

Negative magnetoresistance in Dirac Semimetals from Keldysh technique

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CKT and CME

The renowned theory suggests [Fukushima, Kharzeev, Warringa Phys.Rev.D78 074033 (2008), Li and Kharzeev et al. Nature Physics 12, 550-554 (2016)], that in a system that hosts massless Dirac fermions, chiral anomaly pumps up chiral density

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which via CME

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yields (actually, observed!) magnetoconductivity

$$\sigma_{CME}^{ij} = \frac{3}{2} N_f \frac{H^i H^j}{4\pi^2} \frac{v_F^3}{\pi^2 T^2 + \mu^2} \tau_5 \quad (4)$$

The gap in real materials

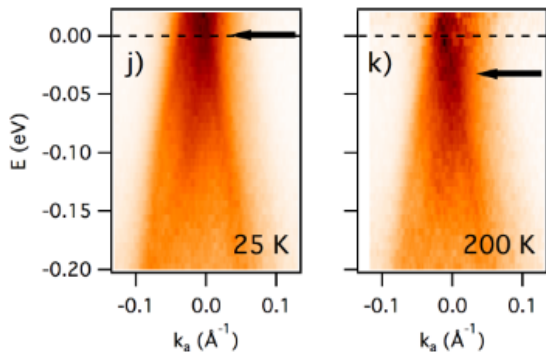


Figure: ARPES for ZrTe_5 [Li and Kharzeev et al. Nature Physics 12, 550-554 (2016)]

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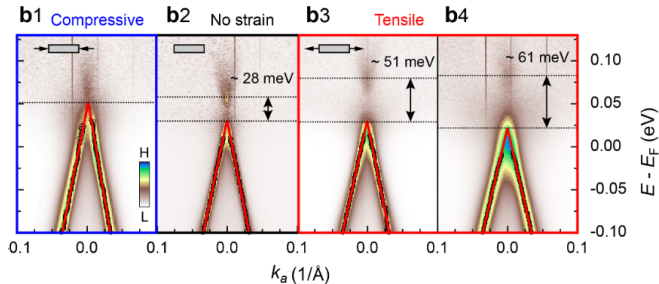


Figure: ARPES for $ZrTe_5$ [Zhang et al. Nature Communications 12, 406 (2021)]

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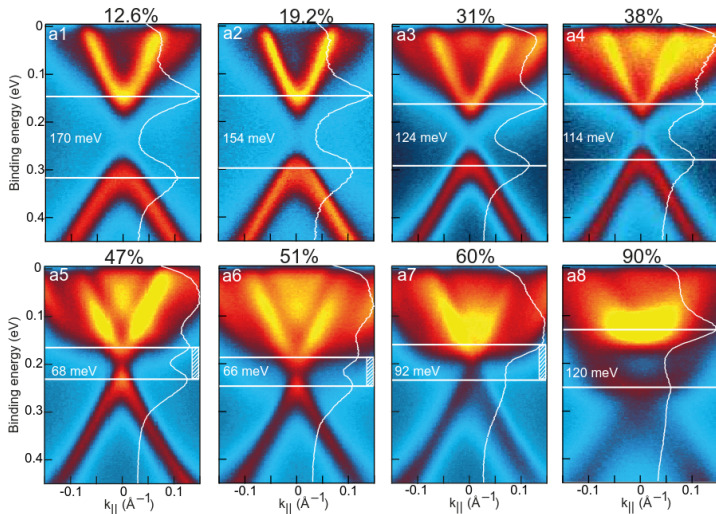


Figure: ARPES for $Mn_{1-x}Ge_xBi_2Te_4$ [Shikin et al. arXiv:2406.15065]

An idea for the chirality relaxation mechanism

Though the gap $2m$ can be negligible in comparison to the chemical potential μ (the non-ideal Fermi-point is deep in the Fermi sphere), the gap indicates the chiral symmetry breaking, which, with some energy dissipation mechanism, can constitute a chirality relaxation mechanism

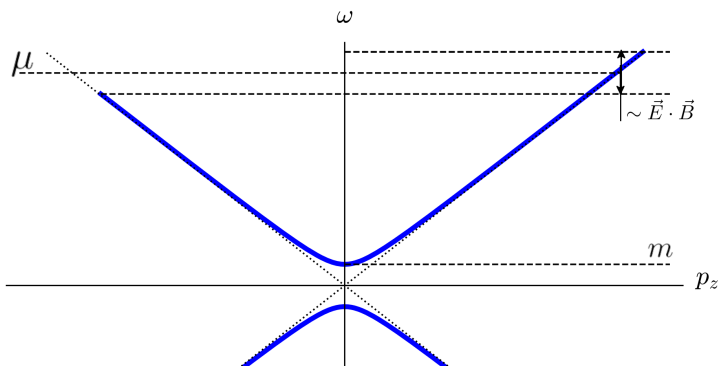


Figure: A sketch of the non-equilibrium stationary population density that accompanies the magnetoconductivity

Non-equilibrium Diagram Technique for relativistic fermions

For Dirac fermions (let $v_F, e \rightarrow 1$, etc.)

$$\mathcal{L}_0 = \bar{\psi}(\gamma^\mu(\partial_\mu - ieA_\mu) - m)\psi, \quad (5)$$

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Keldysh-Green's functions satisfy

$$\hat{Q} = \begin{pmatrix} Q^R & Q^< \\ 0 & Q^A \end{pmatrix}, \quad \hat{G} = \begin{pmatrix} G^R & G^< \\ 0 & G^A \end{pmatrix} \equiv \hat{Q}^{-1}, \quad (6)$$

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$$Q^{R(A)} = p_0 \pm i\epsilon - (\alpha_i p^i + \gamma_0 m), \quad Q^< = -2i\epsilon n(p_0), \quad (7)$$

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$$G^{R(A)} = Q^{R(A)-1} = (p_0 \pm i\epsilon - (\alpha_i p^i + \gamma_0 m))^{-1}, \quad (8)$$

$$G^< = -G^R Q^< G^A = (G^A - G^R)n(p_0) \quad (9)$$

Non-equilibrium Diagram Technique for electron-phonon interaction

$$\hat{H}_{\text{e-ph}} = \int d^3x \frac{w \hat{\rho}'}{\rho_0} \hat{\psi}^\dagger \hat{\psi}, \quad (10)$$

where w , ρ_0 — crystal's elastic properties.

Non-equilibrium Diagram Technique for electron-phonon interaction

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$$N_{\mathbf{k}} = (e^{\beta u k} - 1)^{-1}, \text{ where } u = \frac{s}{v_F} \ll 1 \quad (11)$$

is speed of sound for the acoustic phonons, so the effective temperature for the phonons $T_{\text{eff}} = \frac{T}{u} \gg T$ than the electrons temperature.

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is speed of sound for the acoustic phonons, so the effective temperature for the phonons $T_{eff} = \frac{T}{u} \gg T$ than the electrons temperature. And the Keldysh-Green's functions

$$D^R = D^{--} - D^{-+} = \frac{\rho_0 k^2}{(\omega + i\epsilon)^2 - u^2 k^2}, \quad (12)$$

$$D^< = D^{-+} = -i\pi \frac{\rho_0 k}{u} (N_k \delta(\omega - uk) + (1 + N_{-k}) \delta(\omega + uk))$$

Magnetoconductivity from NDT

$$j_i^{(2)} = \gamma_i \left(\begin{array}{c} \pm \\ \pm \\ \pm \end{array} \right) + \text{dozens of diagrams}$$

$$\sim E_j H^j H_i,$$

Magnetoconductivity from NDT

$$j_i^{(2)} = \gamma_i \left[\begin{array}{c} \text{---} \oplus \text{---} \\ \text{---} \oplus \text{---} \\ \text{---} \oplus \text{---} \end{array} \right] + \text{dozens of diagrams}$$

$$\approx \gamma_i \left[\begin{array}{c} \text{---} \oplus \text{---} \\ \text{---} \oplus \text{---} \end{array} \right] E_j \sim E_j H^j H_i,$$

$$\longrightarrow = \begin{array}{c} H \otimes \dots \otimes H \\ \text{---} \longrightarrow \end{array},$$

Effective diagram technique. Constant width

$$\hat{G}_{(1)} = \rightarrow_{\pm} \sum_{n=0}^{+\infty} \left(\begin{array}{c} \text{---} \text{---} \text{---} \\ \text{---} \text{---} \text{---} \\ \text{---} \text{---} \text{---} \end{array} \right)^n = (\hat{Q} - \hat{\Sigma})^{-1}, \quad (13)$$

$$G^R = (p_0 \pm i\epsilon - \mathcal{H})^{-1}, \quad (14)$$

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Electron coupling to the thermal bath of acoustic phonons provides an energy dissipation mechanism.

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Electron coupling to the thermal bath of acoustic phonons provides an energy dissipation mechanism. We estimate the energy dissipation rate at the Fermi surface

$$\epsilon \approx -\text{Im} \frac{1}{4} \text{tr} \Sigma_R(p_0 \sim \mu, |\mathbf{p}| \sim \mu) \quad (15)$$

$$= -\text{Im} \frac{1}{4} \text{tr} \left[ig^2 G^R D^< \right] \Big|_{(p_0 \sim \mu, |\mathbf{p}| \sim \mu)} \quad (16)$$

$$\dots \approx \frac{1}{2\pi} \frac{w^2}{\rho_0} \frac{\mu^2 T}{u^2}, \quad \frac{u\mu}{T} \ll 1. \quad (17)$$

Effective diagram technique. Weak magnetic field

(18)

$$G_H^{R(A)} \gamma_0 = \text{---} \rightarrow \text{---} \begin{array}{c} H \otimes \\ \text{wavy} \\ \end{array} \dots \begin{array}{c} \otimes H \\ \text{wavy} \\ \end{array}$$

(19)

(21)

Effective diagram technique. Weak magnetic field

$$\not{D}^2 = D^2 + \frac{1}{2} \sigma_{\mu\nu} i \mathcal{F}^{\mu\nu} \quad (18)$$

$$G_H^{R(A)} \gamma_0 = \text{---} \text{---} \text{---} \text{---} \text{---} \text{---} \quad (19)$$


A Feynman diagram representing the Green's function $G_H^{R(A)} \gamma_0$. It consists of a horizontal solid line with an arrow pointing to the right. Above this line, there are two vertices, each marked with a circled cross (\otimes). Wavy lines connect these two vertices. An ellipsis (\dots) is placed between the two vertices, indicating a continuation of the chain.

(21)

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$$G_H^{R(A)}\gamma_0 = \begin{array}{c} H \otimes \dots \otimes H \\ \downarrow \quad \quad \quad \downarrow \\ \text{---} \longrightarrow \text{---} \end{array} \quad (19)$$

$$= (\not{D}_\pm + m)(D_\pm^2 - m^2 - \Sigma H)^{-1} \quad (20)$$

$$= (\not{D}_\pm + m)\left((D_\pm^2 - m^2)\left(1 - \frac{\Sigma H}{D_\pm^2 - m^2}\right)\right)^{-1} \\ \approx G^{R(A)}\gamma_0\left(1 + \frac{\Sigma H}{D_\pm^2 - m^2} + \frac{\frac{1}{4}H^2}{(D_\pm^2 - m^2)^2}\right), \quad (21)$$

where $D_\pm^\mu = \partial^\mu - ie\mathcal{A}_H^\mu \pm \delta^{\mu 0}\epsilon$.

Effective diagram technique. Weak magnetic field

$$\not{D}^2 = D^2 + \frac{1}{2}\sigma_{\mu\nu}i\mathcal{F}^{\mu\nu} \rightarrow -\not{p}^2 + \Sigma H, \quad (18)$$

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where $D_{\pm}^{\mu} = \partial^{\mu} - ie\mathcal{A}_H^{\mu} \pm \delta^{\mu 0}\epsilon$.

Magnetoconductivity and chiral density from NDT

$$j^\mu = \text{tr } \gamma^0 \gamma^\mu iG_1^<(x, x) \sim \gamma^\mu \text{ (loop diagram) } \otimes E_j, \quad (22)$$

the same for $\rho_5 = \text{tr } \gamma_5 iG_1^<(x, x)$.

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The diagram shows a fermion loop with two vertices. The left vertex is a circle with a dot, and the right vertex is a circle with a cross. A wavy line connects the two vertices, labeled with a cross and the index j . The loop has two arrows indicating a clockwise direction.

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$$G_{p+q,p}^{1-+} = - \text{ (diagram) } + = G_{p+q}^{--} U_q^- G_p^{-+} + G_{p+q}^{-+} U_q^+ G_p^{++}.$$

The diagram shows a fermion line with a wavy line loop. The wavy line is labeled U_q^\pm and has a cross. The fermion line has a cross on the left and a dot on the right. The line is labeled with $-$ on the left and $+$ on the right.

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The diagram shows a fermion loop with two vertices. The left vertex is a small circle with a dot, and the right vertex is a wavy line with a circle containing a cross. The loop is labeled with a \pm sign.

the same for $\rho_5 = \text{tr } \gamma_5 iG_1^<(x, x)$.

$$G_{p+q,p}^{1--} = - \text{ (diagram) } + = G_{p+q}^{--} U_q^- G_p^{-+} + G_{p+q}^{-+} U_q^+ G_p^{++}.$$

The diagram shows a fermion line with a wavy line loop. The wavy line is labeled U_q^\pm and has a circle with a cross. The fermion line has a \pm sign below it.

$$U_q^\pm = \pm \gamma_0 \gamma_j \mathcal{A}^j(q), \quad \mathcal{A}^j(q) = -iE^j (2\pi)^4 \delta(q) \delta'(q_0), \quad (23)$$

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$$U_q^\pm = \pm \gamma_0 \gamma_j \mathcal{A}^j(q), \quad \mathcal{A}^j(q) = -iE^j (2\pi)^4 \delta(q) \delta'(q_0), \quad (23)$$

$$G_{1p}^< = -2iE^j \text{Re}(\partial_0 G_p^< \alpha_j G_p^A) \quad (24)$$

Magnetoconductivity and chiral density from NDT

With the Pauli-Villars regularization

$$\rho_5 = 2 \operatorname{Re} \int_p^{(R)} \operatorname{tr} \gamma_5 E^j \partial_0 G_H^< \alpha_j G_H^A \quad (25)$$

(27)

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$$\dots \approx \frac{E^j H_j}{4\pi^2 \epsilon} \int_0^{+\infty} P^2 dP \left(\frac{\Theta(\mu - E)}{E^3} + \frac{1}{E^3} - \frac{1}{E_R^3} \right) \quad (26)$$

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$$\approx \frac{E^j H_j}{4\pi^2} \frac{1}{2\epsilon} \ln \left(4 \frac{\mu^2}{m^2} \right), \quad \mu^2 \gg m^2. \quad (27)$$

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$$\sigma_{jk} = 2 \operatorname{Re} \int_p^{(R)} \operatorname{tr} \alpha_k \partial_0 G_H^< \alpha_j G_H^A \rightarrow \sigma_{jk}^{(1)} = 0, \quad \sigma_{jk}^{(2)}. \quad (28)$$

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$$\sigma_{jk}^{(2)} \approx \frac{3}{2} \frac{H_j H_k}{4\pi^2} \frac{1}{\epsilon^2} \frac{1}{2\epsilon} \ln \left(\frac{4\mu^2}{m^2} \right) \quad (29)$$

Magnetoconductivity and chiral density from NDT

In the limit $\mu^2 \gg m^2$, $\frac{u\mu}{T} \ll 1$ with the dissipation rate ϵ substituted

$$\rho_5 \approx \frac{E^j H_j}{4\pi^2} \frac{2\pi\rho_0 u^2}{w^2 \mu^2 T} \frac{1}{2} \ln\left(4 \frac{\mu^2}{m^2}\right), \quad (30)$$

$$\sigma_{jk}^{(2)} \approx \frac{3}{2} \frac{H_j H_k}{4\pi^2} \left(\frac{2\pi\rho_0 u^2}{w^2 \mu^2 T}\right)^3 \frac{1}{2} \ln\left(\frac{4\mu^2}{m^2}\right) \quad (31)$$

Results summary

CKT & CME

our NDT calculation

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$$\rho_5 = \frac{E^j H_j}{4\pi^2} \tau_5$$

our NDT calculation

$$\rho_5 \approx \frac{E^j H_j}{4\pi^2} \frac{1}{2\epsilon} \ln \left(4 \frac{\mu^2}{m^2} \right)$$

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CKT & CME

$$\rho_5 = \frac{E^j H_j}{4\pi^2} \tau_5$$

$$\sigma_{ij}^{CME} = \frac{3}{2} \frac{H_i H_j}{4\pi^2} \frac{v_F^3}{\pi^2 T^2 + \mu^2} \tau_5$$

our NDT calculation

$$\rho_5 \approx \frac{E^j H_j}{4\pi^2} \frac{1}{2\epsilon} \ln \left(4 \frac{\mu^2}{m^2} \right)$$

$$\sigma_{jk}^{(2)} \approx \frac{3}{2} \frac{H_j H_k}{4\pi^2} \frac{1}{\epsilon^2} \frac{1}{2\epsilon} \ln \left(\frac{4\mu^2}{m^2} \right)$$

$$\text{where } \epsilon \approx \frac{1}{2\pi} \frac{w^2}{\rho_0} \frac{\mu^2 T}{u^2},$$

$$\text{at } m^2 \ll \mu^2 \text{ and } \frac{u\mu}{T} \ll 1$$

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our NDT calculation

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$$\text{where } \epsilon \approx \frac{1}{2\pi} \frac{w^2}{\rho_0} \frac{\mu^2 T}{u^2},$$

$$\text{at } m^2 \ll \mu^2 \text{ and } \frac{u\mu}{T} \ll 1$$

$$\tau_5 \sim \frac{1}{2\epsilon} \ln \left(\frac{4\mu^2}{m^2} \right) \sim \frac{\pi \rho_0 u^2}{w^2 \mu^2 T} \ln \left(\frac{4\mu^2}{m^2} \right) \quad (32)$$

Backup Slides. Strong magnetic field limit

At $2eH > \mu^2 \gg m^2$

$$\hat{G}_{p_0 p_2 p_3}(x_1, x_2) \approx \hat{G}_{p_0 p_2 p_3}^{(LLL)}(x_1, x_2) =$$

(33)

Backup Slides. Strong magnetic field limit

At $2eH > \mu^2 \gg m^2$

$$\begin{aligned}\hat{G}_{p_0 p_2 p_3}(x_1, x_2) &\approx \hat{G}_{p_0 p_2 p_3}^{(LLL)}(x_1, x_2) = \\ &= \hat{G}_{p_0 p_3} \mathcal{O}^{-\sqrt{\frac{H}{\pi}}} e^{-\frac{H}{2} \left((x_1 - \frac{p_2}{H})^2 + (x_2 - \frac{p_2}{H})^2 \right)},\end{aligned}\quad (33)$$

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where

$$O^- = \frac{1}{2}(1 + i\gamma_1\gamma_2 \text{sign}(eH_3))\quad (34)$$

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The reduced 2D Keldysh-Green functions are

$$\tilde{G}^{R(A)} = (p_0 \pm i\epsilon_H - \alpha_3 p_3 - \gamma_0 m)^{-1},\quad (35)$$

$$\tilde{G}^< = (\tilde{G}^A - \tilde{G}^R)n(p_0), \quad \tilde{E} = \sqrt{p_3^2 + m^2}\quad (36)$$

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Then the result reads

$$\rho_5 = \frac{E_j H^j}{2\pi^2} \frac{1}{2\epsilon_H} \ln \left(4 \frac{\mu^2}{m^2} \right), \quad (37)$$

$$\sigma_{kj} = \frac{H_j H_k}{2\pi^2 |\vec{H}|} \frac{1}{2\epsilon_H} \ln \left(4 \frac{\mu^2}{m^2} \right), \quad 2eH > \mu^2 \gg m^2, \quad (38)$$

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where to fix the dissipation rate expression, the density of states at the Fermi sphere surface is to be replaced with the density of state at the Landau level, $\mu^2 \rightarrow |H|$

$$\epsilon_H \approx \frac{w^2}{\rho_0} \frac{T}{u^2} \frac{|H|}{4\sqrt{2}}. \quad (39)$$