中国科学院大学
University of Chinese Academy of Sciences

The 8th International Conference on Chirality, Vorticity and Magnetic Field in Quantum Matter

Magnetic catalysis and diamagnetism from pion fluctuations

arXiv: 2402.19193 (paper submitting)

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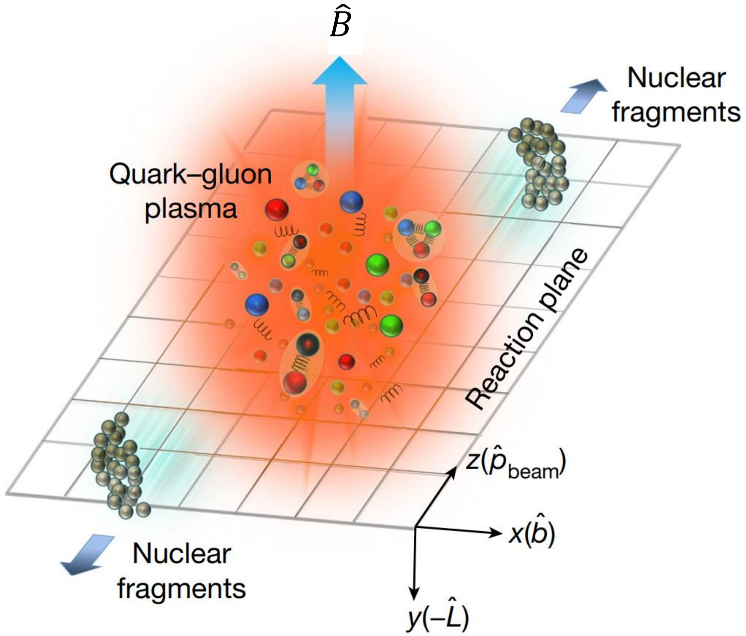
2024.7.25

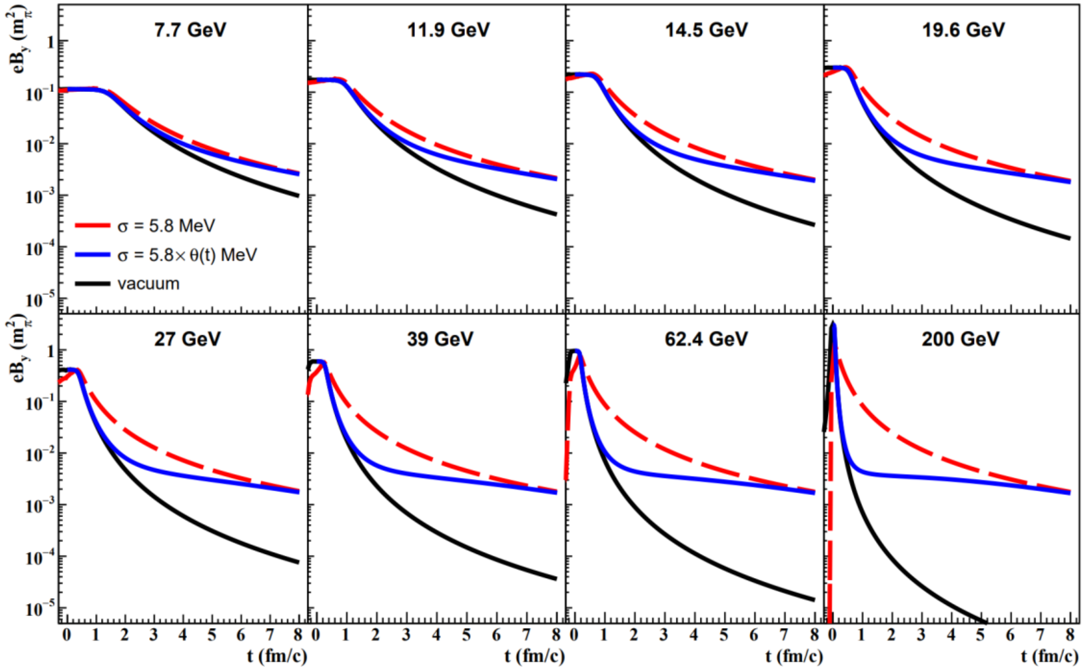
Outline

- Motivation
- Formalism
  - Formalism in Mean-Field Level (pure quark)
  - Formalism beyond Mean-Field Level (feedback from mesons)
- Numerical Results
- Conclusion

Motivation

- In the initial stage of non-central HIC, an intense but transient magnetic field can be generated.



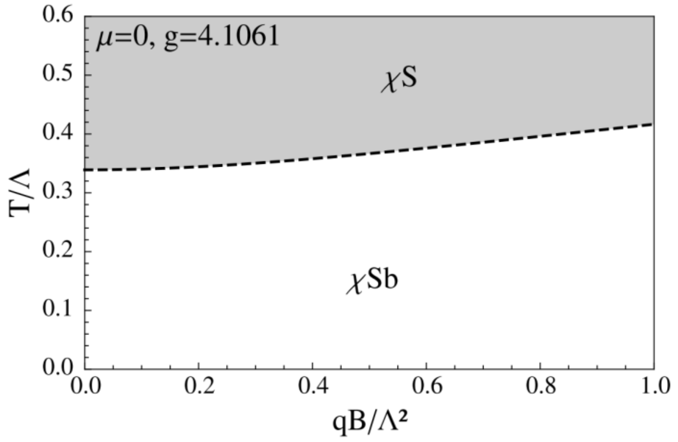
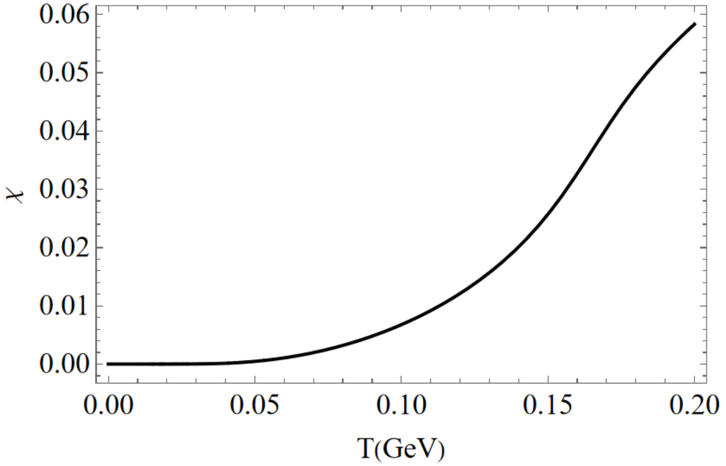
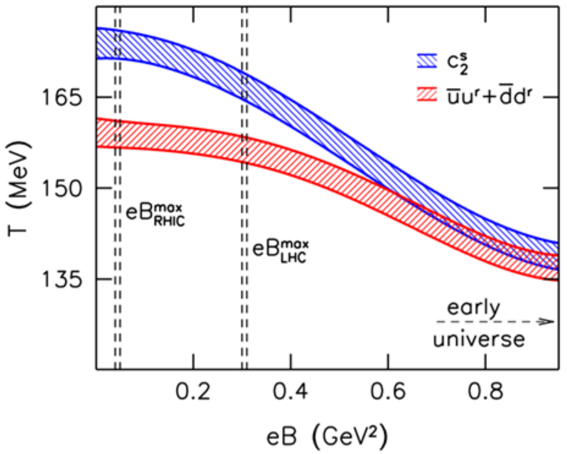
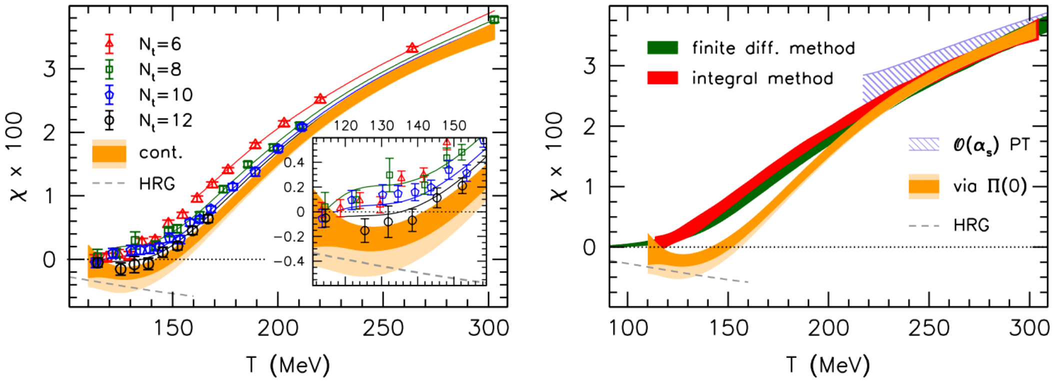


Strength: 10^{18-20}Gauss
Life time of eB: $\sim 10^{-23}$s
Life time of QGP: $\sim 10^{-23}$s

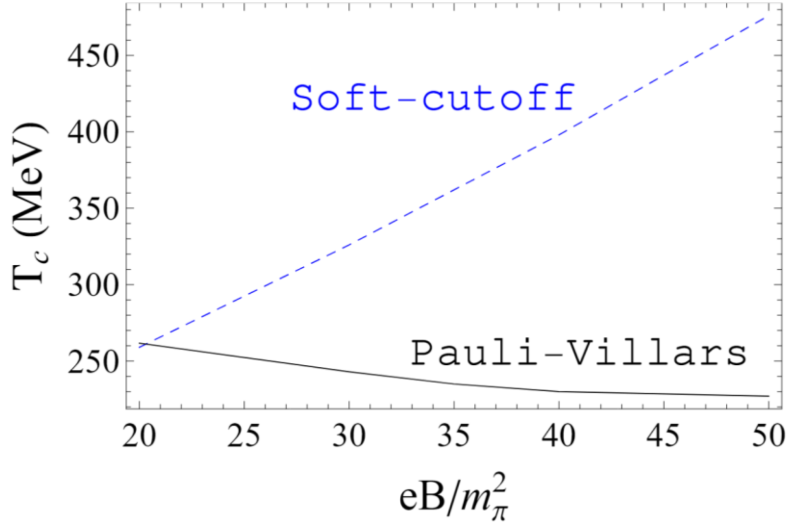
Hui Li and Xiao-Liang Xia and Xu-Guang Huang and Huan Zhong Huang, Phys. Rev. C 108 (2023) 4, 044902

- Methods: Lattice QCD, HQCD, Effective Theories (NJL Model, QM Model), FRG Approach…

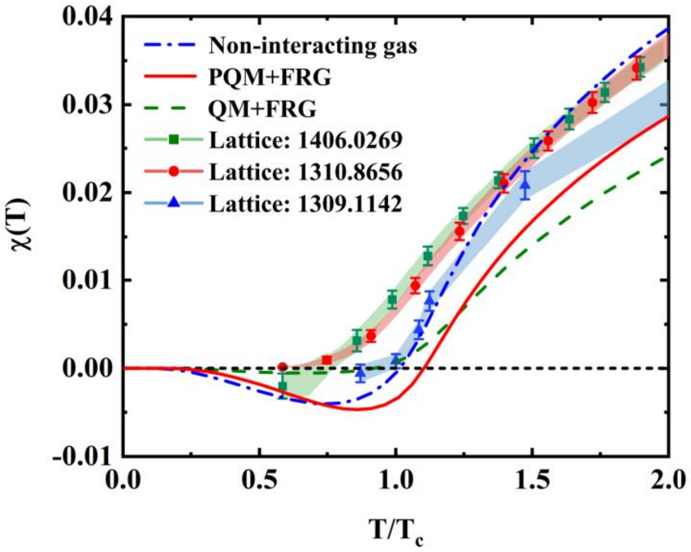
Motivation

	T_c versus eB	Magnetic Susceptibility
Effective Model Prediction	 R. Gatto and M. Ruggieri, Lect. Notes Phys. 871, 87 (2013).	
LQCD	 G. S. Bali et al, J. High Energy Phys. 02 (2012) 044	 G. S. Bali et al, J. High Energy Phys. 07 (2020) 183

Motivation



S. Mao, Physics Letters B **758**, 195 (2016)



X. Li, W. Fu, and Y. Liu, Physical Review D **99**, (2019)

Previous work [1] suggests that the inclusion of π^0 with finite velocity can lead to inverse magnetic catalysis. Calculations in chiral limit also gives the same result.

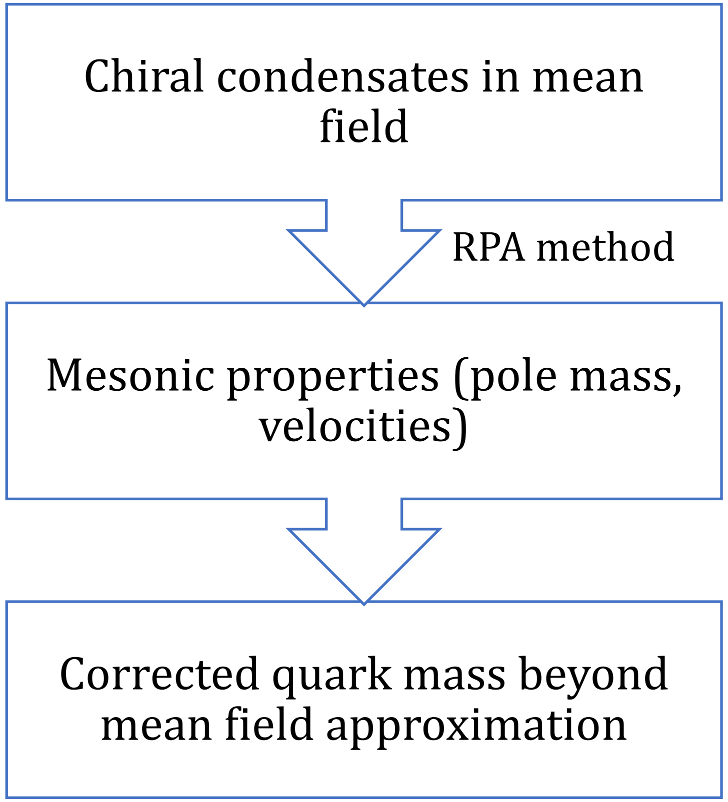
[1] K. Fukushima and Y. Hidaka, Phys. Rev. Lett. 110, 031601(2013)

In FRG-QM model, diamagnetism is achieved, where they believe $\pi^\pm$ contributes to this negative magnetic susceptibility.

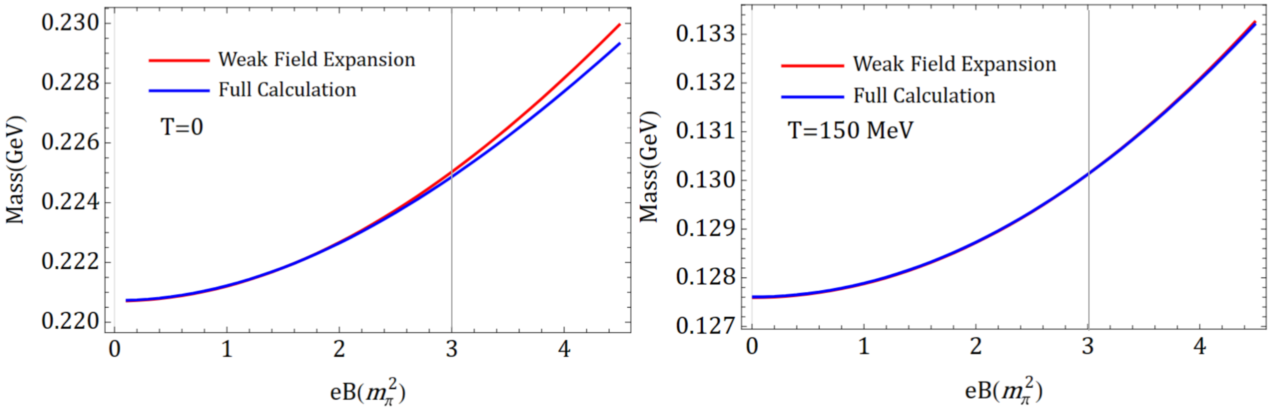
We want to identify the role of π^0 and $\pi^\pm$ in these phenomenon.

Formalism

Procedure of BMF-NJL



- To ensure that π_0 and $\pi_\pm$ share the similar strength of effect on the system, we work in weak-field limit ($eB \leq 3m_\pi^2$)
- We also adopt weak-field expansion method, which is proved to be effective at low magnetic field region.



Quark mass in weak-field expansion and full calculation versus eB

Formalism in Mean-Field Level

- The Lagrangian of magnetized SU(2) Nambu-Jona-Lasinio model

$$\mathcal{L} = \bar{\psi}\left(i\gamma^{\mu}D_{\mu} - \hat{m}\right)\psi + G\left[\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi}i\gamma_5\vec{\tau}\psi\right)^2\right]$$

- The thermodynamic potential in mean field approximation

$$\Omega_{mf}(T,B) = \frac{(M-m_0)^2}{4G} + \Omega_q(T,B),$$

$$\Omega_q(T,B) = \mathrm{Tr}_{\{c,f,s,x\}} \ln\left(\frac{1}{T}S^{-1}(x,x)\right)$$

- Here, the quark propagator

$$S_f\left(x,y\right) = e^{i\Phi_f(x_\perp,y_\perp)}\int\frac{d^4p}{(2\pi)^4}e^{-ip(x-y)}\tilde{S}_f\left(p_\perp,p_\parallel\right)$$

$$\tilde{S}_f\left(p_\perp,p_\parallel\right) = \int_0^\infty d\tau\ \exp\left[-\tau\left(M_f^2 + p_\parallel^2 + \frac{\tanh(\tau B_f)}{\tau B_f}\,p_\perp^2\ - i\epsilon\right)\right]\times$$

$$\left\{\left(M_f - p_\parallel\cdot\gamma_\parallel\right)\left[1 + is_f\,\gamma_1\gamma_2\,\tanh(\tau B_f)\right] - \frac{p_\perp\cdot\gamma_\perp}{\cosh^2(\tau B_f)}\right\}$$

Formalism in Mean-Field Level

- In weak field approximation, the translational invariant part of quark propagator can be expanded in power of $(q_f B)$

$$
\begin{aligned}
i\tilde{S}_f(k_\perp, k_\parallel) &= i\frac{M_f + \not{k}}{k^2 - M_f^2} - (q_f B)\frac{\gamma_1\gamma_2\left(M_f + \not{k}_\parallel\right)}{(k^2 - M_f^2)^2} \\
&\quad -2i\left(q_f B\right)^2 \frac{k_\perp^2(M_f + \not{k}_\parallel) + \not{k}_\perp(M_f^2 - k_\parallel^2)}{(k^2 - M_f^2)^4} \\
&= i\tilde{S}^{(0)} + (q_f B) i\tilde{S}^{(1)} + (q_f B)^2 i\tilde{S}^{(2)}
\end{aligned}
$$

- By locating the minimal value of thermodynamic potential, we can calculate the chiral condensate in mean-field approximation

$$
M(1 - 2GI_1^{(B=0)} - 2GI_1^{(B^2)}) = m_0
$$

$$
\begin{aligned}
I_1^{(B=0)} &= -4N_c N_f \int \frac{d^3k}{(2\pi)^3} \frac{\tanh(\frac{E_q}{2T})}{2E_q} \\
&= N_c N_f \int \frac{d^3k}{(2\pi)^3} f(M^2),
\end{aligned}
$$

$$
I_1^{(B^2)} = -\frac{N_c}{3}\sum_f (q_f B)^2 \int \frac{d^2k_\perp}{(2\pi)^2}\frac{dk_3}{2\pi} k_\perp^2 \frac{\partial^3 f(M^2)}{\partial(M^2)^3}
$$

Formalism beyond Mean-Field Level

- In NJL model, mesons are treated as the fluctuation above the mean field, and can be "constructed" through the RPA method

$$\Pi_M(k) = i\int \frac{d^4p}{(2\pi)^4} \mathrm{Tr}_{\{c,f,s\}} \left[\Gamma_M^* \tilde{S}(p) \Gamma_M \tilde{S}(p-k)\right] \qquad \Gamma_M = \begin{cases} 1, & M = \sigma \\ i\gamma_5\tau_+, & M = \pi_+ \\ i\gamma_5\tau_-, & M = \pi_- \\ i\gamma_5\tau_3, & M = \pi_0, \end{cases}$$

- Similarly, substituting the weak-field form of quark propagator, the polarization function

$$\Pi_{\pi_0}(k) = N_c \sum_f \left[\Pi^{00} + (q_f B)^2 \left(2\Pi^{20} + \Pi^{11}\right)\right]$$

$$\Pi_{\pi_\pm}(k) = 2N_c \left[\Pi^{00} + (q_u B)(q_d B)\Pi^{11} + \sum_f (q_f B)^2 \Pi^{20}\right]$$

Quark loop without eB

- With the definition

$$\Pi^{lm}(k) = i\int \frac{d^4p}{(2\pi)^4} \mathrm{Tr}_{\{s\}} \left[i\gamma_5 \tilde{S}^{(l)}(p) i\gamma_5 \tilde{S}^{(m)}(p-k)\right]$$

Formalism beyond Mean-Field Level

- The pole and screening masses in specific direction of meson can be obtained by solving

$$1 - 2G\Pi_M(k_0^2 = m_{pole}^2, 0) = 0$$

$$1 - 2G\Pi_M(0, k_i^2 = -m_{scr,i}^2) = 0.$$

- The propagating velocities in specific direction is defined by

$$v_i = \frac{m_{M,pole}}{m_{M,scr,i}}$$

- In this work, we calculate the neutral and charged pions' contribution on the critical temperature with and without considering the finite pion propagating velocities

- Thermodynamic potential BMF: $\Omega = \frac{(M - m_0)^2}{4G} + \Omega_q + \sum_M \Omega_M$

Formalism beyond Mean-Field Level

Thermodynamic potential BMF $\Omega = \frac{(M-m_0)^2}{4G} + \Omega_q + \sum_M \Omega_M$

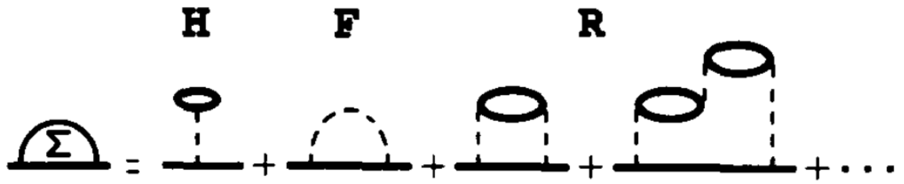
$$\Omega_{\pi_0} = \int \frac{d^3k}{(2\pi)^3}\left[\frac{E_{\pi_0}}{2} + T\ln\left(1-e^{-E_{\pi_0}/T}\right)\right] \qquad E_{\pi_0} = \sqrt{m^2_{pole,\pi_0} + v_\perp^2 k_\perp^2 + v_\parallel^2 k_3^2}$$

$$\Omega_{\pi_\pm} = \sum_{n=0}^{\infty} \frac{|eB|}{2\pi}\int_{-\infty}^{\infty}\frac{dk_3}{2\pi}\left[\frac{E_{\pi_\pm}}{2} + T\ln\left(1-e^{-E_{\pi_\pm}/T}\right)\right] \qquad E_{\pi_\pm} = \sqrt{m^2_{\pi_\pm} + v_\perp^2(2n+1)|eB| + v_\parallel^2 k_3^2}.$$

- Gap eq. with feed back from mesons

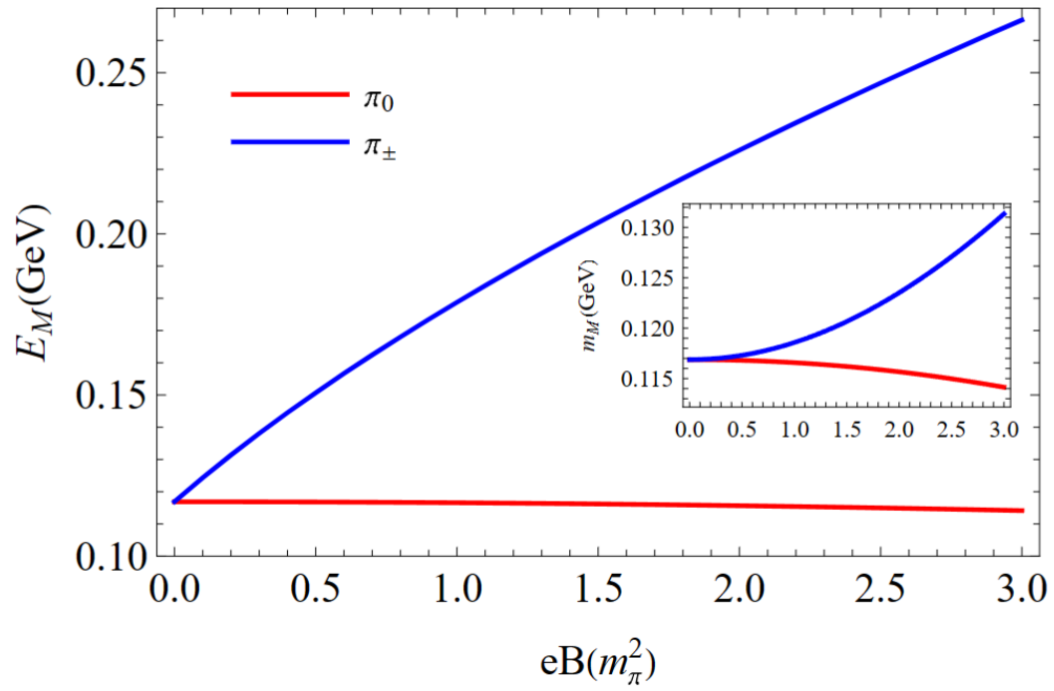
$$m_q\left(\frac{1}{4G} + \frac{\partial\Omega_q}{\partial m_q^2} + \sum_M \frac{\partial \Omega_M}{\partial m_q^2}\right) = \frac{m_0}{4G}$$

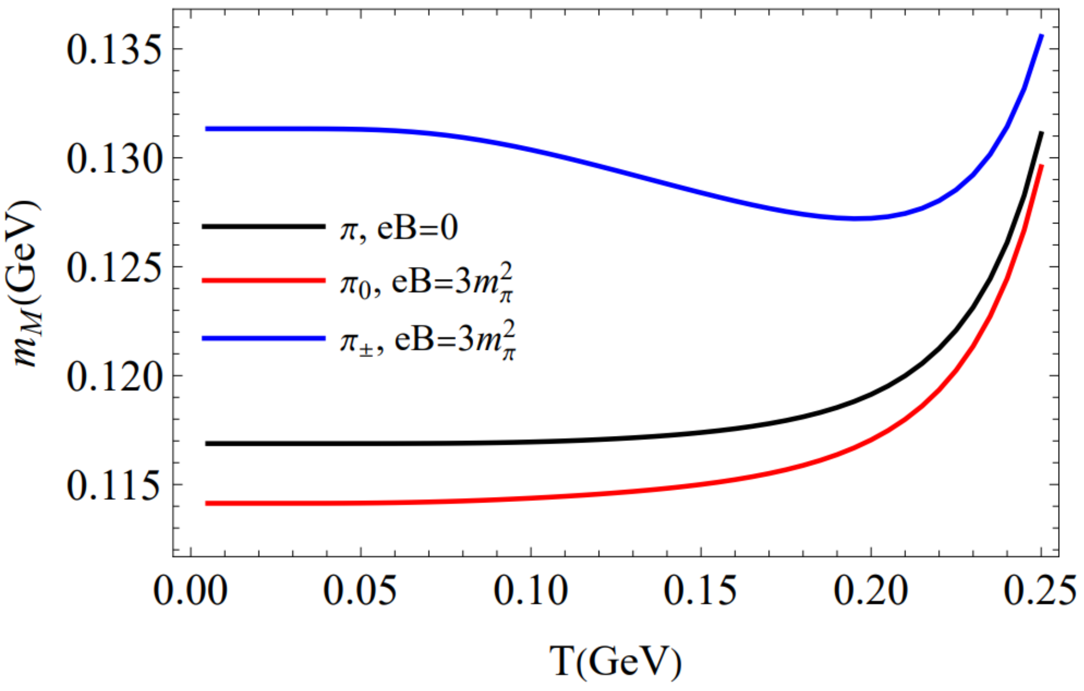
$$\frac{1}{4\tilde{G}} \equiv \frac{1}{4G} + \sum_M \frac{\partial\Omega_M}{\partial m_q^2}\Big|_{m_q^2 = m^2_{\rm MF}}$$



- Effective coupling $\tilde{G}(T, eB)$

Numerical Results

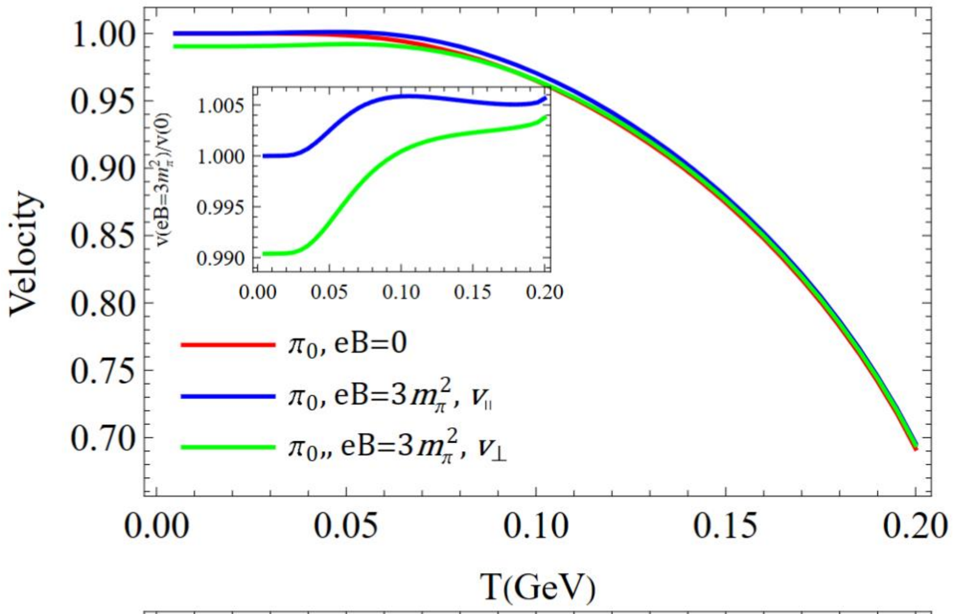


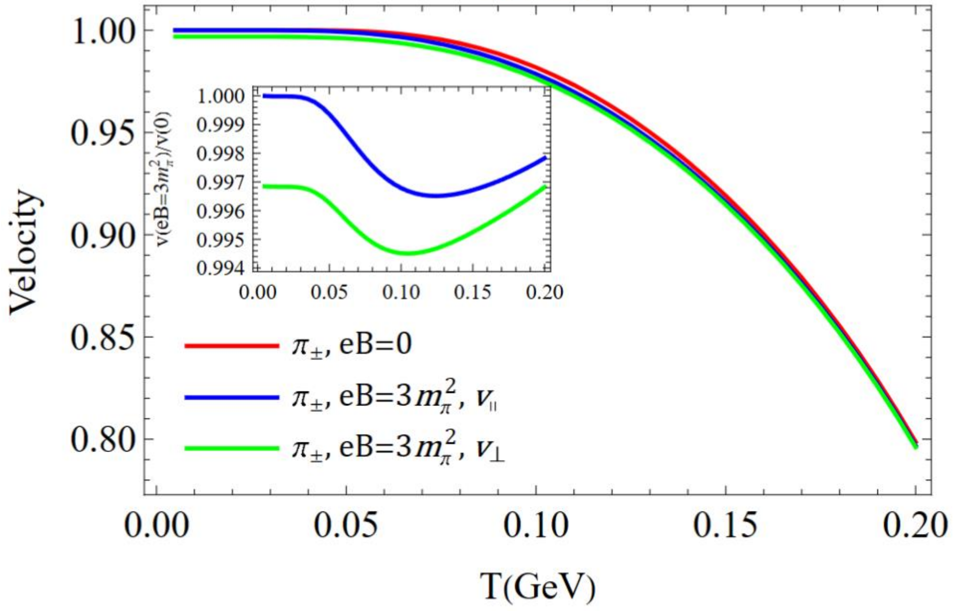


- π^0 and $\pi^\pm$ have opposite response to eB
- At weak-eB region π_0 and $\pi_\pm$ have similar masses

- The mass difference between π^0 and $\pi^\pm$ is weakened at finite temperature region.

Numerical Results

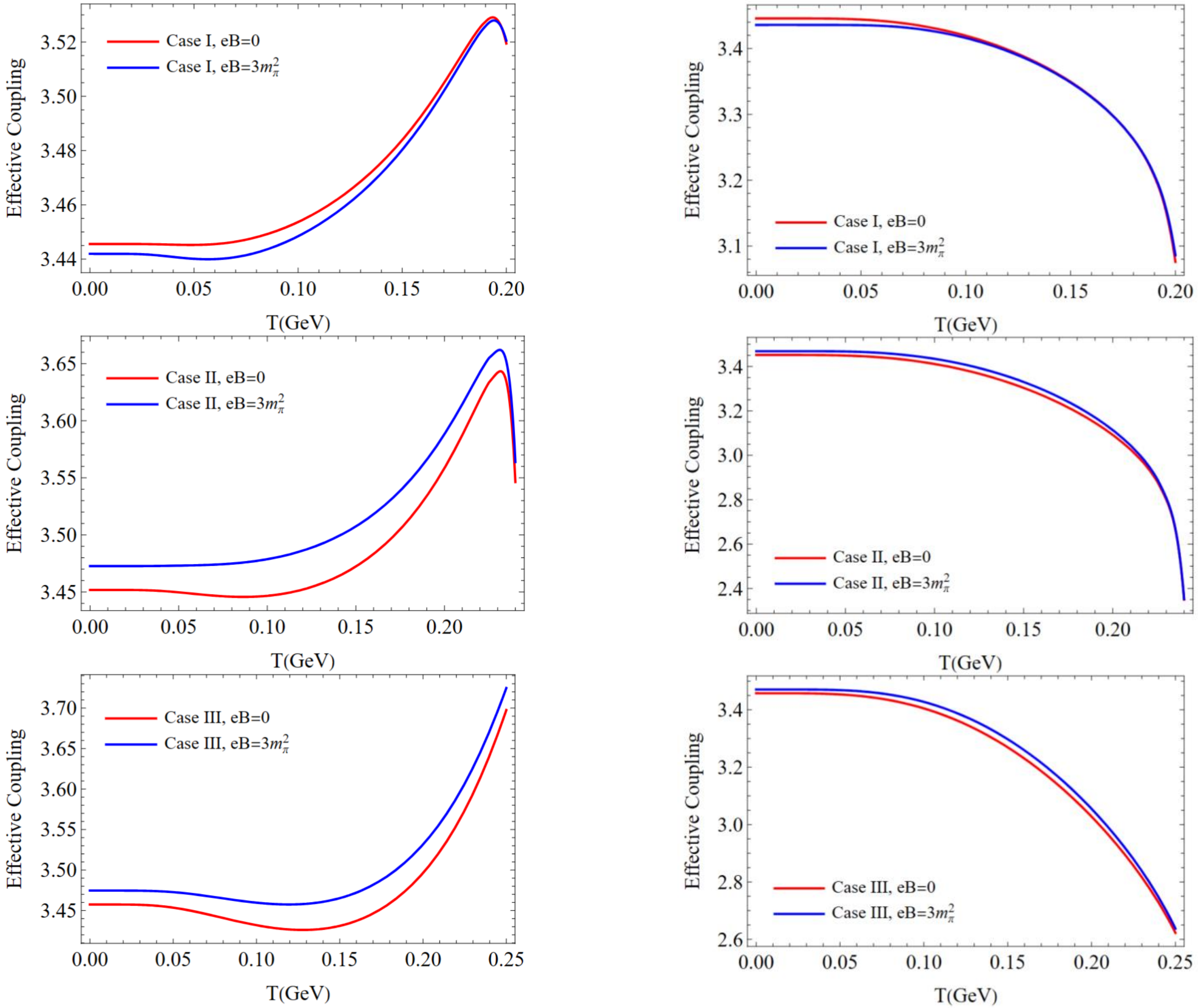




- In previous analysis, IMC or magnetic inhibition is from the splitting between transverse and longitudinal velocities;
- Calculation in chiral limit shows that the splitting is enhanced as temperature increases;
- In our calculation, this splitting effect is thermalized when the temperature is high enough.

Numerical Results

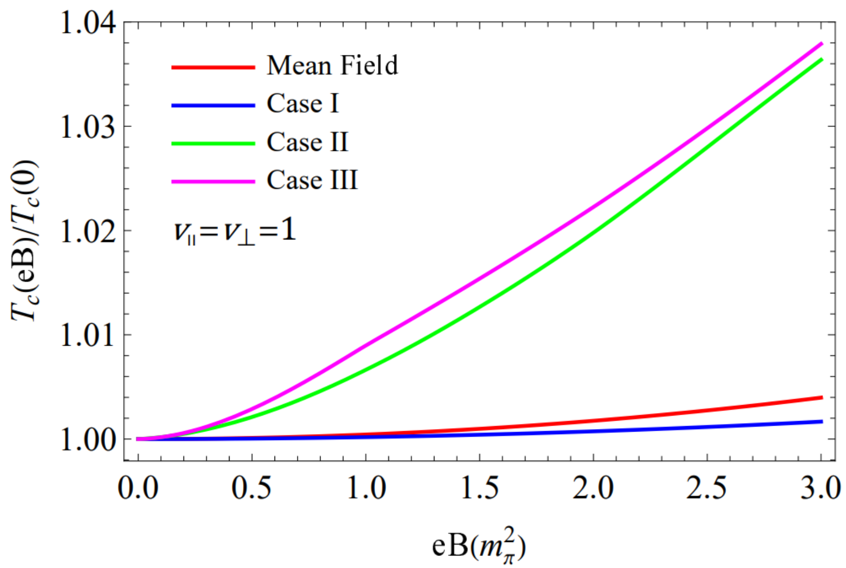
$v_{\parallel} = v_{\perp} = 1$ | finite v

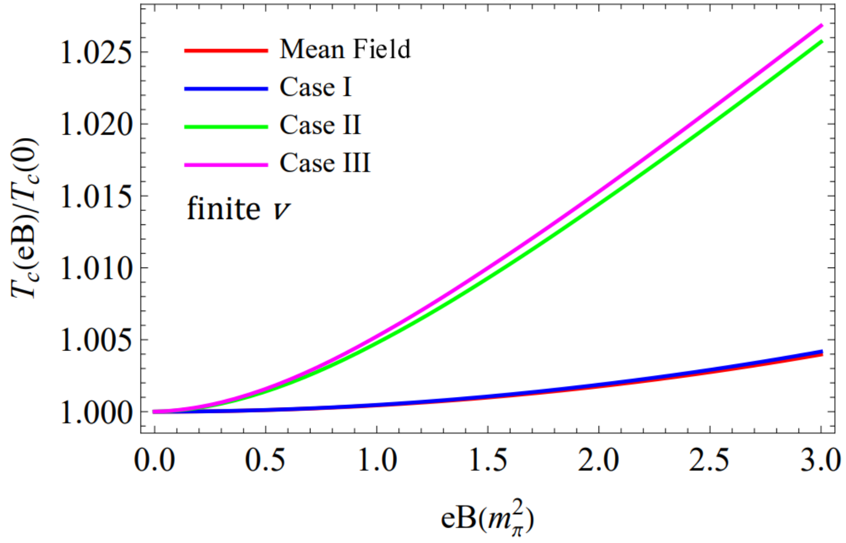


Case I: only π^0
Case II: only $\pi^\pm$
Case III: both π^0 and $\pi^\pm$

- In both case, $\tilde{G}$ tends to be weakened by π_0, but enhanced by $\pi_\pm$;
- Opposite response of $\tilde{G}$ to T;
- Effect from feedback of meson weakened by finite v.

Numerical Results



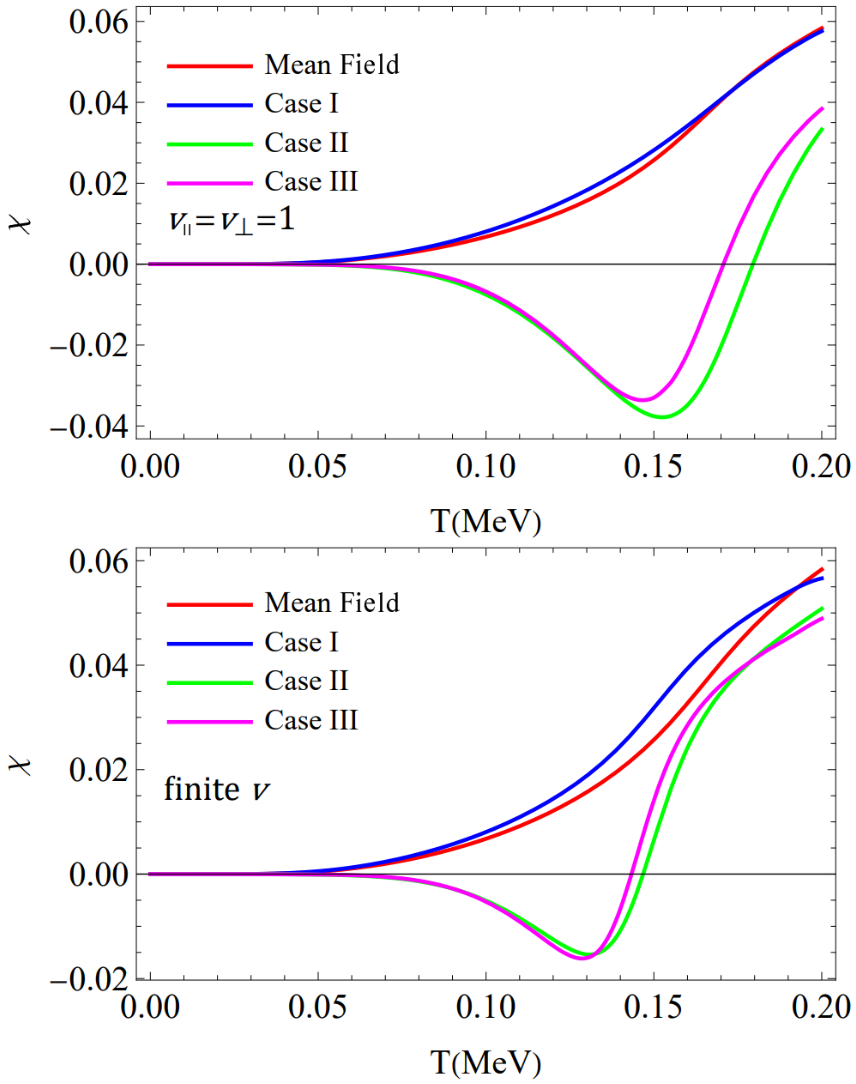


Case I: only π^0
Case II: only $\pi^\pm$
Case III: both π^0 and $\pi^\pm$

Similar conclusion as in $\tilde{G}$ analysis

- T_c tends to be weakened by π_0, but enhanced by $\pi_\pm$, due to their opposite response to eB;
- Effect from $\pi_\pm$ are more dominant (as in Case III);
- Effect from feedback of meson weakened by finite v.

Numerical Results



Case I: only π^0
Case II: only $\pi^{\pm}$
Case III: both π^0 and $\pi^{\pm}$

Magnetic susceptibility

$$\chi(T) = \bar{\chi}(T) - \bar{\chi}(0) = \frac{\partial^2 \Delta(T, eB)}{\partial eB^2}\Big|_{eB=0}$$

$$\Delta(T, eB) = \Omega(0, eB) - \Omega(T, eB)$$

- Diamagnetism in low temperature region. Because in low temperature, the dominant $\pi^{\pm}$ only contributes to Landau diamagnetism;
- Results in Case II and III qualitatively consistent with Lattice result.

Conclusion

- In this work, we calculate the critical temperature and magnetic susceptibility in Nambu-Jona-Lasinio model with feedback from both π_0 and $\pi_\pm$;
- When considering a non-zero current quark mass, the velocity difference between transverse and longitudinal directions will be thermalized at high temperature region.
- Due to opposite response of π_0 and $\pi_\pm$ to eB, T_c is weakened by π_0 but enhanced by $\pi_\pm$. In chiral symmetry explicitly broken case, system is still MC.
- At low temperature, $\pi_\pm$ being dominant, the system is diamagnetic from Landau diamagnetism; at high temperature, quark being dominant, the system is paramagnetic from Pauli paramagnetism.

Thank you!

Supporting Materials

- The next-to-leading order of polarization functions

$$\Pi^{20}(q_0^2, q_\perp^2 = q_3^2 = 0) = -8T \sum_{n=-\infty}^{\infty} \int \frac{d^3k}{(2\pi)^3} \frac{(i\omega_n) q_0 k_\perp^2}{[(i\omega_n)^2 - E_q^2]^4 \{(i\omega_n - q_0)^2 - E_q^2\}}.$$

$$\Pi^{20}(q_0^2 = q_\perp^2 = 0, q_3^2) = -\frac{T}{8\pi} \sum_{n=-\infty}^{\infty} \int_0^1 dx \frac{x^3(1-x) q_3^2}{(\Delta - (i\omega_n)^2)^{5/2}},$$

$$\Pi^{20}(q_0^2 = q_3^2 = 0, q_\perp^2) = \frac{T}{16\pi} \sum_{n=-\infty}^{\infty} \int_0^1 dx\ x^3(1-x) \left\{ \frac{q_\perp^2}{(\Delta - (i\omega_n)^2)^{5/2}} + \frac{5(m^2 - (i\omega_n)^2) q_\perp^2}{(\Delta - (i\omega_n)^2)^{7/2}} \right\}.$$

$$\Pi^{11}(q_\perp^2 = q_3^2 = 0, q_0^2) = 4T \sum_{n=-\infty}^{\infty} \int \frac{d^3k}{(2\pi)^3} \frac{m^2 + k_3^2 - (i\omega_n)(i\omega_n - q_0)}{[(i\omega_n)^2 - E_q^2]^2 \{(i\omega_n - q_0)^2 - E_q^2\}^2},$$

$$\Pi^{11}(q_0^2 = q_\perp^2 = 0, q_3^2) = \frac{T}{8\pi} \sum_{n=-\infty}^{\infty} \int_0^1 dx\ x(1-x) \left\{ \frac{1}{(\Delta - (i\omega_n)^2)^{3/2}} + \frac{3[m^2 - x(1-x) q_3^2 - (i\omega_n)^2]}{(\Delta - (i\omega_n)^2)^{5/2}} \right\},$$

$$\Pi^{11}(q_0^2 = q_3^2 = 0, q_\perp^2) = \frac{T}{8\pi} \sum_{n=-\infty}^{\infty} \int_0^1 dx\ x(1-x) \left\{ \frac{1}{(\Delta - (i\omega_n)^2)^{3/2}} + \frac{3[m^2 - (i\omega_n)^2]}{(\Delta - (i\omega_n)^2)^{5/2}} \right\}.$$

Supporting Materials

- Consider the 2-point correlation function in a simple meson model

$$\Gamma^{(2)} = Z_0 p_0^2 + Z_{\vec{p}} \vec{p}^2 + m_0^2.$$

Z: wave function renormalizations m_0: bare mesonic mass

- Pole mass and screening mass are defined by

$$\Gamma^{(2)}(p_0 = im_{pole}, \vec{p} = 0) = 0\,,$$
$$\Gamma^{(2)}(p_0 = 0, \vec{p}^2 = -m_{\text{scr}}^2) = 0\,.$$

- The ratio of the temporal and spatial wave function renormalizations is

$$v \equiv \sqrt{\frac{Z_{\vec{p}}}{Z_0}} = \frac{m_{pole}}{m_{\text{scr}}}$$

$$E = \sqrt{m_{pole}^2 + v^2 \vec{p}^2}$$