

Interplay between Epidemic and News Propagation Processes

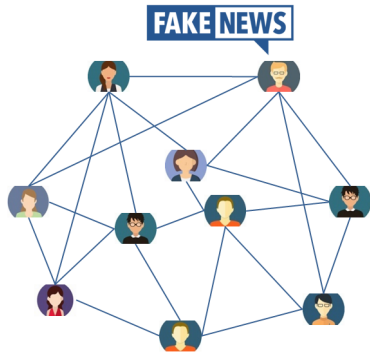
WORKSHOP ON PERFORMANCE EVALUATION

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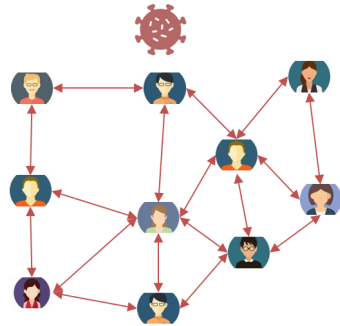
Joint work with Veeraruna Kavitha at IEOR, IIT Bombay
and Chen Peng, Quanyan Zhu at ECE, New York University

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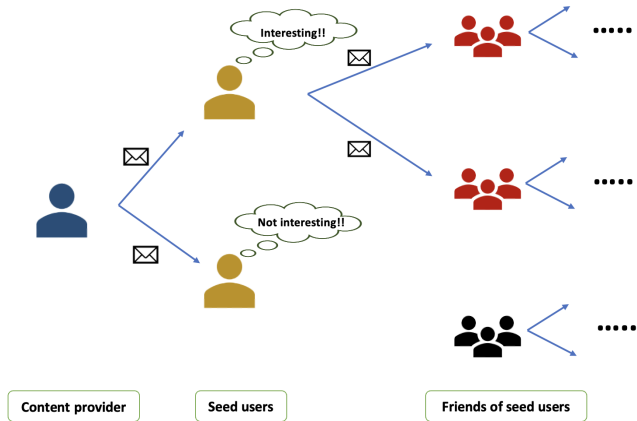


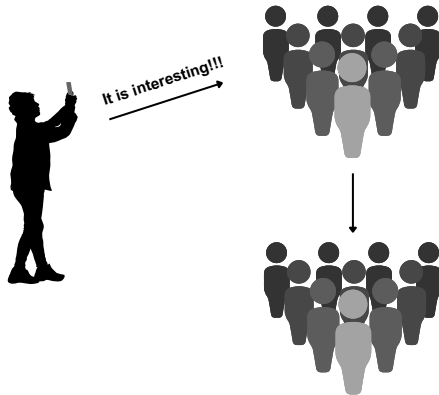


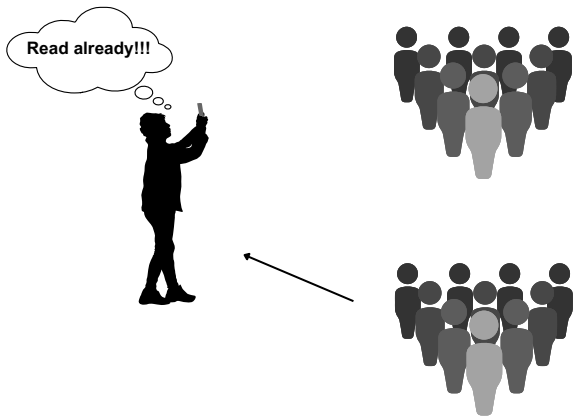
Online Social Network



Epidemic Network







After k -th user forwards the news-item,

$$\# \text{ unread copies, } \Psi_{k+1} = \Psi_k + \xi_{k+1} \underbrace{-1}_{\substack{\text{user read} \\ \text{the post}}} \quad \text{and}$$

$$\# \text{ total copies, } \Theta_{k+1} = \Theta_k + \xi_{k+1}$$

- Ψ_k — no. of unread copies of the post,
- Θ_k — no. of total copies of the post, and
- ξ_{k+1} — additional copies generated by new share/forward.

Aim: A stochastic approximation (SA)-based iterative scheme – to aid analysis

Scaling

$$\psi_k = \frac{\Psi_k}{k}, \quad \text{and} \quad \theta_k = \frac{\Theta_k}{k}, \quad \text{for any } k \geq 1$$

SA-based iterative scheme

$$\begin{aligned}\psi_{k+1} &= \psi_k + \frac{1}{k+1} (\xi_{k+1} - 1 - \psi_k) \\ \theta_{k+1} &= \theta_k + \frac{1}{k+1} (\xi_{k+1} - \theta_k)\end{aligned}$$



Suyog Kapsikar, Indrajit Saha, Khushboo Agarwal, Veeraruna Kavitha, and Quanyan Zhu, “Controlling Fake News by Collective Tagging: A Branching Process Analysis”. IEEE Control Systems Letters, 2020 and ACC 2020

- Any news-item can trend - copies grow exponentially
- After a while, saturates and replaced by new trending topic

- Saturation \mathcal{E}_k^S captured by:
 - unread copies $\psi_k <$ a threshold
 - Total copies $\theta_k >$ a threshold
- $\mathcal{E}_k^S = 1$ iff $\psi_k < \delta_\psi$ and $\theta_k > \delta_\theta$

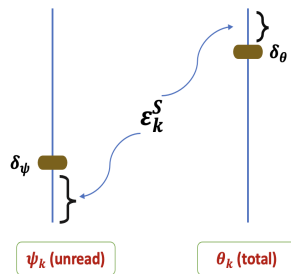


Figure: Saturated Regime

- Extra or fictitious iterates to model saturation
 - total copies reduce drastically – with a big rate, C
 - unread copies also reduce to zero.

The overall SA iteration

$$\begin{aligned}\psi_{k+1} &= \psi_k + \epsilon \left(\underbrace{(1 - \mathcal{E}_k^S)(\xi_{k+1} - 1 - \psi_k)}_{\text{Regular news-item update}} - \underbrace{\mathcal{E}_k^S \psi_k}_{\text{Saturation}} \right), \\ \theta_{k+1} &= \theta_k + \epsilon \left(\underbrace{(1 - \mathcal{E}_k^S)(\xi_{k+1} - \theta_k)}_{\text{Regular news-item update}} - \underbrace{\mathcal{E}_k^S C \theta_k}_{\text{Saturation}} \right).\end{aligned}$$

$$\mathcal{M}(\theta_k) := E_k[\xi_{k+1} | \sigma(\psi_s, \theta_s); s \leq k],$$

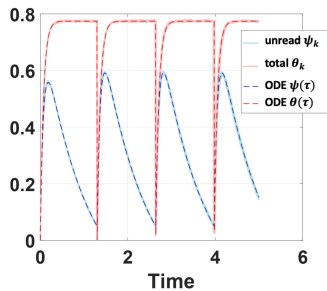
$\mathcal{M}(\theta)$ – expected number of shares, given θ (fraction of total copies)

$$\dot{\psi} = (1 - \mathcal{E}^S) \left(\mathcal{M}(\theta) - 1 - \psi \right) - \mathcal{E}^S \psi,$$

$$\dot{\theta} = (1 - \mathcal{E}^S) \left(\mathcal{M}(\theta) - \theta \right) - \mathcal{E}^S C \theta.$$

Theorem, under certain conditions

The news-update trajectory $\{\theta_k, \psi_k\}$ converges to ODE solution $(\theta(\cdot), \psi(\cdot))$ trajectory over any finite time horizon.



The Monte-Carlo estimates and the ODE solution trajectory are inseparable
 Each cycle – one news-item (start to saturation)

Well-known dynamics

$$\underbrace{\frac{di}{dt} = i(\bar{\beta}(1 - i - r) - \alpha)}_{\text{Infected fraction}}, \quad \underbrace{\frac{dr}{dt} = (i\alpha p_r - rl_i)}_{\text{Recovered fraction}},$$

- $(1 - i(t) - r(t))$ – susceptible fraction
- $\bar{\beta}$ – disease spread rate, α – recovery rate
- p_r – immunized fraction of recovered sub-population
- l_i – rate at which immunized individual loses immunity

Theorem

- When disease spread rate $<$ recovery rate, disease eradicates eventually,
- Else, disease settles to non-zero (i, r) .

News-items can influence public behaviour

- Spread of fake news about masks – influences mask behaviour
- Epidemic spread can increase/decrease
- Or panic can be created etc.

- Every trending post has an explosion phase followed by saturation
- Different people interact with post at different times of its life-time.
- Capture the influence via **the total copies shared**

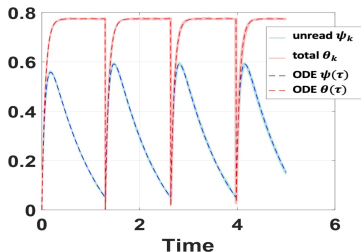


Figure: Limit Cycle

Mean number of shares

$$\mathcal{M}(\theta) = \eta(1 - a\theta),$$

- η – attractiveness factor
- a – proportion factor

- News propagates at **faster** time scale (hours)
- Disease propagates at **slower** time scale (days)
- Influence is not instantaneous – rather over the entire cycle
- Influenced disease spread parameter

$$\beta = \bar{\beta} + w \frac{\eta}{a\eta + 1}$$

Epidemic ODE solution has a limit cycle, solving it

$$\theta_{\infty}^*(\eta) \approx \frac{\eta}{a\eta + 1}$$

Disease spread parameter influenced by one news-item

$$\beta = \bar{\beta} + w \frac{\eta}{a\eta + 1}$$

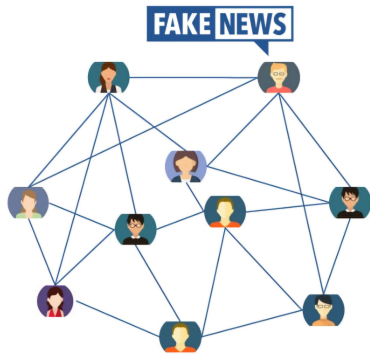
- No influence on recovery α and immunization rate parameters p_r
- $w = w(i)$, $\eta = \eta(i)$ – depend on infected population
 - when infection is high, people are **more sensitive, more attracted** to news-items

Influence of all trending topics, assuming similar post-characteristics

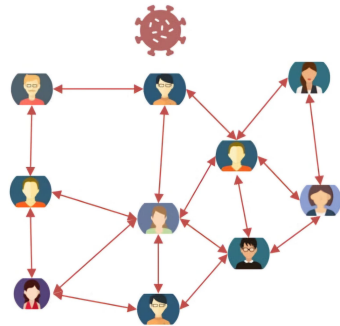
$$\begin{aligned} \beta(i) &= \bar{\beta} + \sum_{m \text{ is trending}} w_m(i) \frac{\eta_m(i)}{a\eta_m(i) + 1} \\ &= \bar{\beta} + \overline{w}(i) \frac{\eta(i)}{a\eta(i) + 1} \end{aligned}$$

$$\underbrace{\frac{di}{dt} = i(\bar{\beta}(i)(1 - i - r) - \alpha)}_{\text{Infected fraction}}, \quad \underbrace{\frac{dr}{dt} = (i\alpha p_r - rl_i)}_{\text{Recovered fraction}},$$

$$\bar{\beta}(i) = \bar{\beta} + \bar{w}(i) \frac{\eta(i)}{a\eta(i) + 1}$$



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Epidemic Network

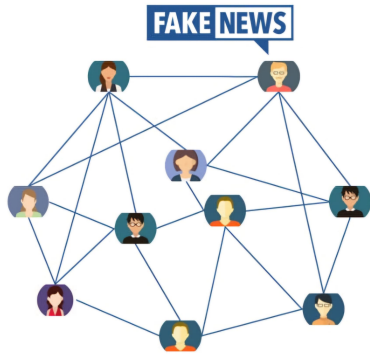
As infection level i increases

- $\eta(i) = \bar{\eta}(pi + q)$ — public show more interest in reading and sharing posts
- $\bar{w}(i) \equiv \bar{w}$ — not reacting significantly over consumed information

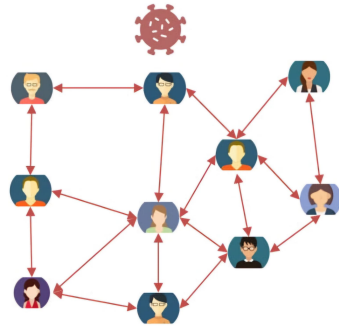
$\beta(0)$ – disease spread rate near zero infected fraction

Theorem

- When $\beta(0) < \alpha$, disease eradicates.
- Otherwise,
 - either disease eradicates – example when $\bar{w} < 0$ (authentic information)
 - or settles to non-zero value.



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As infected fraction increases

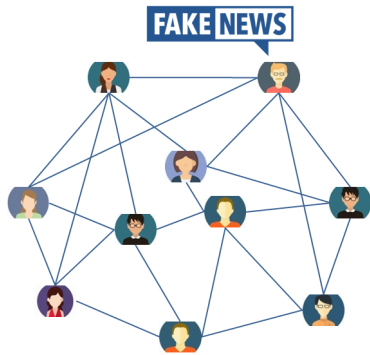
- $\eta(i) \equiv 1$ – interest towards consuming and spreading news remains the same
- $\bar{w}(i) = ui$ (linear influence) – reactions are more pronounced at higher i

Theorem

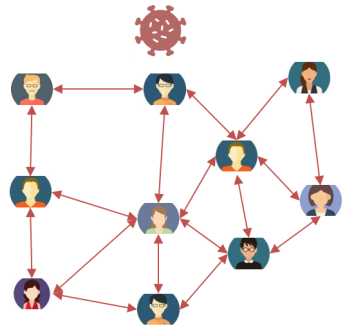
- when $\beta(0) < \alpha$, disease eradicates.
- otherwise, either disease eradicates or settles to non-zero value or leads to **limit cycle**.

When public responds to news more sensitively

Limit cycles are created even without change in disease parameters !



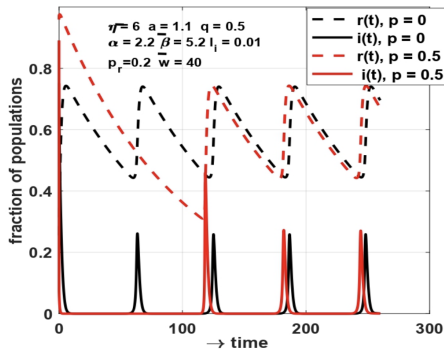
Online Social Network



Epidemic Network

As infected fraction increases

- $\eta(i) = \bar{\eta}(pi + q)$ – more interest in reading and showing posts
- $\bar{w}(i) = \bar{w} + ui$ – more reaction towards posts








$p = 0$ – increasing behavioural influence by News (with higher i)
 $p = 0.5$ – interest towards posts increases as well as the response
 Limit Cycle seen in either case

Figure: IBIN ($p = 0$), IBIN and I3N ($p = 0.5$)

- When the disease characteristics remains the same and epidemic is not influenced
 - the epidemic converges to a fixed fraction of infected, recovered fraction
 - the disease is either eradicated or non-eradicated.
- We developed a integrative two time scale ODE
 - that captures the influence of epidemic on trending topics and vice versa
 - the two time scale is different type than usually considered in literature
- Studied variety of influences
 - Public interest towards posts on OSN increases with increase in disease
 - the number of shares, total number of copies shared grow non-linearly
 - Public react more strongly with higher infection levels
 - the disease spread rates are modulated accordingly.

- The integrated model indicates several interesting phenomenon
 - disease can be cured probably when sufficient authentic posts get viral
 - infection level can increase significantly when fake news spreads
 - More interestingly, one can see limit cycles
 - disease fluctuates between small and large values of infection levels
 - this happens not due to variations in disease characteristic
 - rather due to public response towards available information

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Thank You!