Non-Preemptive Scheduling with Non-Observable Environments

13ème Atelier en Évaluation des Performances

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IRIT/INP-ENSEEIHT

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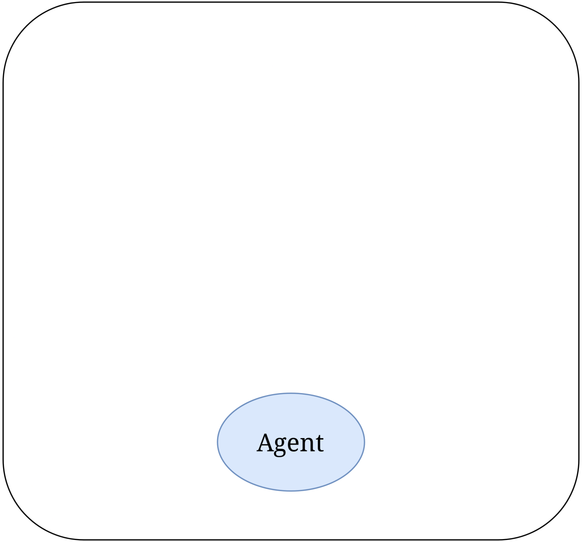
A General Framework

- **An Agent**: The decision-making entity.



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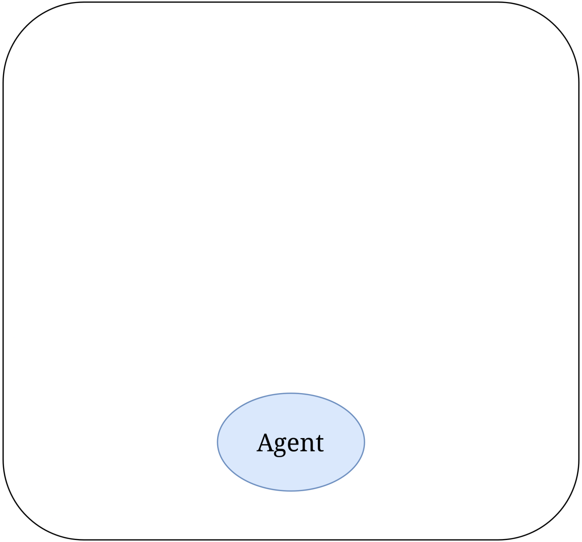

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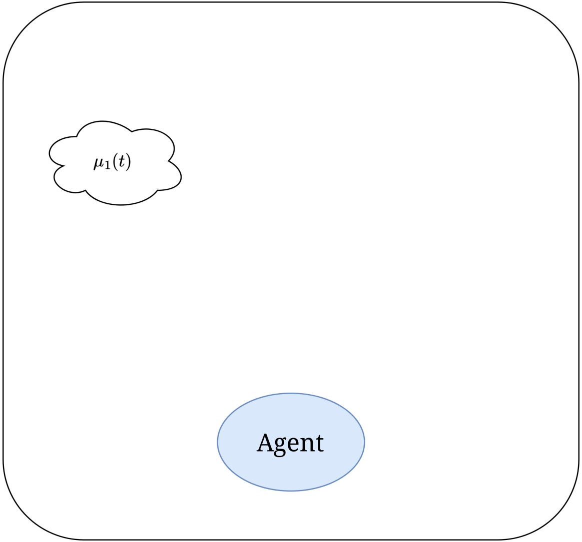

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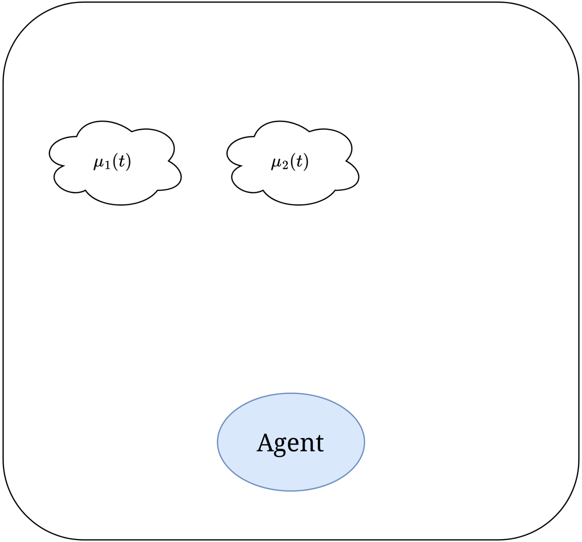

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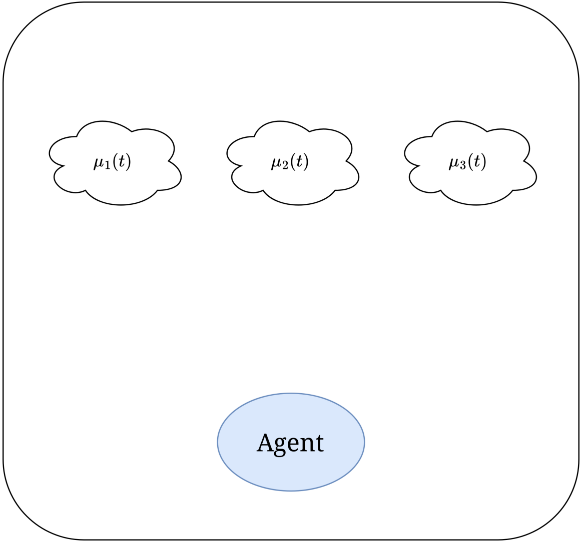

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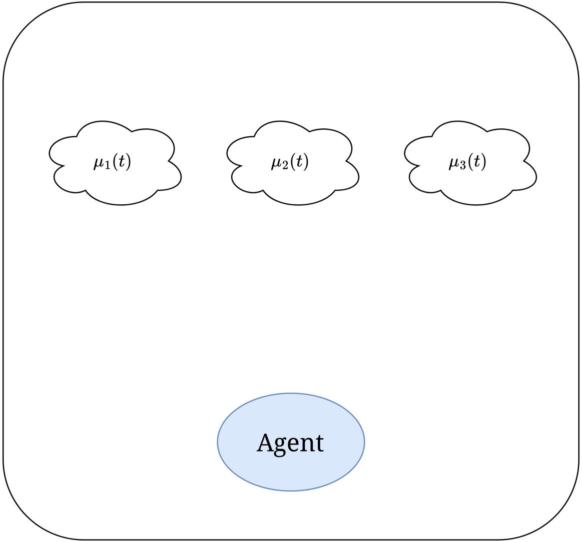

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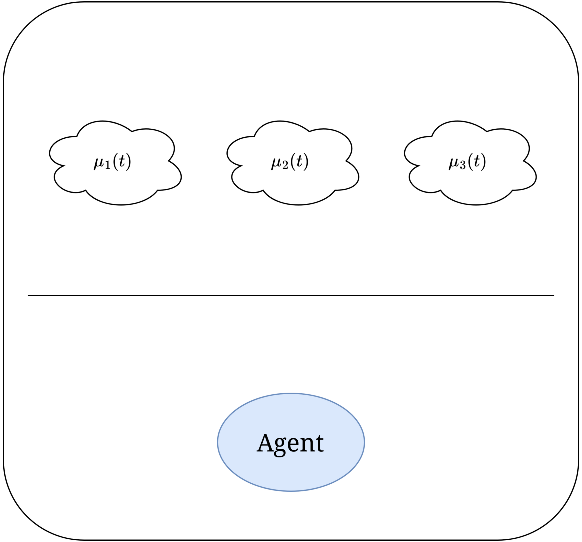

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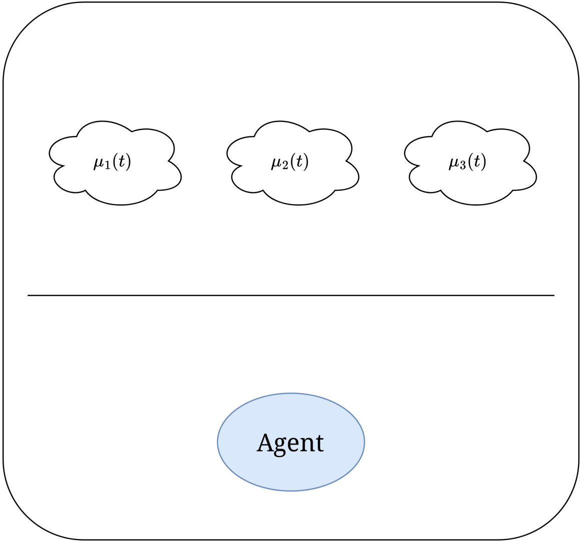

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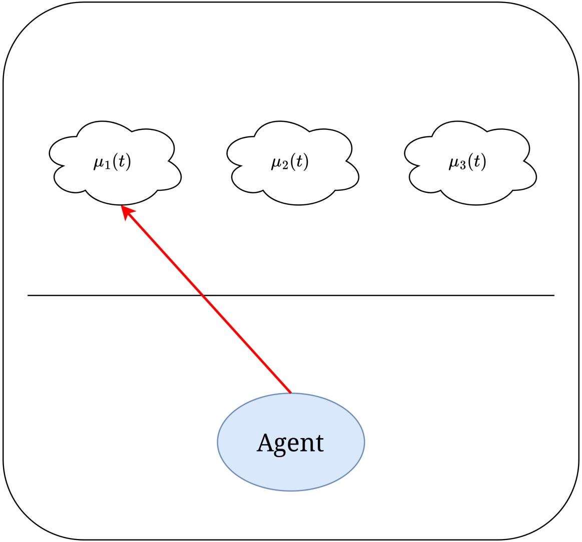

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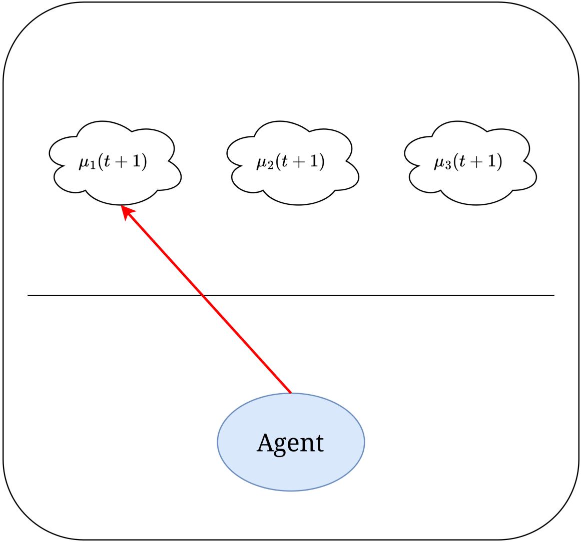

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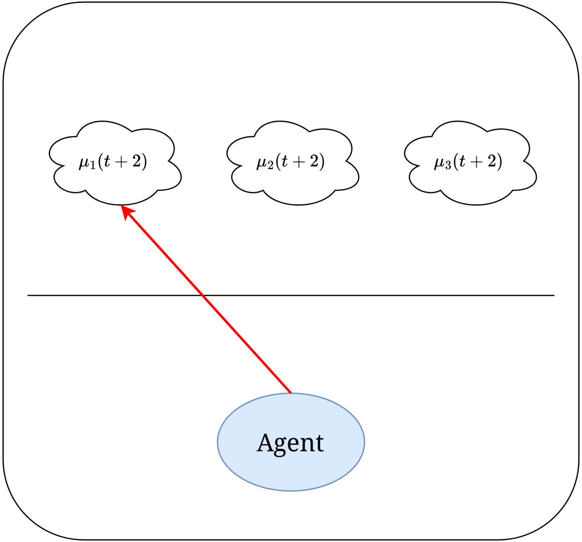

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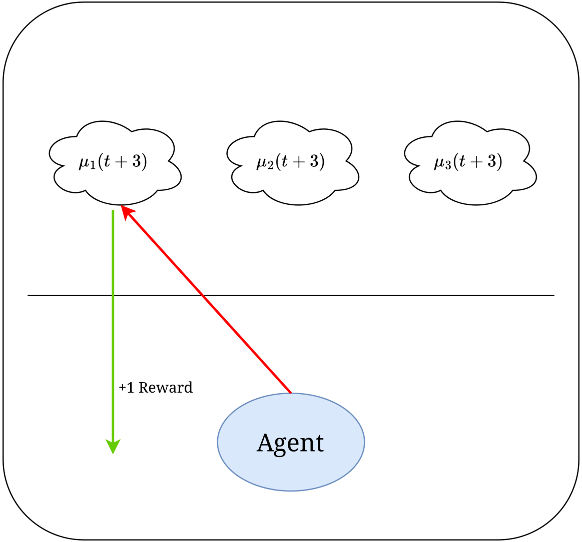

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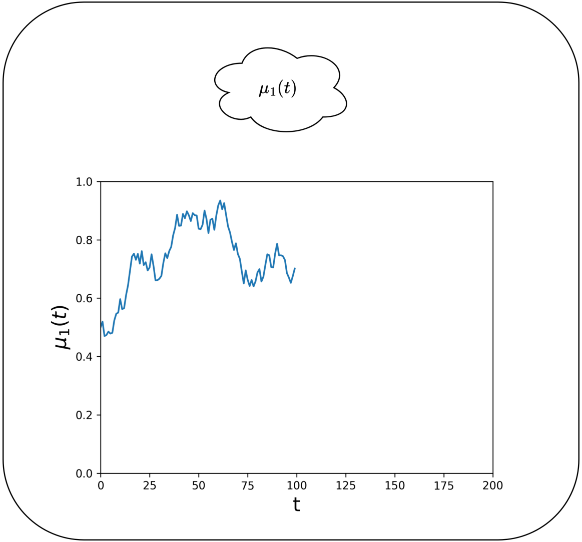

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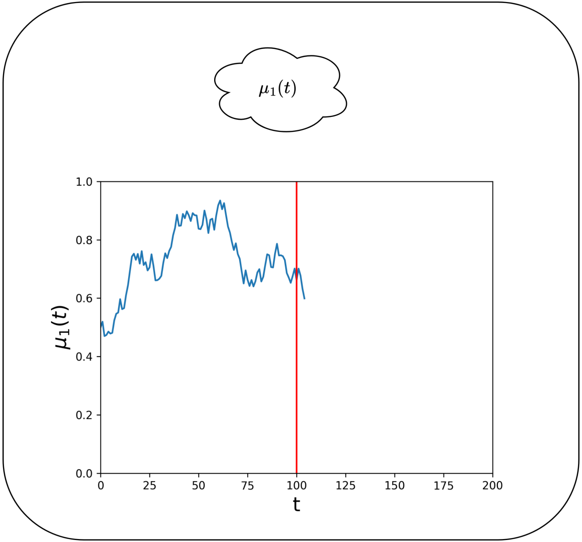

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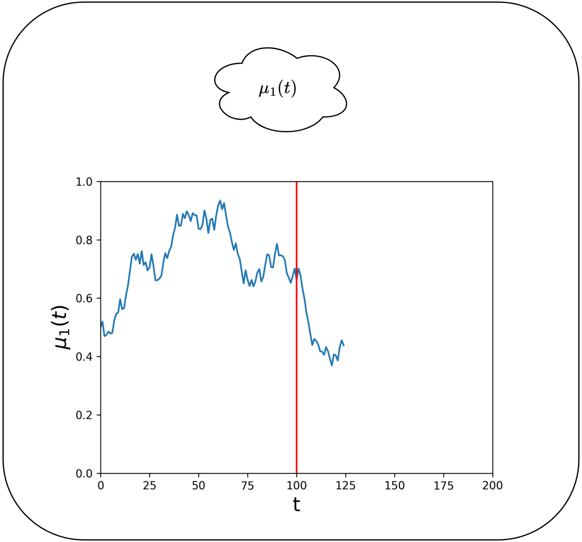

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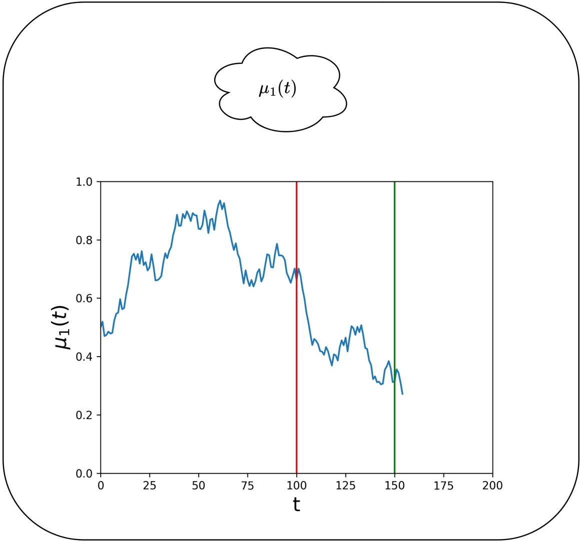

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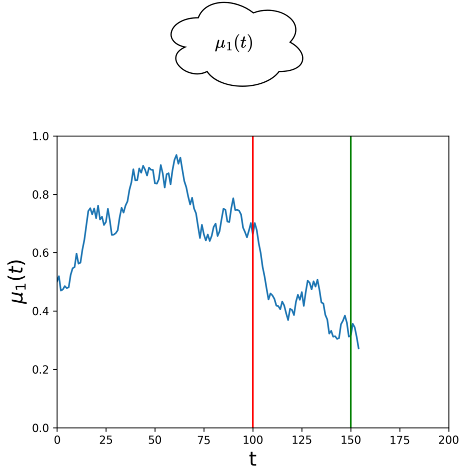
- **The Agent's Goal**:

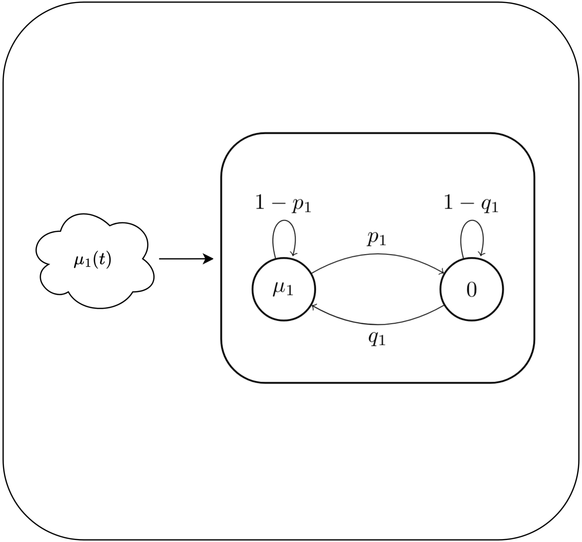
$$\text{Maximize} \quad \lim_{T\to\infty} \frac{1}{T}\mathbb{E}[\sum_{t=1}^{T} R(t)].$$



A Simple Model

In this simple model,

- We consider the environments to be two states Markov chains.
- The agent is aware of the parameters $p = (p_1, \cdots, p_K)$, $q = (q_1, \cdots, q_K)$, $\mu = (\mu_1, \cdots, \mu_K)$.
- We will note the action at time t, $u(t) = \big(u_1(t), \cdots, u_K(t)\big)$ with $u_k(t) \in \{0, 1\}$ such that $\sum_{k=1}^{K} u_k(t) = 1$



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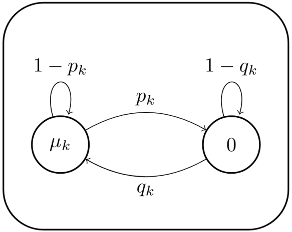
Belief State



The belief state is represented by $(\omega_1, ..., \omega_K) \in [0,1]^K$.

The belief state transitions are governed by the following rule:

$$\omega_k(t+1) = \begin{cases} 1 - p_k & \text{if } R(t) = 1 \\ \bar{T}(\omega_k(t)) & \text{if } R(t) = 0 \text{ and } u_k(t) = 1 \\ T(\omega_k(t)) & \text{if } u_k(t) = 0 \end{cases}$$

with

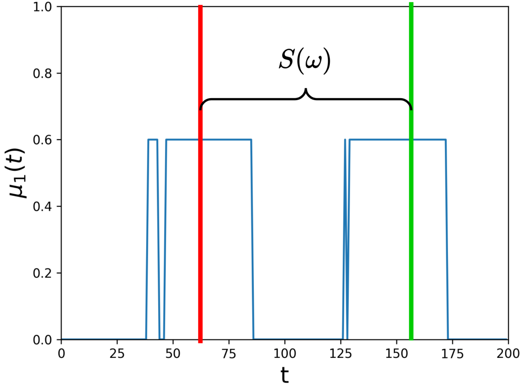
$$T(\omega_k) := \omega_k(1 - p_k) + (1 - \omega_k)q_k \qquad \bar{T}(\omega_k) := \frac{(1-\mu_k)\omega_k(1-p_k)+(1-\omega_k)q_k}{1-\omega_k\mu_k}$$

Service time

For Non-preemptive policy, a critical notion is the "Effective Service Time":

$$S_k(\omega) := \inf\{t \geq 1 \mid R(t) = 1; \omega(0) = \omega\}.$$

It is the time between the decision and the next reward.



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Characterization of the Effective Service Time

Proposition

for $\rho \in \mathbb{R}$,

$$\mathbb{E}[\rho^{S(\omega)}] = C(\rho) + D(\rho)\omega$$

with

- $C(\rho) := \frac{\mu\rho^2 q}{1-\rho(2-p-q-\mu(1-p))+\rho^2(1-p-q)(1-\mu)}$
- $D(\rho) := \frac{\mu\rho(1-\rho)}{1-\rho(2-p-q-\mu(1-p))+\rho^2(1-p-q)(1-\mu)}$

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Corollary

$$\mathbb{E}[S(\omega)] = \frac{p+q+\mu(1-p-\omega)}{\mu q}$$

Positively Autocorrelated Chains/

Theorem

In the non-preemptive case, if all chains are positively auto-correlated, then the optimal policy is to always serve the chain

$$k^* = \underset{k \in \{1,\ldots,K\}}{\operatorname{argmin}} \mathbb{E}[S_k(1 - p_k)] = \underset{k \in \{1,\ldots,K\}}{\operatorname{argmin}} \frac{p_k + q_k}{q_k \mu_k}.$$

and the optimal long-run average reward rate will be

$$\mu_{k^*} \frac{q_{k^*}}{p_{k^*} + q_{k^*}}$$

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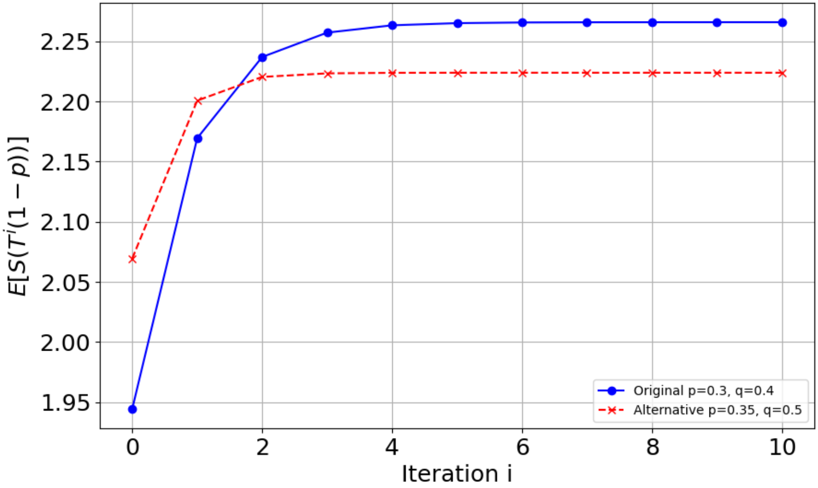
Positively Autocorrelated Case



Negatively Autocorrelated Chains

Theorem

For $K = 2$, symmetric, negatively autocorrelated, selecting the largest belief state is Bellman-optimal.

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For $K = 2$, symmetric, negatively autocorrelated, selecting the largest belief state is Bellman-optimal.

The expected average reward under this policy when $K = 2$ is:

$$g^* = \left(A + B \frac{C + D\frac{q}{p+q}}{1 - (1 - p - \frac{q}{p+q})D} \right)^{-1}.$$

Gain of Observation

Proposition

- *When the environments are symmetric,* $(1-p-q)>0$*, the relative Gain of Observability is*

$$Gain = \frac{g^{obs} - g^{non-obs}}{g^{non-obs}} = \frac{\mu q(\mathbb{E}[S(1)] + \frac{m^{\nu^*}(0)}{(1-q)^K})}{p+q} - 1$$

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- *In the case of two symmetric environments,* $(1-p-q) \leq 0$*, the relative Gain of Observability is*

$$Gain = \left(\mathbb{E}[S(1)] + \frac{m^{\nu^*}(0)}{(1-q)^K}\right)\left(A + B\frac{C + D\frac{q}{p+q}}{1-(1-p-\frac{q}{p+q})D}\right) - 1$$

Gain of Preemptability

Proposition (Informal)

Under some condition and assuming positive autocorrelation, the relative gain of preemptability is bounded:

$$\frac{f\left(c_1\right)\left(p+q\right)}{(1-\left(1-p-f\left(c_1\right)\right)\mu)q}-1 \leq Gain := \frac{g^{pre}-g^{non-pre}}{g^{non-pre}} \leq \frac{1}{1-(1-p-\frac{q}{p+q})\mu}-1$$

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Proposition (Informal)

When the environments are negatively autocorrelated, i.e. $1-p-q<0$ *and under some other condition the relative gain of preemptability is bounded by:*

$$\frac{A+B\frac{C+D\frac{q}{p+q}}{1-(1-p-\frac{q}{p+q})D}}{\left(\frac{(2-y_1)\mu q}{(1-y_1)^2}-x_1\right)h(x_1,y_1,\mu q,a_1,0)+1}-1 \leq Gain \leq \frac{A+B\frac{C+D\frac{q}{p+q}}{1-(1-p-\frac{q}{p+q})D}}{\left(\frac{2-(1-\mu q)(1-y_1)}{(\mu q)^2}-\frac{1}{x_1}\right)h(\frac{1}{x_1},1-\mu q,1-y_1,a_1,3)+1}-1$$

Thank You for Your Attention!