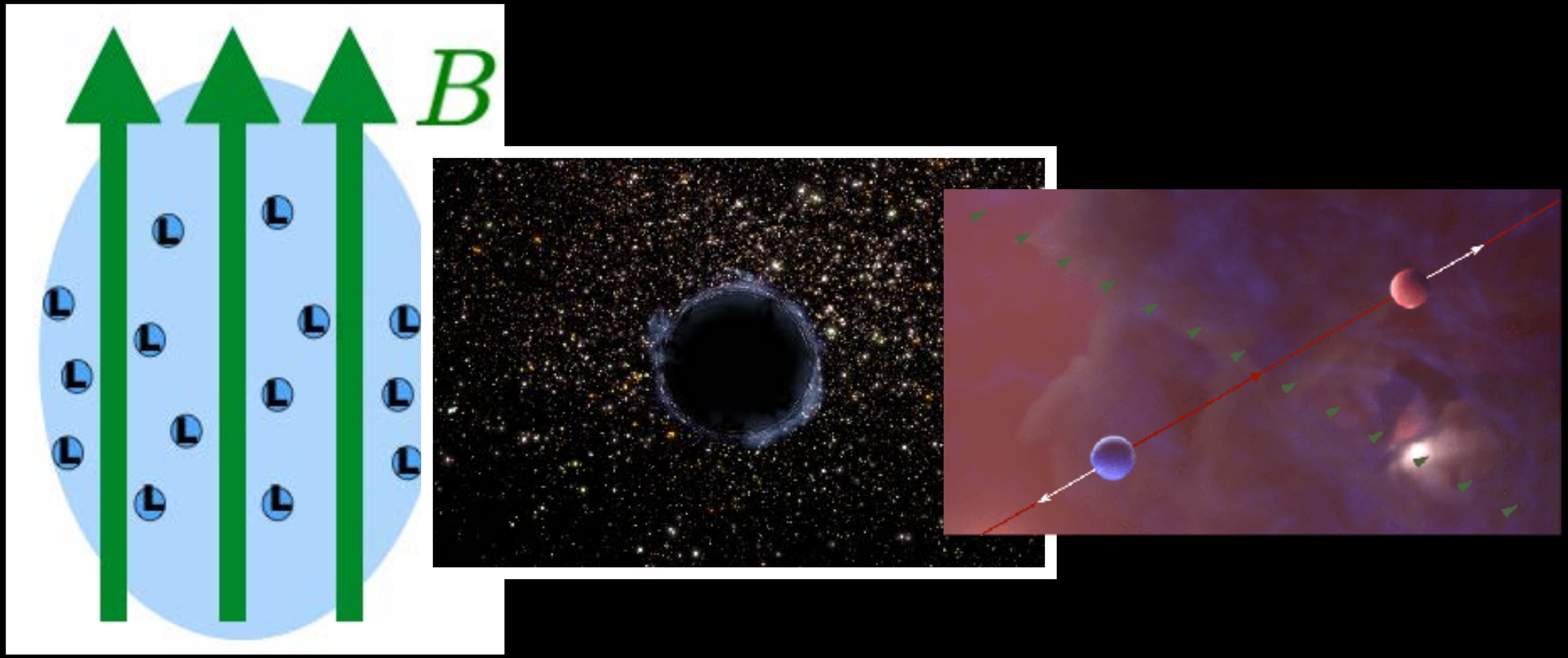
Chirality and a strong magnetic field give rise to novel hydrodynamic transport near and far from equilibrium

8th International Conference on Chirality, Vorticity, and Magnetic Field in Quantum Matter, West University of Timișoara, Romania

July 25th, 2024



[Ammon, Grieninger, Hernandez, Kaminski, Koirala, Leiber, Wu; JHEP (2020)]
[Cartwright, Kaminski, Schenke; PRC (2022)]
[Ammon, Cartwright, Grieninger, Hernandez, Kaminski; PPNP review (to appear)]

Matthias Kaminski
University of Alabama

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tic Field in Quantum Matter, West University of Timișoara, Romania

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U.S. DEPARTMENT OF ENERGY

Chirality and a strong magnetic field give rise to novel hydrodynamic transport

Basic idea

✦ **standard relativistic hydrodynamics is <u>isotropic</u> and <u>parity-invariant</u>**

✦ **but QCD quark gluon plasma *<u>breaks</u>* isotropy and parity**

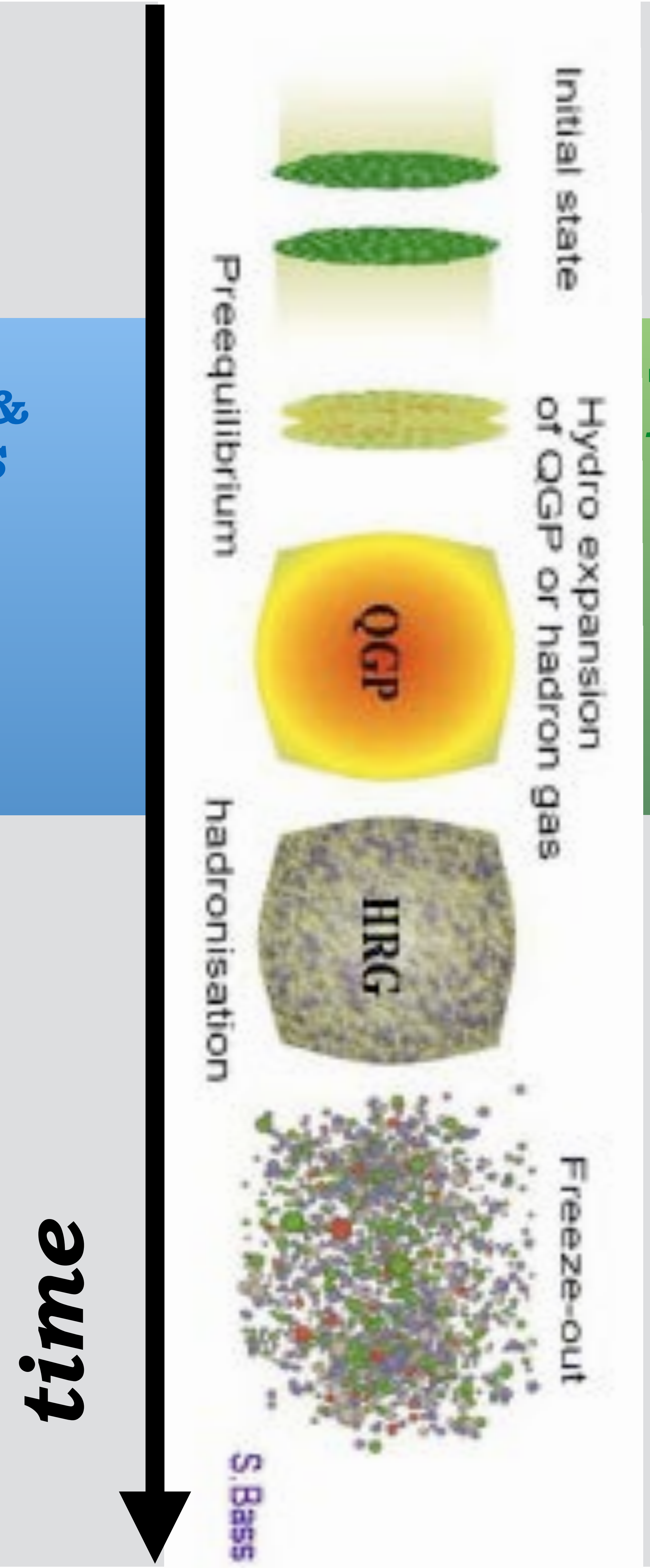
[Adler; Phys.Rev. (1969)]
[Bell, Jackiw; Nuovo Cim.(1969)]
[Blackie; PRL (1960)]

➡ **anisotropic chiral hydro**

➡ **novel transport effects**

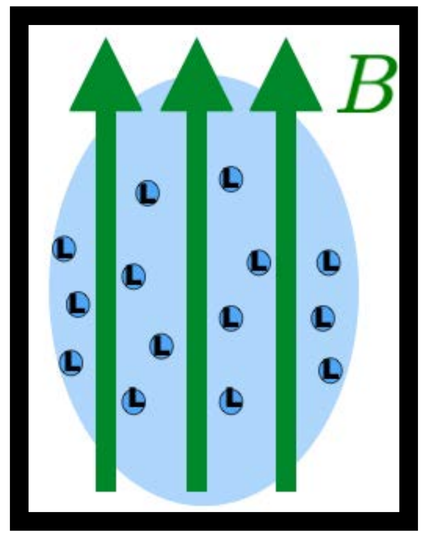
Method

First Calculation: HYDRODYNAMICS & THERMODYNAMICS

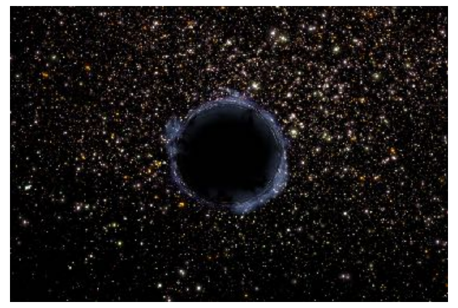


Second Calculation: HOLOGRAPHY

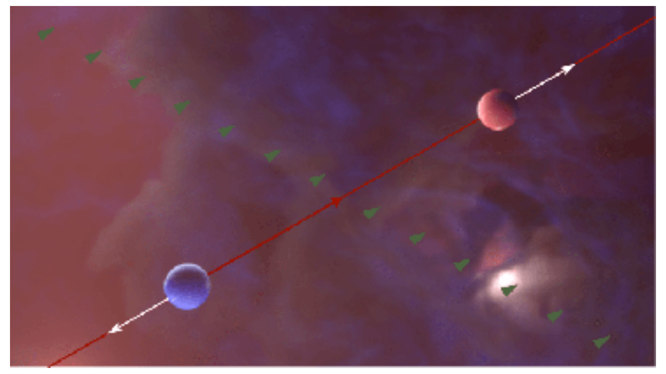
OUTLINE



1. **Novel transport coefficients: deriving chiral hydrodynamics**

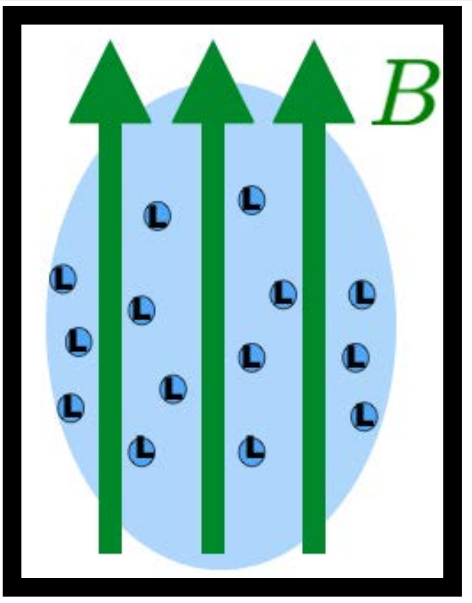


2. **Holographic model of chiral hydrodynamics**



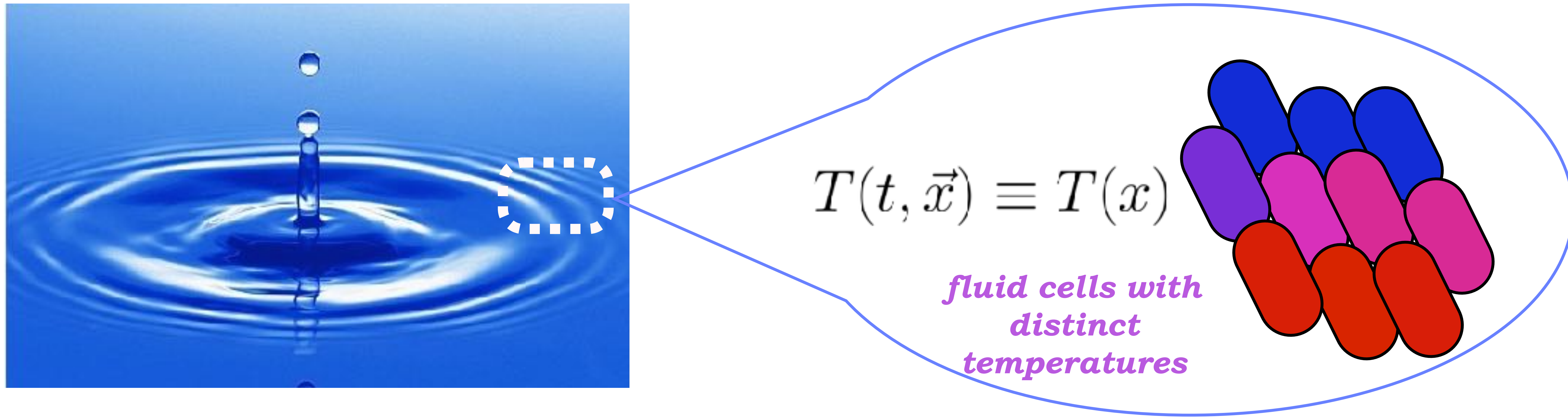
3. **Holographic transport far from equilibrium**

1. Chiral hydrodynamics - Concepts



Hydrodynamics

- **effective field theory**
- expansion in small gradients
- large temperature
- conserved quantities survive



$T(t,\vec{x}) \equiv T(x)$

fluid cells with distinct temperatures

Constitutive equations

$$\langle T^{\mu\nu}\rangle = \epsilon\, u^\mu u^\nu + P\,\Delta^{\mu\nu} + \mathcal{O}(\partial) + \mathcal{O}(\partial^2) + \ldots$$

$$\langle j^\mu_{\text{vector}}\rangle = n u^\mu + \mathcal{O}(\partial) + \mathcal{O}(\partial^2) + \ldots$$

$$\langle j^\mu_{\text{axial}}\rangle = n_a u^\mu + \mathcal{O}(\partial) + \mathcal{O}(\partial^2) + \ldots$$

Scales

Fourier transform hydro fields, e.g. T(x):

$$\partial_t\, e^{-i\omega t} = -i\omega\, e^{-i\omega t}$$

$$\frac{\omega}{T} \ll 1, \qquad \frac{|\vec{k}|}{T} \ll 1$$

$$B \sim \mathcal{O}(1) \qquad B \ll T^2$$

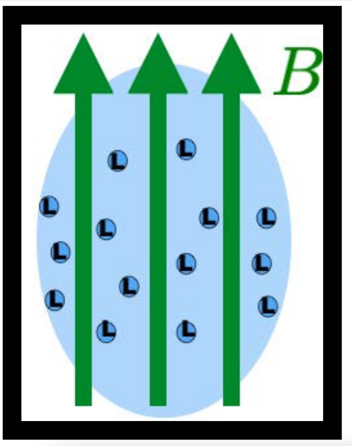
Conservation equations

$$\nabla_\mu T^{\mu\nu} = F^{\mu\nu} j_\mu$$

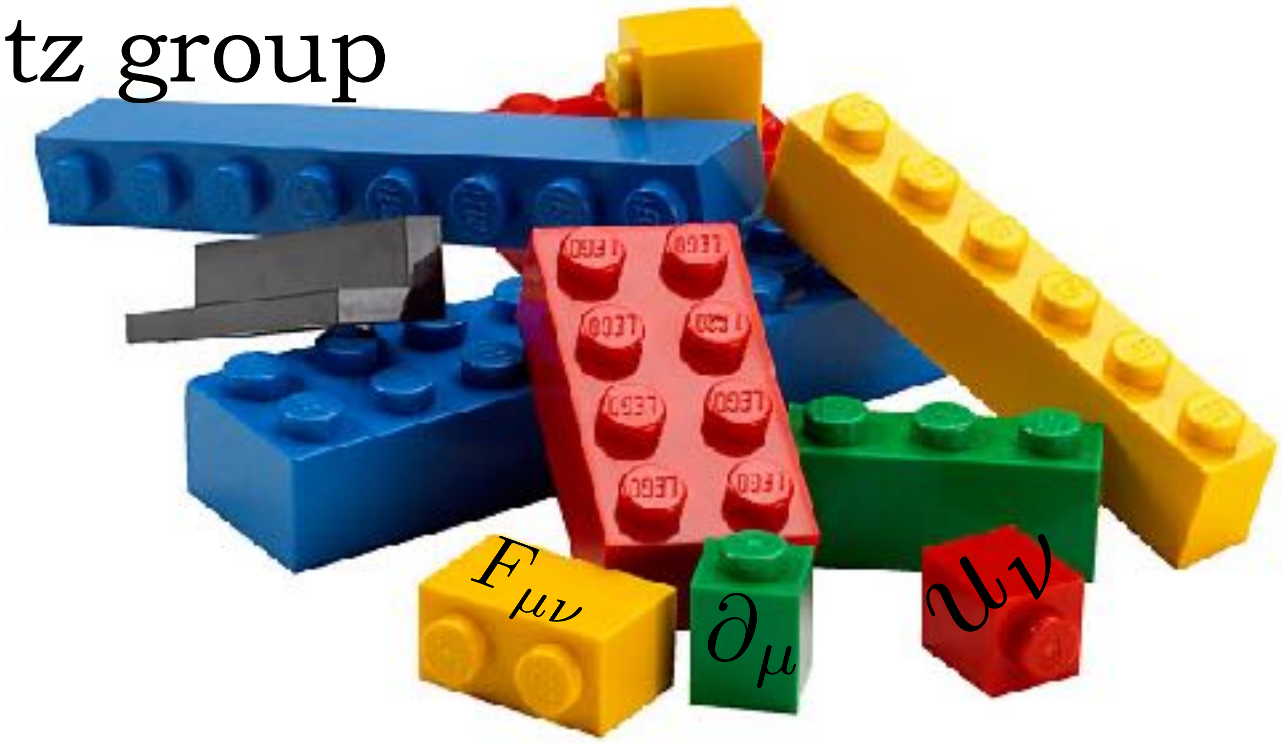
$$\nabla_\mu j^\mu_{\text{vector}} = 0$$

$$\nabla_\mu j^\mu_{\text{axial}} = C\,\vec{E}\cdot\vec{B}$$

1. Chiral hydrodynamics - Construction



1. Construct **constitutive equations or generating functional**: all (pseudo)scalars, (pseudo)vectors and (pseudo)tensors under Lorentz group



$$< j^\mu > = n u^\mu + \mathcal{O}(\partial) + \mathcal{O}(\partial^2) + \ldots$$

Examples at $\mathcal{O}(\partial)$: $\nabla^\mu n$ *charge gradient (covariant derivative)*

vorticity $\omega^\mu = \frac{1}{2}\epsilon^{\mu\nu\lambda\rho} u_\nu \nabla_\lambda u_\rho$

magnetic vorticity $\Omega^\mu_B = \epsilon^{\mu\nu\rho\sigma} u_\nu \nabla_\rho B_\sigma$

Idea: generating functional $W[T,\mu]$

$$\langle T^{\mu\nu} \rangle \sim \frac{\delta W[T,\mu]}{\delta g_{\mu\nu}}$$

$$\langle T^{\mu\nu} T^{\alpha\beta} \rangle \sim \frac{\delta^2 W}{\delta g_{\mu\nu} \delta g_{\alpha\beta}}$$

[Jensen, Kaminski, Kovtun, Meyer, et al.; PRL (2012)] [JHEP (2011)]

[Banerjee et al.; JHEP (2012)]

[Haehl et al.; PRL (2015)]

[Crossley et al.; (2015)]

2. Restricted by conservation equations

Example: $\nabla_\mu j^\mu_{(0)} = \nabla_\mu (n u^\mu) = 0$

3. Further restricted by positivity of local entropy production: *[Landau, Lifshitz]*

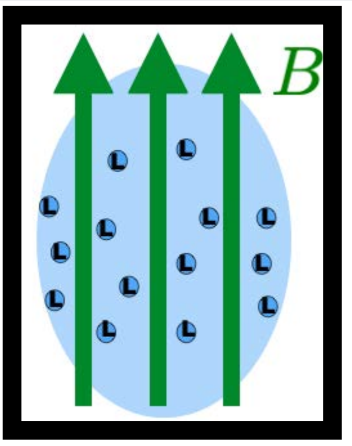
$$\nabla_\mu J^\mu_s \geq 0$$

cf. method reported in talk by J. Liao (used there for spin hydro)

➡ Most general hydrodynamic 1-point functions for chiral charged fluid in strong magnetic field *[Ammon, Kaminski et al.; JHEP (2017)]*

[Ammon, Grieninger, Hernandez, Kaminski, Koirala, Leiber, Wu; JHEP (2020)]

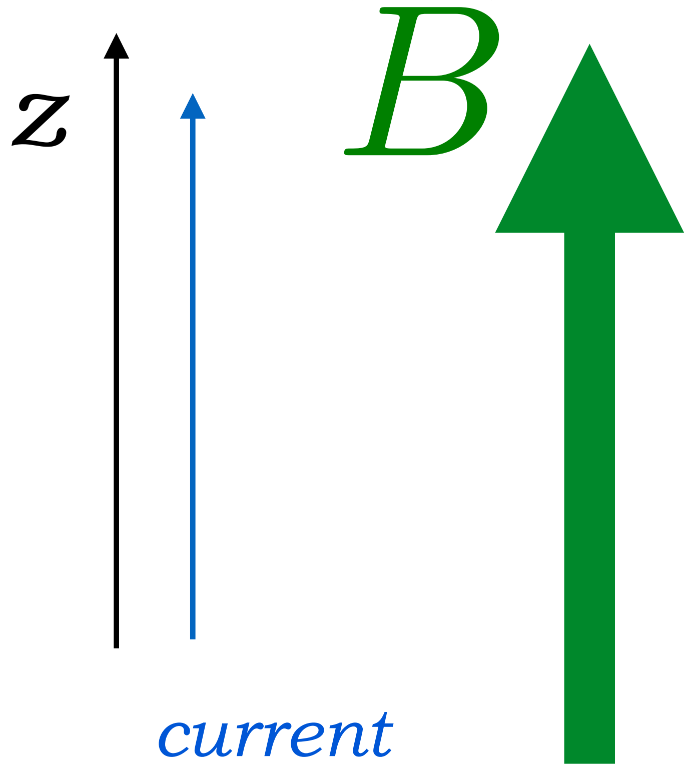
1. Chiral hydrodynamics - conductivity Kubo formulae



[Ammon, Grieninger, Hernandez, Kaminski, Koirala, Leiber, Wu; JHEP (2020)]

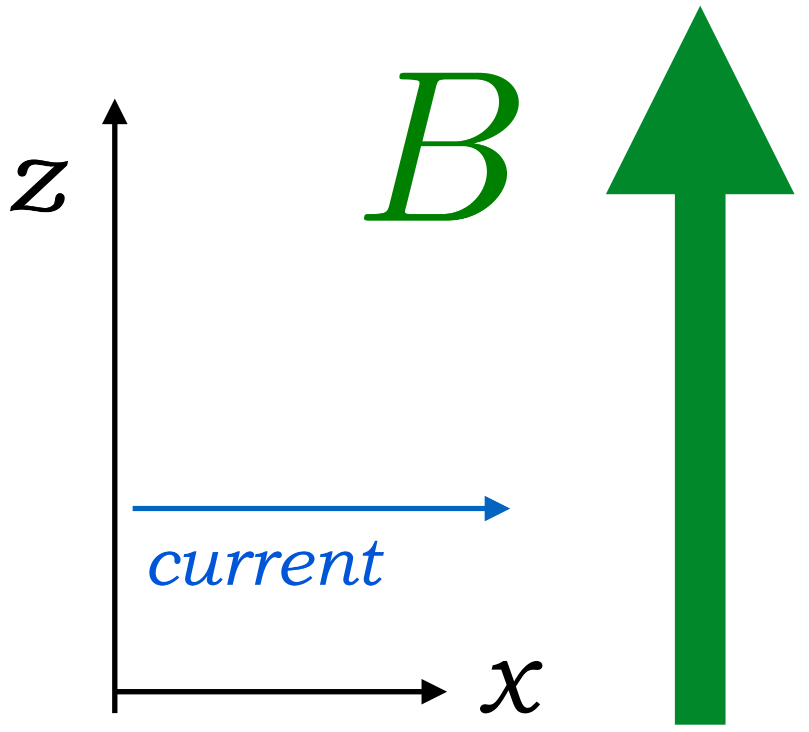
Parallel **conductivity**

$$\lim_{\omega\to 0} \frac{1}{\omega}\mathrm{Im}\langle J^z J^z\rangle(\omega, \mathbf{k}{=}0) = \sigma_{||} + \cdots$$



Perpendicular **resistivity**

$$\lim_{\omega\to 0} \frac{1}{\omega}\mathrm{Im}\,\langle J^x J^x\rangle(\omega, \mathbf{k}{=}0) = \omega^2 \rho_\perp \frac{w_0(w_0 - M_{5,\mu} B_0^2)}{B_0^4}$$



Very different parallel versus perpendicular

$$\langle J^z J^z\rangle(\omega, \mathbf{k} = 0) \sim \sigma_{||}$$

$$\langle J^x J^x\rangle(\omega, \mathbf{k} = 0) \sim \rho_\perp$$

For *any* anisotropic fluid:

➡ also two distinct shear viscosities $\eta_\perp$ and $\eta_{||}$

➡ $\eta_{||}$ has no lower bound

➡ affects hydro modeling !

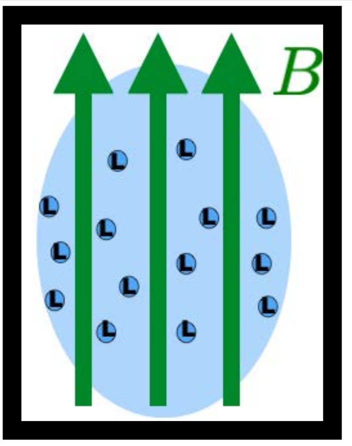
Two Prog.Part.Nucl.Phys. review articles:

[Cartwright,Garbiso-Amano,Kaminski,Wu; PPNP review (accepted version on arXiv)] on ***rotation***

[Ammon,Cartwright,Grieninger,Hernandez, Kaminski; PPNP review (to appear)] on ***strong B***

cf. [Florkowski, Ryblewski; PRC (2010)]
[Martinez, Strickland; Nucl.Phys.A (2010)]

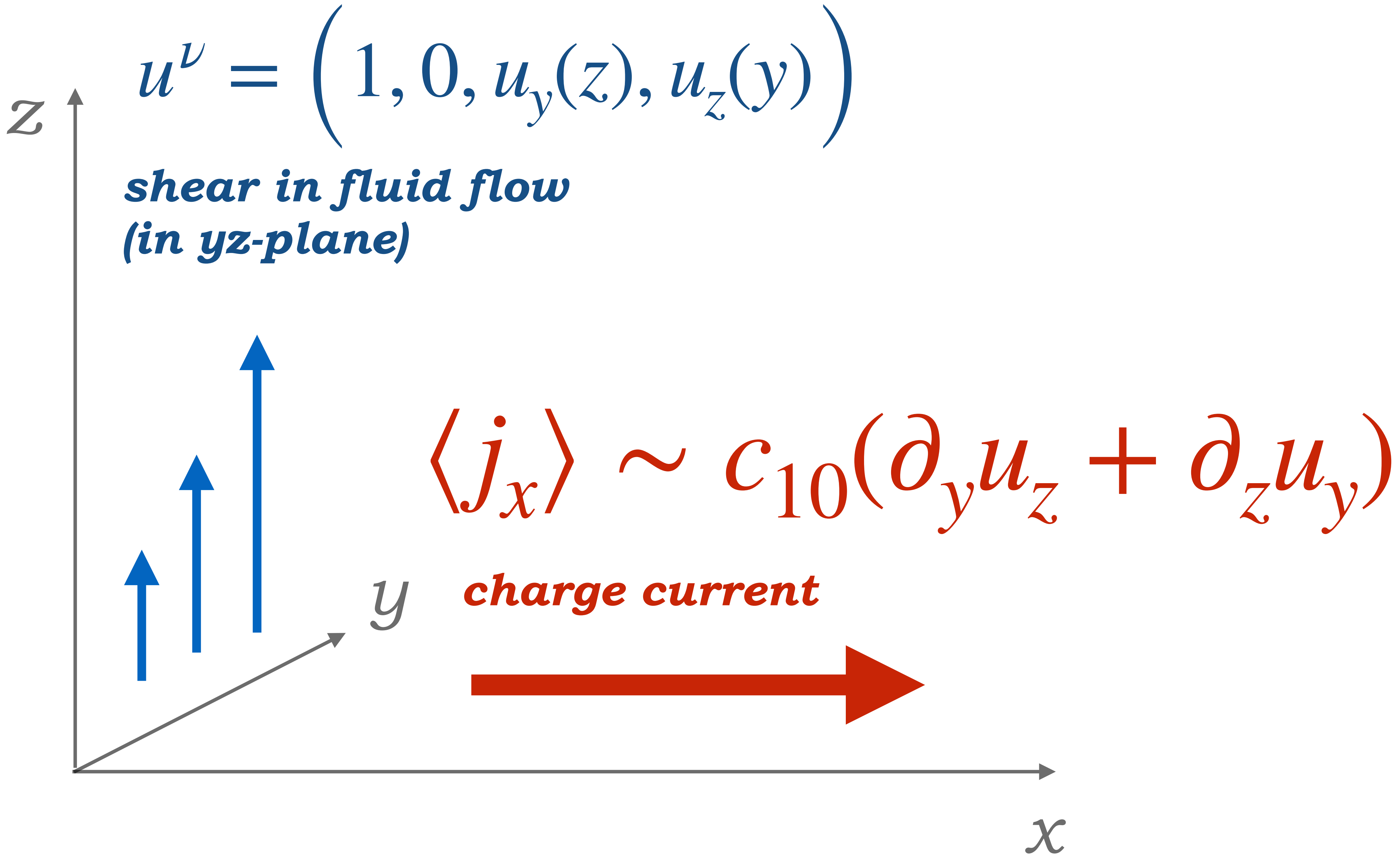
1. Chiral hydrodynamics - novel transport coefficient c_{10}



[Ammon, Grieninger, Hernandez, Kaminski, Koirala, Leiber, Wu; JHEP (2020)]

Shear-induced Hall conductivity c_{10}

$$c_{10} \sim \lim_{\omega\to 0} \frac{1}{\omega} \mathrm{Im}\langle T^{tx}\, T^{yz}\rangle(\omega, \vec{k}=0)$$



$$u^\nu = \left(1, 0, u_y(z), u_z(y)\right)$$

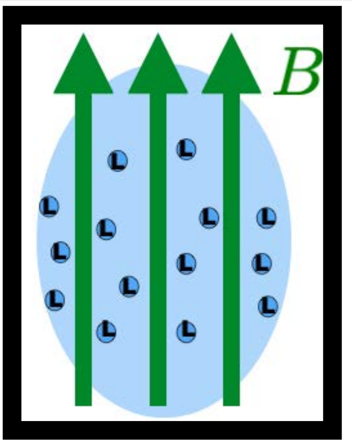
shear in fluid flow (in yz-plane)

$$\langle j_x\rangle \sim c_{10}(\partial_y u_z + \partial_z u_y)$$

charge current

- ➡ **novel Hall response**
- ➡ **non-dissipative**
- ➡ **interplay: shear-charge**

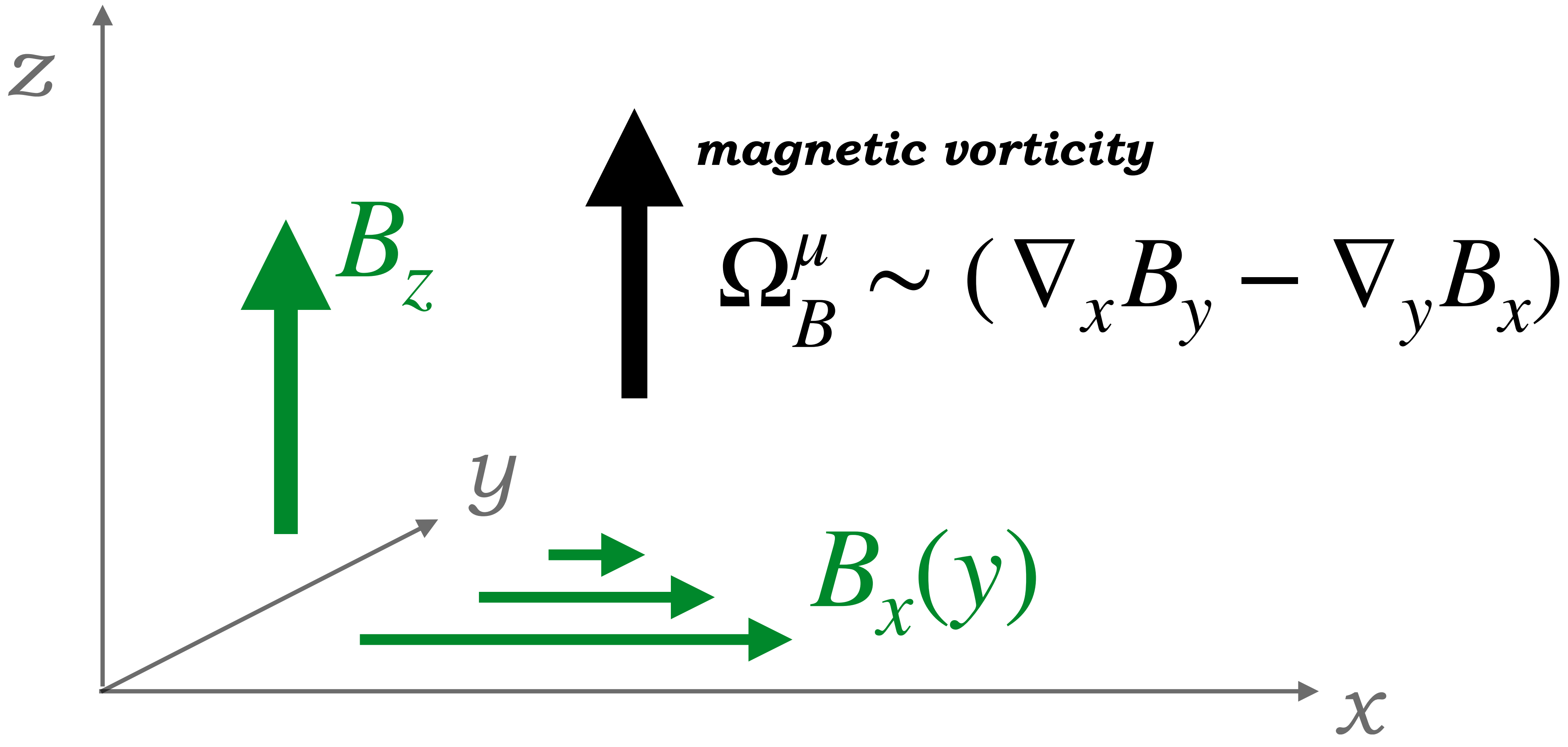
1. Chiral hydrodynamics - novel equilibrium coefficient M_2



[Ammon, Grieninger, Hernandez, Kaminski, Koirala, Leiber, Wu; JHEP (2020)]

Perpendicular magnetic vorticity susceptibility M_2

$$\boxed{M_2 = -\lim_{k_z\to 0}\frac{1}{2k_zB_0^2}\mathrm{Im}\langle T^{xz}T^{yz}\rangle(\omega=0,k_z)}$$



$$\Omega_B^\mu \sim (\nabla_x B_y - \nabla_y B_x)$$

response in energy/pressure :

$$\langle T^{tt}\rangle = \ \mathcal{E}_{\rm eq} \sim \mathcal{P}_{\rm eq} \sim M_2 B\cdot\Omega_B$$

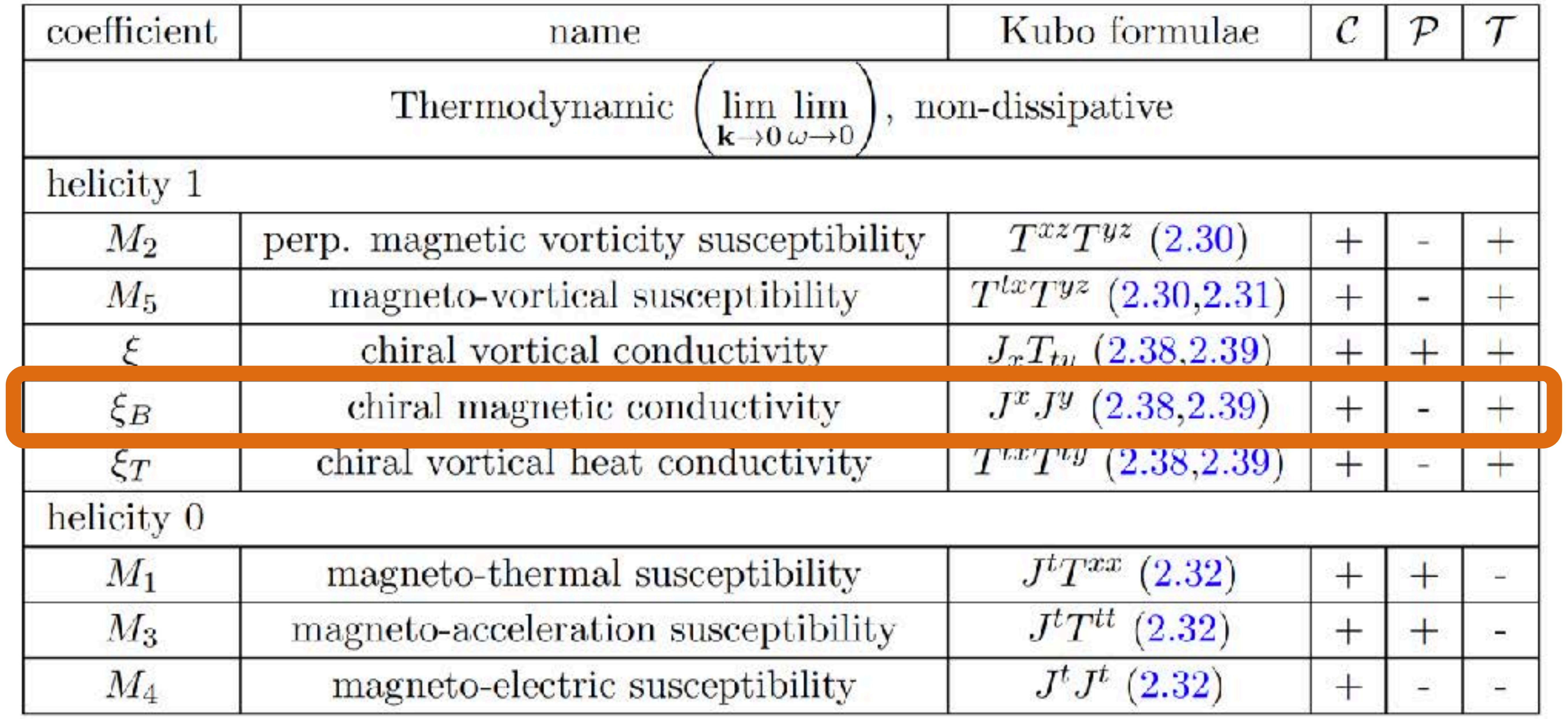
magnetic vorticity : $\Omega_B^\mu = \epsilon^{\mu\nu\rho\sigma}u_\nu\nabla_\rho B_\sigma$

➡ Can we test these Kubo formulae and constitutive relations on the lattice or in experiments?

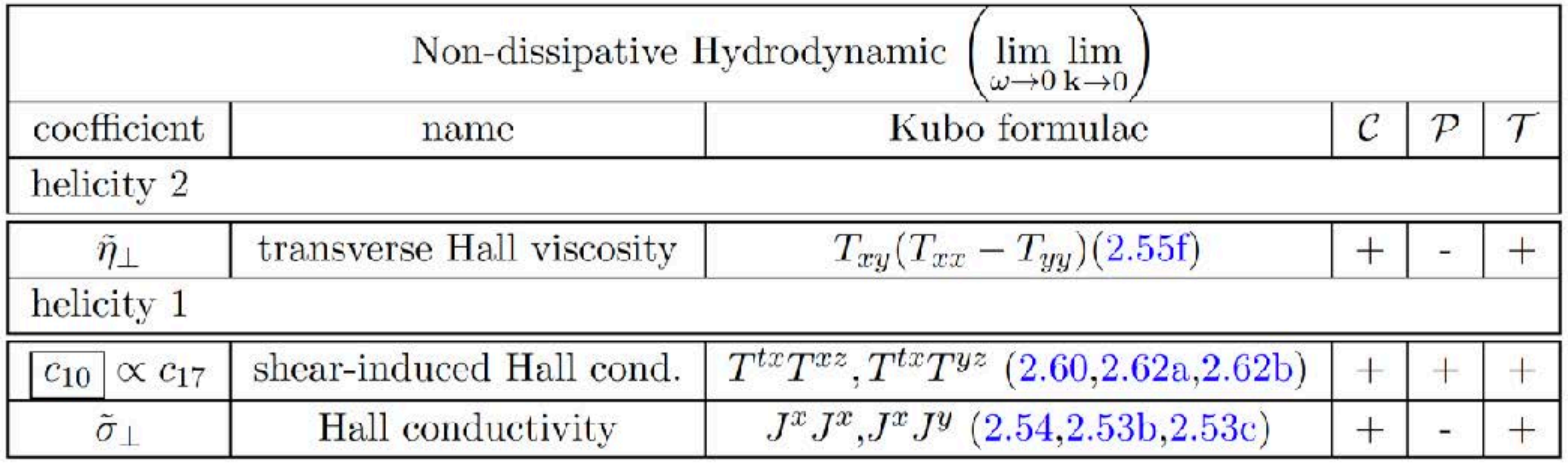
Chiral hydrodynamics - all coefficients

[Ammon, Grieninger, Hernandez, Kaminski, Koirala, Leiber, Wu; JHEP (2020)]

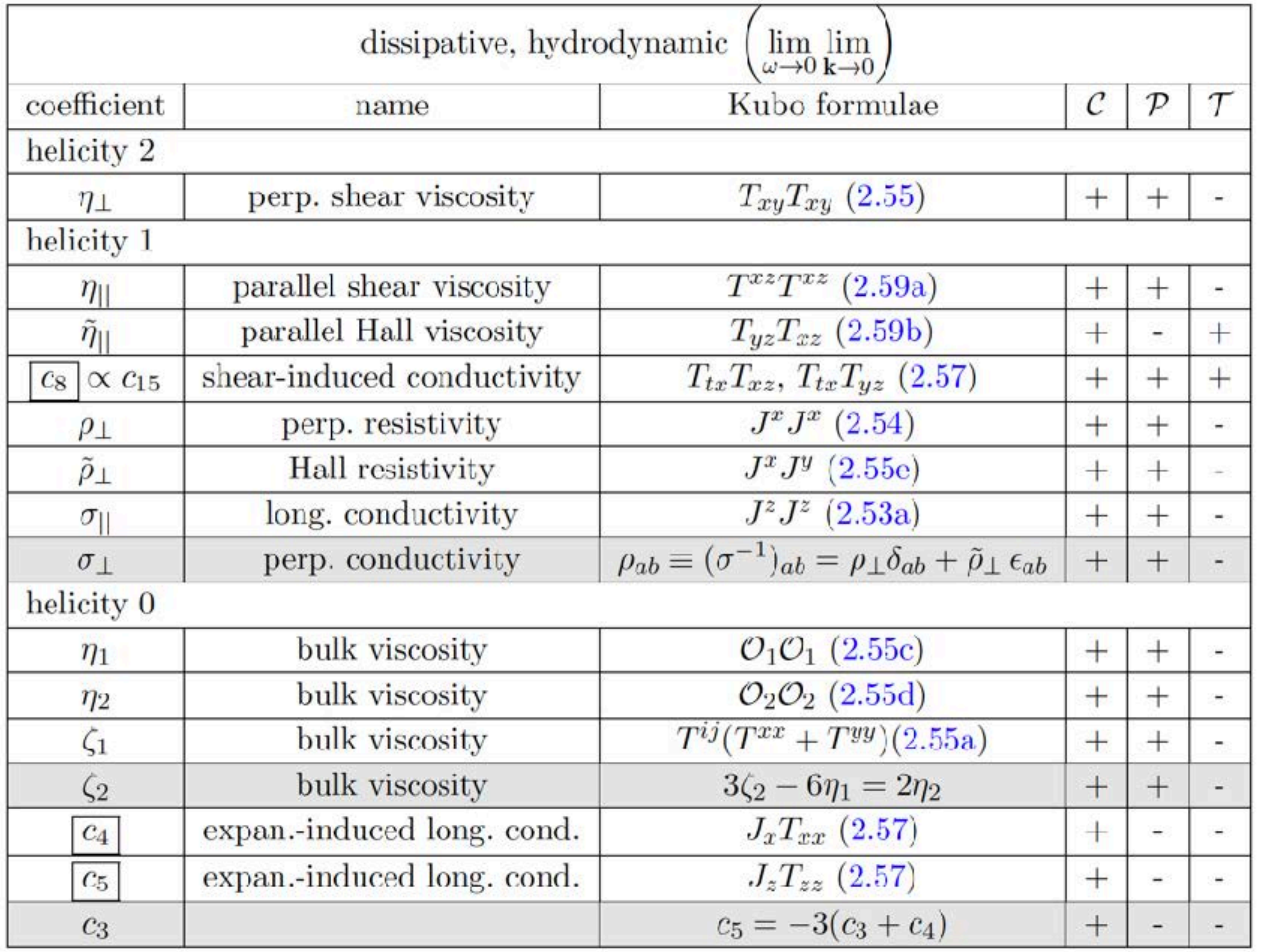
cf. [Hernandez, Kovtun; JHEP (2017)]



coefficient	name	Kubo formulae	$\mathcal{C}$	$\mathcal{P}$	$\mathcal{T}$
Thermodynamic $\left(\lim\limits_{\mathbf{k}\to 0}\lim\limits_{\omega\to 0}\right)$, non-dissipative					
helicity 1					
M_2	perp. magnetic vorticity susceptibility	$T^{xz}T^{yz}$ (2.30)	+	-	+
M_5	magneto-vortical susceptibility	$T^{tx}T^{yz}$ (2.30,2.31)	+	-	+
ξ	chiral vortical conductivity	J_xT_{ty} (2.38,2.39)	+	+	+
ξ_B	chiral magnetic conductivity	J^xJ^y (2.38,2.39)	+	-	+
ξ_T	chiral vortical heat conductivity	$T^{tx}T^{ty}$ (2.38,2.39)	+	-	+
helicity 0					
M_1	magneto-thermal susceptibility	J^tT^{xx} (2.32)	+	+	-
M_3	magneto-acceleration susceptibility	J^tT^{tt} (2.32)	+	+	-
M_4	magneto-electric susceptibility	J^tJ^t (2.32)	+	-	-



Non-dissipative Hydrodynamic $\left(\lim\limits_{\omega\to 0}\lim\limits_{\mathbf{k}\to 0}\right)$					
coefficient	name	Kubo formulae	$\mathcal{C}$	$\mathcal{P}$	$\mathcal{T}$
helicity 2					
$\tilde{\eta}_\perp$	transverse Hall viscosity	$T_{xy}(T_{xx}-T_{yy})$(2.55f)	+	-	+
helicity 1					
$\boxed{c_{10}} \propto c_{17}$	shear-induced Hall cond.	$T^{tx}T^{xz}, T^{tx}T^{yz}$ (2.60,2.62a,2.62b)	+	+	+
$\tilde{\sigma}_\perp$	Hall conductivity	J^xJ^x, J^xJ^y (2.54,2.53b,2.53c)	+	-	+



dissipative, hydrodynamic $\left(\lim\limits_{\omega\to 0}\lim\limits_{\mathbf{k}\to 0}\right)$					
coefficient	name	Kubo formulae	$\mathcal{C}$	$\mathcal{P}$	$\mathcal{T}$
helicity 2					
$\eta_\perp$	perp. shear viscosity	$T_{xy}T_{xy}$ (2.55)	+	+	-
helicity 1					
$\eta_{\vert\vert}$	parallel shear viscosity	$T^{xz}T^{xz}$ (2.59a)	+	+	-
$\tilde{\eta}_{\vert\vert}$	parallel Hall viscosity	$T_{yz}T_{xz}$ (2.59b)	+	-	+
$\boxed{c_8} \propto c_{15}$	shear-induced conductivity	$T_{tx}T_{xz}$, $T_{tx}T_{yz}$ (2.57)	+	+	+
$\rho_\perp$	perp. resistivity	J^xJ^x (2.54)	+	+	-
$\tilde{\rho}_\perp$	Hall resistivity	J^xJ^y (2.55e)	+	+	-
$\sigma_{\vert\vert}$	long. conductivity	J^zJ^z (2.53a)	+	+	-
$\sigma_\perp$	perp. conductivity	$\rho_{ab} \equiv (\sigma^{-1})_{ab} = \rho_\perp\delta_{ab} + \tilde{\rho}_\perp\,\epsilon_{ab}$	+	+	-
helicity 0					
η_1	bulk viscosity	$\mathcal{O}_1\mathcal{O}_1$ (2.55c)	+	+	-
η_2	bulk viscosity	$\mathcal{O}_2\mathcal{O}_2$ (2.55d)	+	+	-
ζ_1	bulk viscosity	$T^{ij}(T^{xx}+T^{yy})$(2.55a)	+	+	-
ζ_2	bulk viscosity	$3\zeta_2 - 6\eta_1 = 2\eta_2$	+	+	-
$\boxed{c_4}$	expan.-induced long. cond.	J_xT_{xx} (2.57)	+	-	-
$\boxed{c_5}$	expan.-induced long. cond.	J_zT_{zz} (2.57)	+	-	-
c_3		$c_5 = -3(c_3+c_4)$	+	-	-

➡ **novel transport relevant for QGP ?!**

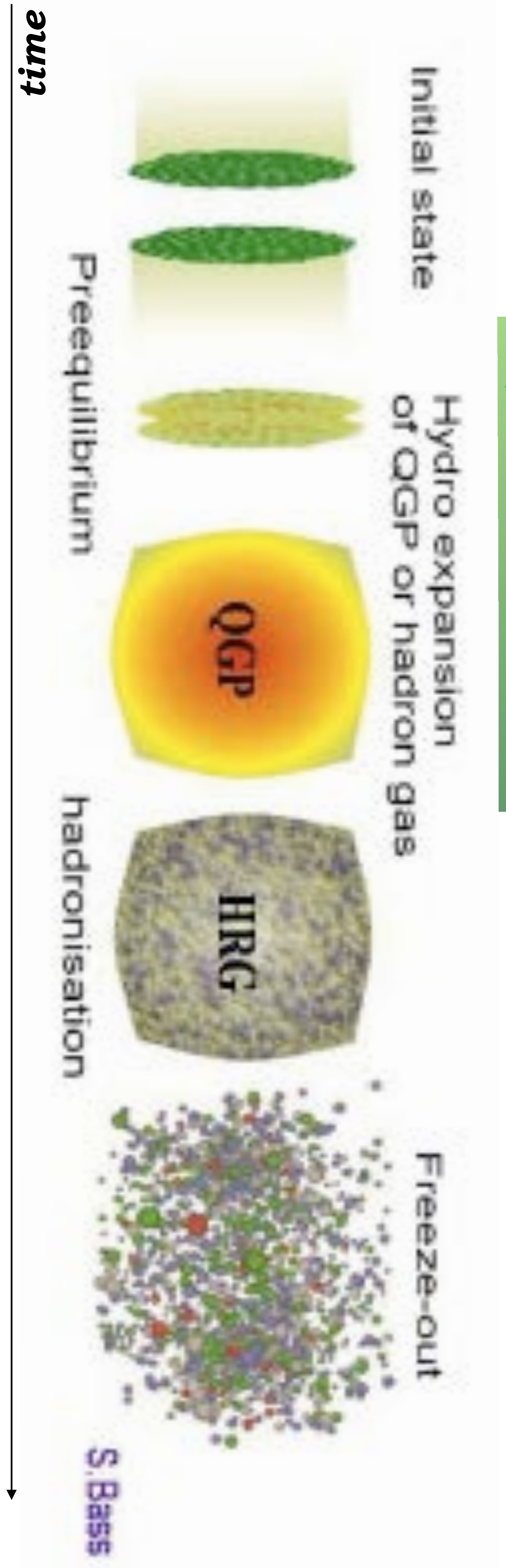
➡ **CME in equilibrium on the lattice ?!**

cf. talk by E. Garnacho-Velasco

cf. poster by E. Garnacho-Velasco as well as by K. Fukushima

A winning team: hydrodynamics and holography in parallel

HYDRODYNAMICS & THERMODYNAMICS



HOLOGRAPHY

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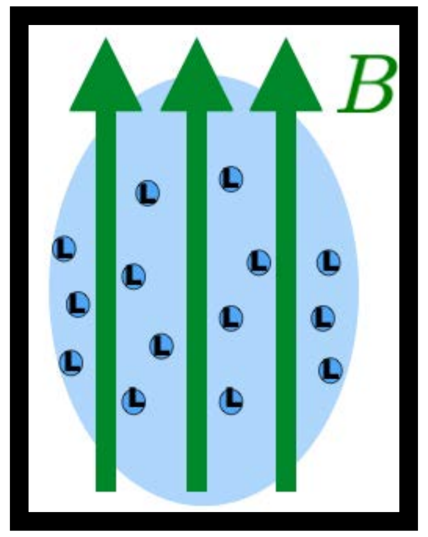
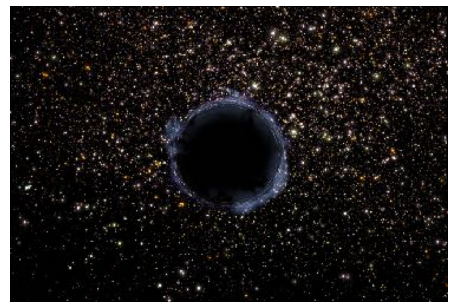
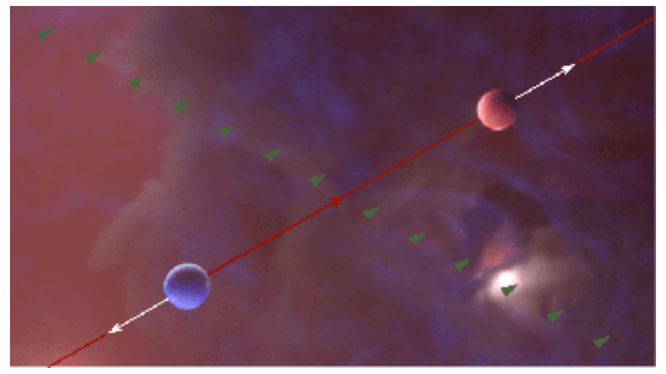


1. Novel transport coefficients: deriving chiral hydrodynamics



2. **Holographic model of chiral hydrodynamics**



3. Holographic transport far from equilibrium

2. Holographic model for chiral hydrodynamics

➡ **Construct holographic dual to charged plasma in strong *B***

➡ **Compute values for all transport coefficients of *N*=4 SYM**

[Ammon, Kaminski et al.; JHEP (2017)]
[Ammon, Grieninger, Hernandez, Kaminski, Koirala, Leiber, Wu; JHEP (2020)]

Einstein-Maxwell-Chern-Simons action dual to *N*=4 Super-Yang-Mills (SYM)

$$S_{grav} = \frac{1}{2\kappa^2}\left[\int_{\mathcal{M}} d^5x\,\sqrt{-g}\left(R + \frac{12}{L^2} - \frac{1}{4}F_{mn}F^{mn}\right) - \frac{\gamma}{6}\int_{\mathcal{M}} A\wedge F\wedge F\right]$$

5-dimensional Einstein-Maxwell term encodes N=4 Super-Yang-Mills (SYM)

5-dimensional Chern-Simons term encodes chiral anomaly of SYM
cf. [Erdmenger,Haack,Kaminski,Yarom; JHEP (2008)]
[Son,Surowka; PRL (2009)]

Charged magnetic black branes dual to charged plasma in strong *B*

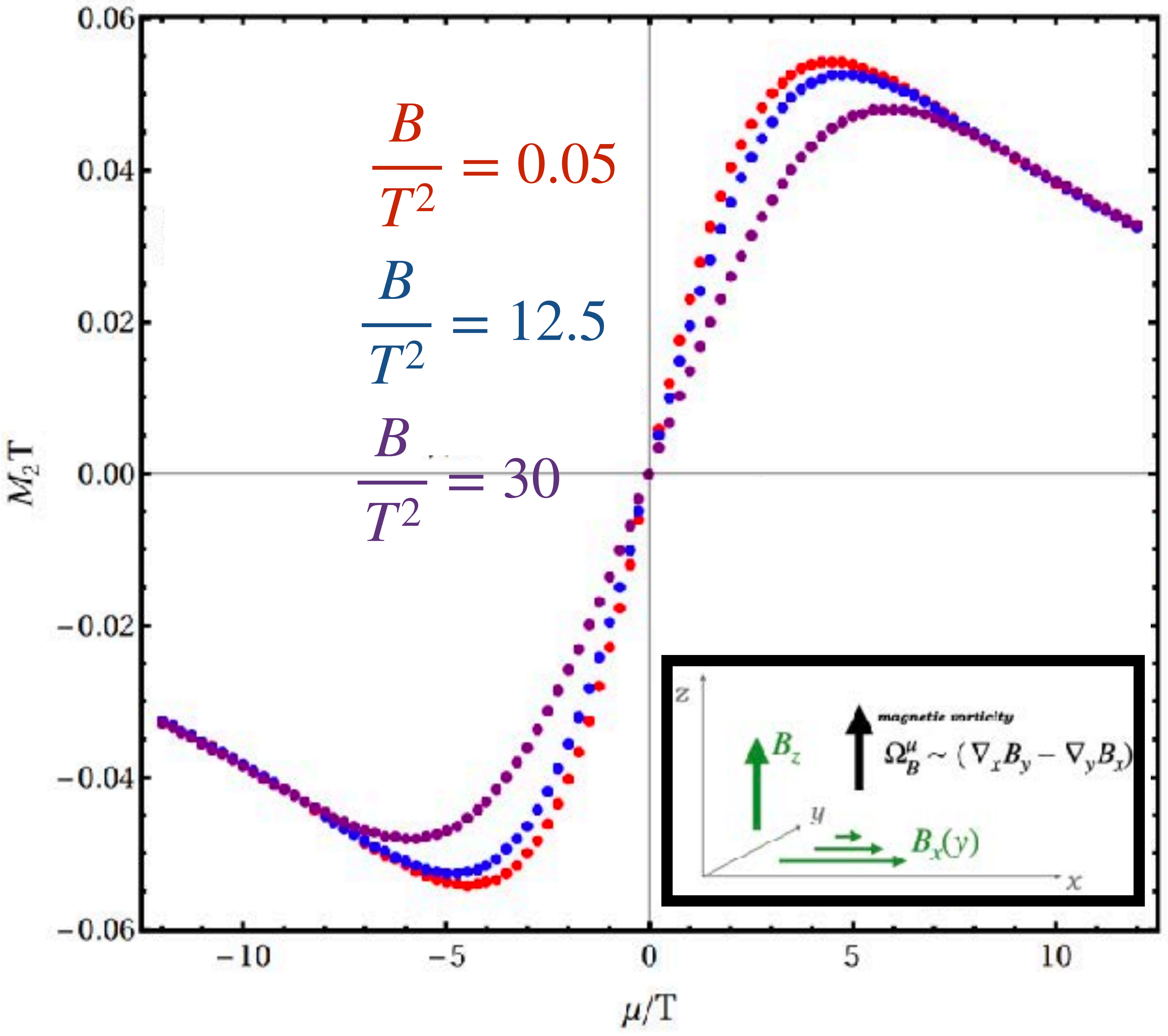
[D'Hoker, Kraus; JHEP (2010)]

- **charged magnetic** analog of Reissner-Nordstrom black brane
- asymptotically AdS_5

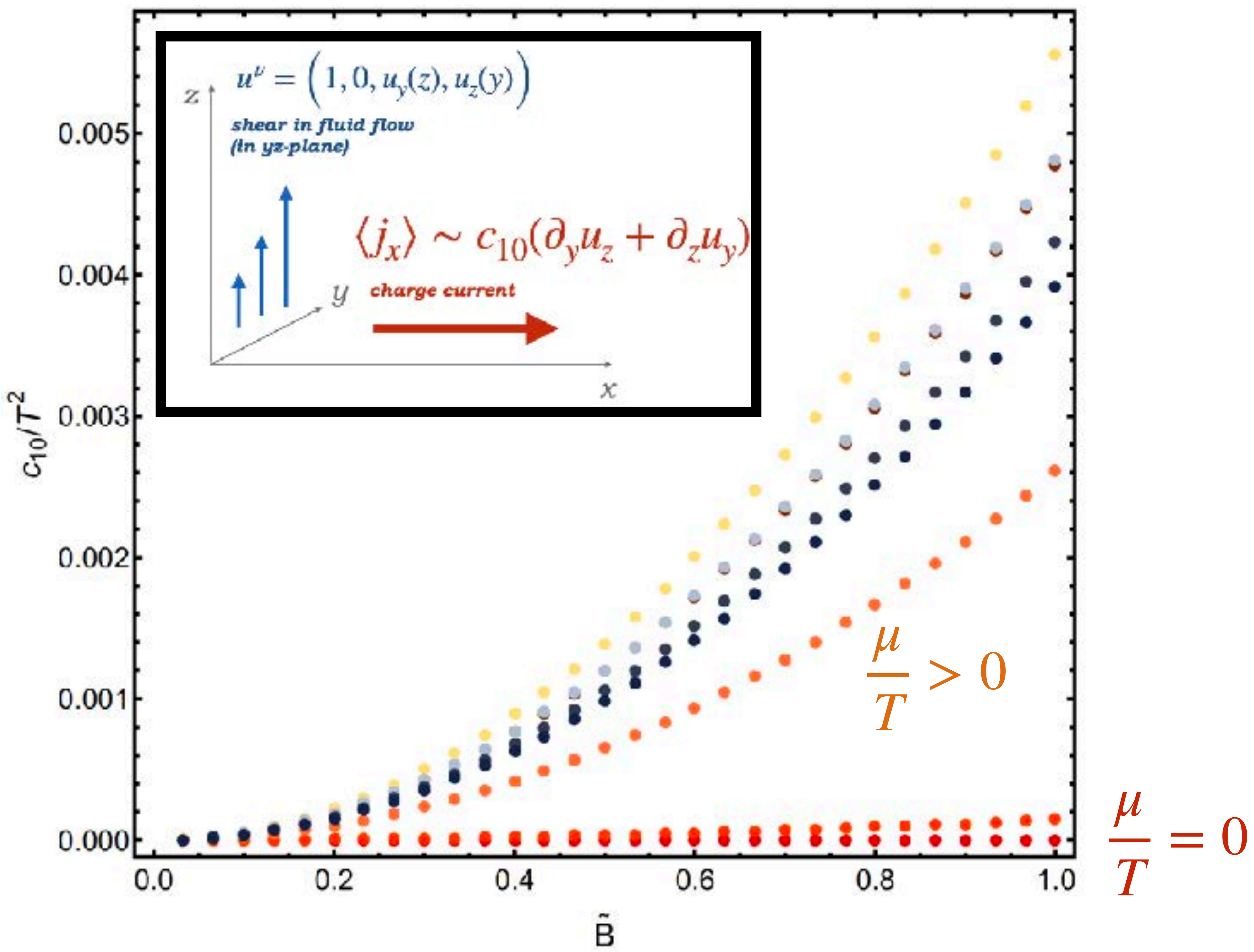
2. Holographic model for chiral hydrodynamics - Results

[Ammon, Grieninger, Hernandez, Kaminski, Koirala, Leiber, Wu; JHEP (2020)]

Perpendicular magnetic vorticity susceptibility M_2



Shear-induced Hall conductivity c_{10}



- ➡ **not zero, finite, Onsager relations satisfied**
- ➡ ***all* Kubo formulae consistent**
- ➡ **transport coefficient relations satisfied**
- ➡ **confirms chiral hydrodynamics**

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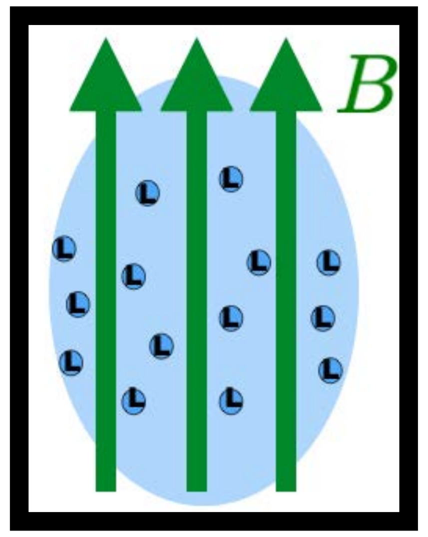
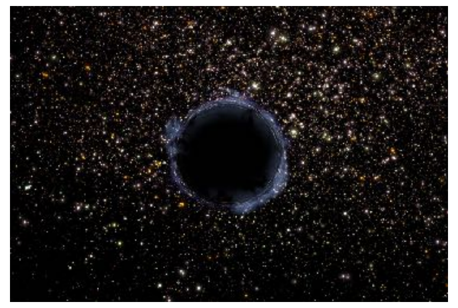
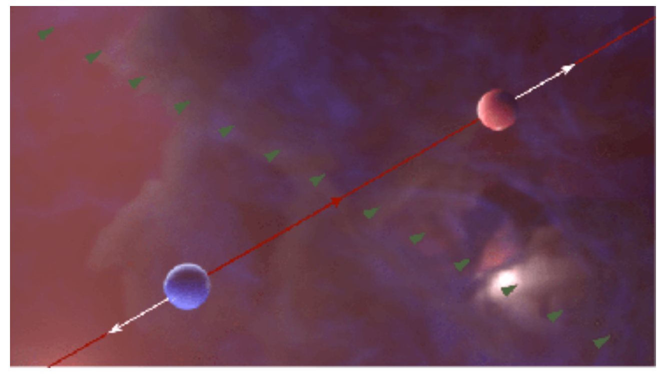


1. Novel transport coefficients: deriving chiral hydrodynamics



2. Holographic model of chiral hydrodynamics



3. **Holographic transport far from equilibrium**

A winning team: hydrodynamics and holography in parallel

DISCLAIMER: More balanced review in my Section 5.2 on Hydrodynamics in White Paper [Sorensen et al.; Prog.Part.Nucl.Phys. (2024)]

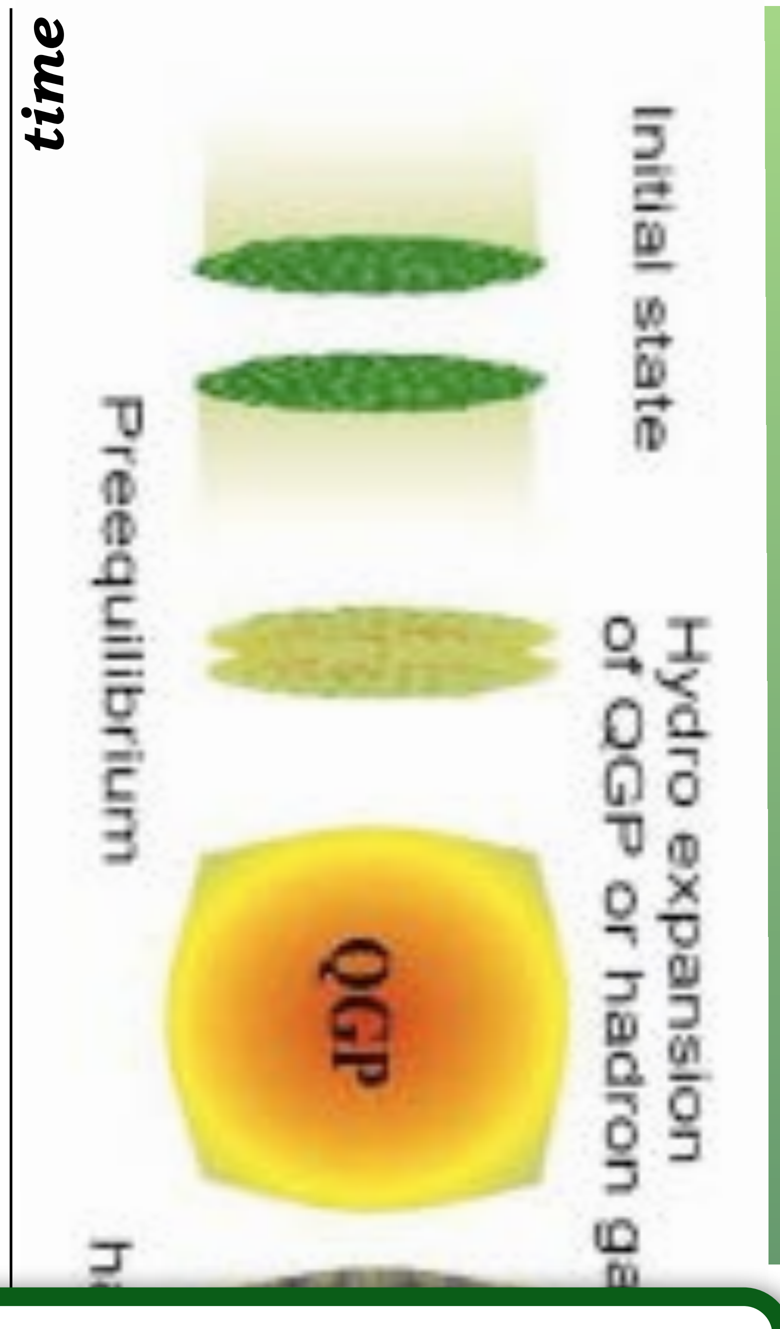
HYDRODYNAMICS & THERMODYNAMICS

Chiral Magnetic Effect (CME) from chiral anomaly

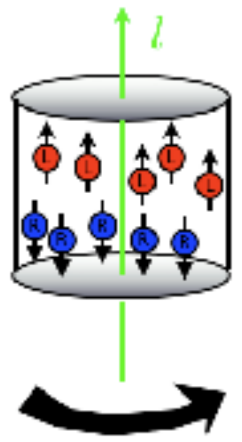
[Kharzeev; PRC (2004)]
[Son,Surowka; PRL (2009)]
[Neiman,Oz; JHEP (2010)]

$$J^\mu_A = \xi_B B$$

hydro and holo in parallel



Chiral Vortical Effect



[Erdmenger,Haack,Kaminski, Yarom; JHEP (2008)]

[Banerjee et al.; JHEP (2011)]

$$J^\mu_A = \xi_V \Omega^\mu$$ ***vorticity***

$$\xi_V \sim C\mu_A^2 + bT^2$$

- **fluid/gravity correspondence**
- **gives constitutive equations**
- **contain *weird* parity-odd term**

[Neiman,Oz; JHEP (2010)]

HOLOGRAPHY

CME far from equilibrium, strong *B*

- **non-expanding plasma**
  [Gosh,Grieninger,Landsteiner,Morales-Tejera; PRD (2021)]
- **expanding plasma**
  [Cartwright,Kaminski,Schenke; PRC (2022)]
  [Grieninger,Morales-Tejera; PRD (2023)]

Frequency dependence of CME

[Amado,Landsteiner,Pena_Benitez; JHEP (2011)]
[Li,Yee; PRD (2018)]
[Koirala; PhD thesis (2020)]

CME near equilibrium (+hydro)

- **weak magnetic field *B***
  [Son,Surowka; PRL (2009)]
  [Kharzeev,Yee; PRD (2011)]
  [Ammon, Kaminski et al.; JHEP (2017)]
- **strong *B*** *[Ammon,Leiber,Macedo; JHEP (2016)]*
  [Ammon, Grieninger, Hernandez, Kaminski, Koirala, Leiber, Wu; JHEP (2021)]

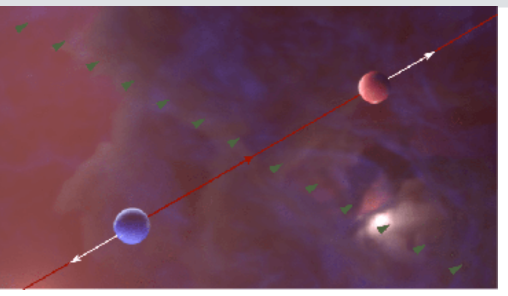
cf. talk by D. Kharzeev

cf. talk by A.F. Dobrin

cf. talk by Huan Huang

cf. talk by S. Grieninger

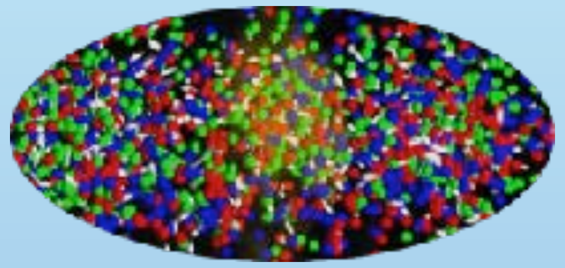
3. Far from equilibrium holography



[DOE Highlight Article; Cartwright,Kaminski, Schenke (2023)]

time

Thermalization in field theory:



T=0 particle “soup”

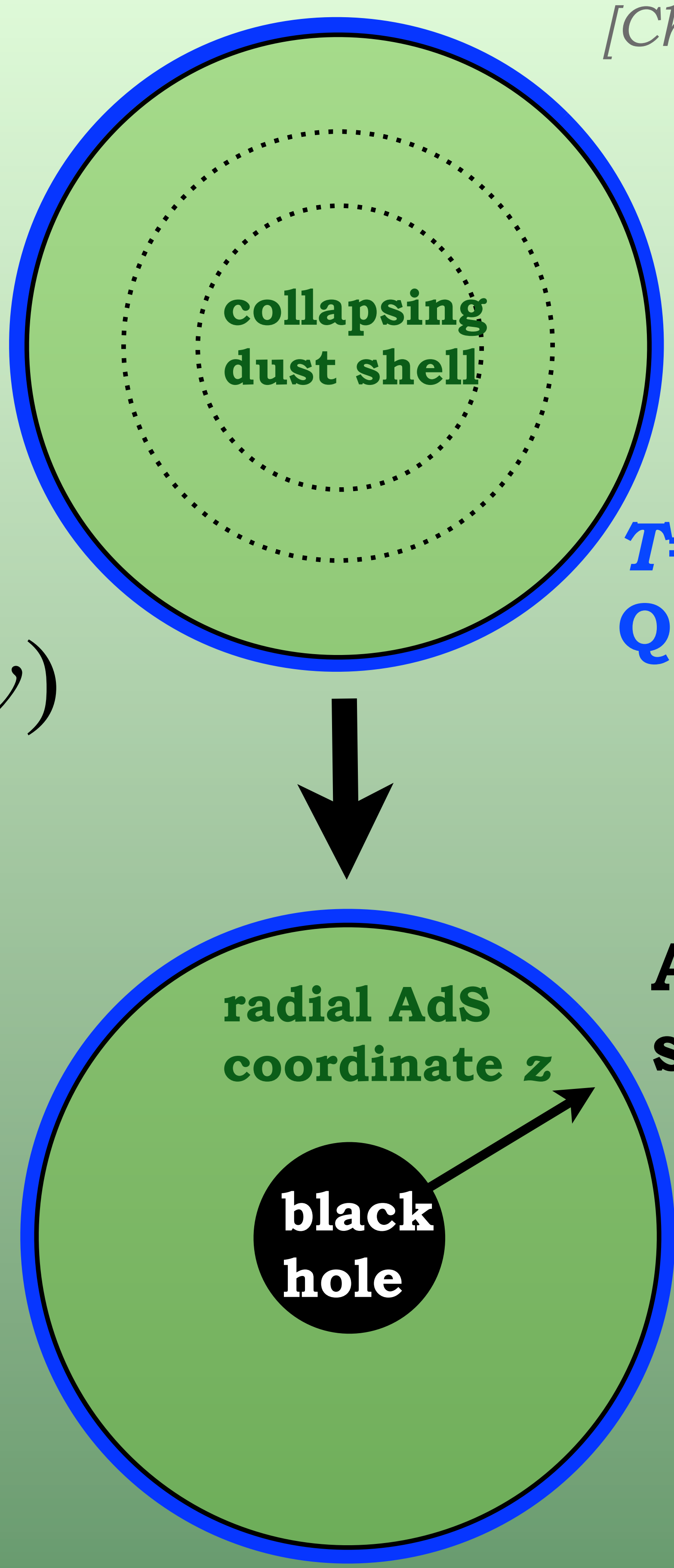
thermal QFT

nonzero T plasma

$\langle T^{\mu\nu}\rangle(v)$ ⟷ **gauge/gravity correspondence** ⟷ $g_{\mu\nu}(v)$

Horizon formation in gravity:

[Janik,Peschanski; PRD (2006)]
[Chesler, Yaffe; PRL (2009)]



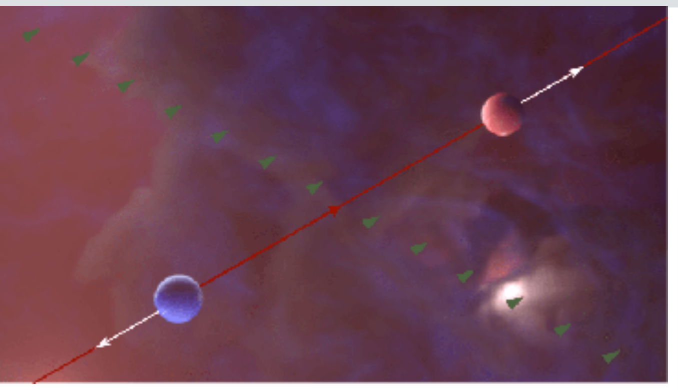
T=0 QFT

Anti-de Sitter space boundary

thermal QFT “lives” here temperature T

➡solve time-dependent Einstein equations

3.1 CME far from equilibrium - Reminder: near equilibrium CME



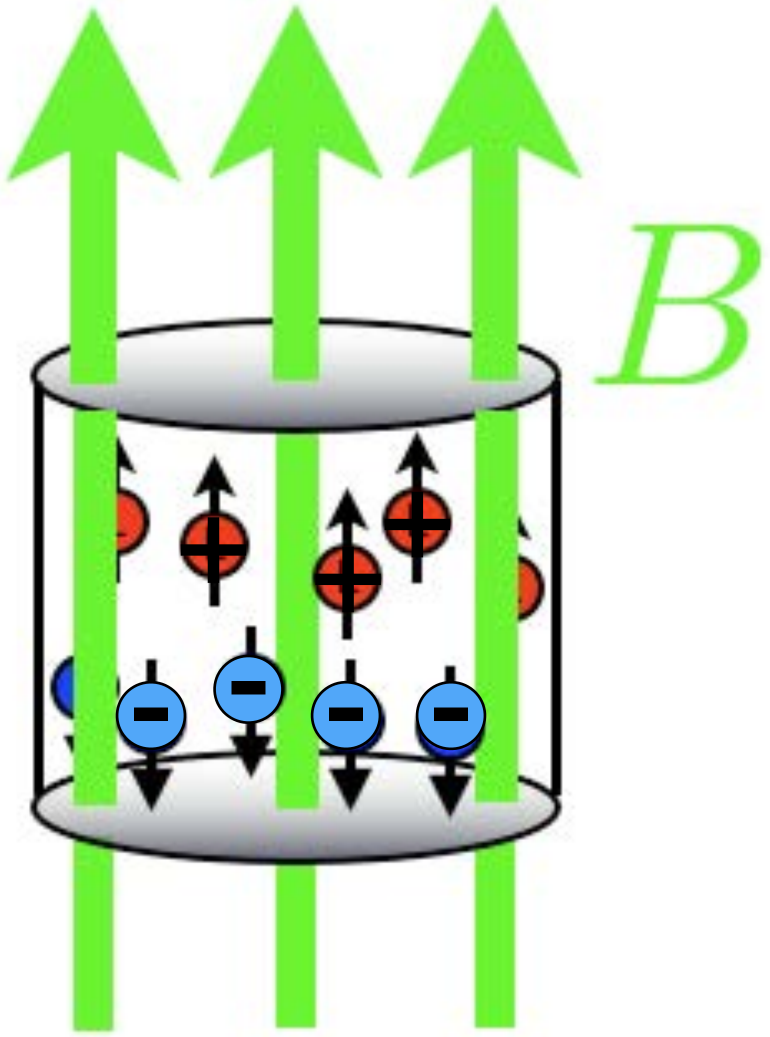
[DOE Highlight Article; Cartwright,Kaminski, Schenke (2023)]

The Chiral Magnetic Effect (CME) caused by chiral anomaly

[Kharzeev; PRC (2004)]
[Fukushima,Kharzeev,Warringa; PRD (2008)]
[Son,Surowka; PRL (2009)]
[Neiman,Oz; JHEP (2010)]

Electric charge current:

$$J^{\mu} = \xi_{\chi} B$$



Chiral magnetic conductivity: $\xi_{\chi} = C\,\mu_A$

Anomalous axial current divergence: $\nabla_{\mu} J^{\mu}_{A} = C\, E \cdot B$

axial charges are generated in aligned E- and B-fields

Needed for observation:

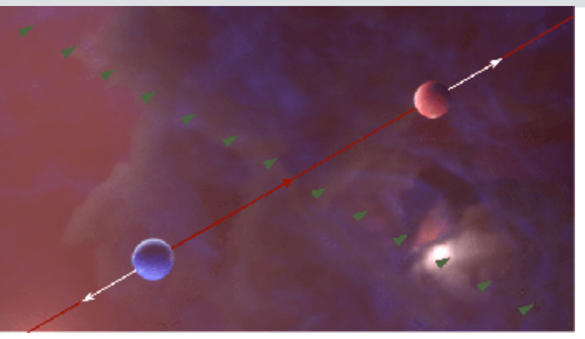
➡chiral anomaly

➡axial charge imbalance

➡magnetic field

➡<u>sufficient life time</u>

3.1 CME far from equilibrium - Bjorken-expanding plasma



[DOE Highlight Article; Cartwright,Kaminski, Schenke (2023)]

[Cartwright,Kaminski,Schenke; PRC (2022)]

Milne coordinates $(\tau, x_1, x_2, \xi; r)$

proper time $\tau = \sqrt{t^2 - x_3^2}$

rapidity $\xi = \frac{1}{2}\ln[(t+x_3)/(t-x_3)]$

Metric Ansatz

AdS radial coordinate r

$$\begin{aligned} ds^2 = &\, 2\mathrm{d}r\mathrm{d}v - A(v,r)\mathrm{d}v^2 + F_1(v,r)\mathrm{d}v\mathrm{d}x_1 \\ &+ S(v,r)^2 e^{H_1(v,r)}\mathrm{d}x_1^2 + S(v,r)^2 e^{H_2(v,r)}\mathrm{d}x_2^2 \\ &+ L^2 S(v,r)^2 e^{-H_1(v,r)-H_2(v,r)}\mathrm{d}\xi^2, \end{aligned}$$

boundary at $r = \infty$ ***has boost invariant Milne metric:***

$$\lim_{r\to\infty} \frac{L^2}{r^2}\mathrm{d}s^2 = -\mathrm{d}\tau^2 + \mathrm{d}x_1^2 + \mathrm{d}x_2^2 + \tau^2\mathrm{d}\xi^2$$

Bjorken flow

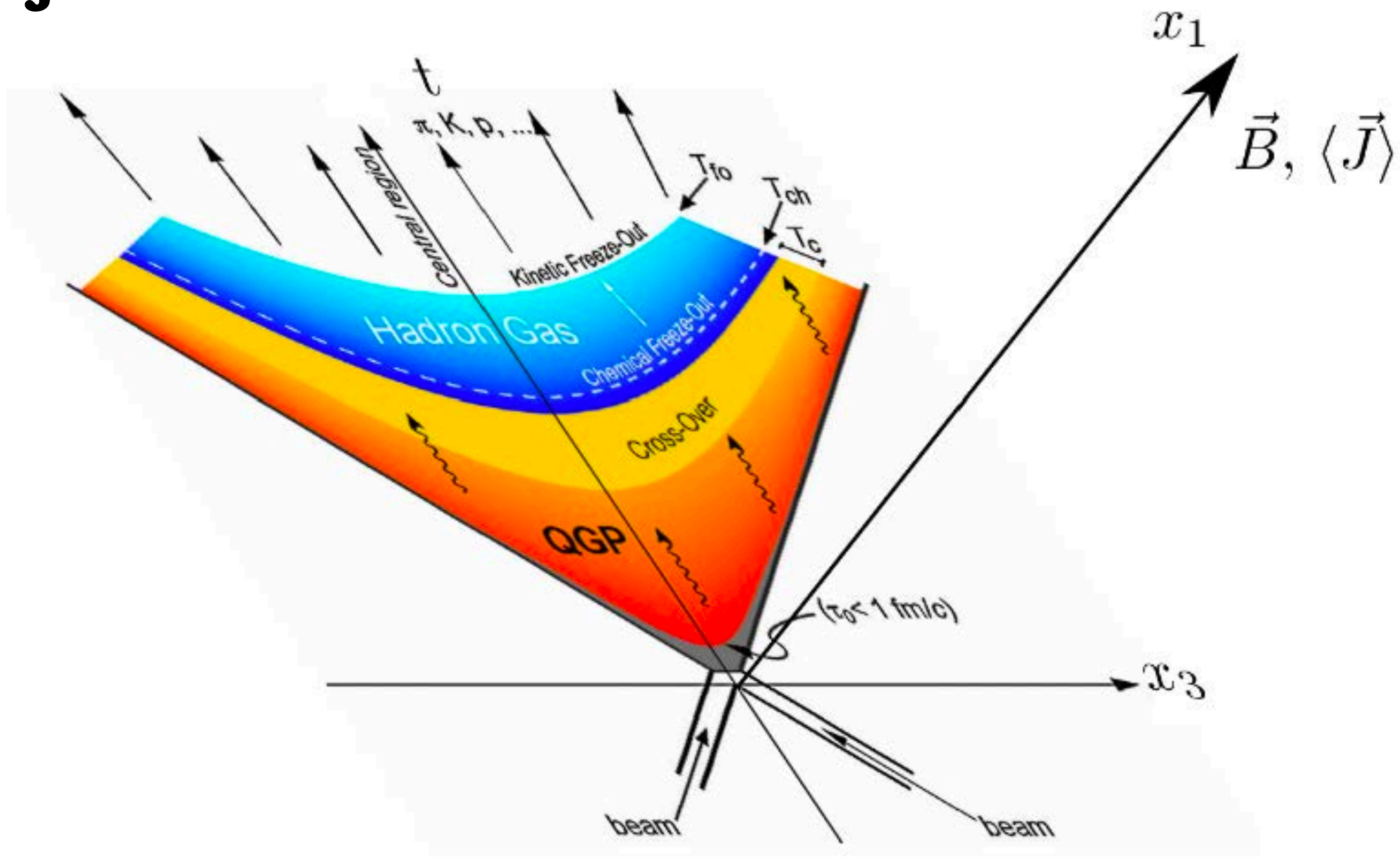
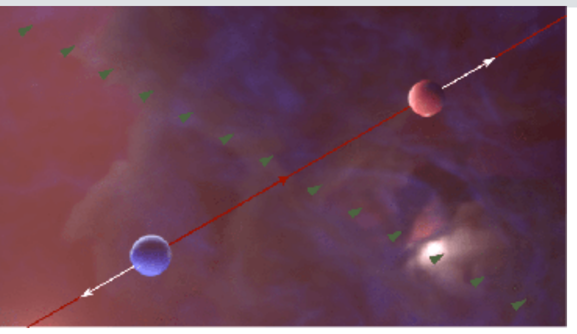


figure generated by Casey Cartwright

3.1 CME far from equilibrium - case I

[Cartwright,Kaminski,Schenke; PRC (2022)]

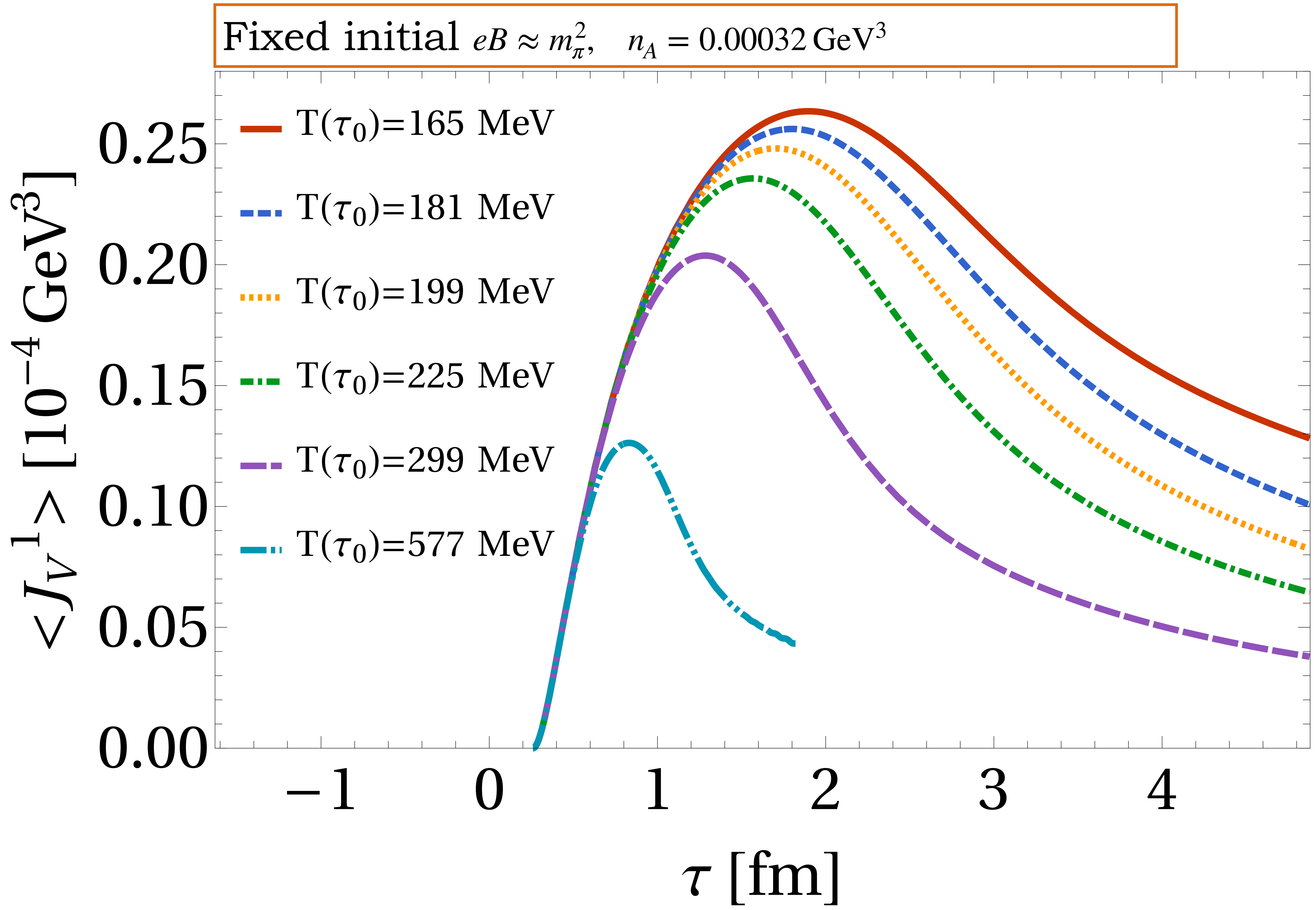


[DOE Highlight Article; Cartwright,Kaminski,Schenke (2023)]

Initial state:
nonzero n_A, B,
pressure anisotropy

time-dependent n_A, B
plasma expanding along beam line

Matching to QCD:
SUSY value for α
L=1fm fixes κ



Fixed initial $eB \approx m_\pi^2, \quad n_A = 0.00032\,\mathrm{GeV}^3$

Near-equilibrium CME

$$J_V^\mu = \xi_\chi B \qquad \xi_\chi = C\,\mu_A$$

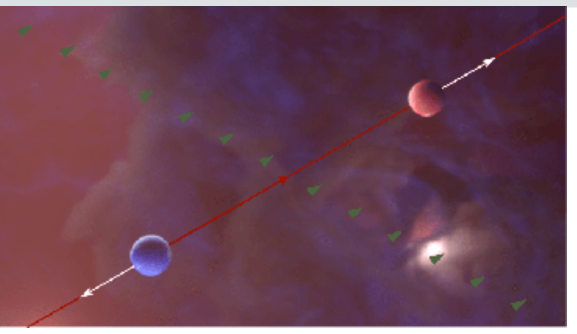
[Kharzeev; PRC (2004)]
[Fukushima,Kharzeev,Warringa; PRD (2008)]
[Son,Surowka; PRL (2009)]

➡CME-current depends strongly on initial conditions (strength of magnetic field vs. axial imbalance vs. collision energy)

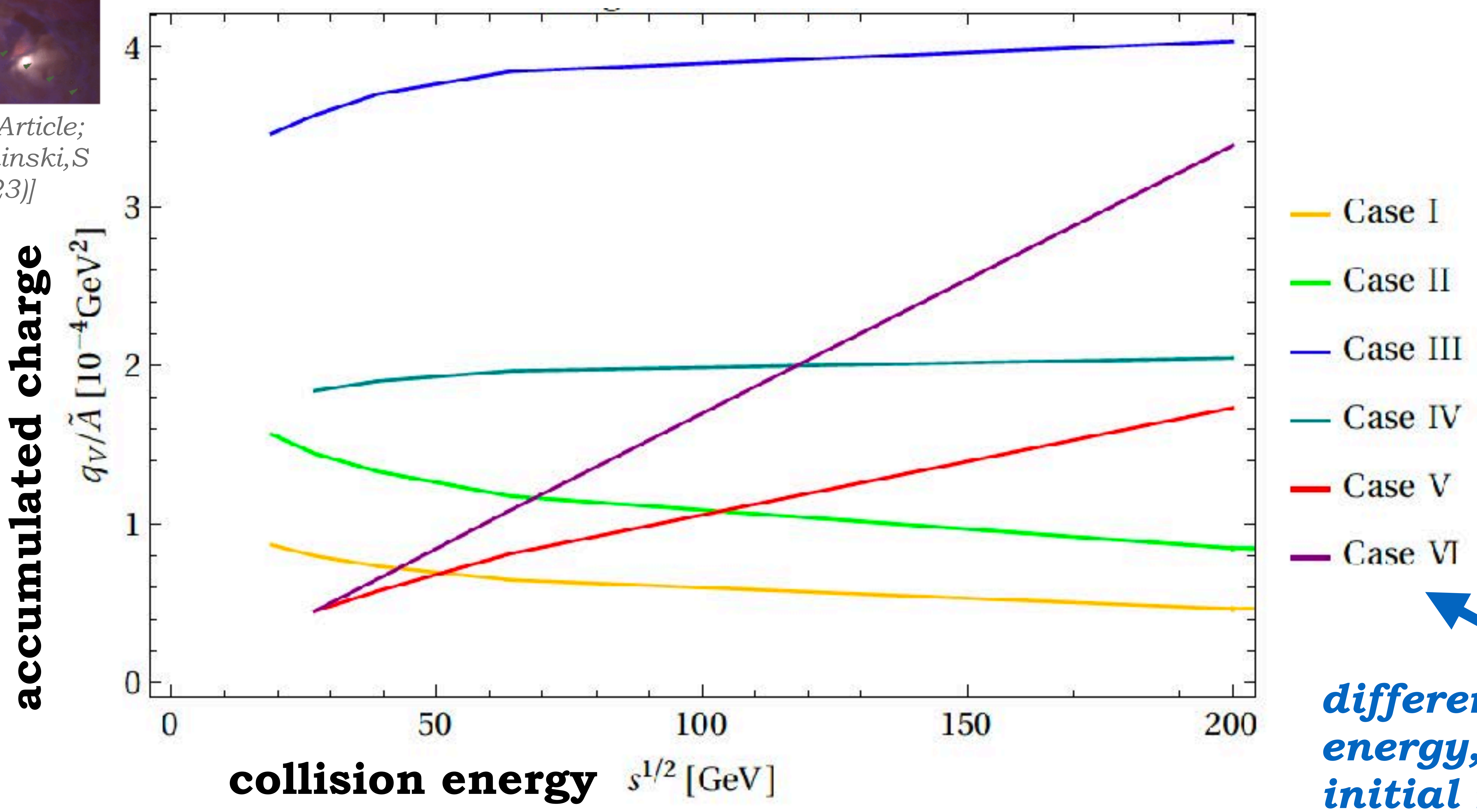
see also the non-expanding holographic model:
[Gosh,Grieninger,Landsteiner,Morales-Tejera; PRD (2021)]

3.1 Chiral Magnetic Effect in Bjorken-expanding plasma

[Cartwright,Kaminski,Schenke; PRC (2022)]



[DOE Highlight Article; Cartwright,Kaminski,Schenke (2023)]



Charge accumulated in detector over time $\Delta\tau = \tau_f - \tau_i$ due to CME

$$q_V/\tilde{A} = \int_{\tau_i}^{\tau_f} \mathrm{d}\tau\tau \langle J^1 \rangle$$

Area: $\tilde{A} = \int \mathrm{d}x_2 \mathrm{d}\xi$

different combinations of initial energy, initial chiral imbalance, initial magnetic field

➡CME-current depends strongly on initial conditions (strength of magnetic field vs. axial imbalance vs. collision energy)

compare: [Gosh,Grieninger,Landsteiner,Morales-Tejera; PRD (2021)]
[Grieninger, Morales-Tejera; PRD (2023)]

Compare to experiments: *top-RHIC energy: [STAR Collaboration; (2021)]*
low-energy update: [STAR Collaboration; (2022)]
high energy update: [ALICE Collaboration; (2022)]

$$\frac{\eta}{s}$$

3.2 Far from equilibrium shear: Results

[Bleicher, Kaminski, Wondrak; Phys.Lett.B (2020)]

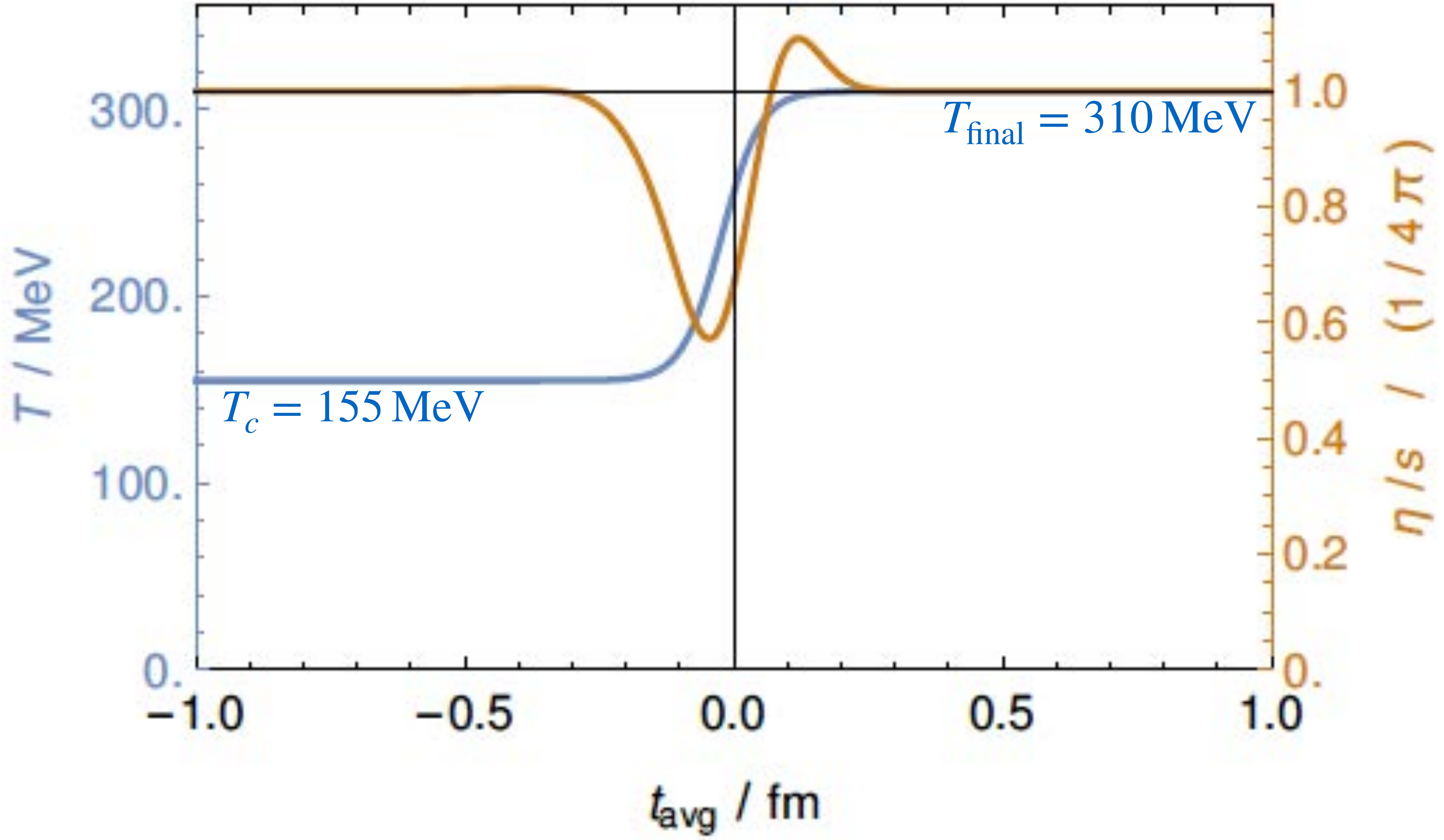
RHIC parameters: $\sqrt{s_{\mathrm{NN}}} = 200\,\mathrm{GeV}$ $\Delta t = 0.3\ \mathrm{fm}$

Temperature

$$T = T_{\mathrm{Hawking}}$$

Entropy density from generating functional

$$s \sim \frac{\partial S^{\mathrm{on-shell}}}{\partial T}$$



KSS equilibrium result

[Kovtun,Son, Starinets; PRL (2005)]

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

No universal bound

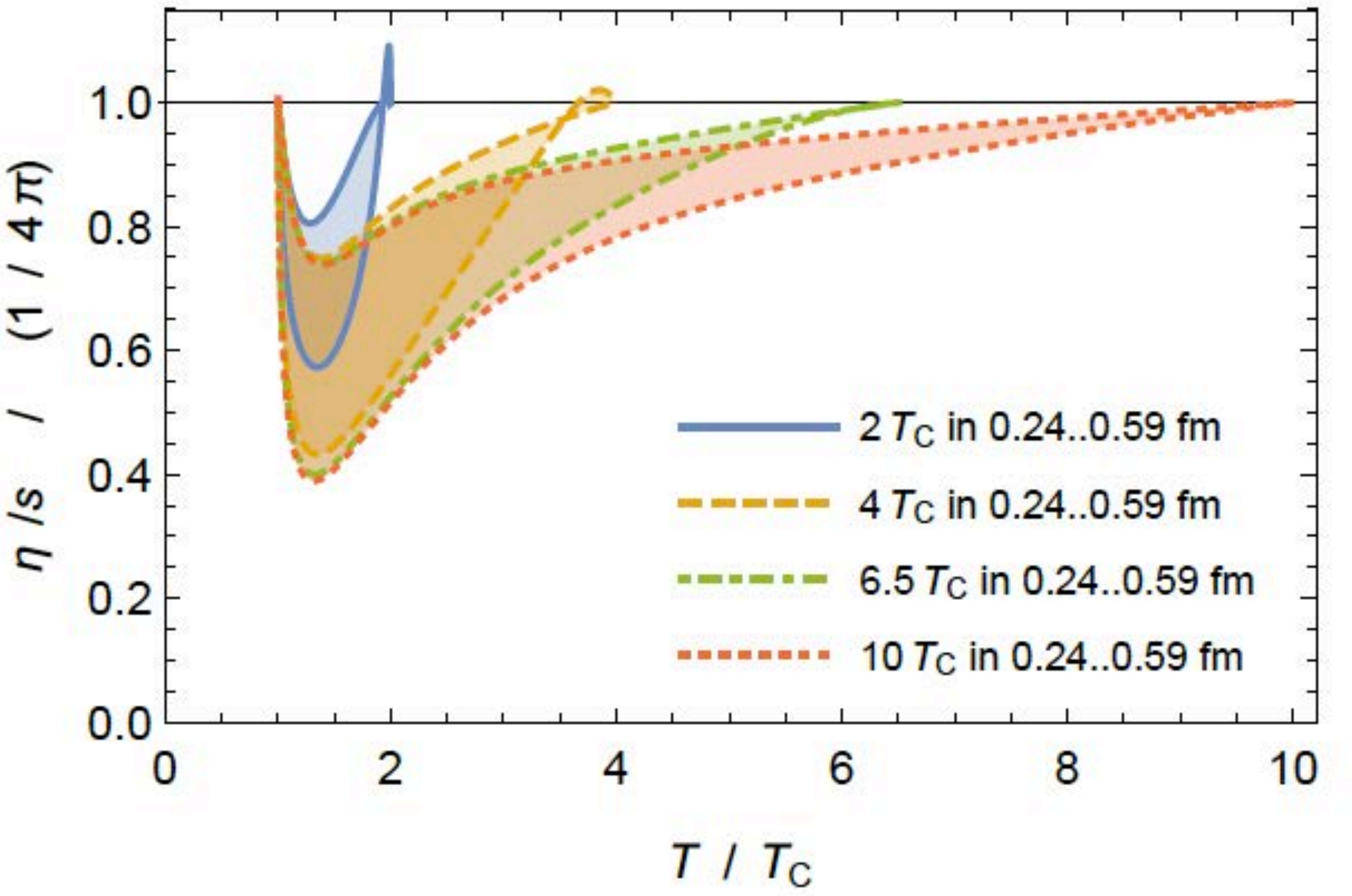
[Buchel, Myers, Sindha; JHEP (2008)]

➡ Shear transport ratio first drops below 60%, then rises above 110% of KSS value 1/(4π)

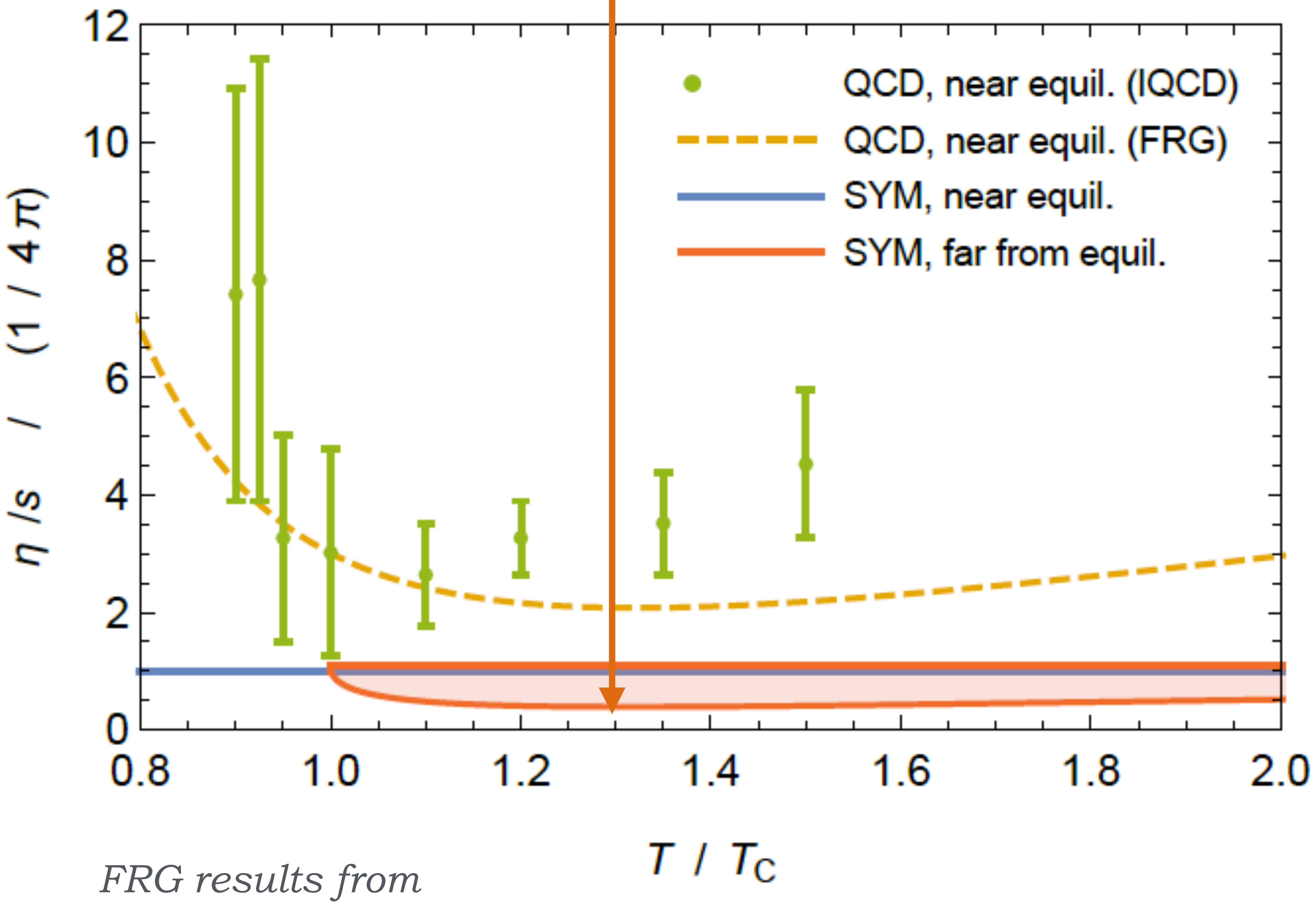
$\frac{\eta}{s}$

3.2 Far from equilibrium shear: Results - continued

[Bleicher, Kaminski, Wondrak; Phys.Lett.B (2020)]



FRG and holography minimum



FRG results from
[Christiansen, Haas, Pawlowski, Strodthoff; PRL (2015)]
Lattice QCD data from
[Astrakhantsev, Braguta, Kotov; JHEP (2017)]

➡ stark contrast: near equilibrium lattice QCD / FRG suggest $\eta/s > 1/(4\pi)$

whereas far from equilibrium Super-Yang-Mills (SYM) plasma suggests $\eta/s < 1/(4\pi)$

➡ currently underestimating flow generated at early times *[Bernhard, Moreland, Bass, Nature (2019)]*

Comments: Spin hydrodynamics & hydrodynamics in rotating fluids

Spin hydrodynamics

[Hongo,Huang,Kaminski,Stephanov,Yee; JHEP (2021)]

- ✦ **spin connection sources spin current, like metric sources energy-momentum tensor**

$$\Sigma^{\mu}_{\hat{a}\hat{b}} = -\frac{2}{e}\frac{\delta W}{\delta \omega_{\mu}^{\hat{a}\hat{b}}}\Bigg|_{\text{torsion}=0}$$

- ✦ **spin connection independent from metric (vielbein e) iff torsion nonzero, dictated by differential geometry:**

$$\boxed{De = \partial e + e * \omega = T\,(\text{torsion})}$$

- ✦ **special case: if zero torsion, $T = 0$, spin current and energy-momentum tensor display <u>pseudogauge dependence</u> because their sources are equivalent**

$$\Rightarrow \omega \sim -(e)^{-1}\partial e$$

cf. talk by M. Buzzegoli

in agreement with [Gallegos,Gursoy,Yarom; SciPost (2021)]
[Gallegos,Gursoy,Yarom; JHEP (2023)]

cf. talk by A. Palermo
cf. talk by Shu Lin
cf. talk by M. Shokri
cf. talk by A. Daher

Hydrodynamics in rotating fluids

- ✦ **analytically known rotating vortical fluid**
  [Bantilan, Ishii, Romatschke; PLB (2018)]
- ✦ **stems from <u>*rotating equilibrium state*</u> on 3-sphere after stereographic projection**
- ✦ **hydrodynamic modes of this rotating fluid <u>*appear as if in boosted fluid*</u>**
  [Garbiso-Amano, Kaminski; JHEP (2019)]
- ✦ **all these results plus applicability of hydrodynamics discussed in <u>review</u>:**

[Cartwright,Garbiso-Amano, Kaminski,Wu; PPNP review (accepted version on arXiv)]

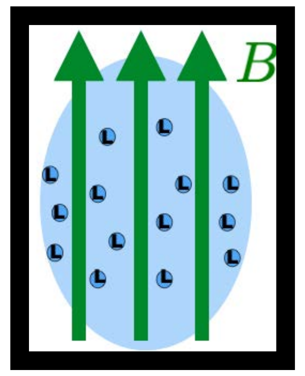
yesterday v2 on arXiv
40 →70 pages!

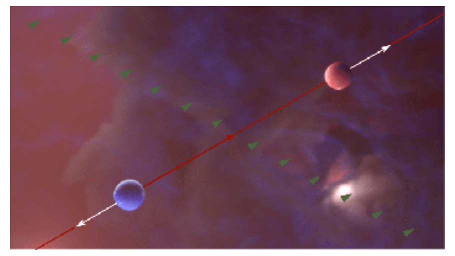
cf. talk by V. Braguta
cf. talk by K. Fukushima

Discussion

Summary

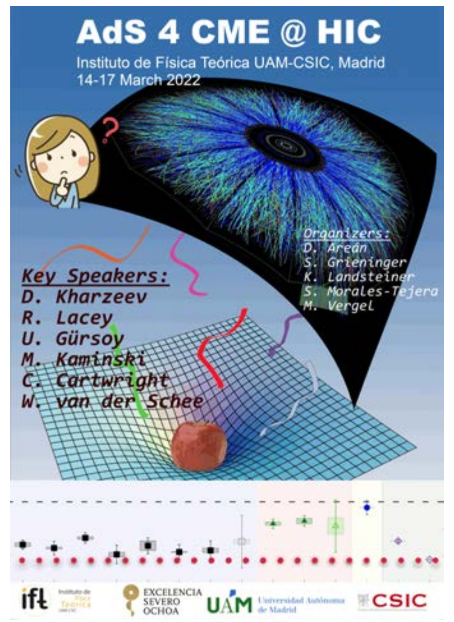
- **chiral hydrodynamics for charged chiral fluids in strong magnetic field**
- derived **Kubo formulae** for 27 transport coefficients (8 novel)
- confirmed Kubo formulae by computation in holographic model
- **Chiral Magnetic Effect** depends on initial values (axial imbalance, B, T, ϵ)
- **Far from equilibrium specific shear viscosity** drops below $1/(4\pi)$





Outlook

- calculate novel transport coefficients **on the lattice** and in **perturbative QCD**
- explore effect on **elliptic flow and hydrodynamic modeling in general**
- construct **chiral magneto-hydrodynamics**
- include **dynamical magnetic field and dynamically created axial imbalance** to model QGP and CME far from equilibrium

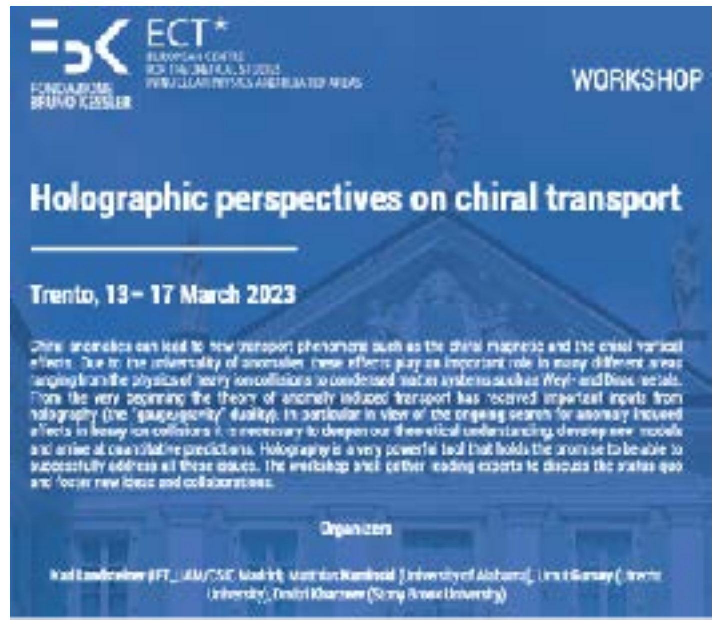


AdS 4 CME @ HIC

Instituto de Física Teórica UAM-CSIC, Madrid
14-17 March 2022

Key Speakers:
D. Kharzeev
R. Lacey
U. Gürsoy
M. Kaminski
C. Cartwright
W. van der Schee

Organizers:
D. Areán
S. Grieninger
K. Landsteiner
S. Morales-Tejera
M. Vergel



ECT*

WORKSHOP

Holographic perspectives on chiral transport

Trento, 13 – 17 March 2023

Organizers

[AdS4CME Collaboration]

Two review articles on <u>rotation</u> and <u>strong B</u> in *Progress in Particle and Nuclear Physics*:

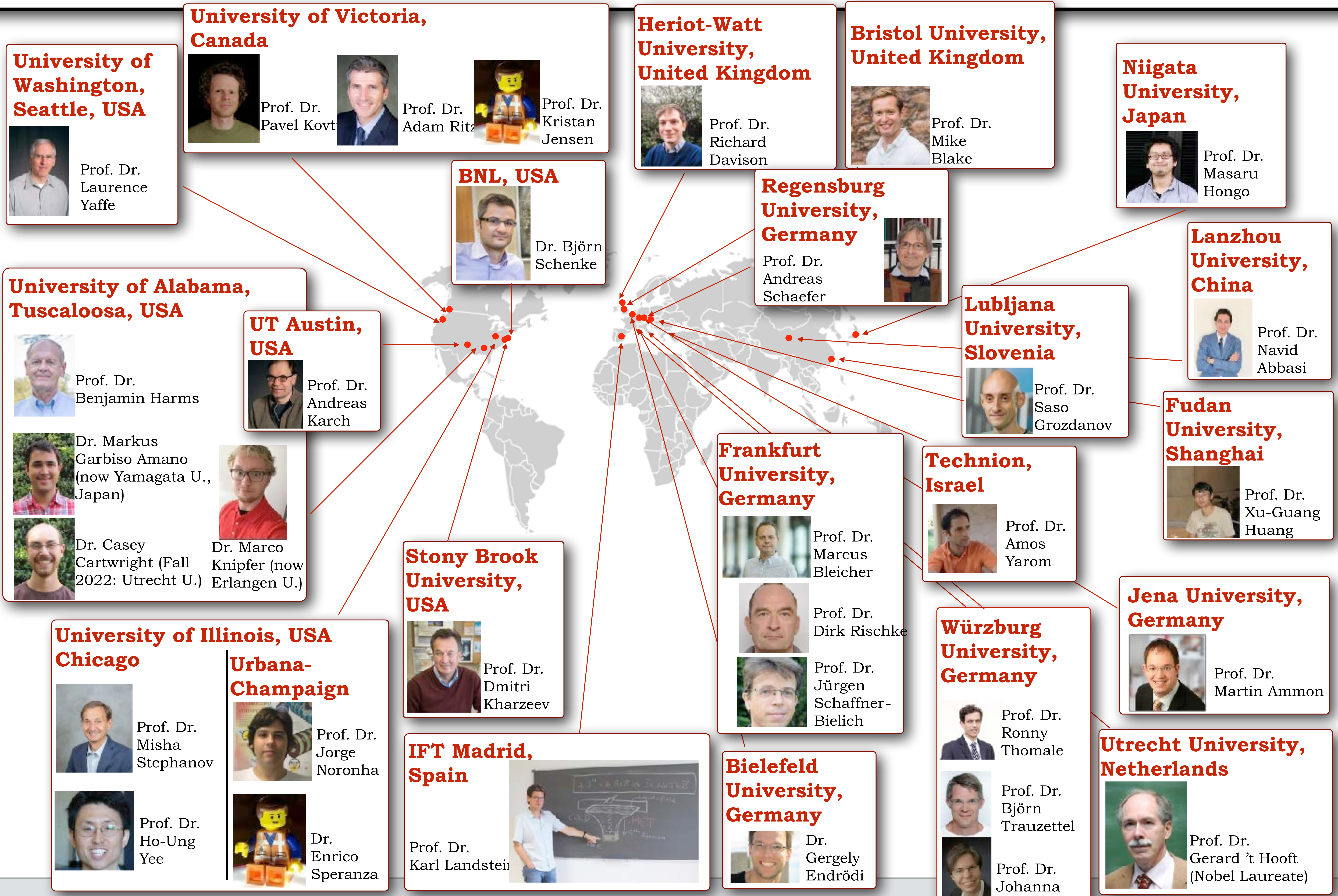
yesterday v2 on arXiv
40 → 70 pages!

[Cartwright, Garbiso-Amano, Kaminski, Wu; PPNP review (accepted version on arXiv)] on ***rotation***

[Ammon, Cartwright, Grieninger, Hernandez, Kaminski; PPNP review (to appear)] on ***strong B***

Our upcoming ECT* Workshop:
March 24-28, 2025 (with Gursoy/Kharzeev/Landsteiner)
Holographic perspectives on chiral transport and spin dynamics

Thanks to my collaborators (since 2012) and Thank You!



University of Washington, Seattle, USA

Prof. Dr. Laurence Yaffe

University of Victoria, Canada

Prof. Dr. Pavel Kovt

Prof. Dr. Adam Ritz

Prof. Dr. Kristan Jensen

BNL, USA

Dr. Björn Schenke

Heriot-Watt University, United Kingdom

Prof. Dr. Richard Davison

Bristol University, United Kingdom

Prof. Dr. Mike Blake

Niigata University, Japan

Prof. Dr. Masaru Hongo

Regensburg University, Germany

Prof. Dr. Andreas Schaefer

Lanzhou University, China

Prof. Dr. Navid Abbasi

University of Alabama, Tuscaloosa, USA

Prof. Dr. Benjamin Harms

Dr. Markus Garbiso Amano (now Yamagata U., Japan)

Dr. Casey Cartwright (Fall 2022: Utrecht U.)

Dr. Marco Knipfer (now Erlangen U.)

UT Austin, USA

Prof. Dr. Andreas Karch

Lubljana University, Slovenia

Prof. Dr. Saso Grozdanov

Fudan University, Shanghai

Prof. Dr. Xu-Guang Huang

Frankfurt University, Germany

Prof. Dr. Marcus Bleicher

Prof. Dr. Dirk Rischke

Prof. Dr. Jürgen Schaffner-Bielich

Technion, Israel

Prof. Dr. Amos Yarom

Stony Brook University, USA

Prof. Dr. Dmitri Kharzeev

Jena University, Germany

Prof. Dr. Martin Ammon

University of Illinois, USA

Chicago

Prof. Dr. Misha Stephanov

Prof. Dr. Ho-Ung Yee

Urbana-Champaign

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Bielefeld University, Germany

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Prof. Dr. Ronny Thomale

Prof. Dr. Björn Trauzettel

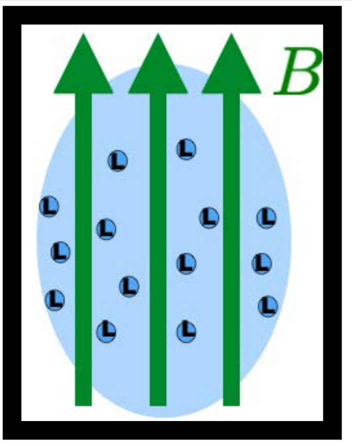
Prof. Dr. Johanna Erdmenger

Utrecht University, Netherlands

Prof. Dr. Gerard 't Hooft (Nobel Laureate)

APPENDIX

APPENDIX: Kubo-formula derivation example: hydrodynamic correlators in 2+1



Simple (non-chiral) example in 2+1 dims:

$$j^\mu = n u^\mu + \sigma \left[E^\mu - T \Delta^{\mu\nu} \partial_\nu \left(\frac{\mu}{T} \right) \right]$$

$$\Delta^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu$$

$$u^\mu = (1,0,0)$$

sources $A_t,\ A_x \propto e^{-i\omega t + ikx}$

fluctuations $n = n(t,x,y) \propto e^{-i\omega t + ikx}$ (fix T and u)

one point functions (use $\nabla_\mu j^\mu = 0$)

$$\langle j^t \rangle = n(\omega, k) = \frac{ik\sigma}{\omega + ik^2 \frac{\sigma}{\chi}} (\omega A_x + k A_t)$$

$$\langle j^x \rangle = \frac{i\omega\sigma}{\omega + ik^2 \frac{\sigma}{\chi}} (\omega A_x + k A_t)$$

$$\langle j^y \rangle = 0$$

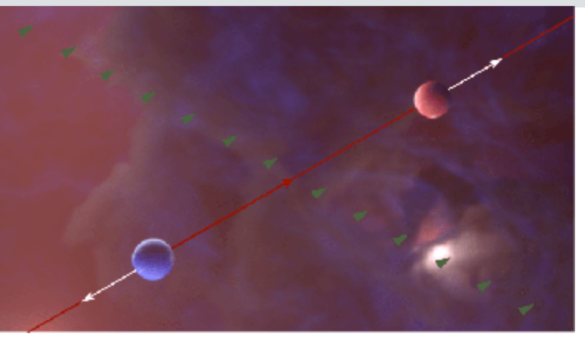
susceptibility: $\chi = \frac{\partial n}{\partial \mu}$

Einstein relation: $D = \frac{\sigma}{\chi}$

$\Longrightarrow$ two point functions $\langle j^x j^x \rangle = \frac{\delta \langle j^x \rangle}{\delta A_x} = \frac{i\omega^2 \sigma}{\omega + iDk^2}$

$\Longrightarrow$ Kubo formula: $\sigma = \lim_{\omega \to 0} \frac{1}{i\omega} \langle j^x j^x \rangle (\omega, k=0)$

APPENDIX: Chiral effects in vector and axial currents



see e.g. [Jensen, Kovtun, Ritz; JHEP (2013)]

Vector current (e.g. QCD electromagnetic *U(1)*)

$$J^\mu_V = \cdots + \xi_V \omega^\mu + \xi_\chi \; B^\mu + \xi_{VA} B^\mu_A$$

chiral
magnetic
effect

Axial current (e.g. QCD axial *U(1)*)

$$J^\mu_A = \cdots + \xi \omega^\mu + \xi_B B^\mu + \xi_{AA} B^\mu_A$$

chiral
vortical
effect

chiral
separation
effect

➡**phenomenology needs *both* currents**

➡**Holographic model with two currents**

APPENDIX: Holographic model with two currents

Einstein-Maxwell-Chern-Simons action with two gauge fields A_μ and V_μ

$$S = \frac{1}{2\kappa^2}\int d^5x\sqrt{-g}\left(R - 2\Lambda - \frac{L^2}{4}F_{\mu\nu}F^{\mu\nu} - \frac{L^2}{4}F^{(5)}_{\mu\nu}F^{\mu\nu}_{(5)} + \frac{\alpha}{3}\epsilon^{\mu\nu\rho\sigma\tau}A_\mu\left(3F_{\nu\rho}F_{\sigma\tau} + F^{(5)}_{\nu\rho}F^{(5)}_{\sigma\tau}\right)\right)$$

[Gosh,Grieninger,Landsteiner,Morales-Tejera; PRD (2021)]

counter term action

$$S_{ct} = \frac{1}{8\pi G}\int \mathrm{d}^d x\sqrt{\gamma}\left(K - \frac{1}{2L}\left(2(1-d)\right.\right.$$
$$\left.\left. - \frac{L^2}{d-2}R(\gamma)\right)\right) + \frac{L^3}{64\pi G}\log(\epsilon)\int \mathrm{d}^d x\sqrt{\gamma_0}F_0^2$$

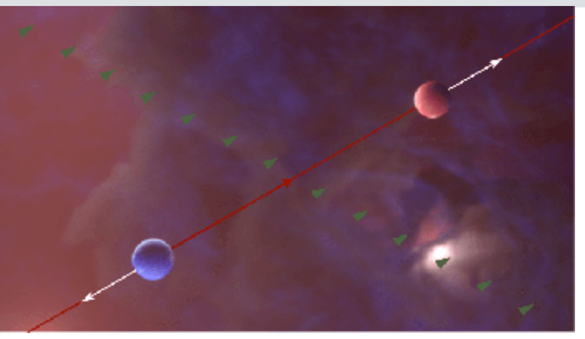
equations of motion

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}(R - 2\Lambda) = \frac{L^2}{2}T_{\mu\nu} + \frac{L^2}{2}T^{(5)}_{\mu\nu}\,,$$
$$\nabla_\mu F^{\mu\nu} = -\frac{2\alpha}{L^2}\epsilon^{\nu\beta\lambda\rho\sigma}F_{\beta\lambda}F^{(5)}_{\rho\sigma}\,,$$
$$\nabla_\mu F^{\mu\nu}_{(5)} = -\frac{\alpha}{L^2}\epsilon^{\nu\beta\lambda\rho\sigma}\left(F_{\beta\lambda}F_{\rho\sigma} + F^{(5)}_{\beta\lambda}F^{(5)}_{\rho\sigma}\right)\,,$$

gravitational Maxwell-CS stress tensors

$$T_{\mu\nu} = F_{\mu\lambda}F_\nu{}^\lambda - \frac{1}{4}g_{\mu\nu}F^2,\ T^{(5)}_{\mu\nu} = F^{(5)}_{\mu\lambda}F^{(5)}{}_\nu{}^\lambda - \frac{1}{4}g_{\mu\nu}F^2_{(5)}$$

APPENDIX: Holographic model with two currents



Einstein-Maxwell-Chern-Simons action
with two gauge fields A_μ and V_μ

[Gosh,Grieninger,Landsteiner,Morales-Tejera; PRD (2021)]

[Grieninger, Morales-Tejera; PRD (2023)]

$$S = \frac{1}{2\kappa^2}\int d^5x\sqrt{-g}\left(R - 2\Lambda - \frac{L^2}{4}F_{\mu\nu}F^{\mu\nu} - \frac{L^2}{4}F^{(5)}_{\mu\nu}F^{\mu\nu}_{(5)} + \frac{\alpha}{3}\epsilon^{\mu\nu\rho\sigma\tau}A_\mu\left(3F_{\nu\rho}F_{\sigma\tau} + F^{(5)}_{\nu\rho}F^{(5)}_{\sigma\tau}\right)\right)$$

gravitational coupling κ

Einstein-Hilbert

Maxwell

“axial Maxwell”

Chern-Simons coupling α

Chern-Simons term encoding chiral anomaly

4D conserved vector current

5D vector gauge field $V_\mu \longleftrightarrow J^\mu_V = \cdots + \xi_V\omega^\mu + \xi_{VV}B^\mu + \xi_{VA}B^\mu_A$

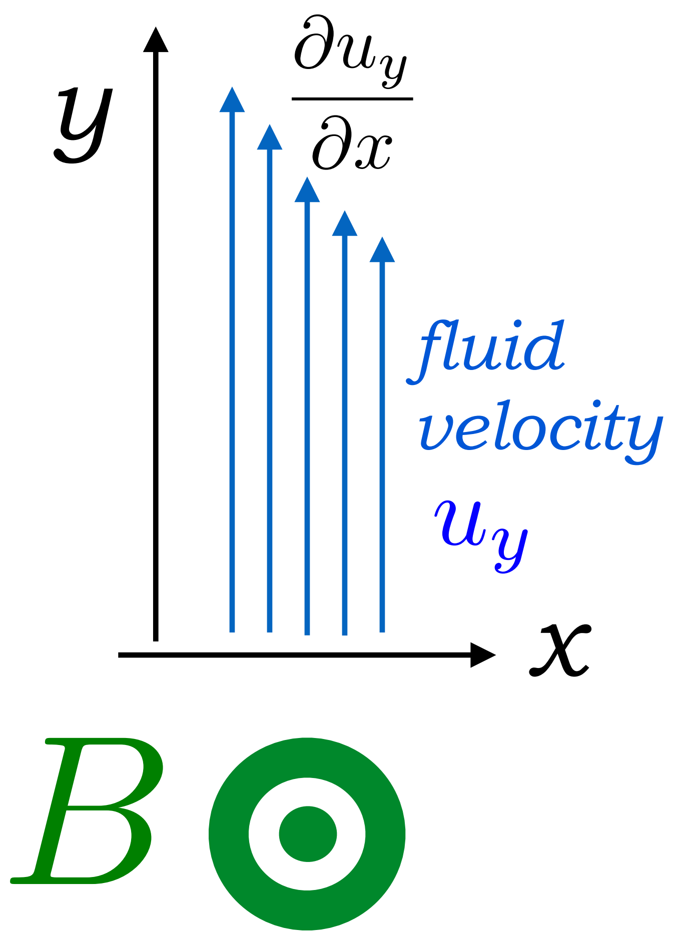
4D anomalous axial current

5D axial gauge field $A_\mu \longleftrightarrow J^\mu_A = \cdots + \xi\omega^\mu + \xi_B B^\mu + \xi_{AA}B^\mu_A$

APPENDIX: Kubo formulae: two shear viscosities

Shear viscosity perpendicular

$$\frac{1}{\omega}\mathrm{Im}\, G_{T^{xy}T^{xy}}(\omega, \mathbf{k}{=}0) = \eta_{\perp}$$

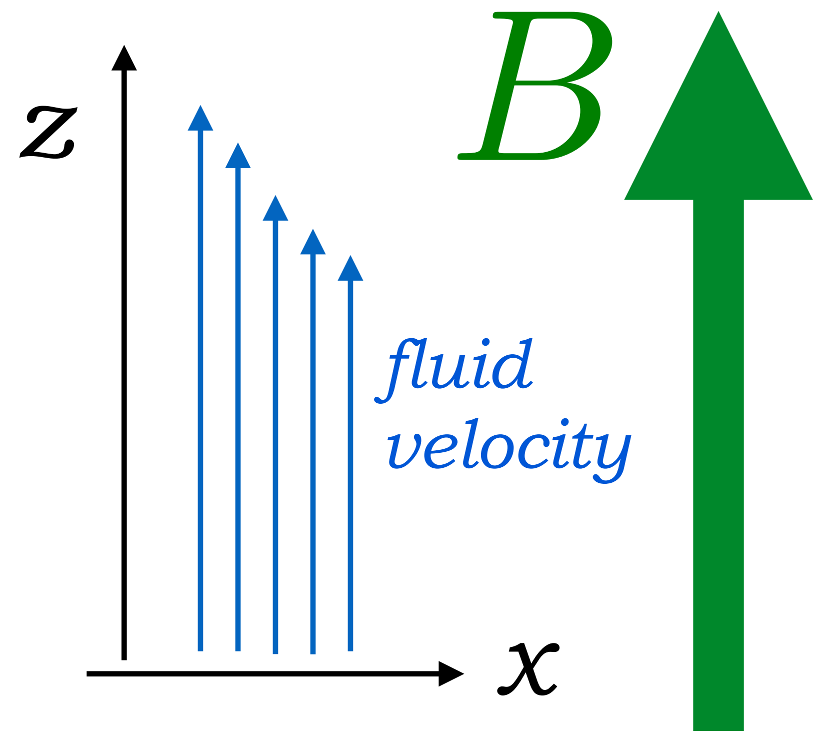


Shear viscosity parallel

$$\frac{1}{\omega}\mathrm{Im}\, G_{T^{xz}T^{xz}}(\omega, \mathrm{k}{=}0) = \eta_{||} + (\bar{c}_8 c_{15} - c_{10}\bar{c}_{17})\rho_{\perp} - (\bar{c}_8\bar{c}_{17} + c_{10}c_{15})\tilde{\rho}_{\perp}$$

perpendicular resistivity *Hall resistivity*

➡ Value of shear viscosity depends on direction of magnetic field
➡ Can lead to creation of flow at early times



$\frac{\eta}{s}$

APPENDIX: Far from equilibrium shear: Perturbations

[Son, Starinets; JHEP (2002)]
[Iqbal, Liu; Fortschr.Phys. (2008)]
[van Rees, Skenderis; PRL (2008)]

Perturb the background metric $g_{\mu\nu} \to g_{\mu\nu} + h_{\mu\nu}$

$$ds^2 = \frac{1}{z^2}\left(-f(v,z)\,\mathrm{d}v^2 - 2\,\mathrm{d}v\,\mathrm{d}z + \mathrm{d}x^2 + \mathrm{d}y^2\right) = g_{\mu\nu}\,dx^\mu dx^\nu \Rightarrow$$ **linearized Einstein equations for** $h_{\mu\nu}$

Near-boundary expansion

$$\underbrace{h_{\mu\nu}}_{\text{metric perturbation}} \sim \underbrace{h^{(0)}_{\mu\nu}}_{\text{source}} + \underbrace{\langle T_{\mu\nu}\rangle}_{\text{one-point function}} z^4 + \ldots$$

Linear response: retarded correlator from metric fluctuation (only shear perturbation h_{xy})

[Bleicher, Kaminski, Wondrak; Phys.Lett.B (2020)]
[Ishii; arXiv:1605.08387]

$$\langle T^{xy}(t_2)\rangle_h = \int \mathrm{d}\tau\, G_{\mathrm{R}}^{xy,xy}(\tau,\,t_2)\, \boxed{\underbrace{h^{(0)}_{xy}(\tau)}_{\propto\,\delta(\tau - t_{\mathrm{p}})}} \propto G_{\mathrm{R}}^{xy,xy}(t_{\mathrm{p}},\,t_2)$$

shear source localized at a time t_{p}

Wigner transform $G_{\mathrm{R}}^{xy,xy}(t_{\mathrm{p}}, t_2) \to G_{\mathrm{R}}^{xy,xy}(t_{\mathrm{avg}}, t_{\mathrm{rel}}) \sim \tilde{G}_{\mathrm{R}}^{xy,xy}(t_{\mathrm{avg}}, \omega)\, e^{-i\omega t_{\mathrm{rel}}}$

$$t_{\mathrm{avg}} = (t_{\mathrm{p}} + t_2)/2$$
$$t_{\mathrm{rel}} = t_{\mathrm{p}} - t_2$$

Equilibrium result

[Kubo formula]

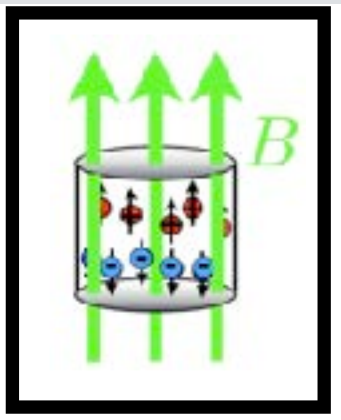
$$\eta = -\lim_{\omega\to 0}\frac{1}{\omega}\Im\,\tilde{G}_R^{xy,xy}(\omega)$$

Generalized Kubo formula for "shear viscosity" far from equilibrium

[Bleicher, Kaminski, Wondrak; Phys.Lett.B (2020)]

$$\eta\,(t_{\mathrm{avg}}) = -\lim_{\omega\to 0}\frac{1}{\omega}\Im\,\tilde{G}_{\mathrm{R}}^{xy,xy}(t_{\mathrm{avg}},\omega)$$

APPENDIX: Bjorken - expanding plasma



[Cartwright,Kaminski,Schenke; PRC (2022)]

Bjorken flow equation

$$\partial_\tau \epsilon + \frac{4}{3}\frac{\epsilon}{\tau} - \frac{4}{3}\frac{\eta}{\tau^2} = 0$$

Holographic Bjorken flow equation

$$-\frac{P_1(\tau)}{\tau} - \frac{P_2(\tau)}{\tau} - \frac{B_1(\tau)^2}{8\tau} + \partial_\tau \epsilon(\tau) + \frac{2\epsilon(\tau)}{\tau} = 0$$

Energy and pressures

$$\epsilon = \langle T_{00}\rangle = \frac{2L^3}{\kappa^2}\left(-\frac{3a_4(\tau)}{4L^4} - \frac{b^2\log(b^{1/2})}{8L^2\tau^2}\right) ,$$

$$P_1 = \langle T_{11}\rangle = \frac{2L^3}{\kappa^2}\left(-\frac{a_4(\tau)}{4L^4} + \frac{h_4^{(1)}(\tau)}{L^4} + \frac{b^2\log(b^{1/2})}{8L^2\tau^2} - \frac{1}{6\tau^4}\right) ,$$

$$P_2 = \langle T_{22}\rangle = \frac{2L^3}{\kappa^2}\left(-\frac{a_4(\tau)}{4L^4} + \frac{h_4^{(2)}(\tau)}{L^4} - \frac{b^2\log(b^{1/2})}{8L^2\tau^2} - \frac{b^2}{16L^2\tau^2} - \frac{1}{6\tau^4}\right) ,$$

$$\tau^2 P_\xi = \langle T_{\xi\xi}\rangle = \frac{2L^3\tau^2}{\kappa^2}\left(-\frac{a_4(\tau)}{4L^4} - \frac{h_4^{(1)}(\tau)}{L^4} - \frac{h_4^{(2)}(\tau)}{L^4} - \frac{b^2\log(b^{1/2})}{8L^2\tau^2} - \frac{b^2}{16L^2\tau^2} + \frac{1}{3\tau^4}\right)$$

$$\langle J^a_{(5)}\rangle = \frac{1}{2\kappa^2}\left(\boxed{\frac{q_5 L}{\tau}}, 0, 0, 0\right) ,$$

$$\langle J^a\rangle = \frac{1}{2\kappa^2}\left(0, 2V_2(\tau), 0, 0\right) ;$$

➡CME current

➡time-dependent axial charge and *B*

$$B^a = \frac{1}{2}\epsilon^{abcd}u_b F_{cd} \quad \Rightarrow \quad \boxed{B^1 = \frac{b}{L\tau}}$$

Recall the metric:

$$\begin{aligned} ds^2 &= 2\mathrm{d}r\mathrm{d}v - A(v,r)\mathrm{d}v^2 + F_1(v,r)\mathrm{d}v\mathrm{d}x_1 \\ &+ S(v,r)^2 e^{H_1(v,r)}\mathrm{d}x_1^2 + S(v,r)^2 e^{H_2(v,r)}\mathrm{d}x_2^2 \\ &+ L^2 S(v,r)^2 e^{-H_1(v,r)-H_2(v,r)}\mathrm{d}\xi^2 , \end{aligned}$$

APPENDIX: Holographic Bjorken - expanding plasma

[Cartwright,Kaminski,Knipfer; (2022)]

Metric Ansatz :

$$\mathrm{d}s^2 = 2\mathrm{d}r\mathrm{d}v - A(v,r)\mathrm{d}v^2 + e^{B(v,r)}S(v,r)^2(\mathrm{d}x_1^2 + \mathrm{d}x_2^2) + S(v,r)^2 e^{-2B(v,r)}\mathrm{d}\xi^2$$

$$\lim_{r\to\infty} \frac{1}{r^2}\mathrm{d}s^2 = -\mathrm{d}\tau^2 + \mathrm{d}x_1^2 + \mathrm{d}x_2^2 + \tau^2\mathrm{d}\xi^2$$

Anisotropy function :

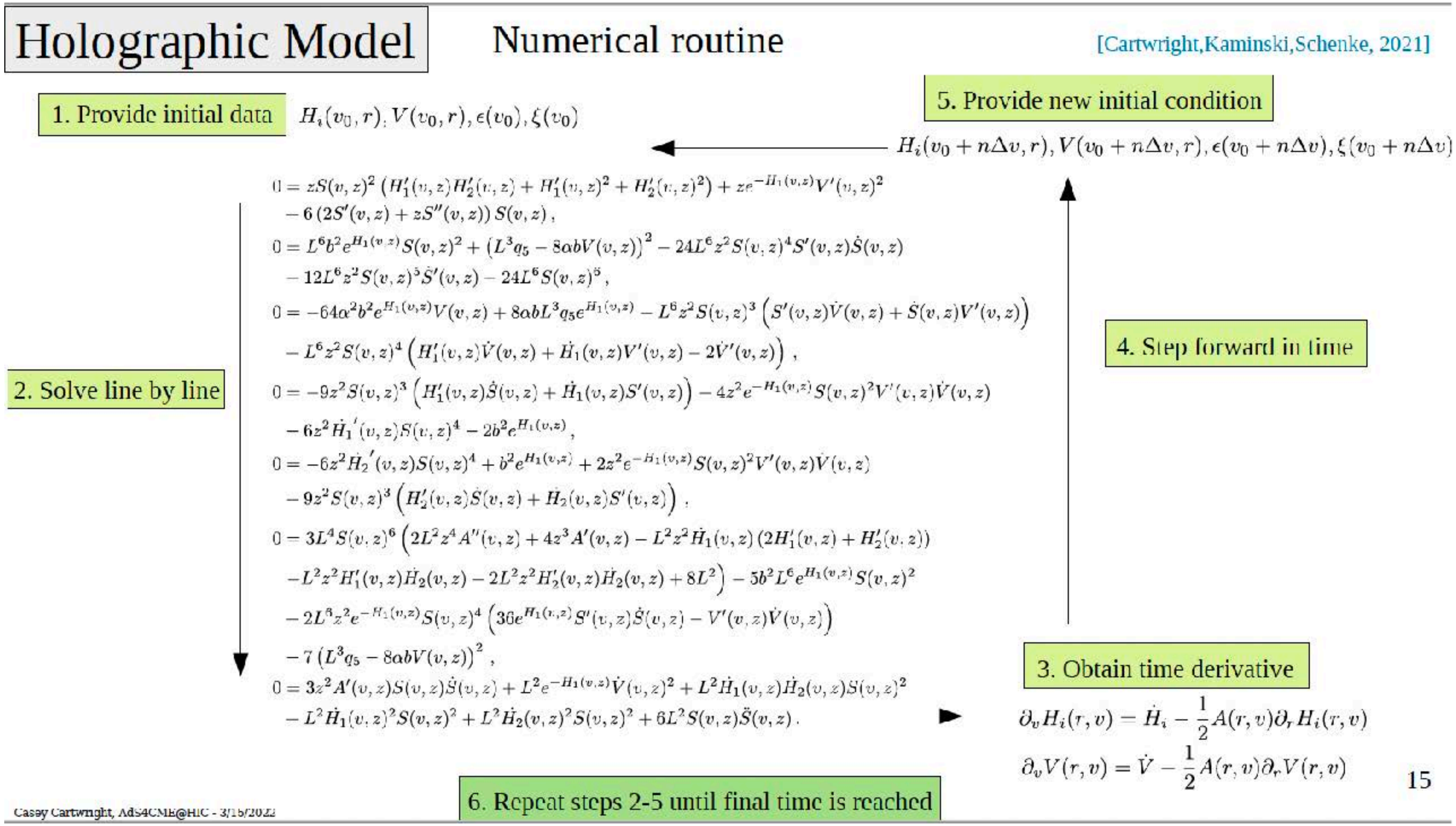
$$B = z^4 B_\mathrm{s} + \Delta_B$$

Initial conditions :

$$B_\mathrm{s}(z, v_0) = \Omega_1 \cos(\gamma_1 z) + \Omega_2 \tan(\gamma_2 z) + \Omega_3 \sin(\gamma_3 z) + \sum_{i=0}^{5} \beta_i z^i$$
$$+ \frac{\alpha}{z^4}\left[-\frac{2}{3}\ln\left(1 + \frac{z}{v_0}\right) + \frac{2z^3}{9v_0^3} - \frac{z^2}{3v_0^2} + \frac{2z}{3v_0}\right],$$

APPENDIX: Bjorken - expanding plasma: C^3-code

[Cartwright,Kaminski,Schenke; PRC (2022)]



Holographic Model

Numerical routine

[Cartwright,Kaminski,Schenke, 2021]

1. Provide initial data $H_i(v_0,r), V(v_0,r), \epsilon(v_0), \xi(v_0)$

5. Provide new initial condition $H_i(v_0+n\Delta v,r), V(v_0+n\Delta v,r), \epsilon(v_0+n\Delta v), \xi(v_0+n\Delta v)$

2. Solve line by line

$$
\begin{aligned}
0 &= zS(v,z)^2\left(H_1'(v,z)H_2'(v,z)+H_1'(v,z)^2+H_2'(v,z)^2\right)+ze^{-H_1(v,z)}V'(v,z)^2\\
&-6\left(2S'(v,z)+zS''(v,z)\right)S(v,z)\,,\\
0 &= L^6b^2e^{H_1(v,z)}S(v,z)^2+\left(L^3q_5-8\alpha bV(v,z)\right)^2-24L^6z^2S(v,z)^4S'(v,z)\dot{S}(v,z)\\
&-12L^6z^2S(v,z)^5\dot{S}'(v,z)-24L^6S(v,z)^5\,,\\
0 &= -64\alpha^2b^2e^{H_1(v,z)}V(v,z)+8\alpha bL^3q_5e^{H_1(v,z)}-L^6z^2S(v,z)^3\left(S'(v,z)\dot{V}(v,z)+\dot{S}(v,z)V'(v,z)\right)\\
&-L^6z^2S(v,z)^4\left(H_1'(v,z)\dot{V}(v,z)+\dot{H}_1(v,z)V'(v,z)-2\dot{V}'(v,z)\right)\,,\\
0 &= -9z^2S(v,z)^3\left(H_1'(v,z)\dot{S}(v,z)+\dot{H}_1(v,z)S'(v,z)\right)-4z^2e^{-H_1(v,z)}S(v,z)^2V'(v,z)\dot{V}(v,z)\\
&-6z^2\dot{H}_1{}'(v,z)S(v,z)^4-2b^2e^{H_1(v,z)}\,,\\
0 &= -6z^2\dot{H}_2{}'(v,z)S(v,z)^4+b^2e^{H_1(v,z)}+2z^2e^{-H_1(v,z)}S(v,z)^2V'(v,z)\dot{V}(v,z)\\
&-9z^2S(v,z)^3\left(H_2'(v,z)\dot{S}(v,z)+\dot{H}_2(v,z)S'(v,z)\right)\,,\\
0 &= 3L^4S(v,z)^6\left(2L^2z^4A''(v,z)+4z^3A'(v,z)-L^2z^2\dot{H}_1(v,z)\left(2H_1'(v,z)+H_2'(v,z)\right)\right.\\
&\left.-L^2z^2H_1'(v,z)\dot{H}_2(v,z)-2L^2z^2H_2'(v,z)\dot{H}_2(v,z)+8L^2\right)-5b^2L^6e^{H_1(v,z)}S(v,z)^2\\
&-2L^6z^2e^{-H_1(v,z)}S(v,z)^4\left(36e^{H_1(v,z)}S'(v,z)\dot{S}(v,z)-V'(v,z)\dot{V}(v,z)\right)\\
&-7\left(L^3q_5-8\alpha bV(v,z)\right)^2\,,\\
0 &= 3z^2A'(v,z)S(v,z)\dot{S}(v,z)+L^2e^{-H_1(v,z)}\dot{V}(v,z)^2+L^2\dot{H}_1(v,z)\dot{H}_2(v,z)S(v,z)^2\\
&-L^2\dot{H}_1(v,z)^2S(v,z)^2+L^2\dot{H}_2(v,z)^2S(v,z)^2+6L^2S(v,z)\ddot{S}(v,z)\,.
\end{aligned}
$$

3. Obtain time derivative

$$\partial_v H_i(r,v)=\dot{H}_i-\frac{1}{2}A(r,v)\partial_r H_i(r,v)$$

$$\partial_v V(r,v)=\dot{V}-\frac{1}{2}A(r,v)\partial_r V(r,v)$$

4. Step forward in time

6. Repeat steps 2-5 until final time is reached

Casey Cartwright, AdS4CME@HIC - 3/15/2022

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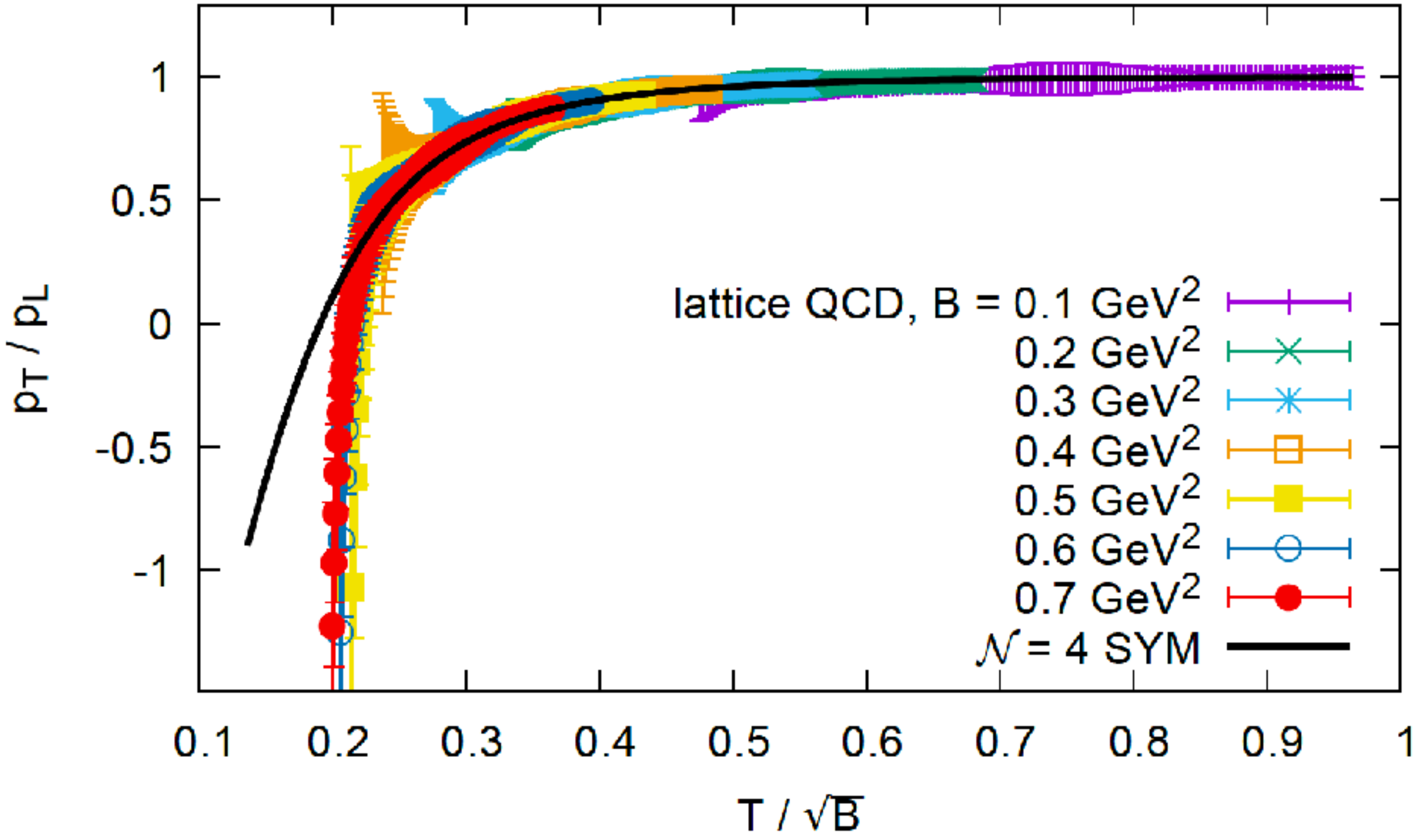
taken from Casey Cartwright's talk

APPENDIX: Same magneto response in LQCD and N=4 SYM with magnetic field

[Endrödi, Kaminski, Schäfer, Wu, Yaffe; JHEP (2018)]



Lattice QCD with 2+1 flavors, dynamical quarks, physical masses

transverse pressure: $p_{\mathrm{T}} = -\frac{L_{\mathrm{T}}}{V}\frac{\partial F_{\mathrm{QCD}}}{\partial L_{\mathrm{T}}}$

longitudinal pressure: $p_{\mathrm{L}} = -\frac{L_{\mathrm{L}}}{V}\frac{\partial F_{\mathrm{QCD}}}{\partial L_{\mathrm{L}}}$

F_{QCD} ... *free energy*

L_{T} ... *transverse system size*

L_{L} ... *longitudinal system size*

V ... *system volume*