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Vortical Waves in a Quantum Fluid with Massless Fermions

The 8th Conference on Chirality, Vorticity and Magnetic Field in Quantum Matter

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Motivation

Motivation: QGP, Anomalous Transport and Helicity

- The quark-gluon plasma (QGP) is a state of deconfined quarks and gluons generated above a critical temperature $T_c \sim 150\, MeV$. Chiral symmetry is restored up to the small values of the quark masses.

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- A variety of anomalous transport phenomena have been studied in the plasma:
  - Chiral Magnetic Effect (CME) $J \sim \mu_5 B$
  - Chiral Separation Effect (CSE) $J^5 \sim \mu B$

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  - Chiral Separation Effect (CSE) $J^5 \sim \mu B$
  - Chiral Vortical Effect (CVE) $J \sim \mu \mu_5 \Omega$
  - Chiral Vortical Separation Effect (CVSE)

    $J^5 \sim (\mu^2 + \mu_5^2 + \#T^2)\Omega$

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- These studies on anomalous transport were focused on the vector (electric) and axial (chiral) degrees of freedom, related to the conserved charges of the $U(1)_V \times U(1)_A$ *symmetry* of the Lagrangian.
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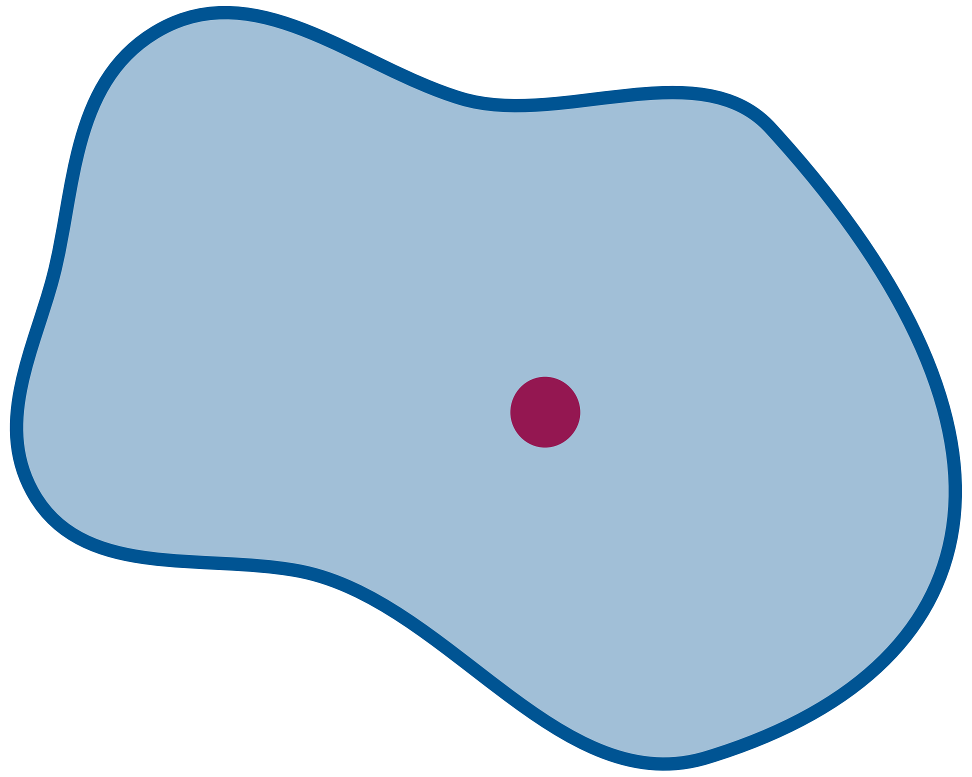

	Particle		Anti-particle	
Chirality	+	-	+	-
Helicity	+	-	-	+

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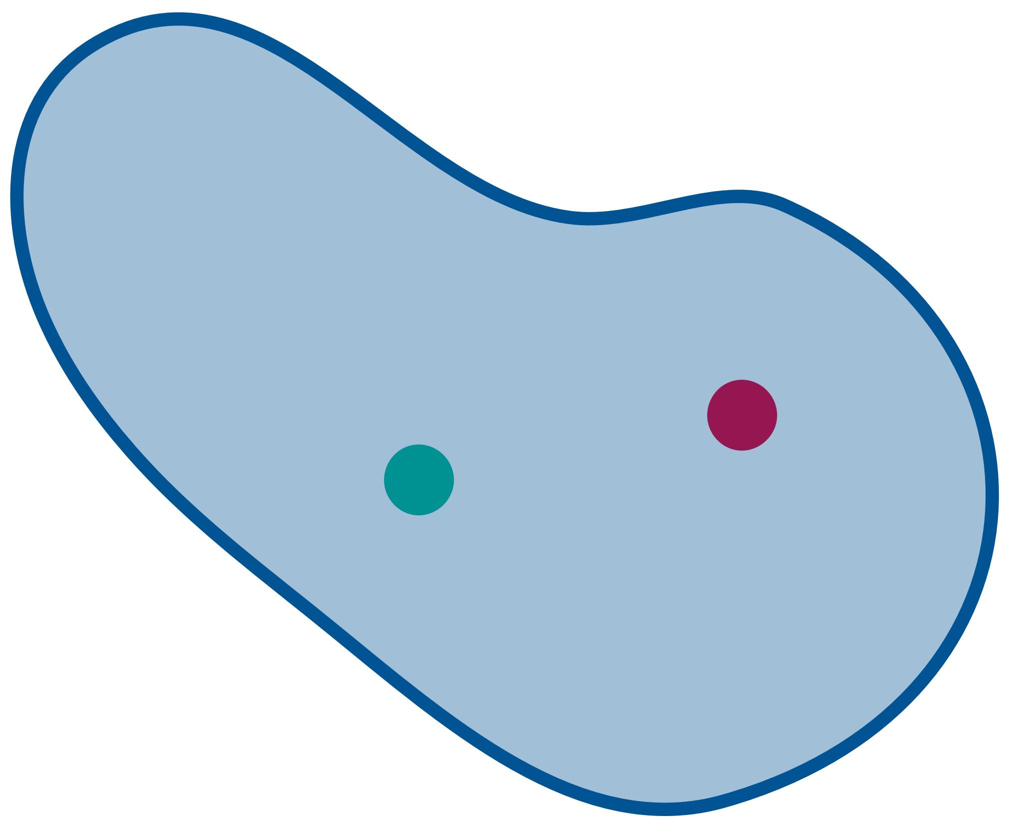

	Particle/Anti-particle			
Chirality	+/+ 2	+/- 0	-/+ 0	-/- -2
Helicity	+/- 0	+/+ 2	-/- -2	-/+ 0

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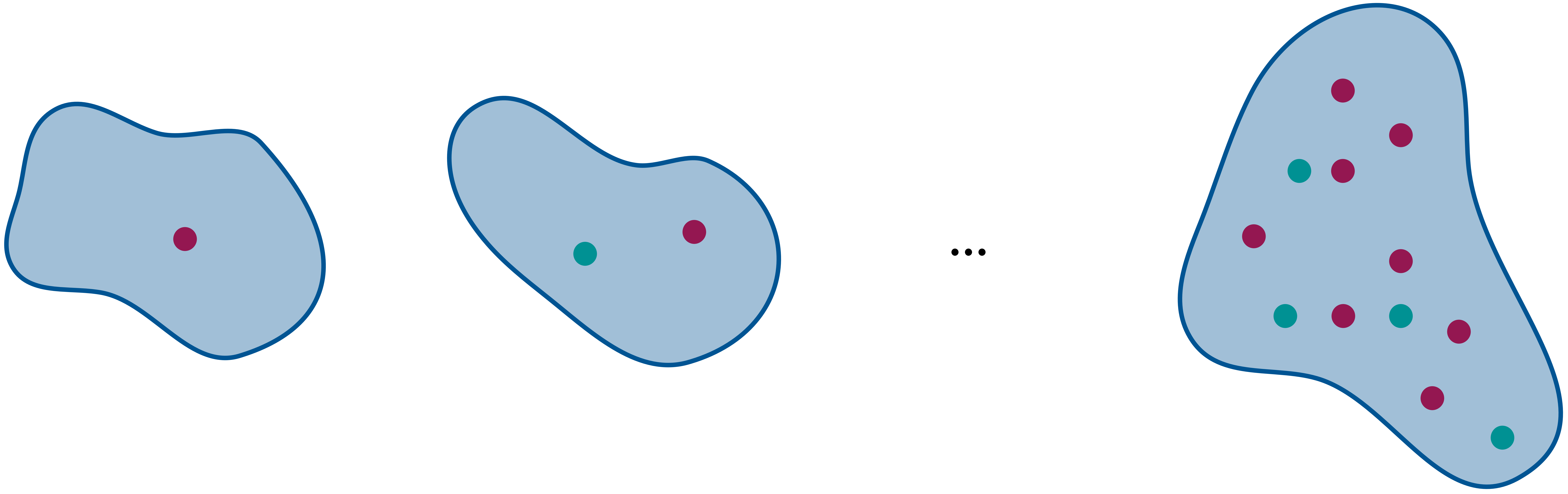

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- These studies on anomalous transport were focused on the vector (electric) and axial (chiral) degrees of freedom, related to the conserved charges of the $U(1)_V \times U(1)_A$ *symmetry* of the Lagrangian.
- There is another conserved charge: Helicity.
- Analogous to the CME, CVE… Transport phenomena has been uncovered involving the helicity current, as the Helical Separation Effect (HSE) $J^H \sim TB$
- The inclusion of the helical degree of freedom is necessary for the study of collective excitations. It modifies the previously discussed CMW and CVW.

Disclaimer: Helicity is not conserved exactly. Its violation comes from pair creation-annihilation processes e.g. $e_R^+ e_L^- \to e_L^+ e_R^-$.

Axial charge is not conserved due to the axial anomaly.

Motivation: QGP, Anomalous Transport and Helicity

- Goal
  Study the collective excitations of a fluid consisting of massless (anti)fermions in the presence of rotation.

Setup

Hydrodynamic setup:
Conservation Equations and Longitudinal Perturbations

- The waves are characterised by the dynamics of the (approximately)-conserved quantities.
- In the problem under consideration energy and momentum are conserved, as well as the vector, axial and helical charges.
- Therefore, we impose the conservation equations:

$$\partial_\mu T^{\mu\nu} = 0 \qquad \partial_\mu J^\mu_V = 0 \qquad \partial_\mu J^\mu_A = 0 \qquad \partial_\mu J^\mu_H = 0$$

- These operators are found by means of the Noether's theorem and for a free massless fermion they are

$$T^{\mu\nu} = \frac{i}{2}\bar{\psi}\gamma^{(\mu}\partial^{\nu)}\psi, \quad J^\mu_V = \bar{\psi}\gamma^\mu\psi, \quad J^\mu_A = \bar{\psi}\gamma^\mu\gamma^5\psi, \quad J^\mu_H = \bar{\psi}\gamma^\mu h\psi + \overline{h\psi}\gamma^\mu\psi.$$

Hydrodynamic setup:
Conservation Equations and Longitudinal Perturbations

- We impose the conservation equations:

$$\partial_\mu T^{\mu\nu} = 0 \qquad \partial_\mu J^\mu_V = 0 \qquad \partial_\mu J^\mu_A = 0 \qquad \partial_\mu J^\mu_H = 0$$

- We evaluate their expectation values $\langle \cdot \rangle = Z^{-1}\ Tr\{\rho \cdot\}$ in a thermal state of rotating massless fermions described by the density operator:

$$\hat{\rho} = \exp\left[-\beta(\widehat{H} - \Omega \widehat{J}^z - \boldsymbol{\mu} \cdot \widehat{\mathbf{Q}}\right] \qquad\qquad \boldsymbol{\mu} = (\mu_V, \mu_A, \mu_H)$$

where we have introduced a chemical potential for each of the conserved U(1) charges.

- The above density operator induces a preferred hydrodynamic frame, the β-frame, defined by the following 4-velocity:

$$u^\mu_\Omega \partial_\mu = \Gamma(\partial_t + \Omega \partial_\varphi), \quad \Gamma_\Omega = \frac{1}{\sqrt{1 - \rho^2 \Omega^2}}$$

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Conservation Equations and Longitudinal Perturbations

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$$u^\mu_\Omega \partial_\mu = \Gamma(\partial_t + \Omega \partial_\varphi), \quad \Gamma_\Omega = \frac{1}{\sqrt{1-\rho^2\Omega^2}}$$

- The velocity allows to define three more (orthogonal) vectors, namely

$$\omega^\mu = \tfrac{1}{2}\varepsilon^{\mu\nu\lambda\sigma} u_\nu \nabla_\lambda u_\sigma = \Omega \Gamma^2_\Omega \partial_z$$

$$a^\mu = u^\nu \nabla_\nu u^\mu = -\rho \Omega^2 \Gamma^2_\Omega \partial_\rho$$

$$\tau^\mu = -\varepsilon^{\mu\nu\lambda\sigma} \omega_\nu a_\lambda u_\sigma = -\rho \Omega^3 \Gamma^5_\Omega (\rho \Omega \partial_t + \rho^{-1} \partial_\varphi)$$

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Hydrodynamic setup:
Conservation Equations and Longitudinal Perturbations

- As is customary in relativistic hydro, we decompose the expectation values in a particular frame.

$$J_\ell^\mu = Q_{\ell;\beta} u_\Omega^\mu + V_{\ell;\beta}^\mu, \quad T^{\mu\nu} = E_\beta u_\Omega^\mu u_\Omega^\nu - (P_\beta + \varpi_\beta)\Delta_\Omega^{\mu\nu} + \pi_\beta^{\mu\nu} + W_\beta^\mu u_\Omega^\nu + W_\beta^\nu u_\Omega^\mu$$

- Conformal fluid ($T_\mu^\mu = 0$) with equation of state $E_\beta = 3P_\beta$. Then $\varpi_\beta = 0$

$$V_{\ell;\beta}^\mu = \sigma_{\ell;\beta}^\omega \omega_\Omega^\mu + \sigma_{\beta;\ell}^\tau \tau_\Omega^\mu, \qquad W_\beta^\mu = \sigma_{\varepsilon;\beta}^\omega \omega_\Omega^\mu + \sigma_{\varepsilon;\beta}^\tau \tau_\Omega^\mu,$$

$$\pi_\beta^{\mu\nu} = \pi_{1;\beta}\left(\tau_\Omega^\mu \tau_\Omega^\nu - \frac{\boldsymbol{\omega}_\Omega^2}{2} a_\Omega^\mu a_\Omega^\nu - \frac{\mathbf{a}_\Omega^2}{2}\omega_\Omega^\mu \omega_\Omega^\nu\right) + \pi_{2;\beta}(\omega_\Omega^\mu \tau_\Omega^\nu + \omega_\Omega^\nu \tau_\Omega^\mu),$$

Quantum corrections to the perfect fluid form. (blue)

Survives to linear order in Ω (pink)

Hydrodynamic setup:
Conservation Equations and Longitudinal Perturbations

- As is customary in relativistic hydro, we decompose the expectation values in a particular frame.

$$J^\mu_\ell \simeq Q_{\ell;\beta} u^\mu_\Omega + \sigma^\omega_{\ell;\beta}\omega^\mu_\Omega, \qquad T^{\mu\nu} \simeq (E_\beta + P_\beta) u^\mu_\Omega u^\nu_\Omega - P_\beta g^{\mu\nu} + \sigma^\omega_{\varepsilon;\beta}(\omega^\mu_\Omega u^\nu_\Omega + u^\mu_\Omega \omega^\nu_\Omega)$$

- The scalar quantities (Energy, Pressure, Charges, Conductivities) can be computed as a classical term (plus quadratic corrections which we neglect). In particular

$$P_{\beta;\mathrm{cl}} = \frac{1}{3}\sum_{\sigma,\lambda}\int dP (p\cdot u) f^{\mathrm{eq};\sigma}_{\mathbf{p},\lambda} = -\frac{T^4}{\pi^2}\sum_{\sigma,\lambda}\mathrm{Li}_4(-e^{\mu_{\sigma,\lambda}/T}),$$

$$Q_{\ell;\beta;\mathrm{cl}} = \frac{\partial P_{\beta;\mathrm{cl}}}{\partial \mu_\ell} \qquad \sigma^\omega_{\varepsilon;\mathrm{cl}} = Q_{A;\mathrm{cl}}$$

$$\sigma^\omega_{\ell;\beta;\mathrm{cl}} = \frac{1}{2}\frac{\partial P}{\partial\mu_A \partial\mu_\ell}$$

$$f^{\mathrm{eq};\sigma}_{\mathbf{p},\lambda} = \frac{1}{(2\pi)^3}\left[\exp\left(\frac{p\cdot u - \mu_{\sigma,\lambda}}{T}\right) + 1\right]^{-1}$$

$$\mu_{\sigma,\lambda} = \boldsymbol{q}\cdot\boldsymbol{\mu} = \sigma\mu_V - 2\sigma\lambda\mu_H - 2\lambda\mu_A,$$

$$\mathrm{Li}_s(z) = \sum_{n=1}^\infty z^n/n^s$$

Hydrodynamic setup:
Conservation Equations and Longitudinal Perturbations

- Two ways to simplify the problem:

I. Go to the Landau frame: $T^{\mu}{}_{\nu} u_L^{\nu} = E_L u_L^{\mu}$

$$T^{\mu\nu} = (E_L + P_L) u_L^{\mu} u_L^{\nu} - P_L g^{\mu\nu} + \pi_L^{\mu\nu} \qquad J_{\ell}^{\mu} = Q_{\ell;L} u_L^{\mu} + V_{\ell;L}^{\mu}$$

$$u_L^{\mu} = u_{\Omega}^{\mu} + \frac{\sigma_{\varepsilon;\beta}^{\omega}}{E+P} \omega_{\Omega}^{\mu}$$

$$\sigma_{\ell}^{\omega} = \sigma_{\ell;\beta}^{\omega} - \frac{\sigma_{\varepsilon;\beta}^{\omega} Q_{\ell}}{E+P}.$$

$$E_L = E_{\beta} \equiv E$$

$$P_L = P_{\beta} \equiv P$$

$$Q_{L;\ell} = Q_{\beta;\ell} \equiv Q_{\ell}$$

II. Perform a Lorentz transformation to remove the ω_{Ω} contribution (i.e. z-th component) from the Landau velocity

$$L^{\mu\nu} = g^{\mu\nu} - \frac{\sigma_{\varepsilon;\beta}^{\omega}}{E+P} (u_{\Omega}^{\mu} \omega_{\Omega}^{\nu} + u_{\Omega}^{\nu} \omega_{\Omega}^{\mu}) \qquad u_L^{\mu} \to L^{\mu}{}_{\nu} u_L^{\nu} = u_{\Omega}^{\mu} + O(\Omega^2) \simeq u_{\Omega}^{\mu}$$

Hydrodynamic setup:
Conservation Equations and Longitudinal Perturbations

- In summary:

$$\partial_\mu T^{\mu\nu} = 0 \quad \partial_\mu J^\mu_V = 0 \quad \partial_\mu J^\mu_A = 0 \quad \partial_\mu J^\mu_H = 0$$

$$T^{\mu\nu} = (E+P)u^\mu u^\nu - P g^{\mu\nu}, \qquad J^\mu_\ell = Q_\ell u^\mu + \sigma^\omega_\ell \omega^\mu$$

$$P = \frac{E}{3} \simeq -\frac{T^4}{\pi^2}\sum_{\sigma,\lambda} \mathrm{Li}_4(-e^{\mu_{\sigma,\lambda}/T}), \quad Q_\ell \simeq \frac{\partial P}{\partial \mu_\ell}, \quad \sigma^\omega_\ell = \frac{1}{2}\frac{\partial^2 P}{\partial \mu_\ell \partial \mu_A} - \frac{1}{4P}\frac{\partial P}{\partial \mu_\ell}\frac{\partial P}{\partial \mu_A}$$

Neglecting quantum corrections of order Ω^2

Hydrodynamic setup:
Conservation Equations and Longitudinal Perturbations

- Upon substitution of the constitutive relations, the previous equations turn into

$$DE + (E+P)\theta = 0$$

$$(E+P)Du^\mu - \nabla^\mu P = 0$$

$$DQ_V + Q_V\theta + \omega^\mu \partial_\mu \sigma_V^\omega + \sigma_V^\omega \partial_\mu \omega^\mu = 0$$

$$DQ_A + Q_A\theta + \omega^\mu \partial_\mu \sigma_A^\omega + \sigma_A^\omega \partial_\mu \omega^\mu = 0$$

$$DQ_H + Q_H\theta + \omega^\mu \partial_\mu \sigma_H^\omega + \sigma_H^\omega \partial_\mu \omega^\mu = 0$$

$$D = u^\mu \partial_\mu$$

$$\nabla^\mu = \Delta^{\mu\nu}\partial_\nu$$

$$\theta = \partial_\mu u^\mu$$

- We study perturbations around the rigidly rotating state along the rotation axis

$$u^\mu \partial_\mu = u^\mu_\Omega \partial_\mu + \delta u e^{-ik(vt-z)}\partial_z,$$

$$Q_\ell = Q_{\ell;0} + \delta Q_\ell e^{-ik(vt-z)},$$

$$P = P_0 + \delta P e^{-ik(vt-z)},$$

$$\omega^\mu \partial_\mu = \Omega\partial_z + \Omega\,\delta u\, e^{-ik(vt-z)}\left(\partial_t - \tfrac{i}{2}kv\rho\partial_\rho\right).$$

$$\delta P = \frac{\partial P}{\partial T}\delta T + \frac{\partial P}{\partial \mu_\ell}\delta\mu_\ell, \quad \delta Q_\ell = \frac{\partial Q_\ell}{\partial T}\delta T + \frac{\partial Q_\ell}{\partial \mu_{\ell'}}\delta\mu_{\ell'}$$

Hydrodynamic setup:
Energy-Momentum sector

$$DE + (E+P)\theta = 0$$

$$(E+P)Du^\mu - \nabla^\mu P = 0$$

- The energy momentum sector gives a closed system for δP and δu

$$\begin{pmatrix} -3v & 4P \\ 1 & -4Pv \end{pmatrix} \begin{pmatrix} \delta P \\ \delta u \end{pmatrix} = 0$$

whose non trivial solutions are the sound modes $v = \pm 1/\sqrt{3}$

- It also admits the trivial solution $\delta u = \delta P = 0$. The non-trivial excitations in the charge sector satisfy this condition.

Hydrodynamic setup:
Charge sector

- The equations for the perturbations in the charge sector read:

$$\left(v\delta Q_\ell - \Omega\delta\sigma_\ell^\omega\right)\Big|_{\delta P=0} = 0$$

- The thermodynamic quantities naturally depend on the chemical potentials, thus we rewrite the equations in terms of $\delta\mu_\ell$

$$\delta Q_\ell = \frac{\partial Q_\ell}{\partial T}\delta T + \frac{\partial Q_\ell}{\partial \mu_{\ell'}}\delta\mu_{\ell'} = \left(\frac{\partial Q_\ell}{\partial \mu_{\ell'}} - \frac{Q_{\ell'}}{s}\frac{\partial Q_\ell}{\partial T}\right)\delta\mu_{\ell'}$$

$$\delta P = \frac{\partial P}{\partial T}\delta T + \frac{\partial P}{\partial \mu_\ell}\delta\mu_\ell = s\delta T + \textstyle\sum_\ell Q_\ell\delta\mu_\ell \quad \Rightarrow \quad \delta T = -\displaystyle\sum_\ell \frac{Q_\ell}{s}\delta\mu_\ell$$

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$$\frac{\partial Q_\ell}{\partial T} = \frac{\partial s}{\partial \mu_\ell} = \frac{3Q_\ell}{T} - \frac{\vec{\mu}}{T}\cdot\frac{\partial Q_\ell}{\partial \vec{\mu}} \qquad s = (e + P - \vec{\mu}\cdot\vec{Q})/T$$

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Hydrodynamic setup: Charge sector

- The equations for the perturbations in the charge sector read:

$$\left(v\delta Q_\ell - \Omega\delta\sigma_\ell^\omega\right)\Big|_{\delta P=0} = 0$$

- The thermodynamic quantities naturally depend on the chemical potentials, thus we rewrite the equations in terms of $\delta\mu_\ell$

$$\mathbb{M}_{\ell\ell'}\delta\mu_{\ell'} = 0$$

- The problem reduces to diagonalising $\mathbb{M}$.

$$\mathbb{M}_{\ell\ell'} = \underbrace{v\left(\frac{\partial Q_\ell}{\partial\mu_{\ell'}} - \frac{3Q_\ell Q_{\ell'}}{sT} + \frac{Q_{\ell'}\vec{\mu}}{sT}\cdot\frac{\partial Q_\ell}{\partial\vec{\mu}}\right)}_{\mathbb{M}_v} - \underbrace{\Omega\left(\frac{\partial\sigma_\ell^\omega}{\partial\mu_{\ell'}} - \frac{2\sigma_\ell^\omega Q_{\ell'}}{sT} + \frac{Q_{\ell'}\vec{\mu}}{sT}\cdot\frac{\partial\sigma_\ell^\omega}{\partial\vec{\mu}}\right)}_{\mathbb{M}_\Omega}$$

Results I

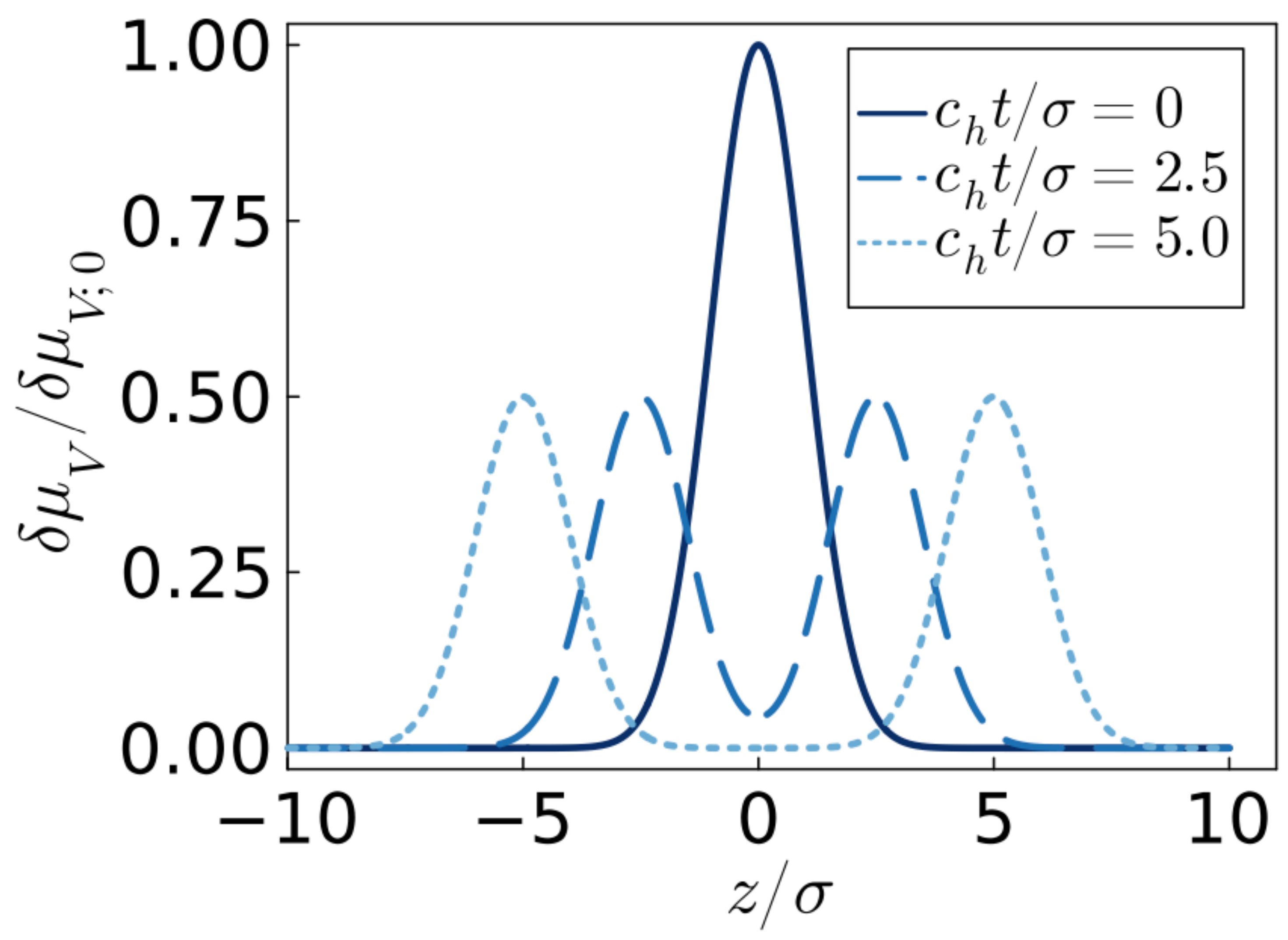
CASE I:
Conserved Charges

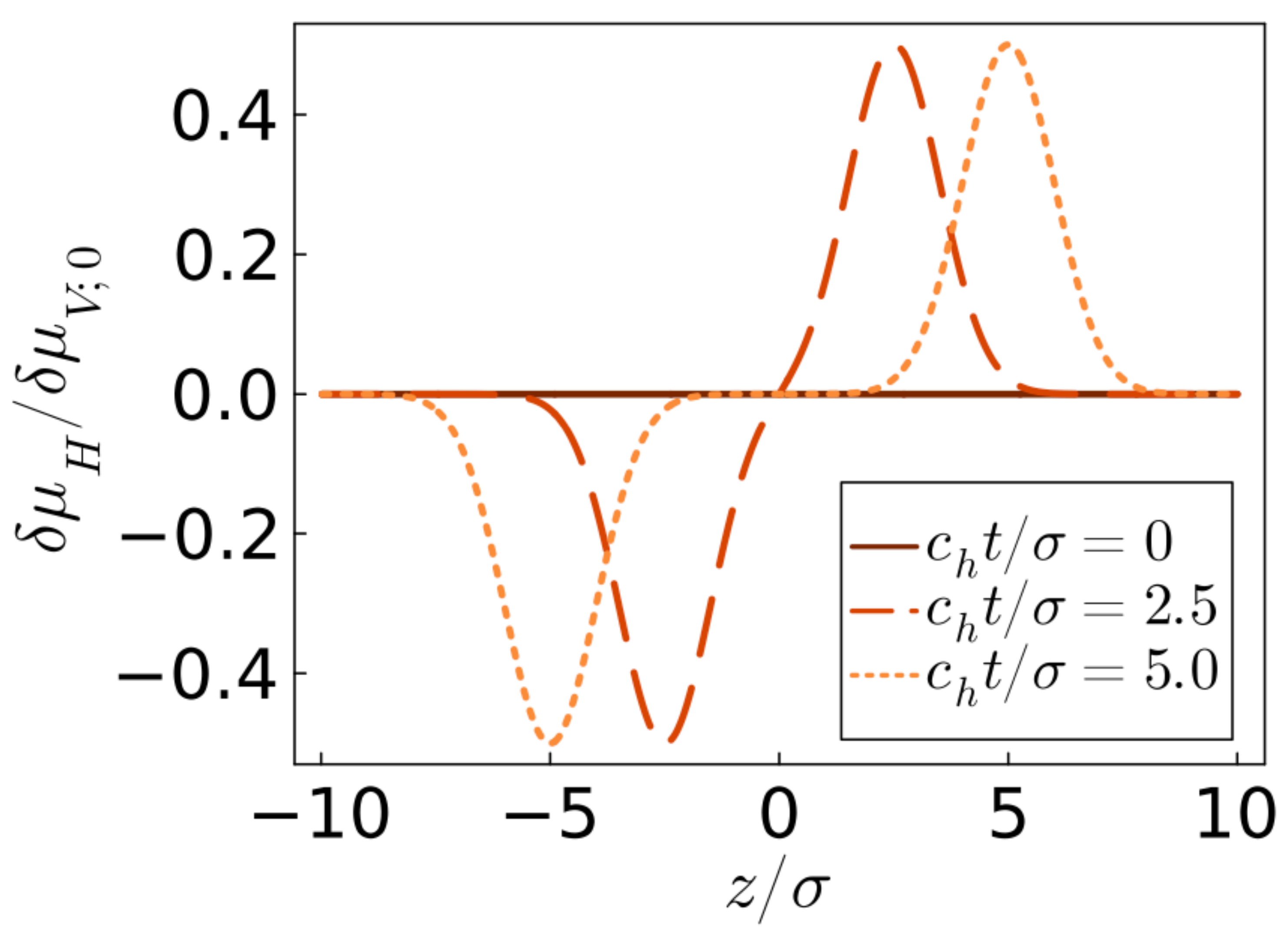
- Large Temperature Limit
  A. Helical Vortical Mode
  (Non-reciprocal!)

c_H

$$v_H^{\pm} = \pm \frac{6\ln 2}{\pi^2}\frac{\Omega}{T} - \frac{6}{7\pi^2}\left[\frac{84}{\pi^2}(\ln 2)^2 - 1\right]\frac{\Omega\mu_A}{T^2} + O(T^{-3})$$

$$\delta\mu_H^{\pm} = \pm\,\delta\mu_V^{\pm} \qquad \delta\mu_A^{\pm} = -\left[\frac{84(\ln 2)^2}{\pi^2} - 1\right]\frac{\mu_H \pm \mu_V}{7T\ln 2}\delta\mu_V^{\pm} + O(T^{-2})$$





$\mu_V = \mu_A = \mu_H = 0$
$T = 300$ MeV
$\Omega = 6.6$ MeV

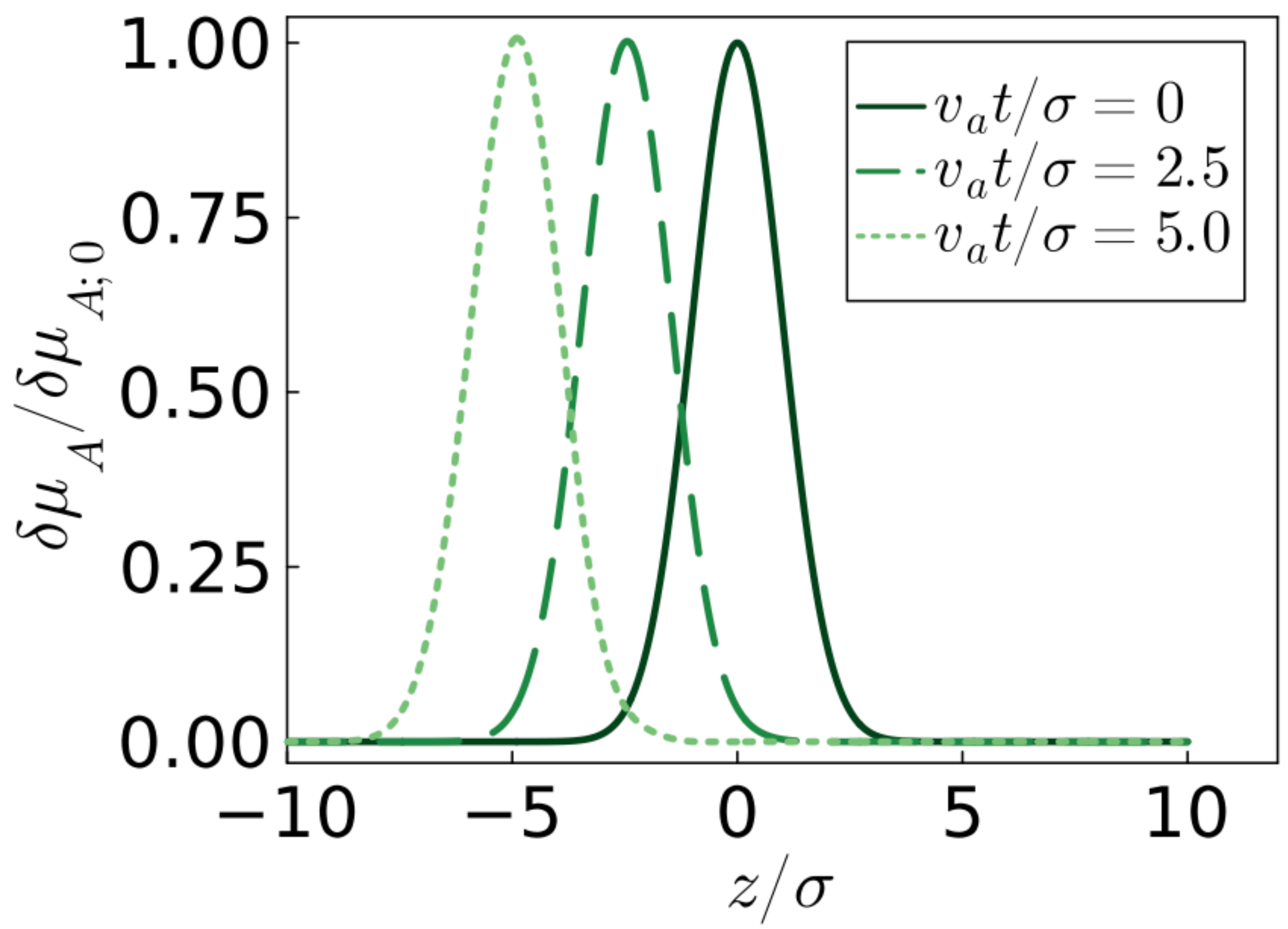
$T^{-2})$

CASE I:
Conserved Charges

- Large Temperature Limit

A. Helical Vortical Mode

(Non-reciprocal!)



$\mu_V = \mu_H = 0$

$\mu_A = 30$ MeV

$T = 300$ MeV

$\Omega = 6.6$ MeV

B. Axial Vortical Mode

(Uni-directional!)

$$v_A = -\frac{24}{7\pi^2}\frac{\mu_A \Omega}{T^2} + O(T^{-3})$$

$$\delta\mu^A_{V/H,0} = -\frac{\mu_{H/V}}{7T\ln 2}\delta\mu^A_{A,0} + O(T^{-2})$$

Setup II

Non-Conservation of Charges

- Non-conservation can be implemented in the RTA approximation. Typically

$$\partial_\mu J^\mu = -\frac{\delta Q}{\tau} \quad \equiv \quad \partial_\mu J^\mu = -\frac{\chi \delta\mu}{\tau}$$

- If there are more than one charges in the system, both approaches are in general inequivalent due to non-vanishing crossed susceptibilities

$$\partial_\mu J^\mu = -\frac{\delta Q}{\tau} \quad \not\equiv \quad \boxed{\partial_\mu J^\mu = -\frac{\chi \delta\mu}{\tau}}$$

- We implement dissipation on the basis of the chemical potential:

  A. On the one hand, a chemical potential is not well defined, in a thermodynamic sense, for non-conserved charges. It is reasonable to demand that it is absent in the final equilibrium state. (Note: The equilibrium equations are solved only for the **unpolarised plasma** $\mu_A = \mu_H = 0$)

  B. Secondly, it can be shown that the other prescription can give rise to unphysical instabilities.

Non-Conservation of Charges

- Helicity is not conserved in the presence of interactions. For example, in processes like $e_R^+ e_L^- \to e_L^+ e_R^-$ chirality is conserved ($\Delta Q_A = 0$) while helicity is violated ($\Delta Q_H = 2$).
- Such processes will take place both in QED and in QCD, leading to 'relaxation of helicity'. QCD weak-coupling estimation given in [Ambrus, Chernodub (Mar 2023)]

$$\frac{dQ_H}{dt} = -16\beta\mu_H g^2 \int dP dK dP' dK' s (2\pi)^6 \times \delta^4(p + k - p' - k') f_{0\mathbf{p}} f_{0\mathbf{k}} \tilde{f}_{0\mathbf{p}'} \tilde{f}_{0\mathbf{k}'} (\tilde{f}_{0\mathbf{p}} + f_{0\mathbf{p}'}) \times \frac{\alpha_{QCD}^2}{18 E_{\rm cm}^2} (1 - \cos\theta_{cm})^2.$$

$\rightarrow$

$$\frac{1}{2}\tau_H \simeq \left(\frac{250 \text{ MeV}}{k_B T}\right) \left(\frac{1}{\alpha_{\rm QCD}}\right)^2 \left(\frac{2}{N_f}\right) \times 2.54 \text{ fm}/c.$$

- The chiral symmetry is actually anomalous (i.e. broken by quantum effects) and the axial current is not truly conserved: $\partial_\mu J_5^\mu = -\frac{1}{12\pi^2} \epsilon^{\mu\nu\rho\sigma} Tr\{G_{\mu\nu} G_{\rho\sigma}\}$

Results II

CASE II:

Non-Conserved Helicity ($0 \leq \tau_H < \infty$)

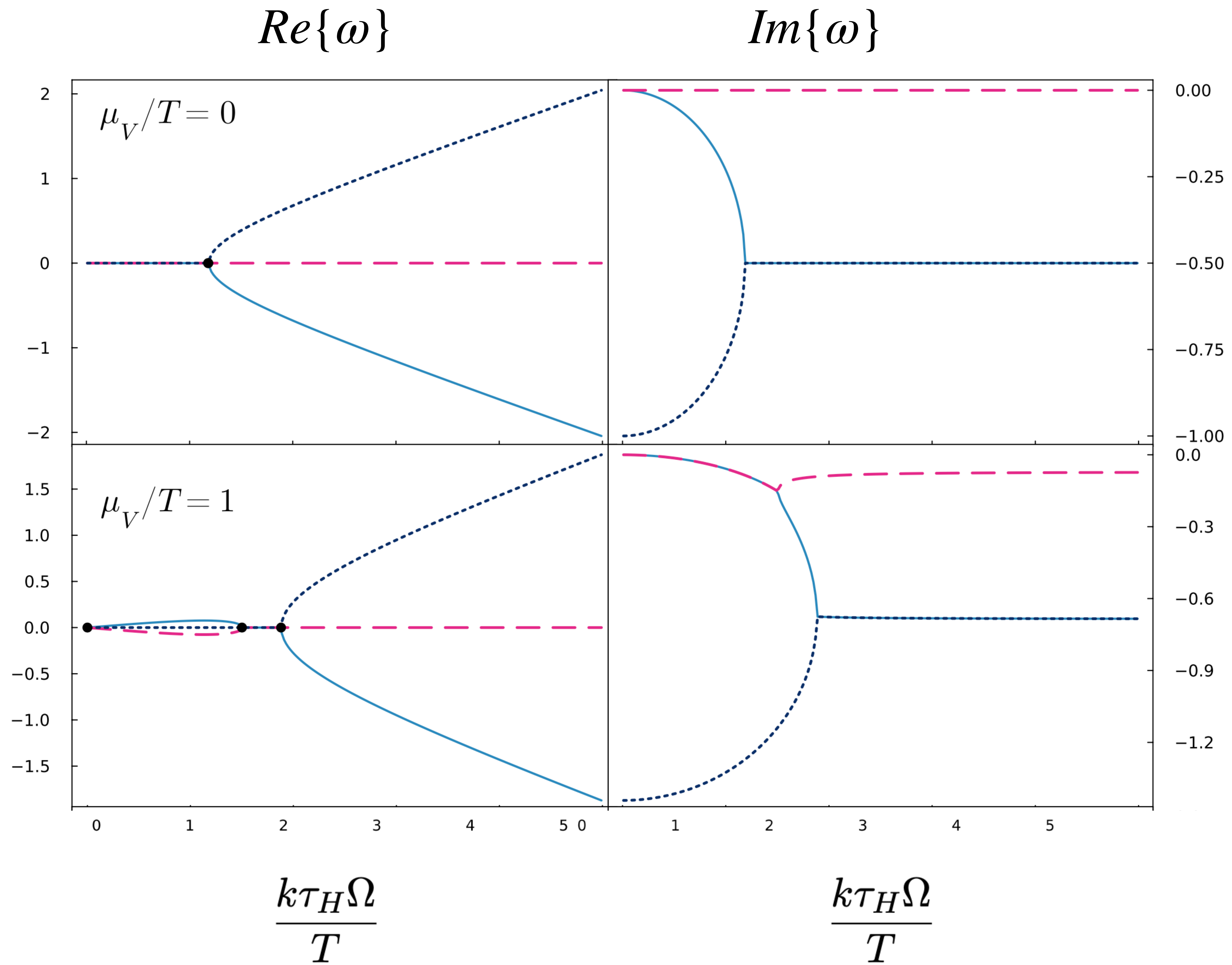
- At small chemical potential we **recover** the **Helical Vortical Wave** [Ambrus, Chernodub (Mar 2023)]:

$$\omega^{\pm}_{h;0} = -\frac{i}{2\tau_H} \pm k c_h \sqrt{1 - \frac{k_{\rm th}^2}{k^2}}, \qquad k_{\rm th} = \frac{1}{2\tau_H c_h} = \frac{\pi^2}{12\ln 2}\frac{T}{\Omega \tau_H}.$$

- The wave propagates for wavenumbers above the threshold k_{th}. Thus, the non-conservation of the helical charge **inhibits propagation of the IR modes.**
- This mode **exists with vanishing chemical potentials**! $\mu_V = \mu_A = \mu_H = 0$. Contrary to the traditional CVW which requires $\mu_V \neq 0$.

CASE II:
Non-Conserved Helicity ($0 \leq \tau_H < \infty$)

$w = \tau_H k v$



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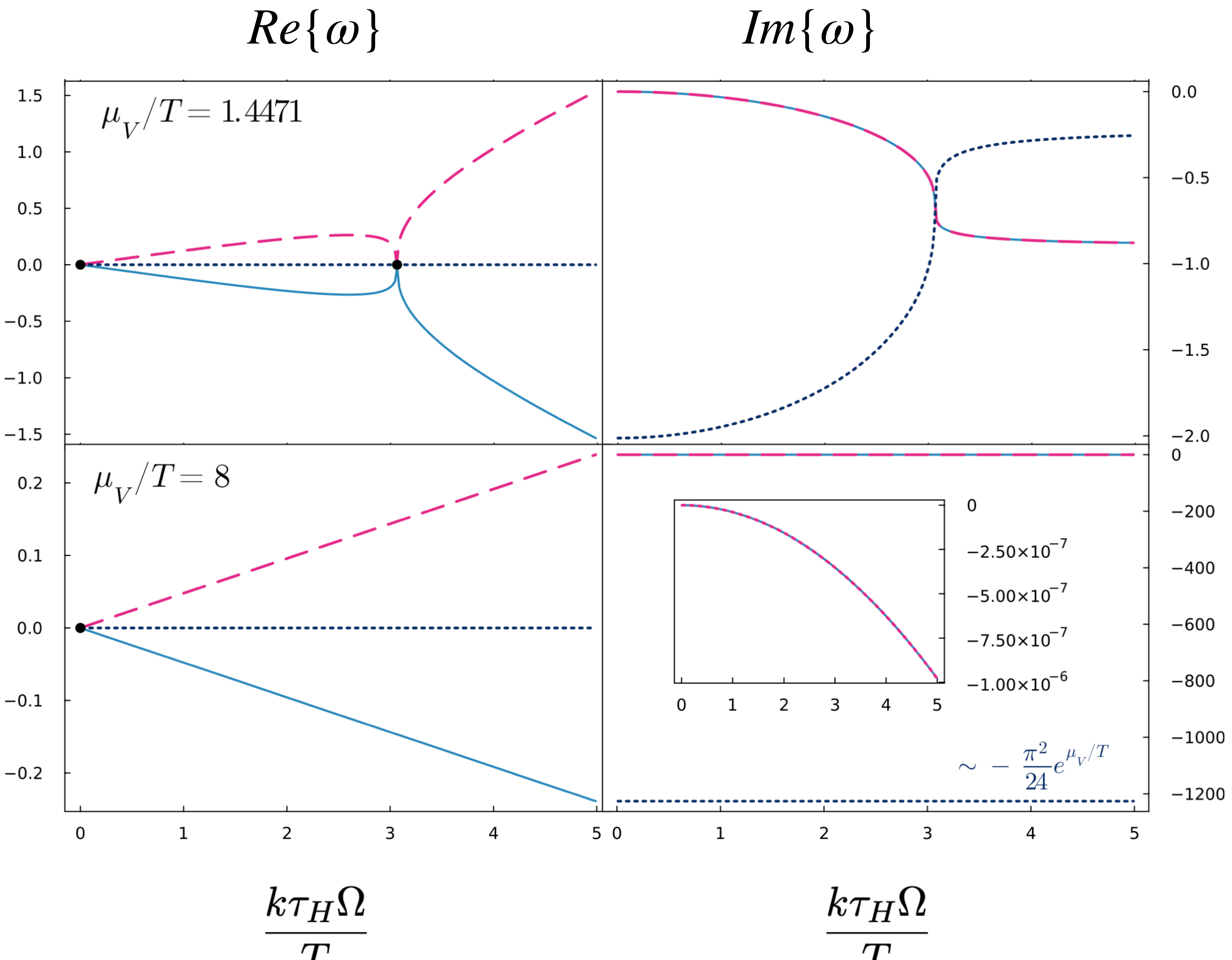

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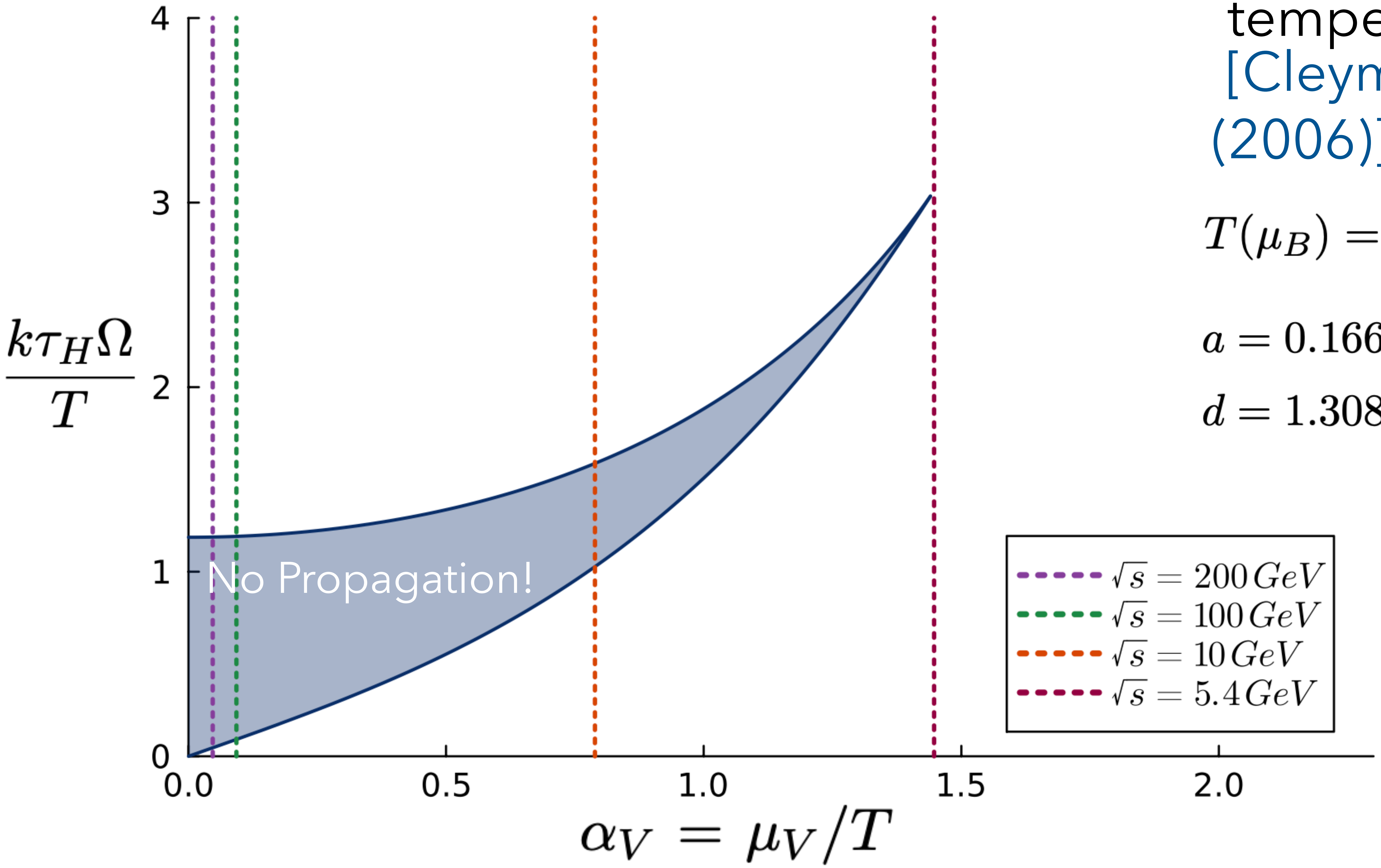


- Dependence of chemical potential and temperature with collision energy: [Cleymans, Oeschler, Redlich, Wheaton (2006)]

$$T(\mu_B) = a - b\mu_B^2 - c\mu_B^4 \qquad \mu_B(\sqrt{s}) = \frac{d}{1 + f\sqrt{s}}$$

$$a = 0.166(2)\,\text{GeV} \quad b = 0.139(16)\,\text{GeV} \quad c = 0.053(21)\,\text{GeV}$$

$$d = 1.308(28)\,\text{GeV} \quad f = 0.273(8)\,\text{GeV}^{-1}\,.$$

CASE II:

Non-Conserved Helicity ($0 \leq \tau_H < \infty$)

- In the limit when helical charge dissipates very quickly ($\tau_H \to 0$) the helical degree of freedom is effectively frozen and we **recover** the traditional **Chiral Vortical Wave** [Jiang, Huang, Liao (2015)]

$$\omega_h = -\frac{iT^2\sigma_A^\omega}{6\tau_H[(\sigma_A^\omega)^2 - (\sigma_H^\omega)^2]}, \qquad \omega_{\mathfrak{a}}^{\pm} = \pm\frac{AT^2}{2\sigma_A^\omega\sqrt{H}}\left(1 - \frac{T^2\Delta H}{3\sigma_A^\omega}\right)^{-1/2}\frac{k\Omega}{T}.$$

$$\alpha_V = \mu_V/T$$

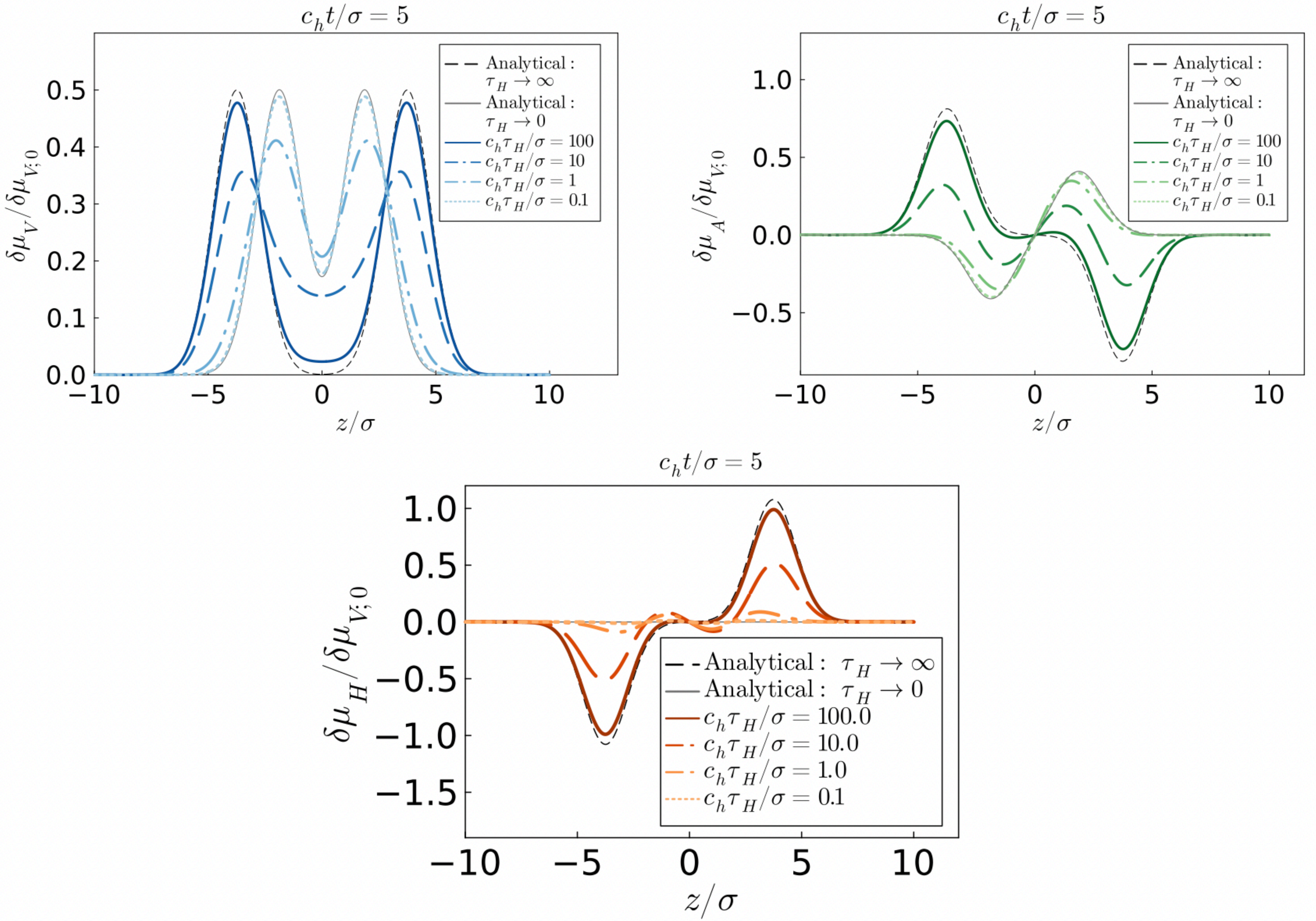
$$H = (e+P)/sT = 1 + \mu_V Q_V/sT$$

$$\Delta H = H - 1$$

$$A = \frac{\alpha_V}{\pi^2} - \frac{Q_V}{3s}$$

$$B = \frac{HL}{\pi^2} - \frac{2Q_V}{sT^2}\sigma_H^\omega$$

$$L = 2\ln\left(2\cosh\frac{\mu_V}{2T}\right)$$



$c_h t/\sigma = 5$
$\delta\mu_V/\delta\mu_{V;0}$
z/σ
Analytical : $\tau_H \to \infty$
Analytical : $\tau_H \to 0$
$c_h\tau_H/\sigma = 100$
$c_h\tau_H/\sigma = 10$
$c_h\tau_H/\sigma = 1$
$c_h\tau_H/\sigma = 0.1$
$c_h t/\sigma = 5$
$\delta\mu_A/\delta\mu_{V;0}$
z/σ
Analytical : $\tau_H \to \infty$
Analytical : $\tau_H \to 0$
$c_h\tau_H/\sigma = 100$
$c_h\tau_H/\sigma = 10$
$c_h\tau_H/\sigma = 1$
$c_h\tau_H/\sigma = 0.1$
$c_h t/\sigma = 5$
$\delta\mu_H/\delta\mu_{V;0}$
z/σ
Analytical : $\tau_H \to \infty$
Analytical : $\tau_H \to 0$
$c_h\tau_H/\sigma = 100.0$
$c_h\tau_H/\sigma = 10.0$
$c_h\tau_H/\sigma = 1.0$
$c_h\tau_H/\sigma = 0.1$

CASE III:

Non-Conserved Helicity and Chirality ($0 \leq \tau_H, \tau_A < \infty$)

- There are two remarkable novelties when axial charge is also dissipating:

A. The $\tau_H \to 0$ limit gives the generalisation of the CVW (Vector-Axial) when axial charge dissipates. The presence of τ_A also **eliminates the propagation of IR modes.**

$$\omega_{\mathfrak{a}}^{\pm} \simeq \frac{T^2}{12\sigma_A^\omega \tau_A}\left(-i \pm \sqrt{\frac{36A^2\sigma_A^\omega}{H(\sigma_A^\omega - \frac{T^2}{3}\Delta H)}\tau_A^2\kappa_\Omega^2 - 1}\right)$$

B. The $\tau_A \to 0$ limit, equivalent to neglecting/freezing the axial degree of freedom, gives a HVW (Vector-Helical). Note that $\tau_A \sim 0.25$ fm/c $\ll \tau_{QGP} \sim 10$ fm/c

$$\omega_{h;0}^{\pm} \simeq \frac{T^2}{12\tau_H \sigma_A^\omega}\left(-i \pm \sqrt{\frac{36B^2\sigma_A^\omega}{H(\sigma_A^\omega - \frac{T^2}{3}\Delta H)}\tau_H^2\kappa_\Omega^2 - 1}\right)$$

[Phys. Rev. D **102**, 054516 (2020)]

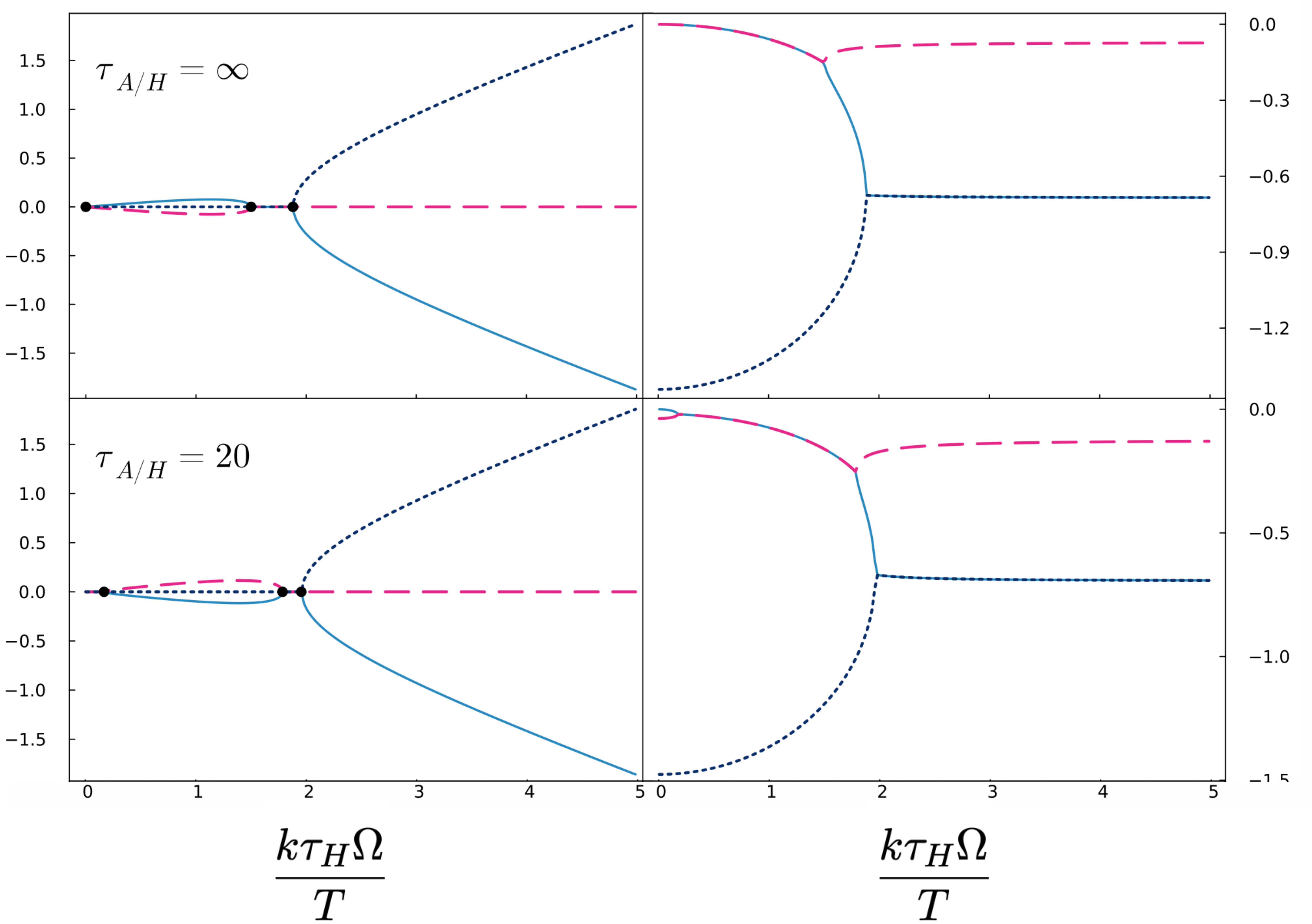
$\mu_V/T = 1$

$w = \tau_H k v$

CASE III:

Non-Conserved Helicity and Chirality ($0 \leq \tau_H, \tau_A < \infty$)

$Re\{\omega\}$ $Im\{\omega\}$



$\mu_V/T = 1$

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Non-Conserved Helicity and Chirality ($0 \leq \tau_H, \tau_A < \infty$)

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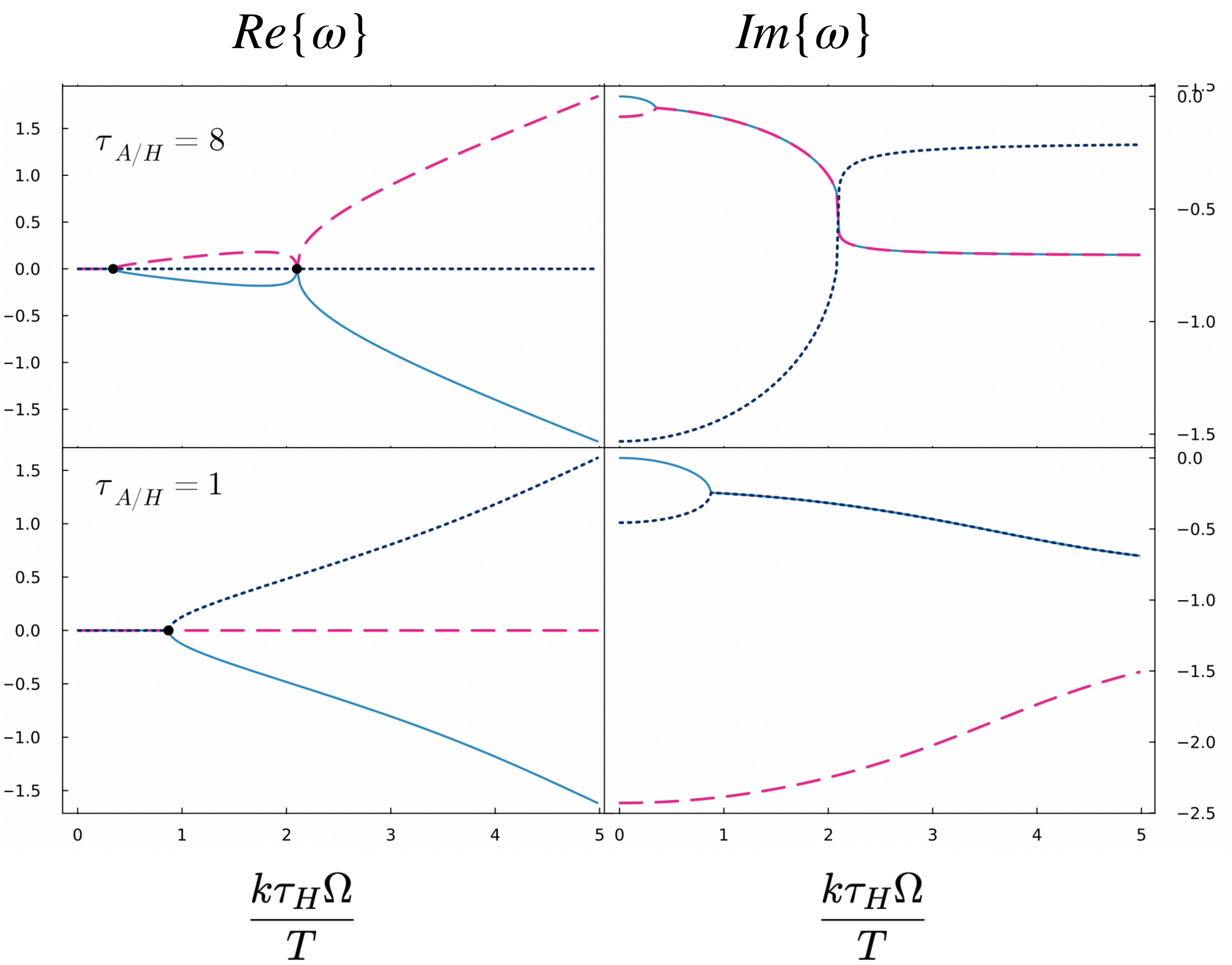

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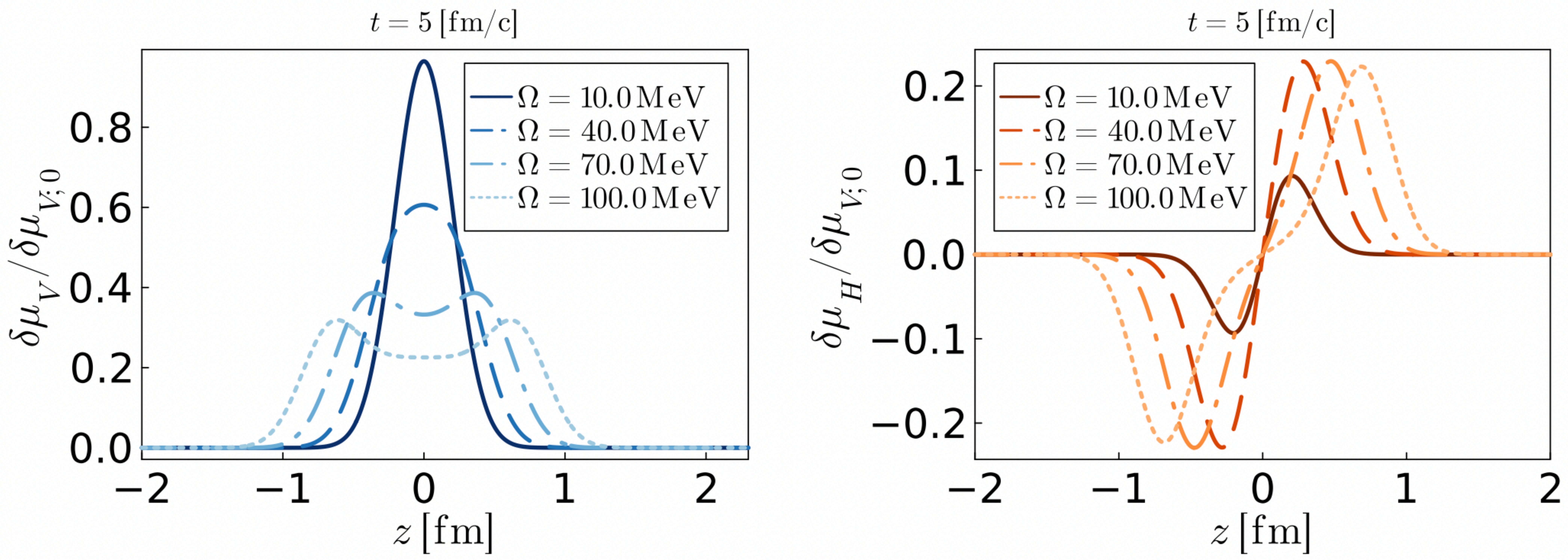


$\tau_H = 2.7$ fm/c $\tau_A = 0.25$ fm/c, $\tau_R = 0$

$\mu_V = \mu_A = \mu_H = 0$ $T = 300$ MeV

Dissipative Effects

- Finally, the presence of interactions gives rise to kinetic dissipative effects. Again these are implemented in the RTA and parametrically controlled by τ_R.
- The dissipative contributions appear as deviations from the perfect fluid form.

$$T^{\mu\nu} \to T^{\mu\nu} = T_0^{\mu\nu} + \pi_d^{\mu\nu} - \Pi_d \Delta^{\mu\nu}, \quad J_\ell^\mu \to J_\ell^\mu = J_{\ell;0}^\mu + V_{\ell;d}^\mu$$

These are unconstrained and must be specified by constitutive relations. In 1st order hydro:

$$V_{\ell;d}^\mu = \sum_{\ell'} \kappa_{\ell,\ell'} \nabla^\mu \alpha_{\ell'}, \qquad \pi_d^{\mu\nu} = 2\eta\sigma^{\mu\nu}$$

A causal and stable theory requires 2nd order hydro.

- The expectation values are now computed with the distribution function that solves the kinetic equation

$$p^\mu \partial_\mu f^\sigma_{\mathbf{p},\lambda} = -\frac{E_\mathbf{p} \cdot u}{\tau_R} \left(f^\sigma_{\mathbf{p},\lambda} - f^{\mathrm{eq};\sigma}_{\mathbf{p},\lambda} \right)$$

$$f^{\mathrm{eq};\sigma}_{\mathbf{p},\lambda} = \left[\exp\left(\frac{p \cdot u - \mu_{\sigma,\lambda}}{T} \right) + 1 \right]^{-1}$$

Dissipative Effects

- After some manipulations one finds

$$V_\ell^\mu = \tau_R \left(\frac{1}{3} \nabla^\mu Q_\ell - \frac{Q_\ell \nabla^\mu P}{E+P} \right) \qquad \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu}$$

$$\sigma^{\mu\nu} = \left(\tfrac{1}{2}\Delta^{\mu\lambda}\Delta^{\nu\kappa} + \tfrac{1}{2}\Delta^{\nu\lambda}\Delta^{\mu\kappa} - \tfrac{1}{3}\Delta^{\mu\nu}\Delta^{\lambda\kappa}\right)\partial_\lambda u_\kappa$$

- The diffusion Matrix becomes $\kappa_{\ell\ell'} = \tau_R \left(\frac{1}{3} \frac{\partial Q_\ell}{\partial \alpha_{\ell'}} - \frac{Q_\ell Q_{\ell'} T}{E+P} \right)$ and the shear viscosity $\eta = \frac{4}{5}\tau_R P.$
- Fluctuations around rotating state + Fourier transform give for $T^{\mu\nu}$ sector

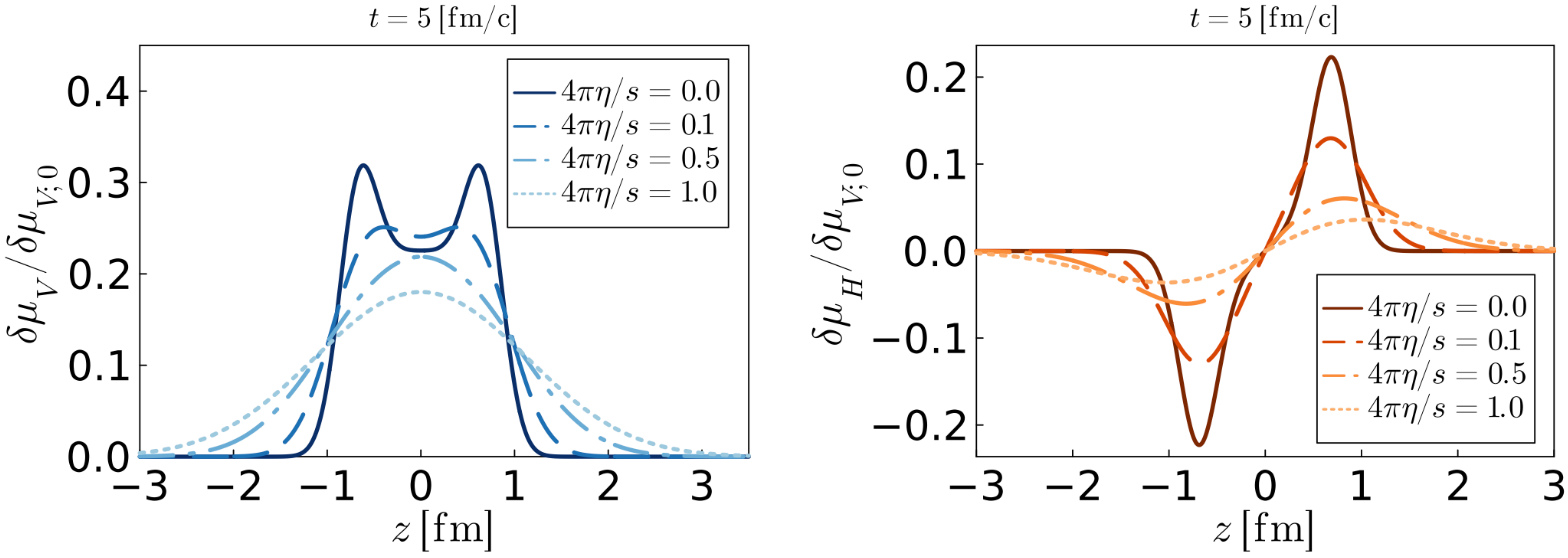
$$\begin{pmatrix} -3v & 4P \\ 1 & -4Pv - \frac{4}{3}\eta i k \end{pmatrix} \begin{pmatrix} \delta P \\ \delta u \end{pmatrix} = 0. \quad \Longrightarrow \quad \omega^\pm_{R;\text{ac.}} = \pm k c_s(\eta) - \frac{ik^2\eta}{6P}, \qquad c_s(\eta) = \frac{1}{\sqrt{3}}\sqrt{1 - \frac{k^2\eta^2}{12P^2}}$$

- The modes from the charge sector get a constant imaginary shift.

$$\left[\left(\omega + \frac{ik^2\tau_R}{3} \right) \delta Q_\ell - k\Omega\delta\sigma^\omega_\ell \right] \Big|_{\delta P=0} = -\frac{iT^2\delta\mu_\ell}{3\tau_\ell} \quad \Longrightarrow \quad \omega_R = \omega - \frac{ik^2\tau_R}{3}$$

Kinetic dissipation
damps UV modes.

Final Case: Dissipation and charge non-conservation



$\tau_H = 2.7$ fm/c $\tau_A = 0.25$ fm/c

$\mu_V = \mu_A = \mu_H = 0$ $T = 300$ MeV $\Omega = 100$ MeV

Summary

Summary & Outlook

- A fluid with vector axial and helical charges shows a rich variety of wave-like excitations.
- In a neutral unpolarised plasma the V, H d.o.f.s propagate as the Helical Vortical Wave.
- In a realistic plasma, axial and helical charges are not conserved ➡ $0 < \tau_A, \tau_H < \infty$

  ➡ IR (large wavelength) propagation cutoff.
- Conversely, kinetic dissipation damps UV modes.
- The traditional Chiral Vortical Wave arises only in the particular limit $\tau_H \to 0$.
- Both non-reciprocity and uni-directionality appear in a chirally imbalanced medium.
- The waves are unlikely to give a phenomenological imprint in the QGP at HIC.

I. Employ the framework for polarisation measurements in HIC.

II. Extension to more realistic EOS.

III. Investigate kinetic theory framework for fluids with these three charges.

Thank you!

Acknowledgments:

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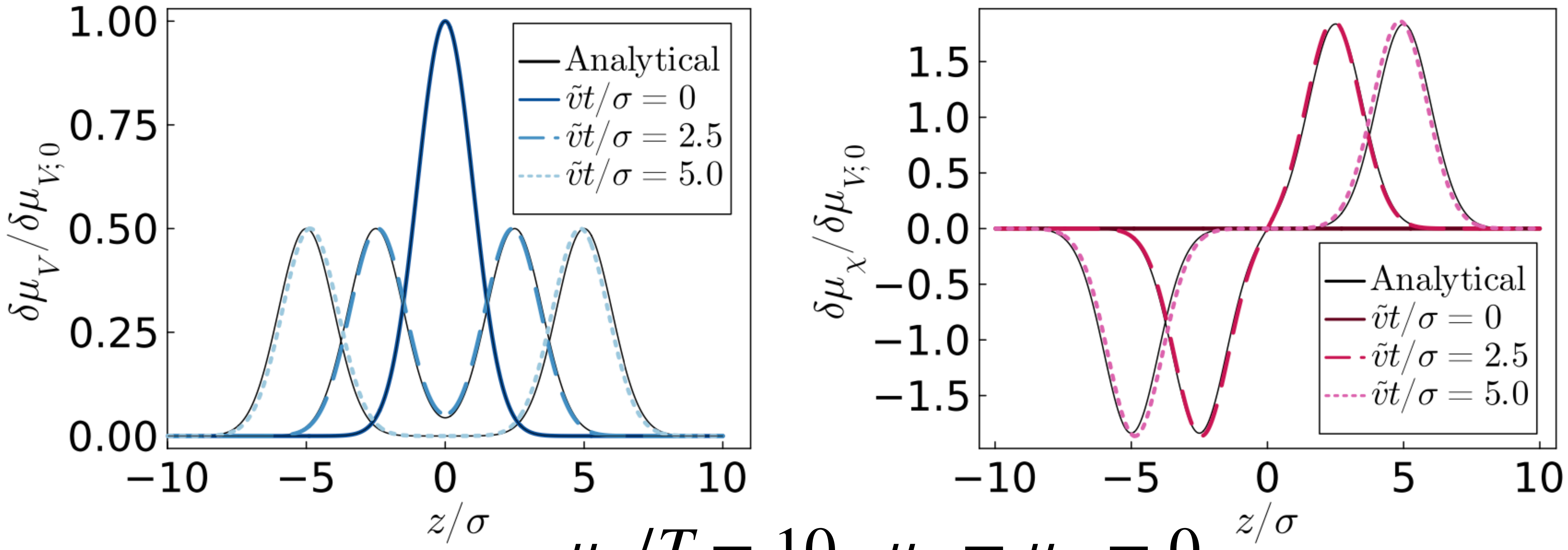
CASE I:
Conserved Charges

- Degenerate Limit: $|\mu_V| \gg T\,, |\mu_V| \gg |\mu_A \pm \mu_H|\,.$
- The system composed of only particles (or antiparticles) up to exponentially small corrections. Hence, the helical and axial degrees of freedom are not independent and the solutions depend only on $\mu_\chi = \mu_A + \mu_H\,.$
- Neglecting exponentially suppressed corrections one finds two modes:

$$\tilde{v}_\pm = -\frac{5\Omega\mu_\chi}{\mu_V^2} \pm \frac{\Omega}{\mu_V^2}\sqrt{\frac{4\pi^2T^2}{3} + 9\mu_\chi^2}$$

$$\widetilde{\delta\mu}_V^\pm = \frac{3\widetilde{\delta\mu}_\chi^\pm}{2\mu_V}\left(\mu_\chi \pm \sqrt{\frac{4\pi^2T^2}{3} + 9\mu_\chi^2}\right)$$



$\mu_V/T = 10\,,\ \mu_A = \mu_H = 0$

CASE I:
Conserved Charges

- **Unpolarised Plasma:** $\mu_A = \mu_H = 0$.

$$\frac{1}{T^2}\mathbb{M} = v\mathbb{M}_v(\alpha_V) - \frac{\Omega}{T}\mathbb{M}_\Omega(\alpha_V)$$

$$\mathbb{M}_v = \frac{2}{T^2}\begin{pmatrix} \sigma_A^\omega - \frac{T^2}{3}\Delta H & 0 & 0 \\ 0 & \sigma_A^\omega & \sigma_H^\omega \\ 0 & \sigma_H^\omega & \sigma_A^\omega \end{pmatrix}$$

$$\mathbb{M}_\Omega = \begin{pmatrix} 0 & \frac{1}{H}A & \frac{1}{H}B \\ A & 0 & 0 \\ B & 0 & 0 \end{pmatrix}$$

$$\alpha_V = \mu_V/T$$

$$H = (e+P)/sT = 1 + \mu_V Q_V/sT$$

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A. Axial-Helical Mode: $v = 0$

$$\delta\mu_V = 0$$

$$A\delta\mu_A + B\delta\mu_H = 0$$

B. Helical Vortical Mode:

$$v_H^\pm = \pm\frac{\Omega T}{2}\sqrt{\frac{\sigma_A^\omega(A^2+B^2) - 2AB\sigma_H^\omega}{H(\sigma_A^\omega - \frac{T^2}{3}\Delta H)[(\sigma_A^\omega)^2 - (\sigma_H^\omega)^2]}}.$$

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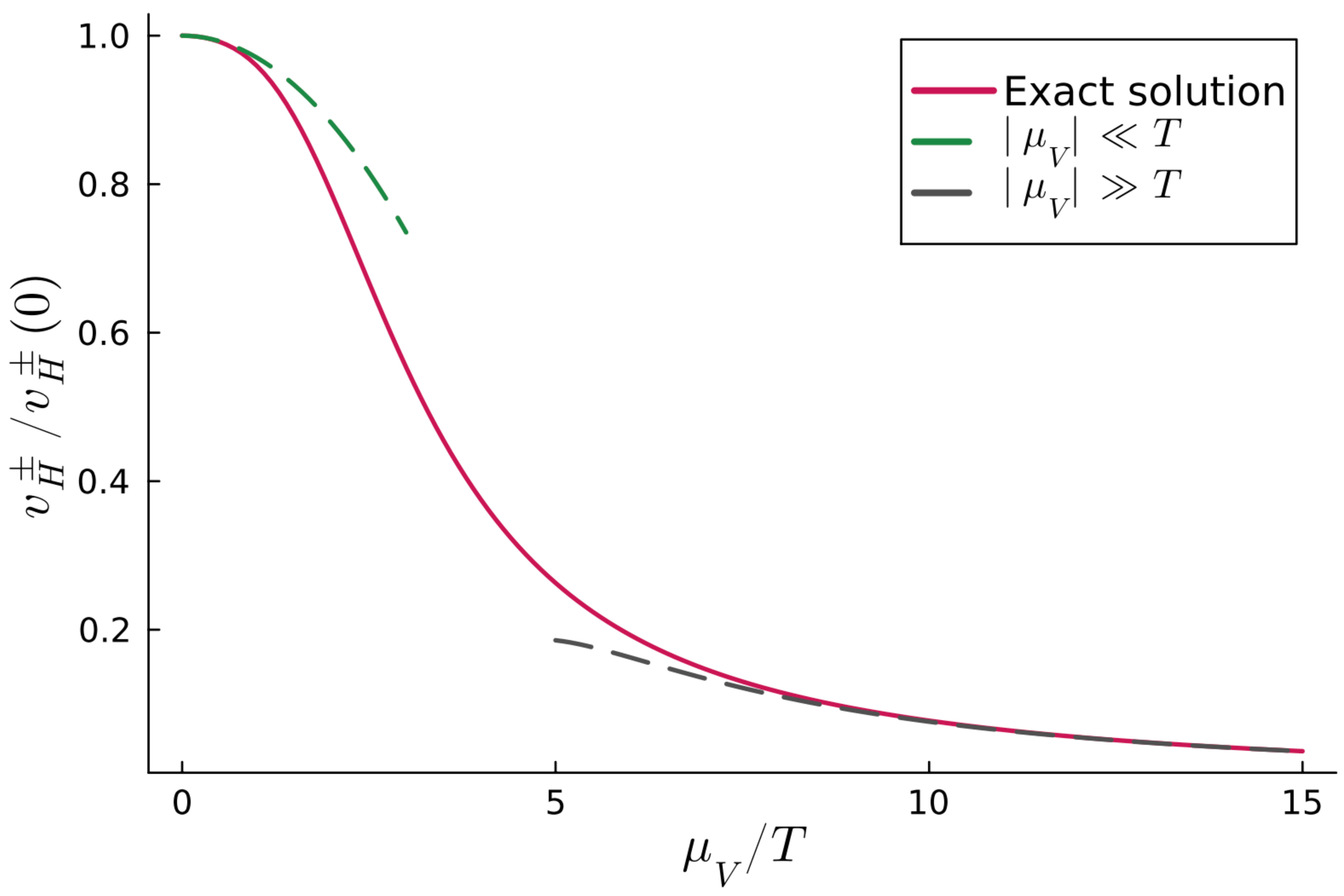

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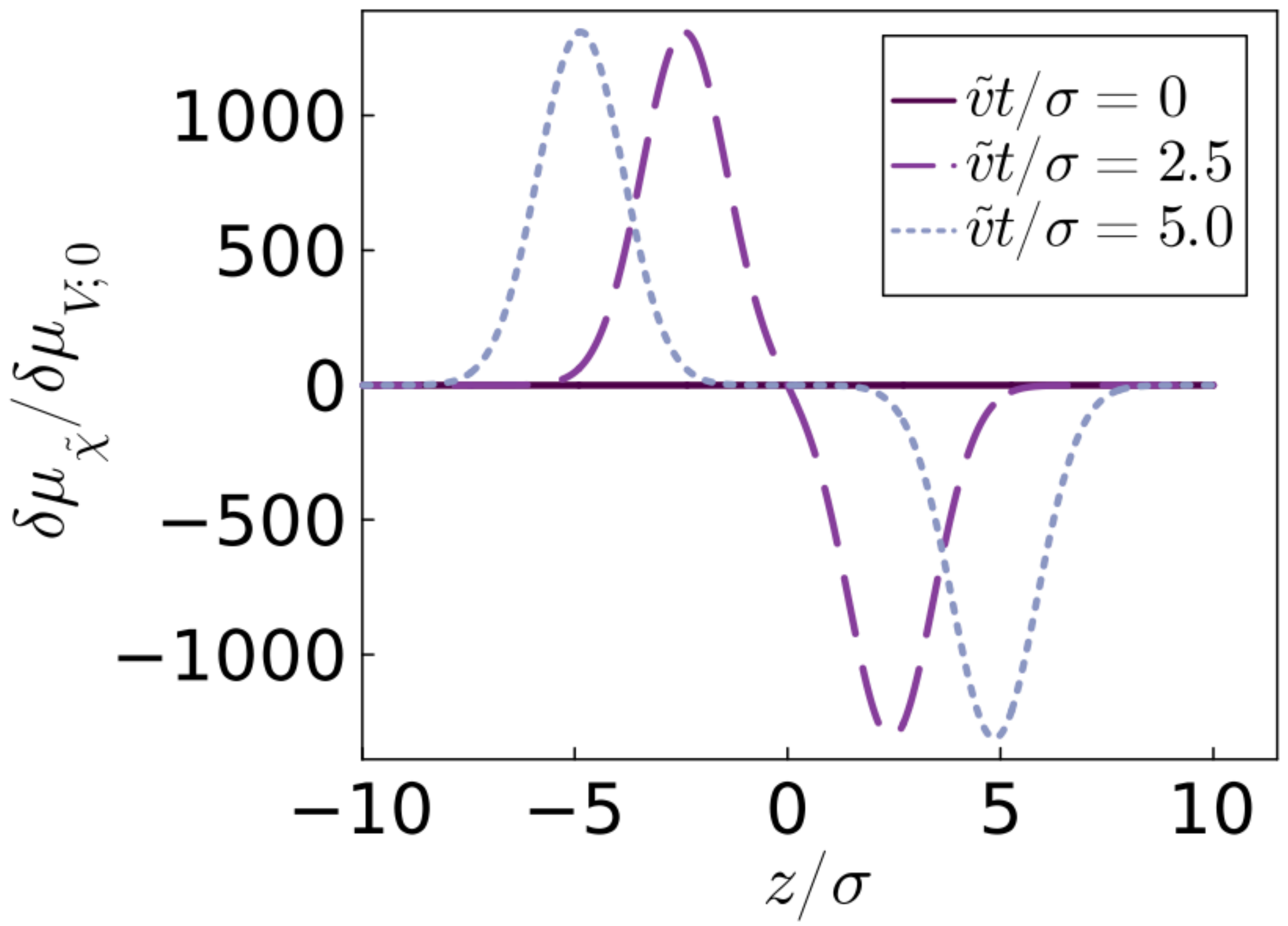


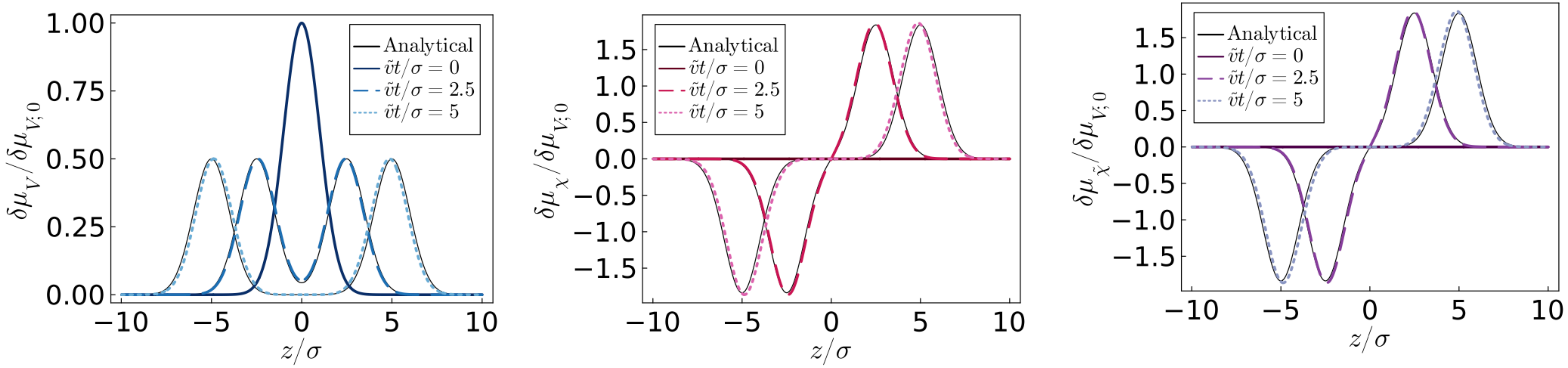
$\delta\mu_{\tilde{\chi}} \sim (\mu_V/T)^4$

The linear perturbation regime is unreliable in this limit. Including dissipation will fix this problem.

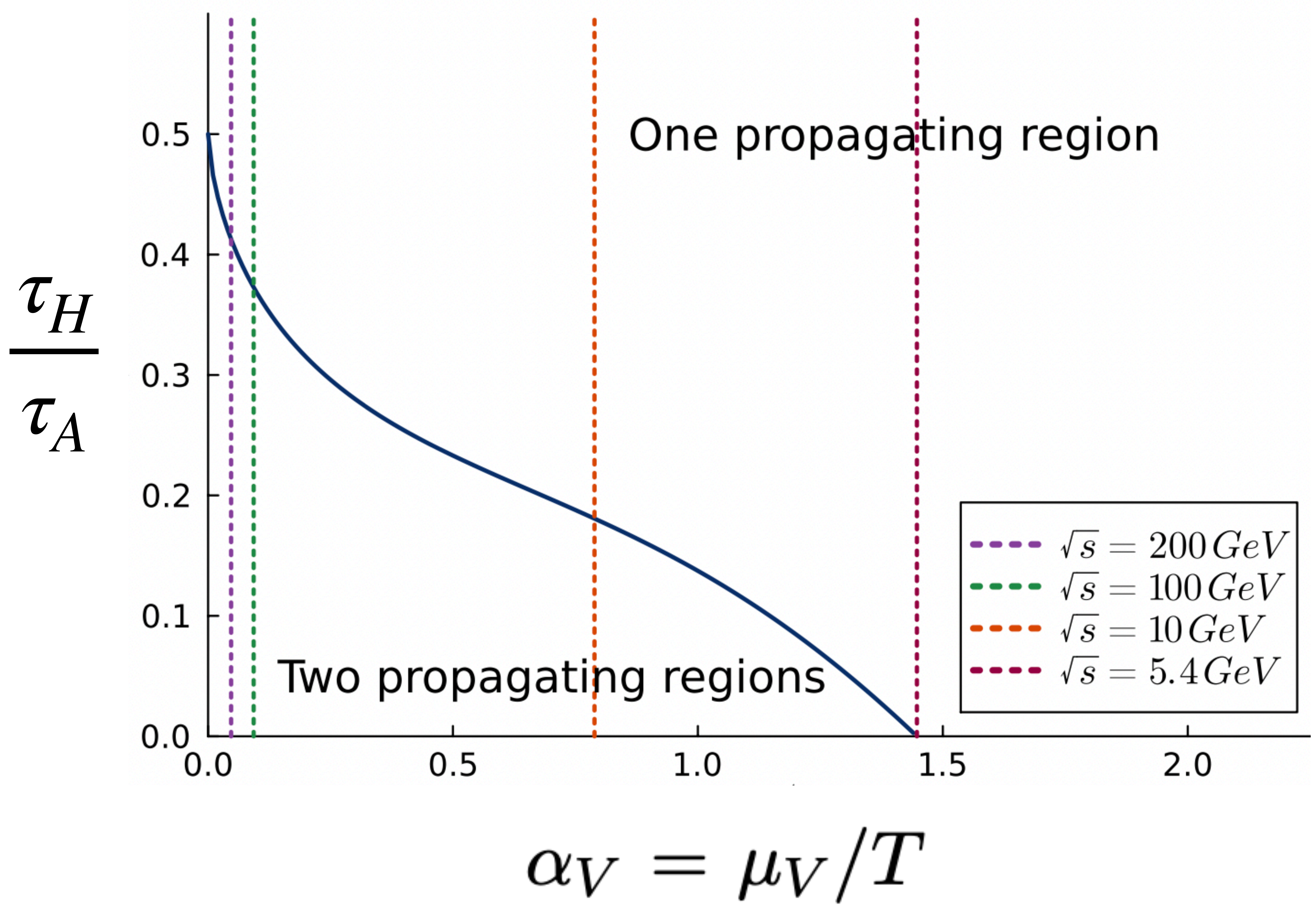
CASE II:

Non-Conserved Helicity ($0 \leq \tau_H < \infty$)

- In the degenerate limit ($\mu_V/T \gg 1$), the finite lifetime τ_H acts as a regulator and no perturbation grows to unreasonably large values.



CASE III:
Non-Conserved Helicity and Chirality ($0 \leq \tau_H, \tau_A < \infty$)



Dissipative Effects

- The currents and the energy-momentum tensor get dissipative contributions, whose dynamics is not fixed by the conservation equations.
- A causal theory calls for second order hydro. For illustrative purposes we use 1st order.

$$\sum_q^{\{B,Q,S\}} \tau_{q'q} \dot{V}_q^{\langle\mu\rangle} + V_{q'}^\mu = \sum_q^{\{B,Q,S\}} \kappa_{q'q} \nabla^\mu \alpha_q - \sum_q^{\{B,Q,S\}} \tau_{q'q} V_{q,\nu}\omega^{\nu\mu} - \sum_q^{\{B,Q,S\}} \delta_{VV}^{(q',q)} V_q^\mu \theta - \sum_q^{\{B,Q,S\}} \lambda_{VV}^{(q',q)} V_{q,\nu}\sigma^{\mu\nu}$$
$$- \ell_{V\Pi}^{(q')} \nabla^\mu \Pi + \ell_{V\pi}^{(q')} \Delta^{\mu\nu}\nabla_\lambda \pi_\nu^\lambda + \tau_{V\Pi}^{(q')} \Pi \dot{u}^\mu - \tau_{V\pi}^{(q')} \pi^{\mu\nu}\dot{u}_\nu$$
$$+ \sum_q^{\{B,Q,S\}} \lambda_{V\Pi}^{(q',q)} \Pi \nabla^\mu \alpha_q - \sum_q^{\{B,Q,S\}} \lambda_{V\pi}^{(q',q)} \pi^{\mu\nu}\nabla_\nu \alpha_q \ ,$$

[Fotakis, Molnar, Niemi, Greiner, Rischke (2022)]

- We implement dissipation through relaxation time approximation $\delta f_{\sigma,\lambda} = -\frac{\tau_R}{p\cdot u} p^\mu \partial_\mu f_{\sigma,\lambda}.$

and the diss. terms under $f \to \delta f$

$$J_\ell^\mu = \sum_{\sigma,\lambda} q_{\sigma,\lambda}^\ell Q_{\sigma,\lambda}, \qquad Q_{\sigma,\lambda} = \int \frac{d^3p}{p^0} f_{\sigma,\lambda} p^\mu,$$

$$T^{\mu\nu} = \sum_{\sigma,\lambda} E_{\sigma,\lambda}, \qquad E_{\sigma,\lambda} = \int \frac{d^3p}{p^0} f_{\sigma,\lambda} p^\mu p^\nu,$$