilon + \dots = -\frac{M_1 - \bar{M}_1}{\tau_0} - \frac{n - \bar{n}}{\tau_1} - \frac{\varepsilon - \bar{\varepsilon}}{\tau_2} - \frac{M_4 - \bar{M}_4}{\tau_3} + \dots$$

$$\partial_t n + \dots = -\frac{M_0 - \bar{M}_0}{\tau_0} - \frac{M_1 - \bar{M}_1}{\tau_1} - \frac{n - \bar{n}}{\tau_2} - \frac{\varepsilon - \bar{\varepsilon}}{\tau_3} + \dots$$

Linearize and identify

$$\frac{1}{\tau_{nn}} = \frac{\partial M_0}{\partial n}\frac{1}{\tau_0} + \frac{\partial M_1}{\partial n}\frac{1}{\tau_1} + \frac{1}{\tau_2} + \dots \qquad \frac{1}{\tau_{n\varepsilon}} = \dots$$

BKG model: I_{ee}

Write

$$f_{\mathbf{p}} = f^{(0)} + \delta f_{\mathbf{p}} = f^{(0)}(1 + h_{\mathbf{p}})$$

And linearize collision integral $I_{ee} \simeq L_{ee} h_{\mathbf{p}} + \mathcal{O}(\delta^2)$. It obeys

$$L_{ee} 1 = 0 \qquad L_{ee} \mathbf{p} = 0 \qquad L_{ee} \varepsilon_{\mathbf{p}} = 0$$

which imply energy, momentum and charge conservation. Its RTA form

$$L_{ee} h_{\mathbf{p}} \approx -f^{(0)} \frac{h_{\mathbf{p}}}{\tau}$$

does not conserve energy and charge $\Rightarrow$ BKG model, i.e. RTA on the subspace orthogonal to zero modes.

BKG model: generalized relaxations

Write

$$f^{(0)} = \bar{f}^{(0)} + \delta f = \bar{f}^{(0)}(1+h)$$

$I_{e\gamma}$ conserves charge, while its RTA form does not. Then, BKG model

$$L_{e\gamma} \approx L_* = -\frac{\bar{f}^{(0)}}{\tau} \sum_{i,j} a_{i,j} \psi^i \tilde{\psi}^j$$

with $\psi \sim 1$ $\psi^2 \sim \varepsilon_{\mathbf{p}}$ charge and energy eigenmodes of I_{ee}.

Charge conservation implies $a_{1,i} = 0$

$$\partial_t f^{(0)} + \cdots = -\frac{1}{\tau}\left[f^{(0)} - \frac{n}{\bar{n}}\bar{f}^{(0)} + \tilde{\alpha}_2 \bar{f}^{(0)}(n-\bar{n}) + \tilde{\alpha}_1 \bar{f}^{(0)}(\bar{\varepsilon} n - \bar{n}\varepsilon)\right]$$

$\Rightarrow$ charge identically conserved, energy has generalized relaxations.