Electromagnetic field induction in quark-gluon plasma due to thermoelectric effects

Presented by: **Jayanta Dey**

Based on- arXiv: **2405.12510 [hep-ph]**

25 July 2024

The 8th International Conference on Chirality, Vorticity, and Magnetic Field in Quantum Matter, July 22 - 26, 2024.

Overview of the talk

Objective and Motivation

Thermoelectric/ Seebeck effect: **Induction of electric field due to thermal gradient**.

Being made of electrically charged particles, QGP shows thermoelectric phenomena.

How strong can an electromagnetic field be produced due to such phenomena?

Is the field strong enough to interfere with QCD interaction and modify the final state observable?

A Brief Formalism: Kinetic theory

$$
\begin{aligned}
T^{\mu\nu} &= N_s N_f N_c \int \frac{d^3\boldsymbol{k}}{(2\pi)^3} \frac{k^\mu k^\nu}{\omega} (f_Q + f_{\bar{Q}}) \ , \\
j^\mu &= q \times N_s N_f N_c \int \frac{d^3\boldsymbol{k}}{(2\pi)^3} \frac{k^\mu}{\omega} (f_Q - f_{\bar{Q}}).
\end{aligned} \tag{1}
$$

Conservation laws: $\partial_\mu T^{\mu\nu} = 0; \qquad \partial_\mu j^\mu = 0$

Four Momentum: $p^\mu = (\omega, \ \boldsymbol{k})$
Total single particle distribution function: $f = f(x, k)$

For a system, slightly out of equilibrium, $f = f_{\mathrm{ideal}} + f_{\mathrm{dissi}}$,
$f_{\mathrm{ideal}} = \frac{1}{e^{\frac{\omega \mp \mu}{T}} + a}; \qquad a = 0, -1, +1$

To find f_{dissi}, we use the Boltzmann Equation.

Relativistic Boltzmann equation (RBE) in presence of magnetic field can be written as,

$$k^\mu \partial_\mu f + qF^{\mu\nu} k_\nu \frac{\partial f}{\partial k^\mu} = \mathcal{C}(f) = -(u \cdot k)\ \frac{\delta f}{\tau_c}$$

Relaxation time approximation (RTA): $\mathcal{C}(f) = -\frac{k^\mu u_\mu}{\tau_c} \delta f$,
J. L. Anderson and H. R. Witting, Physica 74, 466 (1974)

Depending on the dissipative force we considered an ansatz:
$\delta f_i = (\vec{k_i} \cdot \vec{\Omega}) \frac{\partial f_i^0}{\partial \omega_i}$, with $\vec{\Omega} = \alpha_1 \vec{E} + \alpha_2 \vec{\nabla} T + \alpha_3 \vec{\nabla} \dot{T}$

3rd terms contribute when system cools down rapidly.

Boltzmann equation and RTA

Solving the Boltzmann equation we obtain

$$\delta f_i = -q_i\tau_R^i(\vec{v_i}.\vec{E}) + \frac{(\omega_i - \mathrm{b}_i h)\tau_R^i}{T}\left[\left(\vec{v_i}\cdot\vec{\nabla}T\right) - \tau_R^i\left(\vec{v_i}\cdot\vec{\nabla}\dot{T}\right)\right]\frac{\partial f_i^0}{\partial\omega_i}$$

Plugging it in the expression of current:

$$j^l = \sum_i \frac{q_i g_i}{3}\int \frac{d^3|\vec{k_i}|}{(2\pi)^3} v_i^2\tau_R^i\left[-q_iE^l + \frac{(\omega_i - \mathrm{b}_i h)}{T}\left\{\frac{\partial T}{\partial x^l} - \tau_R^i\frac{\partial \dot{T}}{\partial x^l}\right\}\right]\frac{\partial f_i^0}{\partial\omega_i}.$$

When the net current is zero ($j^l = 0$), we find the **electric field** induced due to temperature gradient. Rearranging the above expression

$$\boldsymbol{\sigma}\ \mathbf{E} = \mathbf{L}\ \mathbf{X}$$

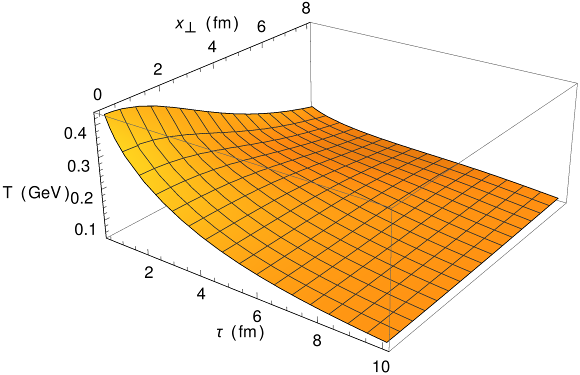
$$\sigma \ \mathbf{E} = \mathbf{L} \ \mathbf{X}$$
$$\Rightarrow \mathbf{E} = (\boldsymbol{\sigma}^{-1}\mathbf{L})\mathbf{X}$$
$$\Rightarrow \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} = \begin{pmatrix} S & 0 & 0 \\ 0 & S & 0 \\ 0 & 0 & S \end{pmatrix} \begin{pmatrix} \frac{dT}{dx} \\ \frac{dT}{dy} \\ \frac{dT}{dz} \end{pmatrix} . \tag{2}$$

Thermoelectric coefficient matrix with **Seebeck coefficient** $S = \frac{\frac{L_{23}}{T^2}}{\frac{\sigma_{el}}{T}}$.

$$L_{23} = \sum_i q_i \frac{g_i}{3T} \int \frac{d^3|\vec{k}_i|}{(2\pi)^3} \frac{\vec{k}_i^2}{\omega_i^2} (\omega_i - \mathrm{b}_i h) \tau_R^i f_i^0 (1 - f_i^0) \ (1 - \tau_R \frac{\partial}{\partial t})$$

Gubser flow

With cooling rate from **Gubser flow** (1+1) D: Transverse flow along with longitudinal boost invariant flow.

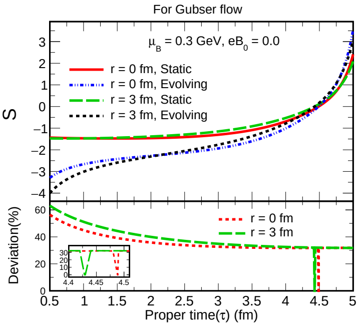


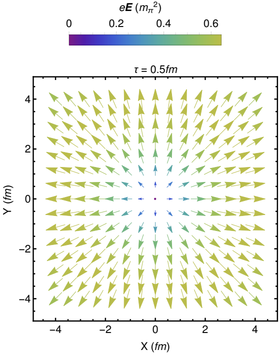
S.S. Gubser, **Phys. Rev. D 82 (2010) 085027**.

Understanding the Electric field vector

For numerical estimation of thermoelectric coefficients, we considered the quasiparticle model by *Gorenstein and Yang* (**Phys. Rev. D 52 (1995) 5206**). For relaxation time, we use the expression for QCD matter from **Nucl. Phys. B 250 (1985) 666** by *A. Hosoya and K. Kajantie*.

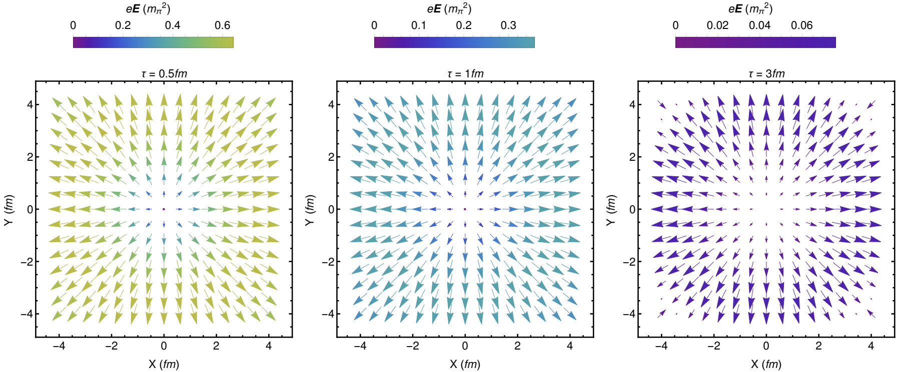
$$\tau_R = \frac{1}{5.1\,T\alpha_s^2\log(1/\alpha_s)[1+0.12(2N_f+1)]}$$





Temperature gradient from low to high T (Surface to center): **Negative**

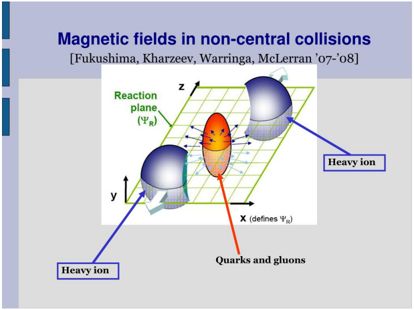
Electric field induction in isotropic QGP



Space time profile of induced electric field

No longitudinal dependency because of **boost invariant** symmetry.

Creation of external magnetic field



Strong transient magnetic field ($eB \approx 15$ m_π^2) produced at the early stage whcih can create momentum or pressure anisotropy.

$$\frac{\partial f_i}{\partial t} + \frac{\vec{k}_i}{\omega_i} \cdot \frac{\partial f_i}{\partial \vec{x}_i} + q_i \left(\vec{E} + \frac{\vec{k}_i}{\omega_i} \times \vec{B} \right) \cdot \frac{\partial f_i}{\partial \vec{k}_i} = -\frac{\delta f_i}{\tau_R^i}$$

Where we introduced Lorentz force due to external magnetic field.

BTE in presence of a magnetic field

Considering time-varying field:

$$B = B_0 \exp\left(-\frac{\tau}{\tau_B}\right), \qquad E = E_0 \exp\left(-\frac{\tau}{\tau_E}\right)$$

Ansatz for $\delta f = (\vec{k_i} \cdot \vec{\Omega}) \frac{\partial f_i^0}{\partial \omega}$, with

$$\vec{\Omega} = \alpha_1 \vec{E} + \alpha_2 \dot{\vec{E}} + \alpha_3 \vec{\nabla} T + \alpha_4 \vec{\nabla} \dot{T} + \alpha_5 (\vec{\nabla} T \times \vec{B}) + \alpha_6 (\vec{\nabla} T \times \dot{\vec{B}}) + \alpha_7 (\vec{\nabla} \dot{T} \times \vec{B}) + \alpha_8 (\vec{E} \times \vec{B}) + \alpha_9 (\vec{E} \times \dot{\vec{B}}) + \alpha_{10} (\dot{\vec{E}} \times \vec{B})$$

Thermoelectric effect in a external magnetic field

$$\mathbf{E} = (\boldsymbol{\sigma}^{-1}\mathbf{L})\mathbf{X}$$

$$\Rightarrow \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} = \begin{pmatrix} S_B & \overline{NB} & NB \\ NB & S_B & \overline{NB} \\ \overline{NB} & NB & S_B \end{pmatrix} \begin{pmatrix} \frac{dT}{dx} \\ \frac{dT}{dy} \\ \frac{dT}{dz} \end{pmatrix}, \tag{3}$$

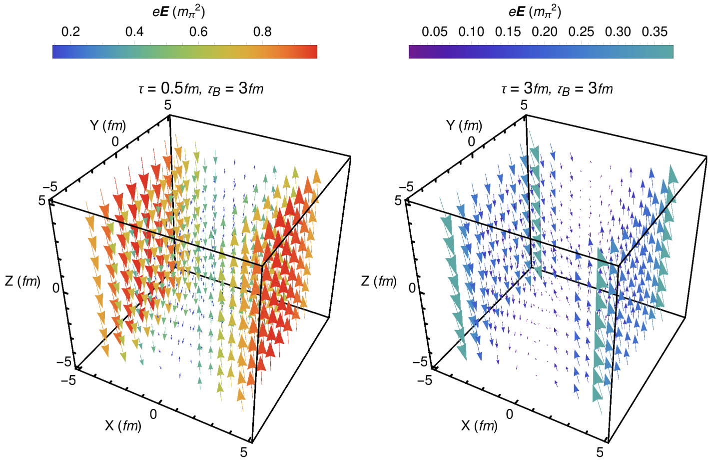
Dimensionless magneto-Seebeck and Nernst coefficients:

$$S_B = \frac{(\sigma_e^2 + \sigma_H^2)L_{34} + (2\sigma_e\sigma_H)L_{56}}{T(\sigma_e^3 + 3\sigma_e\sigma_H^2)},$$

$$\overline{NB} = \frac{\sigma_H(\sigma_e + \sigma_H)L_{34} - \sigma_e(\sigma_e + \sigma_H)L_{56}}{T(\sigma_e^3 + 3\sigma_e\sigma_H^2)},$$

$$NB = \frac{-\sigma_H(\sigma_e - \sigma_H)L_{34} + \sigma_e(\sigma_e - \sigma_H)L_{56}}{T(\sigma_e^3 + 3\sigma_e\sigma_H^2)}. \tag{4}$$

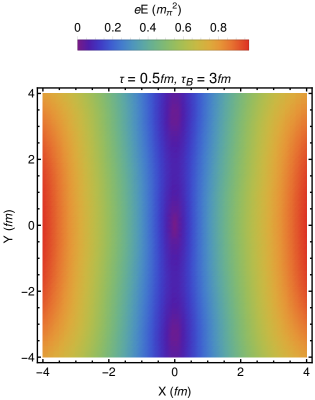
Electric field induction in anisotropic QGP

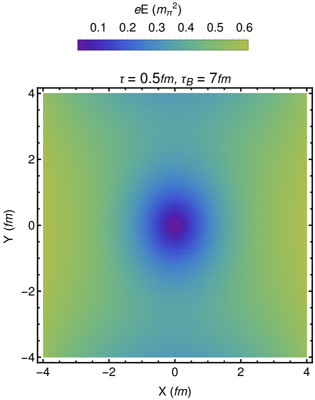


Space time profile of induced electric field

Initial magnetic field $eB_0 = 5\mathrm{m}_\pi^2$

Magnitude Plot





- Initial induced electric field could be as high as 0.9 m_π^2
- magnetic field introduce anisotropy in induce electric field.
- Slower variation of ext. mag. field have less impact and therefore relatively isotropic intensity of induced field.

Summary and Conclusion

- Thermoelectric Phenomena can induce Electromagnetic field in QGP.
- Induce field could be strong enough, close to QCD energy scale.
- (3+1) D hydrodynamic simulation can incorporate this phenomena to estimate the effect in flow coefficients and other observable.
- Magnetic field created in peripheral collisions not only introduce anisotropy in the induced field also enhances its strength.

Thank You