Spin correlations in relativistic heavy ion collisions

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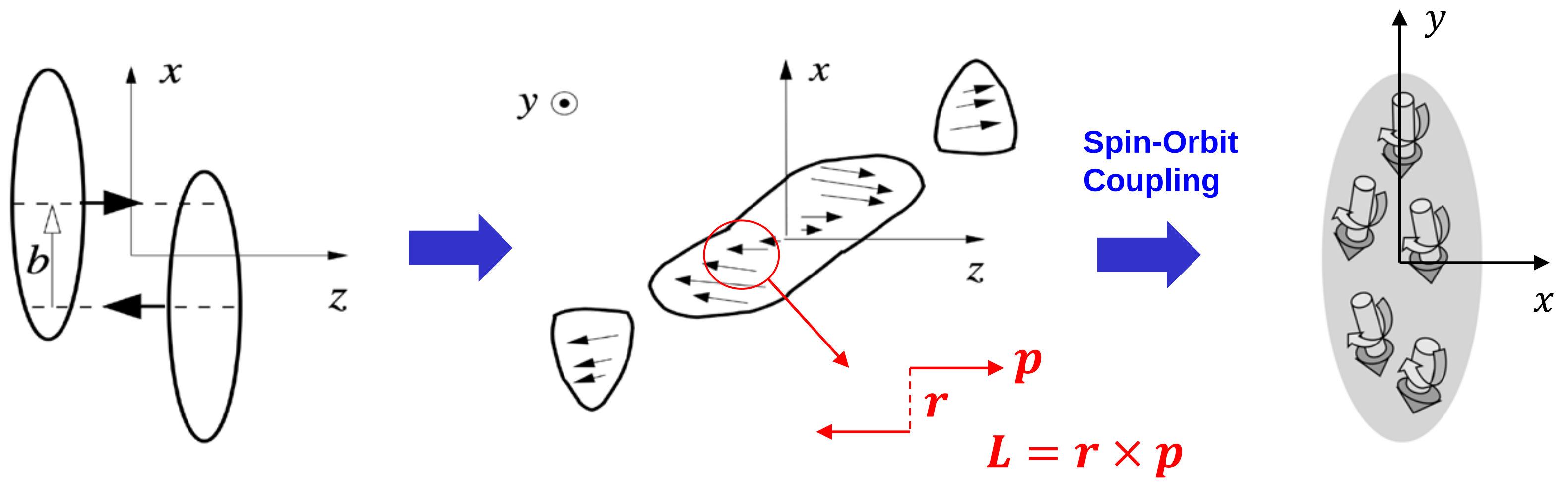
The 8th International Conference on Chirality, Vorticity and Magnetic Field in Quantum Matter, July 22-26, 2024, Timisoara, Romainia

Outline

- **Introduction to spin polarization phenomena in HIC**
- **Spin correlation and entanglement in $\Lambda\bar{\Lambda}$ system in charmonium decays [few body exclusive process]**
- **Quark and hadron spin correlation in coalescence model in high energy HIC**
- **Two examples of spin correlations in HIC: (a) spin correlation in ϕ meson's spin alignment; (b) $\Lambda\Lambda$'s spin correlation as probe to vortical structure of sQGP**
- **Summary**

Global OAM and polarization in HIC



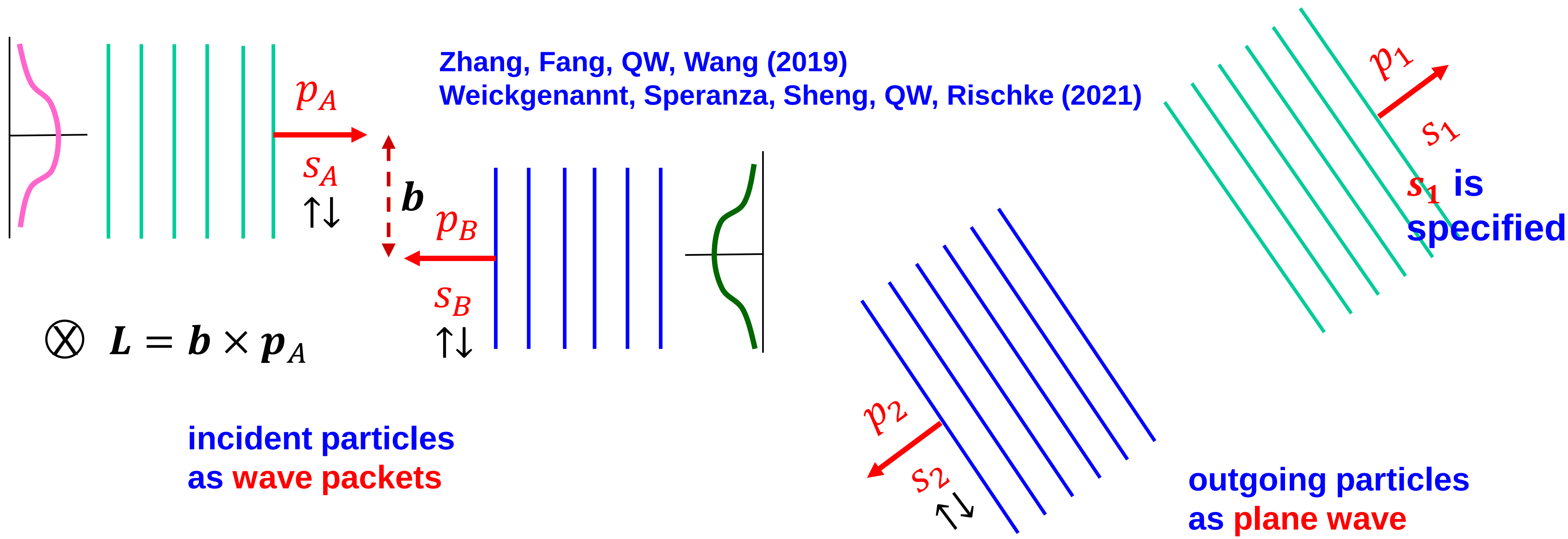
Huge global orbital angular momentum (OAM) is produced in IS of HIC.

Q: How do orbital angular momenta be transferred to the matter in HIC?

A: Part of initial OAM is distributed into the matter in the form of local OAM and then is converted to hadrons' global spin polarization through spin-orbit coupling: e.g. GSP of Λ hyperons [Liang, Wang (2005)]

Non-local collisions: collisions of particles as wave packets

Zhang, Fang, QW, Wang (2019)
Weickgenannt, Speranza, Sheng, QW, Rischke (2021)



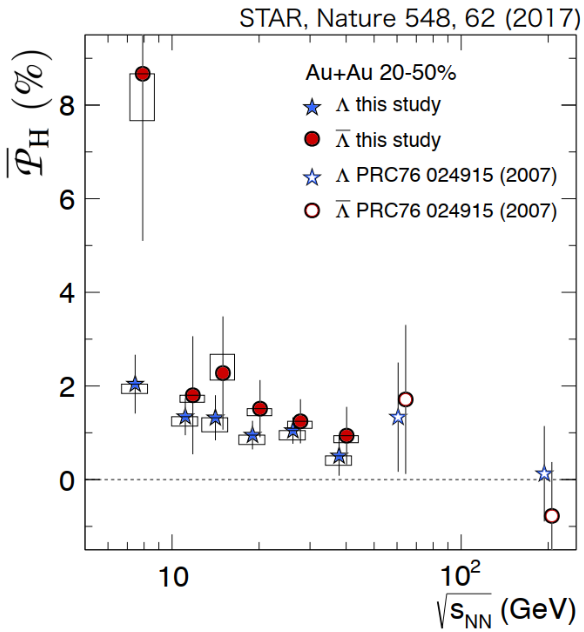
incident particles as wave packets

outgoing particles as plane wave

Particle collisions as wave packets: there is a transverse distance between two wave packets (impact parameter) giving non-vanishing OAM and then the polarization of one final particle

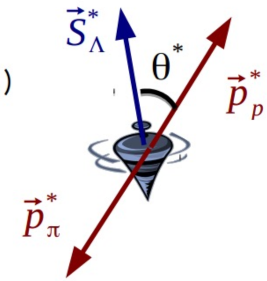
$$\boldsymbol{L} = \boldsymbol{b} \times \boldsymbol{p}_A \Longrightarrow \left(\frac{d\sigma}{d\Omega}\right)_{s_1=\uparrow} \neq \left(\frac{d\sigma}{d\Omega}\right)_{s_1=\downarrow}$$

STAR: global polarization of Λ hyperon



parity-violating decay of hyperons

In case of Λ's decay, daughter proton preferentially decays in the direction of Λ's spin (opposite for anti-Λ)



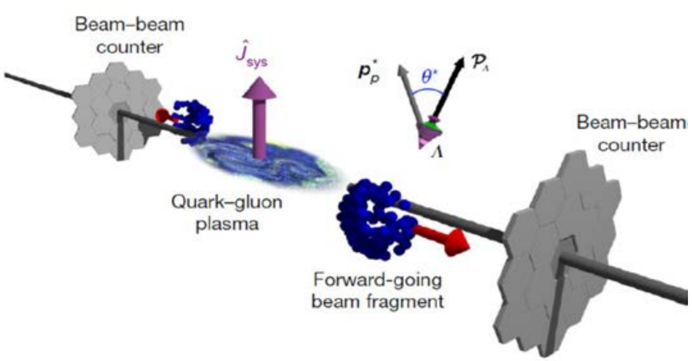
$$\frac{dN}{d\Omega^*} = \frac{1}{4\pi}(1 + \alpha \mathbf{P}_\Lambda \cdot \mathbf{p}_\mathbf{p}^*)$$

α: Λ decay parameter (=0.642±0.013)
P_Λ: Λ polarization
p_p^*: proton momentum in Λ rest frame

$$\Lambda \rightarrow p + \pi^+$$

(BR: 63.9%, cτ~7.9 cm)

Updated by BES III, PRL129, 131801 (2022)

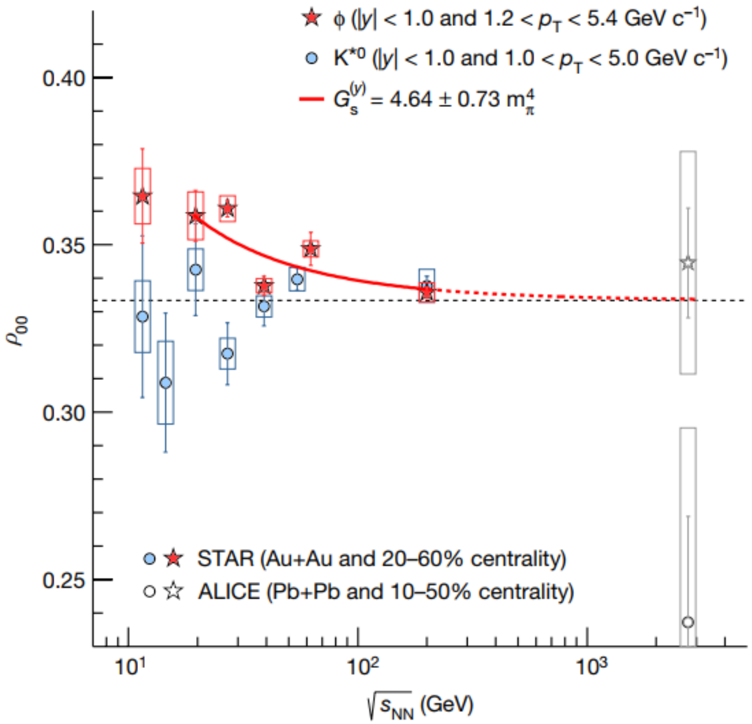


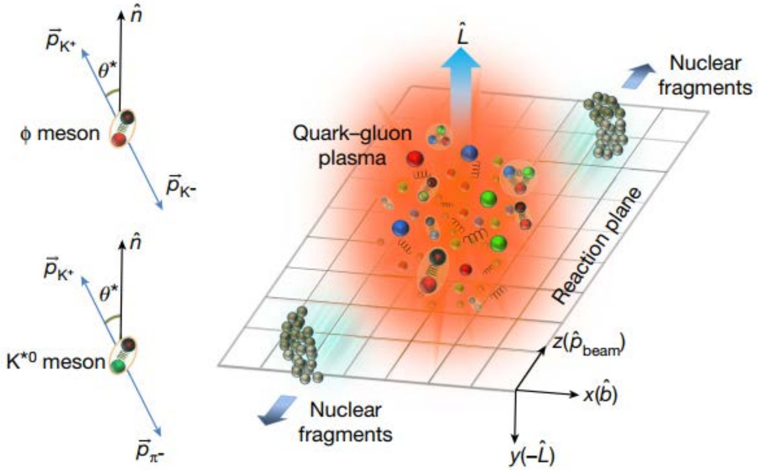
ω = (9 ± 1)x10^{21}/s, the largest angular velocity that has ever been observed in any system

Liang, Wang, PRL (2005)
Betz, Gyulassy, Torrieri, PRC (2007)
Becattini, Piccinini, Rizzo, PRC (2008)
Gao et al., PRC (2008)

STAR: global spin alignments of vector mesons

STAR, Nature 614, 244 (2023);





Implication of correlation or fluctuation of strong force fields

Theory prediction:
Sheng, Oliva, QW (2020);
Sheng, Oliva, et al., (2022).

$$P_\Lambda \sim \langle P_s \rangle, \quad P_{\bar{\Lambda}} \sim \langle P_{\bar{s}} \rangle$$

$$\rho_{00}^{\phi} - \frac{1}{3} \sim \langle P_s P_{\bar{s}} \rangle \neq \langle P_s \rangle \langle P_{\bar{s}} \rangle \sim P_\Lambda P_{\bar{\Lambda}}$$

Some review articles on polarization in HIC

1. Global and local spin polarization in heavy ion collisions: a brief overview, [phenomenology]
   QW, Nucl. Phys. A 967, 225 (2017).

2. Relativistic hydrodynamics for spin-polarized fluids, [theory]
   Florkowski, Kumar, R. Ryblewski, Prog. Part. Nucl. Phys. 108, 103709 (2019).

3. Polarization and Vorticity in the Quark–Gluon Plasma, [phenomenology]
   Becattini, Lisa, Ann. Rev. Nucl. Part. Sci. 70, 395 (2020).

4. Vorticity and Spin Polarization in Heavy Ion Collisions: Transport Models, [phenomenology]
   Huang, Liao, QW, Xia, Lect. Notes Phys. 987, 281 (2021).

5. Global polarization effect and spin-orbit coupling in strong interaction, [phenomenology]
   Gao, Liang, QW, Wang, Lect. Notes Phys. 987, 195 (2021).

6. Spin and polarization: a new direction in relativistic heavy ion physics, [theory+phenom.]
   Becattini, Rept. Prog. Phys. 85, No.12, 122301 (2022)

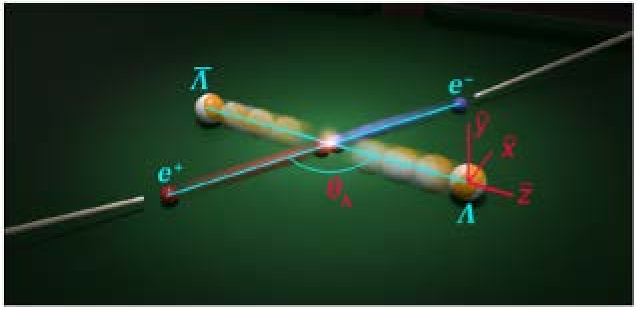
7. Foundations and applications of quantum kinetic theory, [theory]
   Hidaka, Pu, QW, Yang, Prog. Part. Nucl. Phys. 127, 103989 (2022).

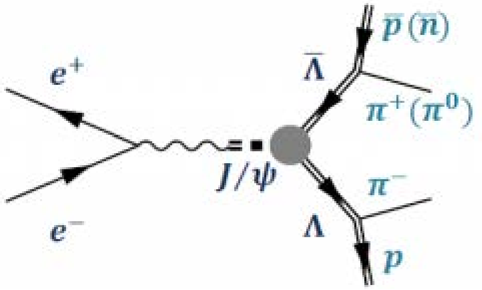
8. Spin polarization in relativistic heavy-ion collisions, [theory+experimet]
   Becattini, Buzzegoli, Niida, Pu, Tang, QW, 2402.04540, to appear in QGP-6.

Spin correlation and entanglement of $\Lambda\bar{\Lambda}$ in charmonium decays

Generalized quantum measurement in spin-correlated hyperon-antihyperon decays,
S.-H. Wu, C. Qian, Y.-G. Yang, QW, 2402.16574

Bell nonlocality and entanglement in $e^+e^- \to Y\bar{Y}$ at BESIII,
S.-H. Wu, C. Qian, QW, 2406.16298





Weak decays of spin-1/2 hyperons

A typical hyperon decay is to another spin-1/2 baryon B' accompanied by a spin-0 meson M denoted as $B \to B'M$ with the decay matrix element

$$\begin{aligned}\mathcal{A}_{B\to B'M} =& G_F m_M^2 \underline{\bar{u}_B}(C_1 - C_2\gamma_5)\underline{u_{B'}} \\ =& G_F m_M^2 \sqrt{2m_B(E_{B'}+m_{B'})} \\ & \times \chi_s^\dagger \left(S + P\boldsymbol{\sigma}\cdot\hat{\mathbf{p}}\right)\chi_{s'}\end{aligned}$$

$$u_{B'}(s',\mathbf{p}) = \sqrt{E_{B'}+m_{B'}}\begin{pmatrix}\chi_{s'} \\ \frac{\boldsymbol{\sigma}\cdot\mathbf{p}}{E_{B'}+m_{B'}}\chi_{s'}\end{pmatrix}$$

$$u_B(s,0) = \sqrt{2m}\begin{pmatrix}\chi_s \\ 0\end{pmatrix}$$

$$\begin{aligned}\sum_{s'}\left|\mathcal{A}_{B\to B'M}\right|^2 =& 8\pi G_F^2 m_M^4 m_B(E_{B'}+m_{B'}) \\ & \times (|S|^2+|P|^2)\mathrm{Tr}\left(M_{\boldsymbol{p}}\underline{\rho_B}M_{\boldsymbol{p}}^\dagger\right)\end{aligned}$$

$$M_{\boldsymbol{p}} = \frac{1}{\sqrt{4\pi\left(|S|^2+|P|^2\right)}}\left(S + P\boldsymbol{\sigma}\cdot\hat{\boldsymbol{p}}\right)$$

$$\rho_B = \frac{1}{2}(1+\boldsymbol{\sigma}\cdot\boldsymbol{s}_B)$$

The normalized differential width in the momentum direction of daughter B'

$$\frac{1}{\Gamma}\frac{d\Gamma}{d\Omega} = \mathrm{Tr}\left(M_{\boldsymbol{p}}\rho_B M_{\boldsymbol{p}}^\dagger\right) = \frac{1}{4\pi}\left(1+\underline{\alpha_B}\boldsymbol{s}_B\cdot\hat{\boldsymbol{p}}\right)$$

$$\alpha_B = \frac{S^*P+SP^*}{|S|^2+|P|^2}$$

Hyperon's decay constant

Spin correlation in $\Lambda\bar{\Lambda}$ system

Spin correlation in $\Lambda\bar{\Lambda}$ system is encoded in the spin density matrix $\rho_{\Lambda\bar{\Lambda}}$

$$\rho_{\Lambda\bar{\Lambda}} = \frac{1}{4}\left(1_4 + \boldsymbol{s}_\Lambda \cdot \boldsymbol{\sigma} \otimes 1_2 + 1_2 \otimes \boldsymbol{s}_{\bar{\Lambda}} \cdot \boldsymbol{\sigma} + \underline{C_{ij}}\sigma_i \otimes \sigma_j\right)$$

3x3 real matrix for spin correlation

The Bell nonlocality can be measured by the observable

$$B_{CHSH} = 2\sqrt{m_1 + m_2}$$

where m_1 and m_2 are two largest eigenvalues of $C^T C$.

In the classical theory we have $0 < m_1 + m_2 \leq 1$. For the quantum state we have $m_1 + m_2 > 1$, in this case we say the state is Bell nonlocal. So a measure for the Bell nonlocality can be defined as

$$m_{12}[\rho_{\Lambda\bar{\Lambda}}] = m_1 + m_2$$

Spin correlation in $\Lambda\bar{\Lambda}$ system

Consider the joint decay $\Lambda\bar{\Lambda} \to (p\pi^-)(\bar{p}\pi^+)$, the joint angular distribution for proton and antiproton is

$$\frac{1}{\Gamma}\frac{d\Gamma(\Lambda\bar{\Lambda}\to p\pi^-\bar{p}\pi^+)}{d\Omega_{\boldsymbol{p}}d\Omega_{\bar{\boldsymbol{p}}}} = \mathrm{Tr}\left[(M_{\boldsymbol{p}}\otimes M_{\bar{\boldsymbol{p}}})\,\rho_{\Lambda\bar{\Lambda}}\left(M_{\boldsymbol{p}}^{\dagger}\otimes M_{\bar{\boldsymbol{p}}}^{\dagger}\right)\right]$$

$$= \frac{1}{(4\pi)^2}\left(1+\alpha_\Lambda s_\Lambda\cdot\hat{\boldsymbol{p}}+\alpha_{\bar{\Lambda}}s_{\bar{\Lambda}}\cdot\hat{\bar{\boldsymbol{p}}}+\alpha_\Lambda\alpha_{\bar{\Lambda}}C_{ij}\hat{\boldsymbol{p}}_i\hat{\bar{\boldsymbol{p}}}_j\right)$$

Now we look at charmonium decay $\eta_c/\chi_{c0} \to \Lambda\bar{\Lambda}$, where η_c/χ_{c0} is in the spin singlet/triplet state of $c\bar{c}$. Spin density matrix for $\Lambda\bar{\Lambda}$:

$$\rho_{\Lambda\bar{\Lambda}}(\eta_c) = \left|\psi^-\right\rangle\left\langle\psi^-\right| = \frac{1}{4}\left(1-\delta_{ij}\sigma_i\otimes\sigma_j\right)$$

$$\rho_{\Lambda\bar{\Lambda}}(\chi_{c0}) = \left|\psi^+\right\rangle\left\langle\psi^+\right| = \frac{1}{4}\left(1+\underline{C_{ij}}\sigma_i\otimes\sigma_j\right)$$

$$C_{ij} = \mathrm{diag}(1,1,-1)$$

$$\boxed{|\psi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle-|10\rangle) \qquad |\psi^+\rangle = \frac{1}{\sqrt{2}}(|01\rangle+|10\rangle)}$$

$$\boxed{|0\rangle = |\uparrow\rangle\,,\ |1\rangle = |\downarrow\rangle}$$

$$\boxed{m_{\Lambda\bar{\Lambda}}(\eta_c) = m_{\Lambda\bar{\Lambda}}(\chi_{c0}) = 2\sqrt{2} > 2}$$

Both states are maximally entangled and nonlocal

Bell nonlocality measure

Spin correlation in $\Lambda\bar{\Lambda}$ system

The joint angular distribution of proton and antiproton in charmonium decay

$$\frac{1}{\Gamma}\frac{d\Gamma(\eta_c)}{d\Omega_p d\Omega_{\bar{p}}} = \frac{1}{(4\pi)^2}\left(1+\alpha_\Lambda\alpha_{\bar{\Lambda}}\hat{\boldsymbol{p}}\cdot\hat{\bar{\boldsymbol{p}}}\right)$$

$$\frac{1}{\Gamma}\frac{d\Gamma(\chi_{c0})}{d\Omega_p d\Omega_{\bar{p}}} = \frac{1}{(4\pi)^2}\left[1+\alpha_\Lambda\alpha_{\bar{\Lambda}}\left(\hat{\boldsymbol{p}}_x\hat{\bar{\boldsymbol{p}}}_x+\hat{\boldsymbol{p}}_y\hat{\bar{\boldsymbol{p}}}_y-\hat{\boldsymbol{p}}_z\hat{\bar{\boldsymbol{p}}}_z\right)\right]$$

The measure of Bell nonlocality is a function of moments that are measured in experiments

$$m_{12}\left[\rho_{\Lambda\bar{\Lambda}}\right] = \left(\frac{9}{\alpha_\Lambda\alpha_{\bar{\Lambda}}}\right)^2\left[\left\langle\hat{\boldsymbol{p}}_i\hat{\bar{\boldsymbol{p}}}_i\right\rangle^2+\left\langle\hat{\boldsymbol{p}}_j\hat{\bar{\boldsymbol{p}}}_j\right\rangle^2\right]$$

where $i \neq j$ (no summation is implied over i and j).

Global quark and hadron spin correlation in coalescence model in HIC

Global quark spin correlations in relativistic heavy-ion collisions,

Lv, Yu, Liang, QW, Wang, PRD(2024); 2402.13721

Also see Xin-Nian Wang's talk on Wednesday

Spin density matrix for quarks

The spin density matrices for one quark and two quarks are

$$\rho_{(q)} = \frac{1}{2}\left(1 + \boldsymbol{P}_q \cdot \boldsymbol{\sigma}\right)$$

$$\rho_{(12)} = \frac{1}{4}\left[1 + \boldsymbol{P}_1 \cdot \boldsymbol{\sigma} + \boldsymbol{P}_2 \cdot \boldsymbol{\sigma} + t_{ij}^{(12)} \sigma_{1i} \otimes \sigma_{2j}\right] \Longrightarrow \text{with shortcoming}$$

$$= \rho_{(1)} \otimes \rho_{(2)} + \frac{1}{4} \underline{c_{ij}^{(12)}} \sigma_{1i} \otimes \sigma_{2j} \Longrightarrow \text{improved}$$

2-body genuine correlation

Similarly, the spin density matrix for three quarks has the form

$$\rho_{(123)} = \rho_{(1)} \otimes \rho_{(2)} \otimes \rho_{(3)} + \frac{1}{2^3} \underline{c_{ijk}^{(123)}} \sigma_{1i} \otimes \sigma_{2j} \otimes \sigma_{3k}$$

3-body correlation

$$+ \frac{1}{2^2}\left[\underline{c_{ij}^{(12)}} \sigma_{1i} \otimes \sigma_{2j} \otimes \rho_{(3)} + \underline{c_{jk}^{(23)}} \rho_{(1)} \otimes \sigma_{2j} \otimes \sigma_{3k} + \underline{c_{ik}^{(13)}} \sigma_{1i} \otimes \rho_{(2)} \otimes \sigma_{3k}\right]$$

2-body correlations

Spin density matrix for quarks

The spin density matrix for four quarks has the form

$$
\begin{aligned}
\rho_{(1234)} = &\rho_{(1)} \otimes \rho_{(2)} \otimes \rho_{(3)} \otimes \rho_{(4)} + \frac{1}{2^4} \underline{c_{ijkl}^{(1234)}} \sigma_{1i} \otimes \sigma_{2j} \otimes \sigma_{3k} \otimes \sigma_{4l} \\
&+ \frac{1}{2^2} \Big[\underline{c_{ij}^{(12)}} \sigma_{1i} \otimes \sigma_{2j} \otimes \rho_{(3)} \otimes \rho_{(4)} + \underline{c_{kl}^{(34)}} \rho_{(1)} \otimes \rho_{(2)} \otimes \sigma_{3k} \otimes \sigma_{4l} \\
&+ \underline{c_{ik}^{(13)}} \sigma_{1i} \otimes \rho_{(2)} \otimes \sigma_{3k} \otimes \rho_{(4)} + \underline{c_{jl}^{(24)}} \rho_{(1)} \otimes \sigma_{2j} \otimes \rho_{(3)} \otimes \sigma_{4l} \\
&+ \underline{c_{il}^{(14)}} \sigma_{1i} \otimes \rho_{(2)} \otimes \rho_{(3)} \otimes \sigma_{4l} + \underline{c_{jk}^{(23)}} \rho_{(1)} \otimes \sigma_{2j} \otimes \sigma_{3k} \otimes \rho_{(4)} \Big] \\
&+ \frac{1}{2^3} \Big[\underline{c_{ijk}^{(123)}} \sigma_{1i} \otimes \sigma_{2j} \otimes \sigma_{3k} \otimes \rho_{(4)} + \underline{c_{ijl}^{(124)}} \sigma_{1i} \otimes \sigma_{2j} \otimes \rho_{(3)} \otimes \sigma_{4l} \\
&+ \underline{c_{ikl}^{(134)}} \sigma_{1i} \otimes \rho_{(2)} \otimes \sigma_{3k} \otimes \sigma_{4l} + \underline{c_{jkl}^{(234)}} \rho_{(1)} \otimes \sigma_{2j} \otimes \sigma_{3k} \otimes \sigma_{4l} \Big]
\end{aligned}
$$

4-body correlation

2-body correlations

3-body correlations

The polarizations and spin correlations can be extracted by taking expectation values of a direct product of Pauli matrices on spin density matrices.

Spin density matrix for vector mesons

We consider the combination process $q_1\bar{q}_2 \to V$. The spin density matrix of the vector meson is given by

$$\rho_V = \underline{\mathcal{M}}\rho_{(q_1\bar{q}_2)}\underline{\mathcal{M}^\dagger}$$

transition matrix for $q_1\bar{q}_2 \to V$

The spin density matrix element of vector meson

$$\left|m_{(12)}\right\rangle \equiv |j_1 m_1, j_2 m_2\rangle$$
$$\left|m'_{(12)}\right\rangle \equiv |j_1 m'_1, j_2 m'_2\rangle$$

$$\begin{aligned}
\rho^V_{mm'} &= \langle jm|\,\mathcal{M}\rho_{(q_1\bar{q}_2)}\mathcal{M}^\dagger\,|jm'\rangle \\
&= \sum_{m_{(12)},m'_{(12)}} \langle jm|\,\mathcal{M}\,\left|m_{(12)}\right\rangle \left\langle \underline{m_{(12)}}\right|\rho_{(q_1\bar{q}_2)}\left|\underline{m'_{(12)}}\right\rangle \left\langle m'_{(12)}\right|\mathcal{M}^\dagger\,|jm'\rangle \\
&\Rightarrow N_V \sum_{m_{(12)},m'_{(12)}} \underbrace{\langle jm|m_{(12)}\rangle}_{\text{CG coefficient}}\langle m_{(12)}|\hat{\rho}_{(q_1\bar{q}_2)}|m'_{(12)}\rangle\underbrace{\langle m'_{(12)}|jm'\rangle}_{\text{CG coefficient}}
\end{aligned}$$

Here we assumed $\langle jm|\mathcal{M}|j'm'\rangle = \delta_{jj'}\delta_{mm'}\langle jm|\mathcal{M}|jm\rangle$ **due to rotational symmetry of the transition matrix.**

Spin density matrix element for vector mesons

Spin density matrix elements of vector mesons are then obtained.
Diagonal elements:

$$\rho_{00}^{V} = \frac{1}{C_V}\left[1 + P_q^x P_{\bar{q}}^x + P_q^y P_{\bar{q}}^y - P_q^z P_{\bar{q}}^z + c_{xx}^{(q\bar{q})} + c_{yy}^{(q\bar{q})} - c_{zz}^{(q\bar{q})}\right]$$

$$\rho_{11}^{V} = \frac{1}{C_V}\left[\left(1 + P_q^z\right)\left(1 + P_{\bar{q}}^z\right) + c_{zz}^{(q\bar{q})}\right]$$

$$\rho_{-1,-1}^{V} = \frac{1}{C_V}\left[\left(1 - P_1^z\right)\left(1 - P_2^z\right) + c_{zz}^{(q\bar{q})}\right]$$

Normalization constant

$$\boxed{C_V = 3 + \boldsymbol{P}_q \cdot \boldsymbol{P}_{\bar{q}} + c_{xx}^{(q\bar{q})} + c_{yy}^{(q\bar{q})} + c_{zz}^{(q\bar{q})}}$$

Off-diagonal elements:

$$\begin{aligned}\rho_{10}^{V} = &\frac{1}{\sqrt{2}C_V}\left\{c_{zx}^{(q\bar{q})} + c_{xz}^{(q\bar{q})} + \left(1 + P_q^z\right)P_{\bar{q}}^x + P_q^x\left(1 + P_{\bar{q}}^z\right)\right.\\ &\left.-i\left[c_{zy}^{(q\bar{q})} + c_{yz}^{(q\bar{q})} + \left(1 + P_q^z\right)P_{\bar{q}}^y + P_q^y\left(1 + P_{\bar{q}}^z\right)\right]\right\}\end{aligned}$$

$$\begin{aligned}\rho_{0,-1}^{V} = &\frac{1}{\sqrt{2}C_V}\left\{-c_{zx}^{(q\bar{q})} - c_{xz}^{(q\bar{q})} + P_q^x\left(1 - P_{\bar{q}}^z\right) + \left(1 - P_q^z\right)P_{\bar{q}}^x\right.\\ &\left.+i\left[c_{zy}^{(q\bar{q})} + c_{yz}^{(q\bar{q})} - P_q^y\left(1 - P_{\bar{q}}^z\right) - \left(1 - P_q^z\right)P_{\bar{q}}^y\right]\right\}\end{aligned}$$

$$\begin{aligned}\rho_{1,-1}^{V} = &\frac{1}{C_V}\left[c_{xx}^{(q\bar{q})} - c_{yy}^{(q\bar{q})} + P_q^x P_{\bar{q}}^x - P_q^y P_{\bar{q}}^y\right.\\ &\left.-i\left(P_q^y P_{\bar{q}}^x + P_q^x P_{\bar{q}}^y + c_{xy}^{(q\bar{q})} + c_{yx}^{(q\bar{q})}\right)\right]\end{aligned}$$

Spin alignment of vector mesons as small effect

If we assume that the polarization and spin correlation are small effects, the average spin alignment is then in the form

$$\langle \rho_{00}^V \rangle \approx \frac{1}{3} + \frac{2}{9}\left[\langle P_q^x P_{\bar{q}}^x \rangle + \langle P_q^y P_{\bar{q}}^y \rangle - 2\langle P_q^z P_{\bar{q}}^z \rangle\right] + \frac{2}{9}\left[\left\langle c_{xx}^{(q\bar{q})}\right\rangle + \left\langle c_{yy}^{(q\bar{q})}\right\rangle - 2\left\langle c_{zz}^{(q\bar{q})}\right\rangle\right]$$

$$\langle \rho_{11}^V \rangle \approx \frac{1}{3} + \langle P_q^z \rangle + \langle P_{\bar{q}}^z \rangle - \frac{1}{9}\left[\langle P_q^x P_{\bar{q}}^x \rangle + \langle P_q^y P_{\bar{q}}^y \rangle - 2\langle P_q^z P_{\bar{q}}^z \rangle\right] - \frac{1}{9}\left[\left\langle c_{xx}^{(q\bar{q})}\right\rangle + \left\langle c_{yy}^{(q\bar{q})}\right\rangle - 2\left\langle c_{zz}^{(q\bar{q})}\right\rangle\right]$$

$$\langle \rho_{-1,-1}^V \rangle \approx \frac{1}{3} - \langle P_q^z \rangle - \langle P_{\bar{q}}^z \rangle - \frac{1}{9}\left[\langle P_q^x P_{\bar{q}}^x \rangle + \langle P_q^y P_{\bar{q}}^y \rangle - 2\langle P_q^z P_{\bar{q}}^z \rangle\right] - \frac{1}{9}\left[\left\langle c_{xx}^{(q\bar{q})}\right\rangle + \left\langle c_{yy}^{(q\bar{q})}\right\rangle - 2\left\langle c_{zz}^{(q\bar{q})}\right\rangle\right]$$

Correlation in spin polarization

Genuine correlation from dynamical processes

quadrupole structure

$$\boxed{\mathrm{Tr}\rho^V = 1}$$

Here the spin quantization direction is along +z direction. The three polarization vectors (direction of the vector field) for the vector meson in the rest frame are

$$\epsilon_0 = \boldsymbol{e}_z, \quad \epsilon_1 = -\frac{1}{\sqrt{2}}(\boldsymbol{e}_x + i\boldsymbol{e}_y), \quad \epsilon_{-1} = \frac{1}{\sqrt{2}}(\boldsymbol{e}_x - i\boldsymbol{e}_y)$$

Spin density matrix for hyperons

We consider the combination process $q_1 q_2 q_3 \to H$. The spin density matrix of the hyperon is

$$\rho_H = \underline{\mathcal{M}}\rho_{(q_1 q_2 q_3)}\underline{\mathcal{M}}^\dagger$$

transition matrix for $q_1 q_2 q_3 \to H$

The spin density matrix element of hyperon

$$\rho^H_{mm'} = N_H \sum_{m_{(123)},m'_{(123)}} \underbrace{\langle jm|m_{(123)}\rangle}_{\text{CG coefficient}} \langle m_{(123)}|\rho_{(q_1 q_2 q_3)}|m'_{(123)}\rangle \underbrace{\langle m'_{(123)}|jm'\rangle}_{\text{CG coefficient}}$$

Here we assumed $\langle jm|\mathcal{M}|j'm'\rangle = \delta_{jj'}\delta_{mm'}\langle jm|\mathcal{M}|jm\rangle$ due to rotational symmetry of the transition matrix. The hyperon polarization is defined as

$$P_H = \frac{A_H}{B_H} = \frac{\rho^H_{\frac{1}{2},\frac{1}{2}} - \rho^H_{-\frac{1}{2},-\frac{1}{2}}}{\rho^H_{\frac{1}{2},\frac{1}{2}} + \rho^H_{-\frac{1}{2},-\frac{1}{2}}}$$

Spin density matrix for hyperons

The result for Λ polarization is

$$P_\Lambda = P_s^z - \frac{\boxed{c_{iiz}^{(uds)} + c_{iz}^{(us)} P_d^i + c_{iz}^{(ds)} P_u^i}}{1 - \boxed{c_{ii}^{(ud)} - \boldsymbol{P}_u \cdot \boldsymbol{P}_d}}$$

$$\approx P_s^z - \left[c_{iiz}^{(uds)} + c_{iz}^{(us)} P_d^i + c_{iz}^{(ds)} P_u^i\right]$$

three-body correlation

two-body correlation

If three-body correlation is much smaller than two-body one

For other spin-1/2 hyperons, we obtain

$$P_{H_{aab}} = \frac{1}{3}(4P_a^z - P_b^z) + \frac{\boxed{\delta A_{H_{aab}}}}{\boxed{B_{H_{aab}}}}$$

$$P_{\Sigma^0} = \frac{1}{3}(2P_u^z + 2P_d^z - P_s^z) + \frac{\boxed{\delta A_{\Sigma^0}}}{\boxed{B_{\Sigma^0}}}$$

three-body correlation

1 + two-body correlation

$\delta A_{H_{aab}}$, $B_{H_{aab}}$, δA_{Σ^0}, B_{Σ^0} are given in Ref. [Lv, Yu, Liang, QW, Wang (2024)]

Spin correlation for hyperon-antihyperon

We consider the combination process $q_1 q_2 q_3 \bar{q}_4 \bar{q}_5 \bar{q}_6 \to H_1 \bar{H}_2$. The spin density matrix of $H_1 \bar{H}_2$ is

$$\rho_{H_1 \bar{H}_2} = \underline{\mathcal{M}} \rho_{(1\cdots 6)} \underline{\mathcal{M}^\dagger}$$

transition matrix for $q_1 q_2 q_3 \bar{q}_4 \bar{q}_5 \bar{q}_6 \to H_1 \bar{H}_2$

The spin density matrix elements of $H_1 \bar{H}_2$ are

$$\begin{aligned}&\rho_{H_1 \bar{H}_2}(m_{H_1} m_{\bar{H}_2}; m'_{H_1} m'_{\bar{H}_2}) \\ =& N_{H_1 \bar{H}_2} \sum_{m_{(1\cdots 6)}, m'_{(1\cdots 6)}} \underbrace{\langle m_{H_1} m_{\bar{H}_2} | m_{(1\cdots 6)} \rangle}_{\text{CG coefficient}} \langle m_{(1\cdots 6)} | \rho_{(1\cdots 6)} | m'_{(1\cdots 6)} \rangle \underbrace{\langle m'_{(1\cdots 6)} | m'_{H_1} m'_{\bar{H}_2} \rangle}_{\text{CG coefficient}}\end{aligned}$$

The spin correlation for $H_1 \bar{H}_2$ is usually defined as

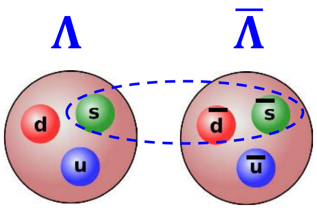
$$c_{nn}^{H_1 \bar{H}_2} = \frac{f^{++}_{H_1 \bar{H}_2} + f^{--}_{H_1 \bar{H}_2} - f^{+-}_{H_1 \bar{H}_2} - f^{-+}_{H_1 \bar{H}_2}}{f^{++}_{H_1 \bar{H}_2} + f^{--}_{H_1 \bar{H}_2} + f^{+-}_{H_1 \bar{H}_2} + f^{-+}_{H_1 \bar{H}_2}}$$

$$\boxed{f^{m_{H_1} m_{\bar{H}_2}}_{H_1 \bar{H}_2} = \langle m_{H_1} m_{\bar{H}_2} | \rho_{H_1 \bar{H}_2} | m_{H_1} m_{\bar{H}_2} \rangle}$$

Spin correlation for $\Lambda\bar{\Lambda}$

When all two-particle spin correlations ($c_{ij}^{(ab)} \neq 0$, all other $c_{i_1\cdots i_n}^{(q_1\cdots q_n)} = 0$ with $3 \leq n \leq 6$) are considered, the result for the spin correlation of $\Lambda\bar{\Lambda}$ is

$$
\begin{aligned}
c_{zz}^{\Lambda\bar{\Lambda}} =& P_s^z P_{\bar{s}}^z + \frac{1}{B_{\Lambda\bar{\Lambda}}}\left(c_{zz}^{(s\bar{s})} + \text{four and six body correlation}\right) \\
\approx& P_s^z P_{\bar{s}}^z + c_{zz}^{(s\bar{s})} + \text{four or more body correlation}
\end{aligned}
$$

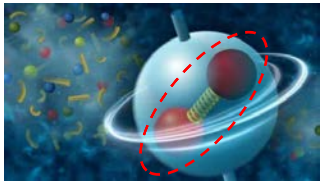


where $B_{\Lambda\bar{\Lambda}}$ is defined as

$$
B_{\Lambda\bar{\Lambda}} = 1 - \boldsymbol{P}_u \cdot \boldsymbol{P}_d - \boldsymbol{P}_{\bar{u}} \cdot \boldsymbol{P}_{\bar{d}} - c_{ii}^{(ud)} - c_{ii}^{(\bar{u}\bar{d})} + \text{four body correlation}
$$

We can take average of $c_{zz}^{\Lambda\bar{\Lambda}}$ over all $\Lambda\bar{\Lambda}$ events

$$
\left\langle c_{zz}^{\Lambda\bar{\Lambda}} \right\rangle \approx \langle P_s^z P_{\bar{s}}^z \rangle + \left\langle c_{zz}^{(s\bar{s})} \right\rangle + \text{four or more body correlation}
$$



In $\Lambda\bar{\Lambda}$ correlation, s is in Λ and $\bar{s}$ is in $\bar{\Lambda}$, i.e. the correlation of constituent quarks in different particles → long range correlation

In spin alignment of ϕ meson $\left\langle \rho_{00}^{\phi} \right\rangle$, there are also $\langle P_s^z P_{\bar{s}}^z \rangle$ and $\left\langle c_{zz}^{(s\bar{s})} \right\rangle$, but the average here is taken inside the ϕ meson → short range correlation

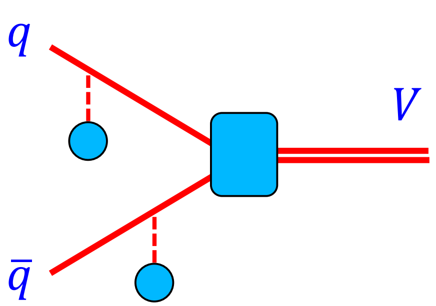
Two examples of spin correlations in HIC

- **Spin correlation in ϕ meson's spin alignment;**
- **$\Lambda\Lambda$'s spin correlation as probe to vortical structure of sQGP**

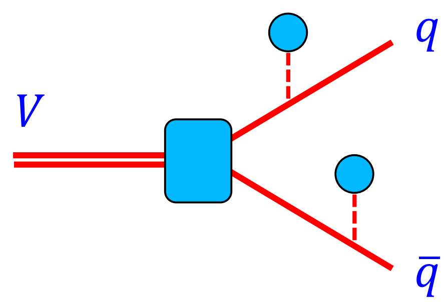
Relativistic Spin Boltzmann (Kinetic) Equation for vector mesons in quark coalescence model

Sheng, Oliva, et al., 2206.05868, 2205.15689

Review on QKE and SKE based on Wigner functions:
Hidaka, Pu, QW, Yang, Prog. Part. Nucl. Phys. 127 (2022) 103989



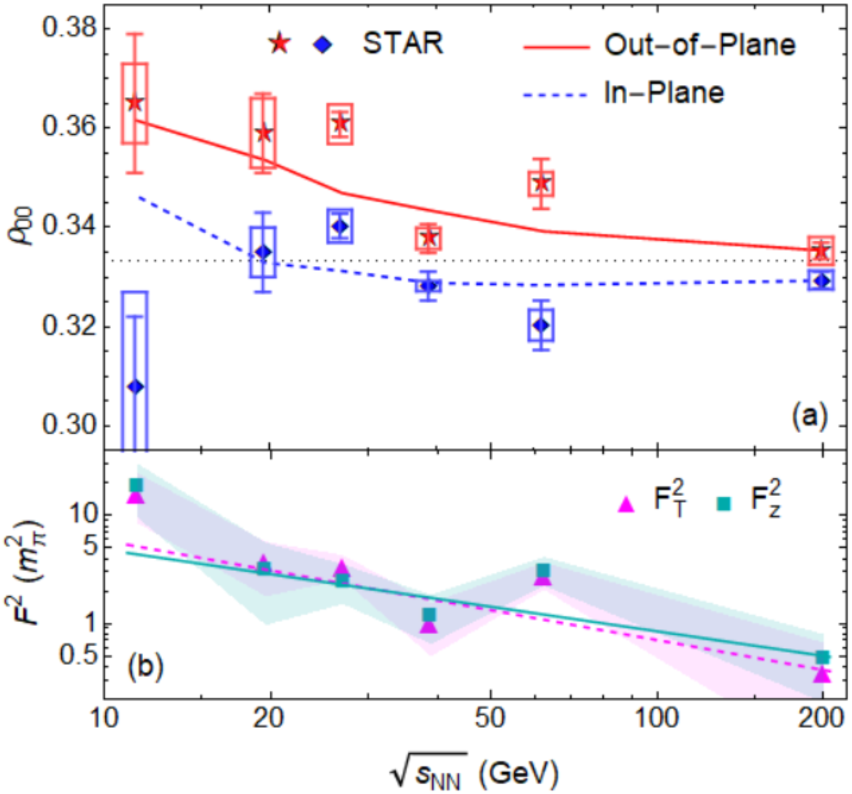
Quark coalescence to V-meson



V-meson dissociation to quarks

Quark coalescence model:
Greco, Ko, Levai (2003);
Fries, Mueller et al (2003);
Yang, Hwa (2003).

Spin density matrix element for vector mesons

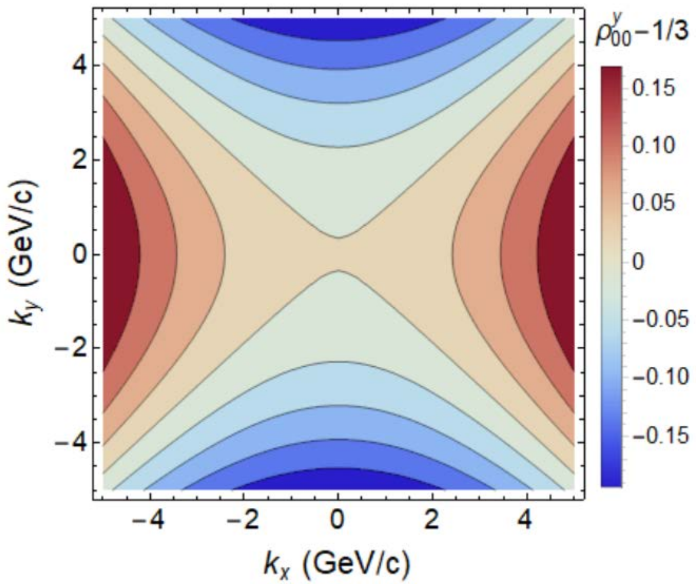


(a) The STAR's data on phi meson's ρ_{00}^{y} (out-of-plane, red stars) and ρ_{00}^{x} (in-plane, blue diamonds) in 0-80% Au+Au collisions as functions of collision energies. The red-solid line and blue-dashed line are calculated with values of F_T^2 and F_z^2 from fitted curves in (b).

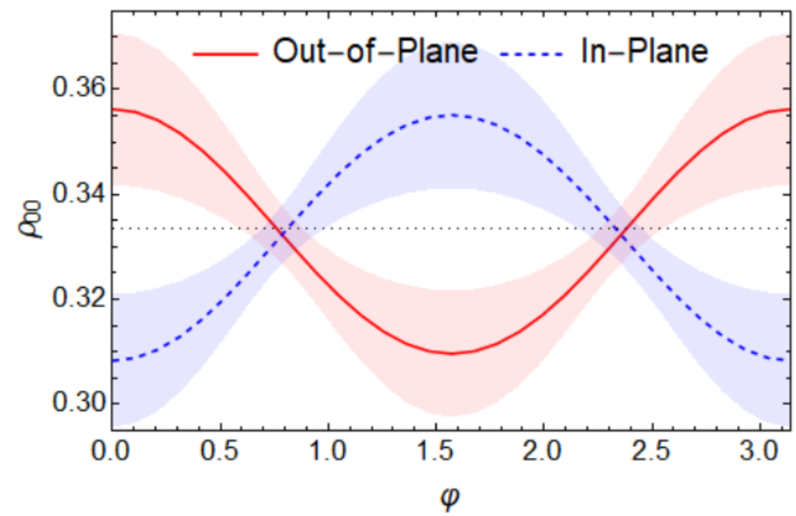
(b) Values of F_T^2 (magenta triangles) and F_z^2 (cyan squares) with shaded error bands extracted from the STAR's data on the phi meson's ρ_{00}^{y} and ρ_{00}^{x} in (a). The magenta-dashed line (cyan-solid line) is a fit to the extracted F_T^2 (F_z^2) as a function of $\sqrt{s_{NN}}$.

Sheng, Lucia, Liang, QW, et al, 2205.15689, 2206.05868

Spin density matrix element for vector mesons

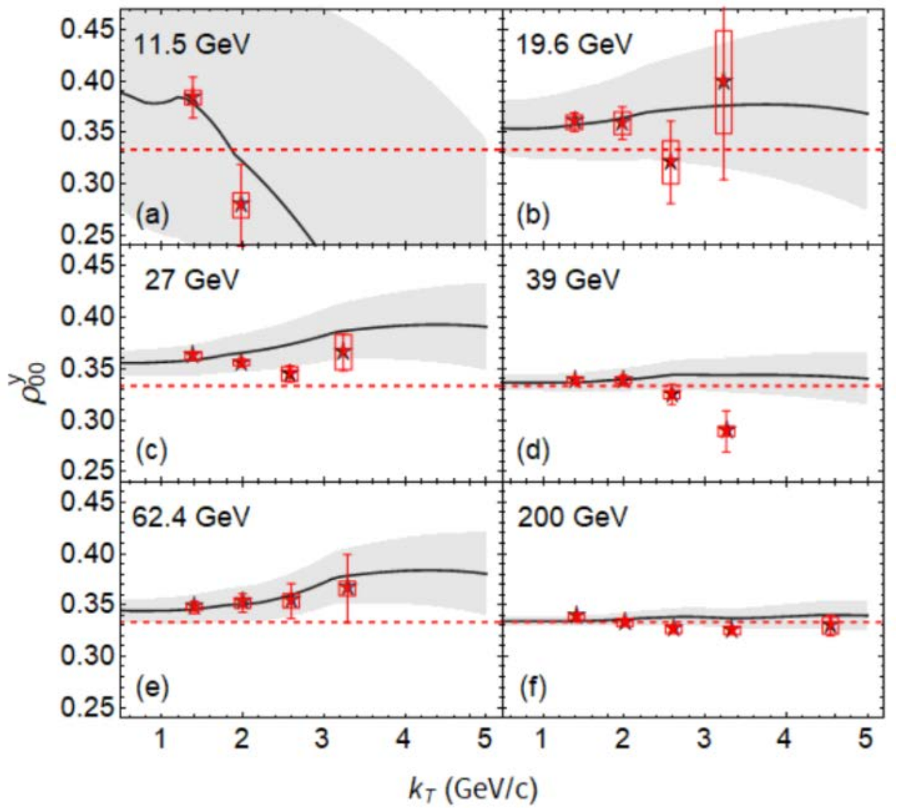


Contour plot of $\rho^y_{00}-1/3$ for ϕ mesons as a function of k_x and k_y in 0-80% Au+Au collisions at $\sqrt{s_{NN}}=200$ GeV.



Calculated ρ^y_{00} (out-of-plane) and ρ^x_{00} (in plane) of ϕ mesons as functions of the azimuthal angle φ in 0-80% Au+Au collisions at $\sqrt{s_{NN}}=200$ GeV. Shaded error bands are from the extracted parameters F^2_T and F^2_z.

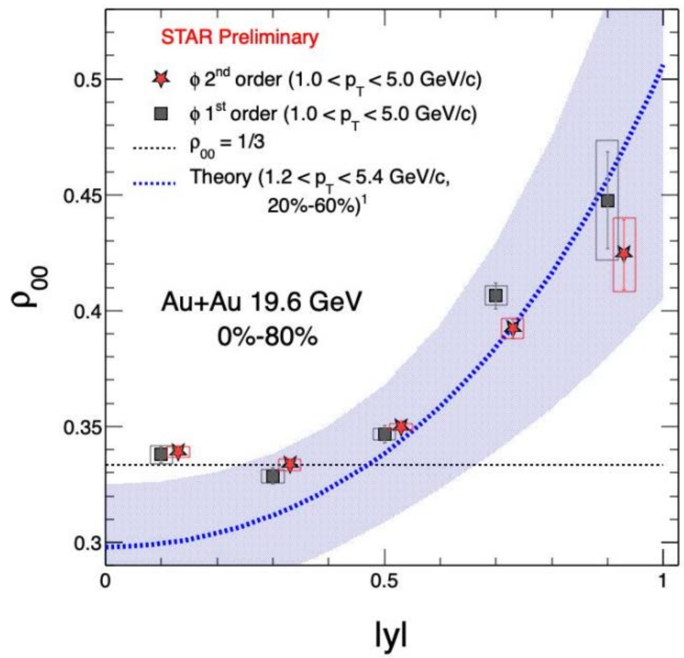
Spin density matrix element for vector mesons

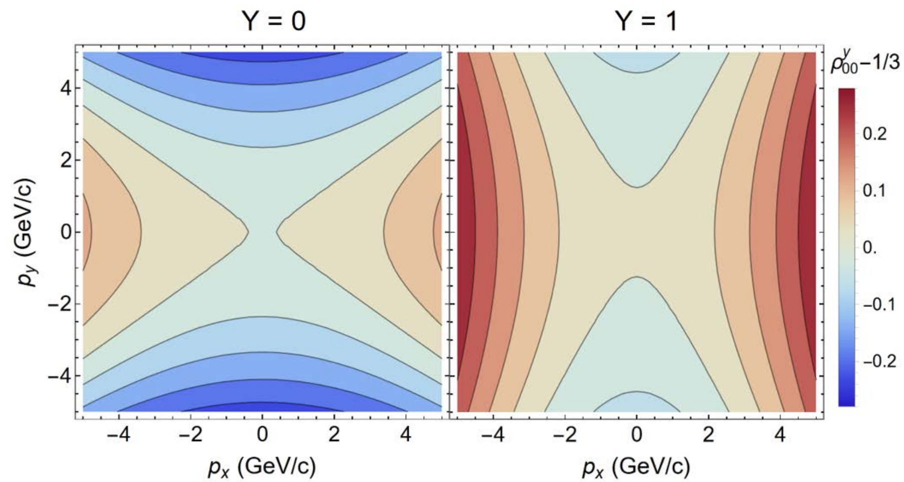


Calculated ρ_{00}^{y} (solid line) of ϕ mesons as functions of transverse momenta in 0-80% Au+Au collisions at different colliding energies in comparison with STAR data. Shaded error bands are from the extracted parameters F_T^2 and F_z^2 .

Sheng, Lucia, Liang, QW, et al, 2205.15689, 2206.05868

Our prediction on rapidity dependence of ρ_{00}^{y}





Sheng, Pu, QW, PRC(2023); 2308.14038

If B^2 and E^2 is isotropic in all directions in lab frame, we have simple formula with clear physics

$$\langle \delta\rho_{00}^{y}\rangle(\mathbf{p}) = \frac{8}{3m_\phi^4}(C_1+C_2)F^2\left(\frac{p_x^2+p_z^2}{2}-p_y^2\right)$$

$$\propto \frac{1}{2}p_T^2\left[3\cos(2\varphi)-1\right]+\left(m_\phi^2+p_T^2\right)\sinh^2 Y$$

Spin correlation of ΛΛ as probe to vortical structure of QGP fluid

PRL **117,** 192301 (2016) PHYSICAL REVIEW LETTERS week ending 4 NOVEMBER 2016

Vortical Fluid and Λ Spin Correlations in High-Energy Heavy-Ion Collisions

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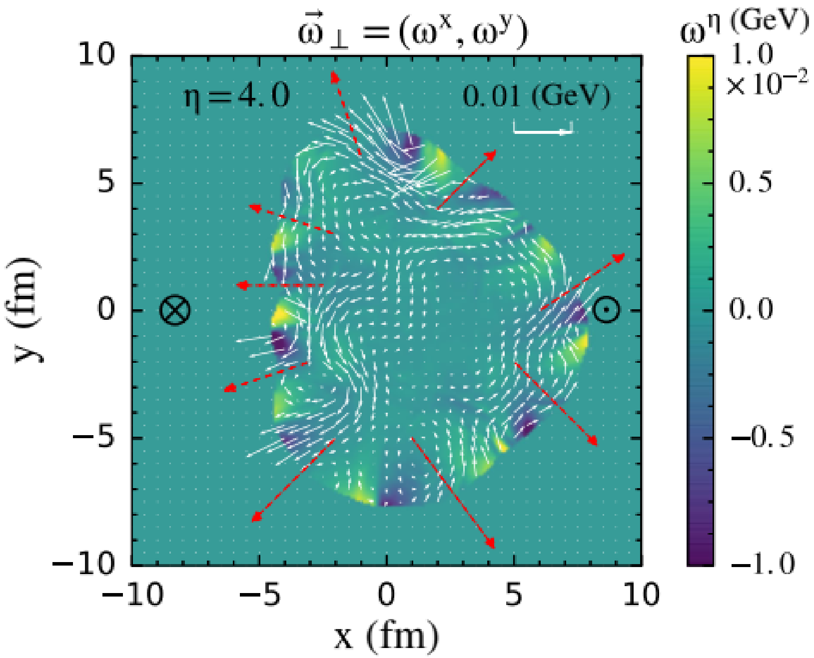
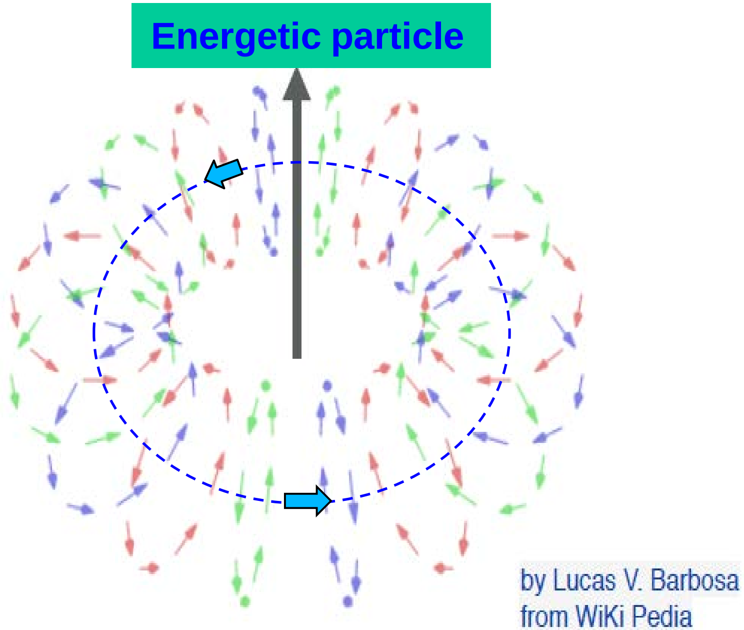
(Received 16 May 2016; published 1 November 2016)

For details, see Xin-Nian Wang's talk on Wednesday

Turbulence and vortex rings in high energy HIC

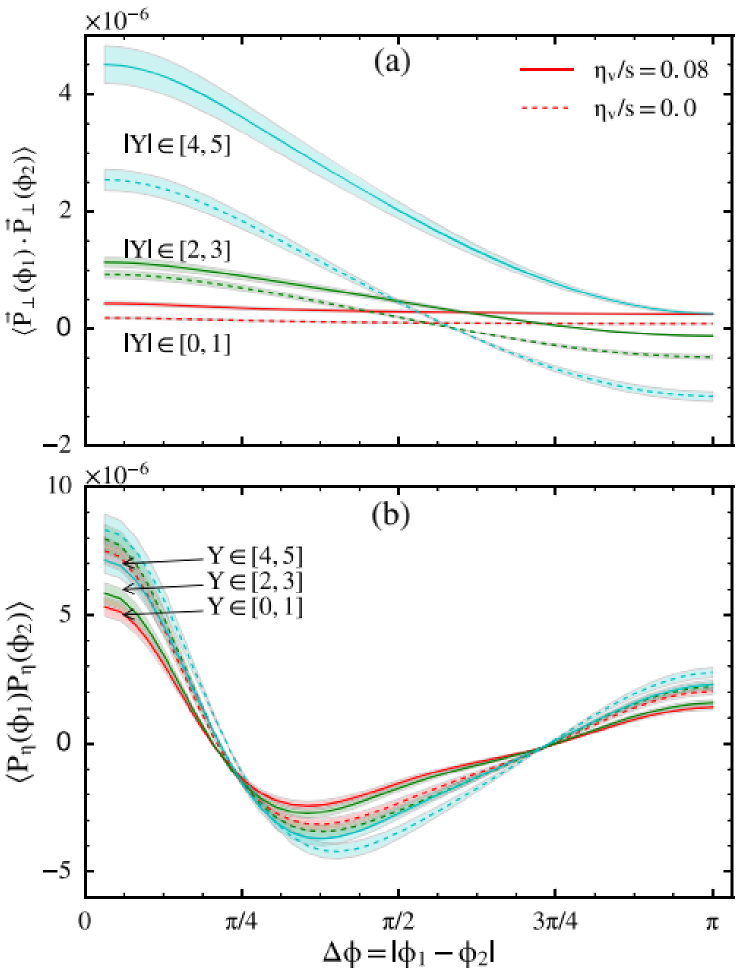


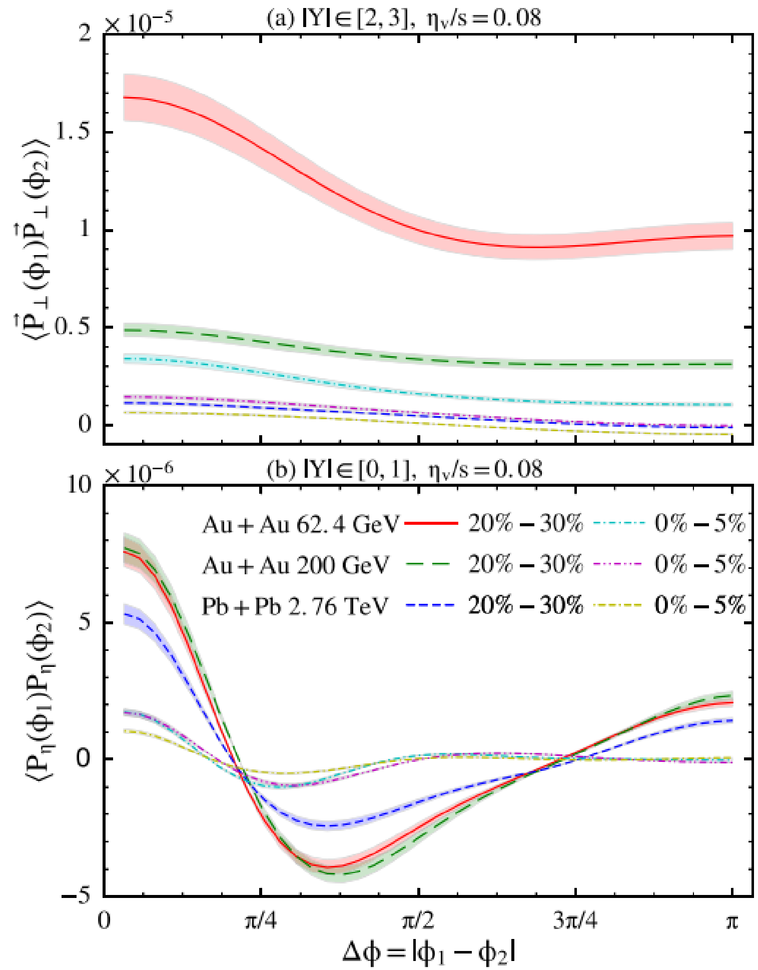
Pang, Petersen, QW, Wang (2016);
Xia, Li, Tang, QW (2018);
Lisa, Barbon, Serenone, Shen (2021)



Spin correlation of Λ Λ can probe the vortical structure in sQGP

Spin correlation of ΛΛ in vortical fluid





Summary

Take-home message:

Spin correlation is a new tool to study quantum properties of sQGP.