Games among selfish and team stations in polling systems

(13ème Atelier en Évaluation des Performance)

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1. Polling systems
2. Cyclic Bernoulli Polling (CBP) system
3. Games in CBP system
4. Analysis
5. Concluding remarks and future directions

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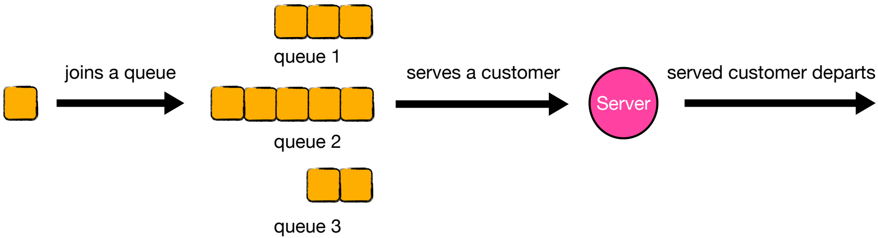
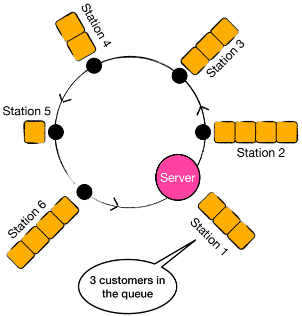
Queuing systems



- key feature: a customer is served as soon as its turn comes
- questions: whether or not to queue, where to queue, when to queue, etc.

Polling systems



- key feature: a customer is served only when the server visits its station
- questions: routing mechanism, choice of service disciplines, etc.

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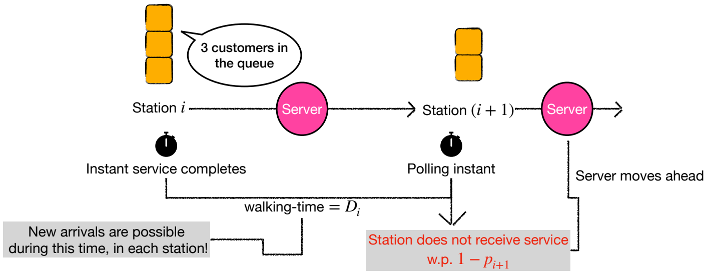
Cyclic Bernoulli Polling (CBP) system [Altman and Yechiali, 1993]

- a single server
- $N < \infty$ number of stations, each with its own queue
- server moves cyclically among the stations to provide the service
- when station i is polled, it is served w.p. $p_i \in (0, 1]$
- some/all waiting customers are served, when polled
- λ_i : arrival rate at station i
- b_i: mean service time at station i
- assumption: $\sum_{i=1}^{N} \rho_i < 1$ for $\rho_i := \lambda_i b_i$
- application: LAN based on token-ring protocol

Dynamics: when station is “not served"



Dynamics: when station is served under “gated" policy

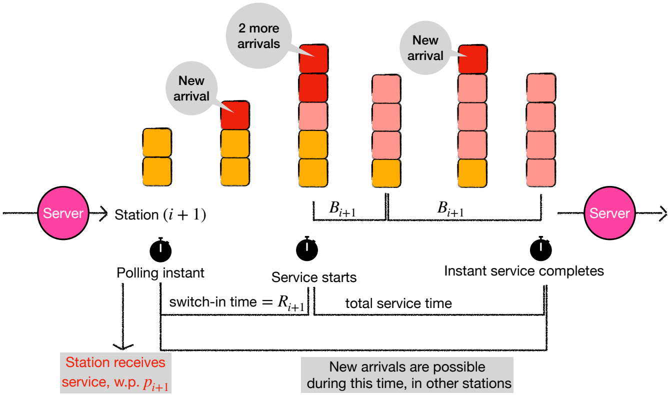


Figure 1: Gated policy (serve only those which are present at the arrival instant of server)

Dynamics: when station is served under "partially-exhaustive" policy

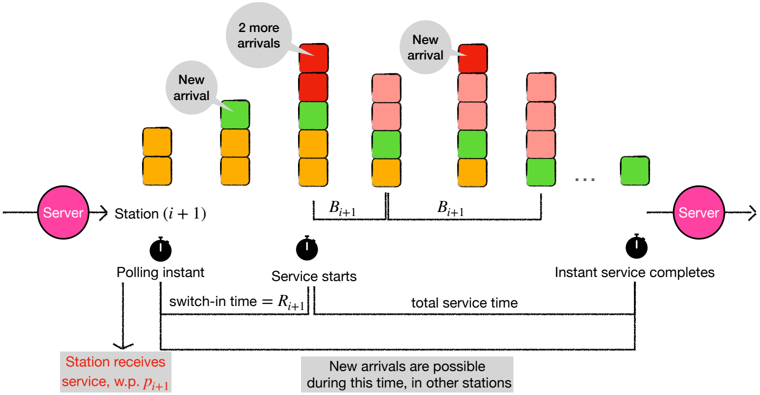


Figure 2: Partially-exhaustive policy (serve all, except those arriving during the switch-in time)

Optimization from server's perspective

- consider a mixed CBP system
  - some stations use gated, some use partially exhaustive, while others use exhaustive discipline
- Q. What are the *optimal switch-in probabilities* to *minimize the expected workload of the system*?

$$\min_{p_1,\ldots,p_N} \sum_{i=1}^{N} \rho_i E[W_i(p_1,\ldots,p_N)]$$

$$\text{subject to:} \quad 0 < p_i \leq 1, \text{ for all } i \in \{1,\ldots,N\}.$$

- pseduo-conservation law $\longrightarrow$ closed-form expression for $\sum_{i=1}^{N} \rho_i E[W_i(p_1,\ldots,p_N)]$

Optimization from server's perspective (contd.)

Paradoxical result [Altman and Yechiali, 1993]

If $N = 1$, then it is optimal to have $p_1^* < 1$, under some conditions!

$\implies$ to minimize the waiting time, it is not the best strategy to serve the queue always!!

Reverse question: from stations' perspective

If the stations could decide to accept or reject the service from the server based on some objective, what will be their individual choice of p_i?

- choice of each station will depend on others' choices
- solution is obtained via a non-cooperative game among stations

Literature survey

- stations in our case are strategic!
- in queuing theory, strategic queuing is a sub-field ([Hassin and Haviv, 2003, Hassin, 2016, Rosokha and Wei, 2024, Gaitonde and Tardos, 2020, Bendel and Haviv, 2018, Burnetas et al., 2017])
- strategic polling can be a sub-field for theory of polling systems:
  - [Adan et al., 2018]: routing game for customers in a two-queue polling system
  - [Dvir et al., 2020]: game between the server and the customers in a tandem queue
    - server decides the operating scheme and the price charged to the customers
    - customers decide whether to join the queue or balk

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Game formulation

- single server, N number of stations cyclic movement, ... (as before)
- walking times, service times, arrivals, switch-in times, ... (as before)

BUT...

- × server decides probabilities to serve or not serve the stations (old)
- ✓ stations decide the probabilities to accept or reject the service (new)

Game formulation (contd.)

Non-cooperative game $\langle \mathcal{N}, (\mathcal{A}_i)_i, (c_i)_i \rangle$

- each station acts as a player $\implies \mathcal{N} := \{1, \ldots, N\}$ is the set of players
- $\mathcal{A}_i := \left[\underline{p}_i, 1\right]$ is set of strategies for station/player i, for some $\underline{p}_i > 0$
  - $p_i \in \mathcal{A}_i$ represents P(station i accepts the service when polled)
- $c_i : \prod_{i=1}^{N} \mathcal{A}_i \to \mathbb{R}$ is the cost function of the station i (in steady state)

Game formulation (contd.)

Common knowledge among stations

- ✓ system parameters (like mean arrival, service, switch-in and walking times)
- ✓ service discipline used by each station
  - $\mathcal{G}$: set of stations using gated discipline
  - $\mathcal{P}$: set of stations using partially-exhaustive discipline
- × position of the server
- × lengths of other queues

Our main questions

- for certain cost functions, what is the (pure/mixed) Nash equilibrium, if it exists?
- is P(station accepts service from the server) < 1 or $= 1$, at equilibrium?

Nash equilibrium

- $\sigma_i : \mathcal{A}_i \to [0,1]$ is a mixed strategy if it assigns to each pure strategy $p_i \in \mathcal{A}_i$, a probability $\sigma_i(p_i)$ such that $\sum_{p_i \in \mathcal{A}_i} \sigma_i(p_i) = 1$.

- a (mixed) strategy profile $(\sigma_1^*, \ldots, \sigma_N^*)$ is called a Nash equilibrium if:

$$\underbrace{c_i(\sigma_i^*, \sigma_{-i}^*) \leq c_i(p_i, \sigma_{-i}^*)}_{\text{better not to deviate alone}}, \text{ for all } p_i \in \mathcal{A}_i, \text{ for all } i \in \mathcal{N}.$$

Three variants of games

Among selfish stations

$$c_i(p_1, \ldots, p_N) = E[W_i(p_1, \ldots, p_N)]$$

(own expected waiting time)

Team approach

$$c_i(p_1, \ldots, p_N) = \sum_{i=1}^{N} \rho_i E[W_i(p_1, \ldots, p_N)]$$

(same as server's objective before)

Among partially-cooperative station

$$c_i(p_1, \ldots, p_N) = \left(\sum_{i=1}^{N} \rho_i E[W_i(p_1, \ldots, p_N)] \right) + Q_i p_i$$

(extra cost, $Q_i \geq 0$)

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Game among selfish stations

- recall, objective is to minimize expected waiting time

$$c_i(p_1, \dots, p_N) = E[W_i(p_1, \dots, p_N)]$$

- either all stations use gated ($\mathcal{N} = \mathcal{G}$) or all use partially exhaustive ($\mathcal{N} = \mathcal{P}$) policy
- game looks simple, but it's NOT ...

Game among selfish stations (contd.)

- X_k^i: number of customers in queue k, when station i is polled (at steady state)
- $f_k(i) = E[X_k^i]$ and $f_i(i,i) = E[(X_i^i)^2]$
- for gated discipline:

$$c_i(p_1,\ldots,p_N) = \frac{1+\rho_i}{2\lambda_i} \frac{f_i(i,i;p_1,\ldots,p_N)}{f_i(i;p_1,\ldots,p_N)} + r_i$$

closed-form expression is available (for $f_i(i;p_1,\ldots,p_N)$)

Game among selfish stations (contd.)

- X_k^i: number of customers in queue k, when station i is polled (at steady state)
- $f_k(i) = E[X_k^i]$ and $f_i(i,i) = E[(X_i^i)^2]$
- for gated discipline:

solution of N^3 linear equations; expression is not direct

$$c_i(p_1, \ldots, p_N) = \frac{1+\rho_i}{2\lambda_i} \frac{f_i(i; p_1, \ldots, p_N)}{f_i(i; p_1, \ldots, p_N)} + r_i$$

closed-form expression is available

- when $k \neq i, k \neq l$:

$$f_{k+1}(i,l) = p_k \Big\{ \lambda_i \lambda_l [d_k^{(2)} + 2 d_k r_k + r_k^{(2)}] + (d_k + r_k)[\lambda_l f_k(i) + \lambda_i f_k(l)] + f_k(k) \lambda_i \lambda_l [2(d_k + r_k) b_k + b_k^{(2)}] + f_k(i,l) + b_k \lambda_l f_k(k,i) + b_k \lambda_i f_k(k,l) + b_k^2 \lambda_i \lambda_l f_k(k,k) \Big\} + (1-p_k) \Big\{ \lambda_i \lambda_l d_k^{(2)} + [\lambda_i f_k(l) + \lambda_l f_k(i)] d_k + f_k(i,l) \Big\}.$$

- when $k \neq l$:

$$f_{k+1}(k,l) = p_k \Big\{ \lambda_k \lambda_l [d_k^{(2)} + 2 d_k r_k + r_k^{(2)}] + (d_k + r_k) \lambda_k f_k(l) + f_k(k) \lambda_k \lambda_l [2(d_k + r_k) b_k + b_k^{(2)}] + b_k \lambda_k f_k(k,l) + b_k^2 \lambda_k \lambda_l f_k(k,k) \Big\} + (1-p_k) \Big\{ \lambda_k \lambda_l d_k^{(2)} + [\lambda_k f_k(l) + \lambda_l f_k(k)] d_k + f_k(k,l) \Big\}.$$

- for any k:

$$f_{k+1}(k,k) = p_k \Big\{ \lambda_k^2 [d_k^{(2)} + 2 d_k r_k + r_k^{(2)}] + f_k(k) \lambda_k^2 [2(d_k + r_k) b_k + b_k^{(2)}] + b_k^2 \lambda_k^2 f_k(k,k) \Big\} + (1-p_k) \Big\{ \lambda_k^2 d_k^{(2)} + 2 \lambda_k d_k f_k(k) + f_k(k,k) \Big\}.$$

Game among selfish stations (contd.)

Theorem

The buffer occupancy (linear) equations admit a unique solution.

Theorem

The expected waiting time $E[W_i(p_1, \ldots, p_N)]$ *is continuous in* $p_1, \ldots, p_N$, *for each station* i.

Theorem

A mixed strategy NE exists.

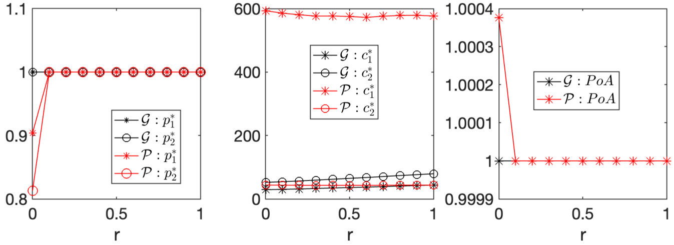
Numerical analysis

- at equilibrium, does a station accepts service from the server w.p. $<$ or $=1$?
- how does the cost vary at equilibrium?
- how does the (in)efficiency of the system compare w.r.t. that at the NE?

$$\text{Price of Anarchy (PoA)} = \frac{\max_{\mathbf{p}^*} \sum_{i=1}^{N} c_i(\mathbf{p}^*)}{\min_{\mathbf{p}} \sum_{i=1}^{N} c_i(\mathbf{p})}$$

Numerical analysis - for ‘selfish’ players

	station 1	station 2
arrival rate	0.393	0.420
E[service time]	0.137	0.292
E[walking time]	0.009	2.210



- even with selfish stations, P(stations reject service) can be 0
- station 2 has higher arrival rate, lower service rate $\implies$ cost under $\mathcal{P}$ is highest
- PoA $= 1 \implies$ best to accept service always
- non-cooperation can lead to $p_i^* < 1$

Game among (partially) cooperative stations

- recall, $c_i(p_1, \ldots, p_N) = \left(\sum_{i=1}^{N} \rho_i E[W_i(p_1, \ldots, p_N)] \right) + Q_i p_i$, where $Q_i \geq 0$
- some/all stations use gated or partially exhaustive policy
- closed-form expression for cost function is available here

Game among (partially) cooperative stations (contd.)

Theorem: when switch-in time equals zero

- if $Q_i = 0$ for all i —

$$p_i^* = 1 \text{ for each } i \text{ is the unique pure strategy NE}$$

- if $Q_i > 0$ at least for some i —

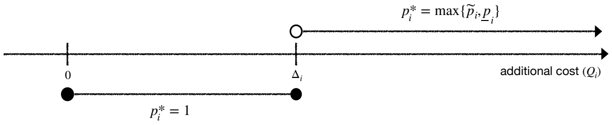


Figure 3: unique pure strategy NE

- in the above, $\widetilde{p}_i$ is some constant, which decreases with Q_i

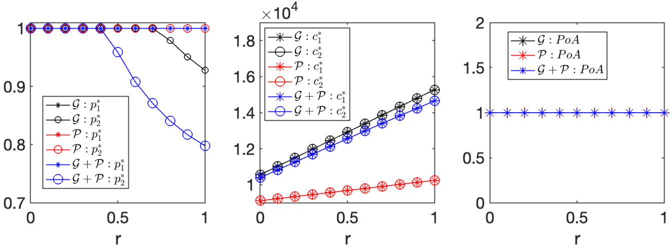
Game among (partially) cooperative stations (contd.)

Theorem: when switch-in time is positive

- there exists a mixed strategy NE
- there exists $\underline{\rho}_i \in (0,1)$ for each i such that for all $\rho_i \geq \underline{\rho}_i$, there exists a unique pure strategy NE.

Numerical analysis - for 'team' players

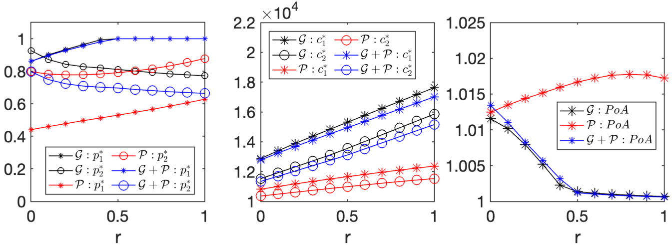
	station 1	station 2
arrival rate	0.153	0.341
E[service time]	0.607	0.192
E[walking time]	4.835	0.766



- again, P(a station reject service) can be > 0
- station 2 with lesser workload rejects service with positive probability
- system is fully efficient at NE

Numerical analysis - for 'partially-cooperative' players

- arrival rate, ... as before, $Q_i = \dfrac{\rho_i d}{1-\rho} + 100$



- $1, 2 \in \mathcal{G}$: station with higher workload accepts with higher probability
- $1, 2 \in \mathcal{P}$: station with less workload accepts with higher probability
- $1 \in \mathcal{G}$, but $2 \in \mathcal{P}$: station 1 accepts with higher probability

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Concluding remarks and future directions

- studied different non-cooperative games among strategic stations in cyclic Bernoulli polling system
- proved the existence of (pure/mixed) strategy Nash equilibrium
- characterized Nash equilibrium, whenever possible
- showed numerically that $P(\text{station reject service}) > 0$ in some cases!
- in future:
    - study alternative service disciplines and routing mechanisms
    - investigate the cooperative counterpart to the non-cooperative games

References I

[Adan et al., 2018] Adan, I. J., Kulkarni, V. G., Lee, N., and Lefeber, E. (2018).
Optimal routeing in two-queue polling systems.
Journal of Applied Probability, 55(3):944–967.

[Altman and Yechiali, 1993] Altman, E. and Yechiali, U. (1993).
Cyclic bernoulli polling.
Zeitschrift für Operations Research, 38:55–76.

[Bendel and Haviv, 2018] Bendel, D. and Haviv, M. (2018).
Cooperation and sharing costs in a tandem queueing network.
European Journal of Operational Research, 271(3):926–933.

[Burnetas et al., 2017] Burnetas, A., Economou, A., and Vasiliadis, G. (2017).
Strategic customer behavior in a queueing system with delayed observations.
Queueing Systems, 86:389–418.

References II

[Dvir et al., 2020] Dvir, N., Hassin, R., and Yechiali, U. (2020).
Strategic behaviour in a tandem queue with alternating server.
Queueing Systems, 96(3):205–244.

[Gaitonde and Tardos, 2020] Gaitonde, J. and Tardos, É. (2020).
Stability and learning in strategic queuing systems.
In *Proceedings of the 21st ACM Conference on Economics and Computation*, pages 319–347.

[Hassin, 2016] Hassin, R. (2016).
Rational queueing.
CRC press.

[Hassin and Haviv, 2003] Hassin, R. and Haviv, M. (2003).
To queue or not to queue: Equilibrium behavior in queueing systems, volume 59.
Springer Science & Business Media.

References III

[Rosokha and Wei, 2024] Rosokha, Y. and Wei, C. (2024).
Cooperation in queueing systems.
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Thank you for your attention!

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