Black holes with electroweak hair

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Brief history of hairy black holes

- No-hair conjecture /Ruffini and Wheeler, 1971/: black holes formed by gravitational collapse are characterized by their mass, angular momentum, and electric charge = the only parameters that can survive the collapse ⇒ all black holes are described by the Kerr-Newman metrics.

- No-hair theorems /Bekenstein, 1972,.../ confirm the conjecture for a number of special cases. No new black holes holes for gravitating massive scalar, spinor, of vector fields, also for a scalar field with a positive potential, etc.

- First explicit counter-example /M.S.V.+ Gal'tsov, 1989/: static black holes with Yang-Mills hair. Triggered an avalanche of discoveries of other hairy black holes.

Black holes with Yang-Millas hair

Non-Abelian Einstein-Yang–Mills black holes

M. S. Volkov and D. V. Gal'tsov
M. V. Lomonosov Moscow State University

(Submitted 7 September 1989)
Pis'ma Zh. Eksp. Teor. Fiz. **50**, No. 7, 312–315 (10 October 1989)

Solutions of the self-consistent system of Einstein-Yang–Mills equations with the SU(2) group are derived to describe black holes with a non-Abelian structure of gauge fields in the external region.

In the case of the electrovacuum, the most general family of solutions describing spherically symmetric black holes is the two-parameter Reissner–Nordström family, which is characterized by a mass M and an electric charge Q. It was recently shown for the Einstein-Yang–Mills systems of equations with the SU(2) group that a corresponding assertion holds when the hold has a nonvanishing color-magnetic charge. In this case the structure of the Yang–Mills hair is effectively Abelian.[1] In the present letter we numerically construct a family of definitely non-Abelian solutions for Einstein-Yang–Mills black holes in the case of zero magnetic charge. These solutions are characterized by metrics which asymptotically approach the Schwarzschild metric far from the horizon but are otherwise distinct from metrics of the Reissner–Nordström family. In addition to the complete Schwarzschild metric, the family of solutions is parametrized by a discrete value of n: the number of nodes of the gauge function. For a

Zoo of hairy black holes

- before 2000: Einstein-Yang-Mills black holes and their generalizations – higher gauge groups, additional fields (Higgs, dilaton), non-spherical solutions, stationary generalizations, Skyrme black holes, Gauss-Bonnet, ...
/M.S.V.+Gal'tsov, Phys.Rep. 319 (1999) 1/

- after 2000: black holes with scalar hair – engineered potential, spinning clouds of massive complex scalar /Herdeiro-Radu/, Horndeski black holes, metric-affine theories, higher dimensions, stringy corrections, hairy black holes with massive gravitons /Gervalle+M.S.V., 2020/, etc, ...
/M.S.V., 1601.0823/

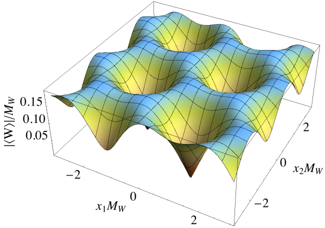
- Which of these solutions are physical ?

Present status of hairy black holes

- All known solutions have been obtained within simplified theoretical models. They are nice theoretically but their physical relevance is not obvious.
- To be physically relevant, the solution should be obtained within the context of the physical theory = Einstein's gravity +Standard Model of fundamental interactions (QCD+electroweak).
- Classical configurations in the QCD sector are destroyed by large quantum corrections ⇒ useless to study. There remains the gravitating electroweak theory = Einstein-Weinberg-Salam. This describes the Kerr-Newman black holes. Does it describe other black holes ?
- Only unphysical limits of the electroweak theory have been analyzed in the black hole context, since in the full theory the spherical symmetry is lost.

Electroweak condensation /Ambjorn-Olesen 1989/

- Homogeneous magnetic field $\vec{\mathcal{B}} = (0, 0, \mathcal{B})$ with constant Higgs is the energy minimum if $\mathcal{B} < m_{\mathrm{w}}^2/e \approx 10^{20}$ T.
- For $m_{\mathrm{h}}^2/e < \mathcal{B} < m_{\mathrm{h}}^2/e$ the energy minimum develops a condensate of W, Z, Φ and becomes inhomogeneous forming a lattice of vortices. Anti-Lenz: the magnetic field is maximal where the condensate is maximal.



- For $B > m_{\mathrm{h}}^2/e$ the energy minimum is again homogeneous but with zero Higgs: symmetric phase.

Magnetically charged black hole

Radial magnetic field and the Reissner-Nordstrom metric

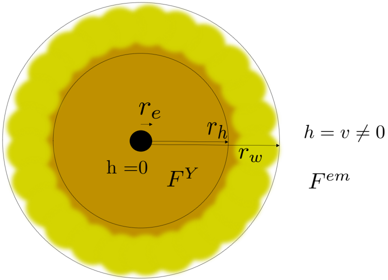
$$\mathcal{B} = P/r^2, \qquad P = \frac{n}{2e}, \qquad n \in \mathbb{Z},$$
$$ds^2 = -N(r)\, dt^2 + \frac{dr^2}{N(r)} + r^2 \left(d\vartheta^2 + \sin^2\vartheta\, d\varphi^2\right),$$
$$N(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2}, \qquad Q^2 = 4\pi G P^2.$$

The event horizon is at $r_h = M + \sqrt{M^2 - Q^2} \geq Q$. The magnetic field at the horizon can be very large,

$$\mathcal{B}(r_h) = \frac{P}{r_h^2} \leq \frac{P}{Q^2} = \frac{1}{\sqrt{4\pi G P}}.$$

If $r_h < \sqrt{eP}/m_{\rm w}$ then the condensate should appear at the horizon.
If $r_h < \sqrt{eP}/m_{\rm h}$ then in addition a bubble of symmetric phase appears.

Magnetic electroweak black hole /Maldacena 2020/



Radial magnetic field near the horizon where Higgs=0, next electroweak "corona" made of vortex pieces, next radial magnetic field in the far field where Higgs=const.

Preliminary analyzis

- The electroweak corona should exist already in flat space around a pointlike magnetic charge. Therefore, one may start by studying electroweak monopoles in flat space.
- Best known are magnetic monopoles of t'Hooft-Polyakov. Their energy is finite but they are obtained within the GUT theories and are not described by the Standard Model.
- Electroweak monopoles were considered by Cho+Maison in 1996 and a lot more extensively in our recent papers Nucl.Phys. B 984 (2022) 115937; B 987 (2023) 116112 together with my student Romain Gervalle. They are superpositions of the pointlike Dirac and extended non-Abelian t'Hooft-Polyakov monopoles. Their energy is infinite because of the pointlike singularity.
- Including gravity converts them into hairy black holes and renders the energy finite: R.Gervalle,M.S.V, arXiv:2406.xxxxx

Einstein-Weinberg-Salam

Einstein-Weinberg-Salam theory

$$\mathcal{L} = \frac{1}{2\kappa} R + \mathcal{L}_{\rm WS}$$

$$\mathcal{L}_{\rm WS} = -\frac{1}{4g^2} \mathrm{W}^a_{\mu\nu} \mathrm{W}^{a\mu\nu} - \frac{1}{4g'^2} B_{\mu\nu} B^{\mu\nu} - (D_\mu \Phi)^\dagger D^\mu \Phi - \frac{\beta}{8} \left(\Phi^\dagger \Phi - 1 \right)^2$$

where Higgs is a complex doublet, $\Phi^{\rm tr} = (\phi_1, \phi_2)$,

$$\begin{aligned} B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu \,, \qquad \mathrm{W}^a_{\mu\nu} = \partial_\mu \mathrm{W}^a_\nu - \partial_\nu \mathrm{W}^a_\mu + \epsilon_{abc} \mathrm{W}^b_\mu \mathrm{W}^c_\nu \,, \\ D_\mu \Phi &= \left(\partial_\mu - \frac{i}{2} B_\mu - \frac{i}{2} \tau^a \mathrm{W}^a_\mu \right) \Phi \,. \end{aligned}$$

The length scale and mass scale are $\boldsymbol{l}_0 =$1.5 × 10^{-16} cm and $\boldsymbol{m}_0 =$128.6 GeV. The couplings

$$g^2 = 0.78, \quad g'^2 = 0.22, \quad \beta = 1.88, \quad \kappa = \frac{4e^2}{\alpha} \frac{m_z^2}{M_{\rm pl}^2} = 5.42 \times 10^{-33}.$$

Electron charge $e = -gg'$, $\alpha = 1/137$. The Z, W, Higgs masses in unites of $\boldsymbol{m}_0$ are $m_z = 1/\sqrt{2}$, $m_w = g m_z$, $m_h = \sqrt{\beta} m_z$.

Electromagnetic field

Nambu:

$$e\mathcal{F}_{\mu\nu} = g^2 B_{\mu\nu} - g'^2 n_a \mathrm{W}^a_{\mu\nu}, \quad n_a = (\Phi^\dagger \tau_a \Phi)/(\Phi^\dagger \Phi)$$

defines conserved electric and magnetic currents

$$4\pi \mathcal{J}^\mu = \nabla_\nu \mathcal{F}^{\mu\nu}, \qquad 4\pi \tilde{\mathcal{J}}^\mu = \nabla_\nu \tilde{\mathcal{F}}^{\mu\nu},$$

magnetic charge

$$P = \int \tilde{\mathcal{J}}^0 \sqrt{-\mathrm{g}} d^3x.$$

t'Hooft:

$$F_{\mu\nu} = \mathcal{F}_{\mu\nu} + \epsilon_{abc} n^a \mathcal{D}_\mu n^b \mathcal{D}_\nu n^c = \partial_\mu A_\nu - \partial_\nu A_\mu,$$

electric current

$$4\pi J^\mu = \nabla_\nu F^{\mu\nu}.$$

30 coupled equations to solve:

Weinberg-Salam:

$$\nabla^\mu B_{\mu\nu} = g'^2 \frac{i}{2} (\Phi^\dagger D_\nu \Phi - (D_\nu\Phi)^\dagger \Phi),$$

$$\mathcal{D}^\mu \mathrm{W}^a_{\mu\nu} = g^2 \frac{i}{2} (\Phi^\dagger \tau^a D_\nu \Phi - (D_\nu\Phi)^\dagger \tau^a \Phi),$$

$$D_\mu D^\mu \Phi - \frac{\beta}{4} (\Phi^\dagger\Phi - 1)\Phi = 0,$$

Einstein:

$$G_{\mu\nu} = \kappa T_{\mu\nu}$$

where $\kappa \sim 10^{-33}$ is very small and

$$T_{\mu\nu} = \frac{1}{g^2} \mathrm{W}^a{}_{\mu\sigma} \mathrm{W}^a{}_\nu{}^\sigma + \frac{1}{g'^2} B_{\mu\sigma} B_\nu{}^\sigma + 2D_{(\mu}\Phi^\dagger D_{\nu)}\Phi + \mathrm{g}_{\mu\nu}\mathcal{L}_{\mathrm{WS}}$$

=30 coupled equations. Vacuum solution:

$$\mathrm{g}_{\mu\nu} = \eta_{\mu\nu}, \quad \mathrm{B} = \mathrm{W} = 0, \ \Phi = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Simplest solutions

Reissner-Nordstrom (RN):

$$
\begin{aligned}
B &= W^3 = \frac{n}{2}\cos\vartheta\, d\varphi, \quad W^1 = W^2 = 0, \quad \Phi = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \\
\vec{\mathcal{B}} &= \frac{P\vec{r}}{r^3}, \qquad P = \frac{n}{2|e|}, \qquad n \in \mathbb{Z}, \\
ds^2 &= -N(r)\, dt^2 + \frac{dr^2}{N(r)} + r^2\left(d\vartheta^2 + \sin^2\vartheta\, d\varphi^2\right), \\
N(r) &= 1 - \frac{2M}{r} + \frac{Q^2}{r^2}, \qquad Q^2 = \frac{\kappa n^2}{8e^2}, \qquad r_h = M + \sqrt{M^2 - Q^2}
\end{aligned}
$$

RN-de Sitter:

$$
\begin{aligned}
B &= \frac{n}{2}\cos\vartheta\, d\varphi, \quad W = \Phi = 0 \quad \Rightarrow \quad \vec{\mathcal{B}} = g^2\frac{P\vec{r}}{r^3}, \\
N(r) &= 1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{\Lambda}{3}r^2, \qquad \Lambda = \frac{\kappa\beta}{8}.
\end{aligned}
$$

Perturbative analysis

Perturbations around RN

Perturbations $w_\mu = \mathrm{W}^1_\mu + i\mathrm{W}^2_\mu$ fulfil the Proca equation

$$D^\mu w_{\mu\nu} + ieF_{\nu\sigma} w^\sigma = m_{\rm w}^2 \, w_\nu \,,$$

with $D_\mu = \nabla_\mu + ieA_\mu$ where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. Solution is

$$w = e^{i\omega t}\psi(r)(\sin\vartheta)^j \sum_{\mathrm{m}\in[-j,j]} c_{\mathrm{m}} \left(\tan\frac{\vartheta}{2}\right)^{\mathrm{m}} e^{i\mathrm{m}\varphi}\,(d\vartheta + i\sin\vartheta d\varphi),$$

where coefficients c_{m} are arbitrary. $j = |n|/2 - 1$ ($|n| > 1$) hence $j > 0$ for $|n| > 2$: perturbations are not spherically symmetric.

$$\left(-\frac{d^2}{dr_\star^2} + N(r)\left[m_{\rm w}^2 - \frac{|n|}{2r^2}\right]\right)\psi(r) = \omega^2\psi(r) \qquad (\star)$$

There are solutions $\psi(r)$ with $\omega^2 < 0$ for small r_h (instability), no such modes for large r_h, and a zero mode $\psi_0(r)$ with $\omega = 0$ for a special value $r_h = r_h^0(n)$ = condensate = static hair.

Perturbative black hole hair

n	2	4	6	10	20	40	100	200
r_H^0	0.89	1.47	1.93	2.69	4.12	6.19	10.33	15.03

One has $r_h^0(n) \approx \sqrt{|n|}/g$ for $n \gg 1$ hence $B(r_h^0) \approx m_{\rm w}^2$, which is the condition for the condensate to appear. The condensate is maximal at the horizon.

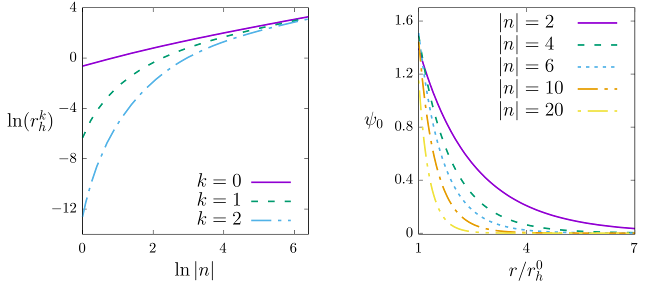


Figure: The RN horizon size r_h^k for which there exists a k-node zero mode $\psi_k(r)$ (left) and the profile of the fundamental zero mode $\psi_0(r)$ (right).

Horizon distribution of vortices

The condensate field w_μ depends on coefficients $c_{\rm m}$ and produces a current $J^\mu = \nabla_\sigma \Im(\bar{w}^\sigma w^\mu)$ tangential to the horizon. The current sources second order corrections for the F, Z, Φ fields forming vortices orthogonal to the horizon. The energy contains the fourth order term $\sim |w_\mu|^4$. To determine the coefficients $c_{\rm m}$ we minimize

$$\langle |w_\mu|^4 \rangle \equiv \int |w_\mu|^4 \sqrt{-{\rm g}}\, d^3x \,,$$

under the condition that the norm should be fixed,

$$\langle |w_\mu|^2 \rangle \equiv \int |w_\mu|^2 \sqrt{-{\rm g}}\, d^3x = const.$$

This leads to the following prescription

Minimization procedure

Minimize with respect to $c_{\rm m}$ and Lagrange multiplier μ the function

$$\begin{aligned} E &= E_4 + \mu\,(E_2 - 1), \\ E_4 &= \sum_{\rm k,m,l\in[-j,j]} A_{2j,\rm k+l}\, c_{\rm m}\, c_{\rm k}\, c_{\rm l}\, c_{\rm k+l-m} \\ E_2 &= \sum_{{\rm m}\in[-j,j]} A_{j,\rm m}\, c_{\rm m}^2 \end{aligned}$$

with $j = |n|/2 - 1$ and

$$A_{j,\rm m} = \int_0^\pi (\sin\vartheta)^{2j+1} \left(\tan\frac{\vartheta}{2}\right)^{2\rm m} d\vartheta = 2^{2j+1} \frac{\Gamma(j+1+{\rm m})\Gamma(j+1-{\rm m})}{\Gamma(2j+2)}\,.$$

This gives values of $c_{\rm m}$ determining positions of $|n| - 2$ vortices homogeneously distributed over the horizon.

Platonic distribution of vortices – corona

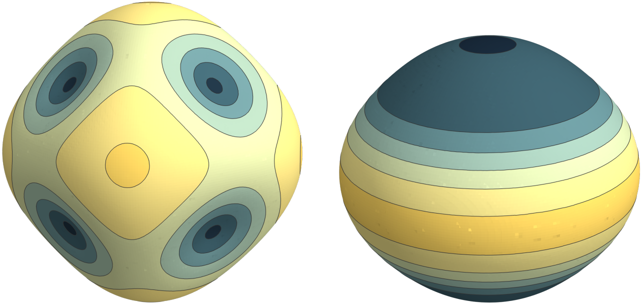


Figure: Left: the horizon distribution of the W-condensate $\bar{w}^{\mu} w_{\mu}$ corresponding to the global energy minimum for $n = 10$. The level lines coincide with the electric current flow forming loops around 8 radial vortices (dark spots) repelling each other and forming a lattice. Right: the same when all vortices merge into two oppositely directed multi-vortices; axial symmetry $c_{\rm m} \sim \delta_{0\rm m}$, also a stationary point.

Non-perturbative analysis

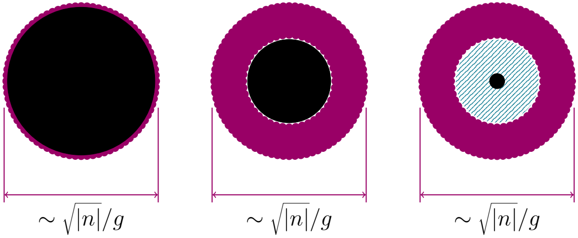
Axial symmetry

$$
\begin{aligned}
ds^2 &= -e^{2\mathrm{U}}\mathrm{N(r)}\,dt^2 + e^{-2\mathrm{U}}\,dl^2, \\
dl^2 &= e^{2\mathrm{K}}\left[\frac{d\mathrm{r}^2}{\mathrm{N(r)}} + \mathrm{r}^2 d\vartheta^2\right] + e^{2\mathrm{S}}\,\mathrm{r}^2\sin^2\vartheta d\varphi^2, \\
\mathrm{W} &= \mathrm{T}_2\,(F_1\,d\mathrm{r} + F_2\,d\vartheta) - \frac{n}{2}\,(\mathrm{T}_3\,F_3 - \mathrm{T}_1 F_4\,)\,d\varphi\,, \\
B &= -(n/2)\,Y\,d\varphi\,, \quad \Phi^{\mathrm{tr}} = (\phi_1, \phi_2).
\end{aligned}
$$

Here $\mathrm{U}, \mathrm{K}, \mathrm{S}, F_1, F_2, F_3, F_4, Y, \phi_1, \phi_2$ are 10 real functions of r, ϑ. For non-extremal solutions $\mathrm{N(r)} = 1 - \mathrm{r_H}/\mathrm{r}$ where $\mathrm{r_H}$ labels the solutions. We require invariance under $\vartheta \to \pi - \vartheta$, obtain 10 elliptic equations fer the 10 functions, and solve them numerically with the FreeFem++ numericatl solver. We solve for values of the gravity coupling $10^{-10} < \kappa < 10^{-2}$ and then extrapolate to the physical value $\kappa \sim 10^{-33}$.

Hairy black holes

Same magnetic charge $P = n/(2|e|)$ as for the RN black hole. We start at $\mathrm{r_H} = \mathrm{r_H^0}(n)$ when hairy solutions just start deviating from RN and then we decrease $\mathrm{r_H}$. The massive hair appears and gets longer as the horizon shrinks. When the field at the horizon increases up to $B(\mathrm{r_H}) = m_\mathrm{h}^2$, the hair stops growing and a bubble of symmetric phase appears. This bubble expands as the horizon shrinks further till reaching the minimal value when it becomes degenerate, surface gravity vanishes, but the area remain finite. The black hole then becomes extremal.



Hair charge

The total magnetic charge P of the black hole splits as

$$P_{\rm h} = \int_{\rm r>r_H} \tilde{\mathcal{J}}^0 \sqrt{-{\rm g}}\, d^3x, \quad P_{\rm H} = P - P_{\rm h},$$

where $P_{\rm h}$ is contained in the hair outside the horizon and $P_{\rm H}$ remains inside. The hair charge $P_{\rm h}$ grows when the horizon shrinks and in the extremal limit one has

$$P_{\rm h} = g'^2 P = 0.22\, P$$

hence 22% of the charge moves to the hair.

ADM mass

is determined from the asymptotic $g_{00} = -1 + 2M/r + \ldots$ or from the formula (same result)

$$M = \frac{k_H A_H}{4\pi} \quad + \quad \frac{\kappa}{8\pi} \int_{r>r_H} \left(-T^0_{\ 0} + T^k_{\ k} \right) \sqrt{-g}\, d^3x,$$

$$\text{surface gravity :} \quad k_H \quad = \quad (1/2)\, N' e^{2U-K} \Big|_{r=r_H}$$

$$\text{horizon area :} \quad A_H \quad = \quad 2\pi r_H^2 \int_0^\pi e^{K+S-2U} \sin\vartheta d\vartheta \Big|_{r=r_H}$$

This can be split as

$$M = M_H + M_h$$

where the "horizon mass" M_H is the mass of the RN black hole with the same area A_H and with the charge P_H. The rest is the "hair mass" $M_h = M - M_H$. When the horizon gets smaller, the hair mass M_h and hair charge P_h increase.

Horizon oblatness

The configurations are not spherical, one can define

$$\begin{aligned}
\text{horizon radius :}\quad r_h &= \sqrt{\mathrm{A_H}/(4\pi)} \\
\text{equatorial radius :}\quad r_{\mathrm{H}}^{\mathrm{eq}} &= \sqrt{\mathrm{g}_{\varphi\varphi}(\mathrm{r_H},\pi/2)} \\
\text{polar radius :}\quad r_{\mathrm{H}}^{\mathrm{pl}} &= (1/\pi)\int_0^{\pi}\sqrt{g_{\vartheta\vartheta}(\mathrm{r_H},\vartheta)}\,d\vartheta \\
\text{horizon oblateness :}\quad \delta &= r_{\mathrm{H}}^{\mathrm{eq}}/r_{\mathrm{H}}^{\mathrm{pl}} - 1
\end{aligned}$$

As the horizon radius decreases, the oblateness δ stars from zero and increases, then reaches a maximum, starts decreasing and approaches zero in the extreme limit. The extremal horizon is perfectly spherical, although the hair is squashed.

Quadrupole moments

Far away from the horizon the theory reduces to electrovacuum, hence one can define the gravitational Q_{G} and magnetic Q_{M} quadrupole moments. They are determined by the asymptotic expansion of the Ernst potentials at the symmetry axis close to the spatial infinity.
As the horizon size decreases and black hole gets more hairy, the quadrupole moments grow.

Non-extremal hairy solutions

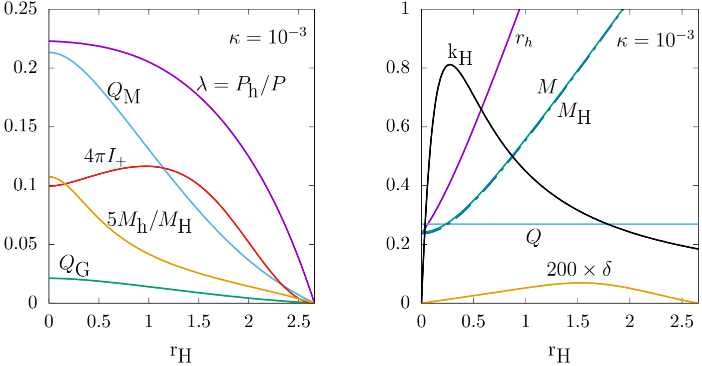


Figure: Parameters of non-extremal solutions with $n = 10$, $\kappa = 10^{-3}$. The M and M_H curves are very close to each other. For $r_H \to 0$ they become extremal, for $r_H \to 2.5$ they loose hair and become RN.

Extremal hairy solutions

They have zero surface gravity and are the most hairy. Depending on the value of their charge parameter

$$Q = \sqrt{\frac{\kappa}{2}}\, P = \sqrt{\frac{\kappa}{8}}\frac{n}{|e|}$$

there are two phases,

$$\text{phase I}: \quad Q < Q_\star, \qquad\qquad \text{phase II}: \quad Q > Q_\star,$$

where

$$Q_\star \approx \frac{0.3}{g\sqrt{\Lambda}}, \qquad \Lambda = \frac{\kappa\beta}{8}$$

In phase I one has $B(r_h) > m_{\mathrm{h}}^2$ and the Higgs field vanishes at the horizon. In phase II one has $B(r_h) < m_{\mathrm{h}}^2$ and the Higgs field deviates from zero at the horizon.

Extremal hairy solutions in phase I ($n = 40$)

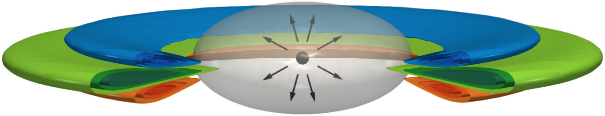


Figure: The extremal solutions contain a small charged black hole inside a bubble of symmetric phase, surrounded by a ring-shaped EW condensate supporting 22 % of the total magnetic charge and two opposite superconducting W-currents. This creates pieces of two magnetic multi-vortices along the positive and negative z-directions. Farther away the condensate disappears and the magnetic field becomes radial.

Extremal hairy solutions in phase I

They have $Q < Q_\star$ and zero horizon oblateness, $\delta = 0$. The Higgs vanishes at the horizon, the horizon geometry coincides with the extreme RN-de Sitter with $r_{\rm ex} \approx g|Q|$

$$
\begin{aligned}
ds^2 &= -N\,dt^2 + \frac{dr^2}{N} + r^2(d\vartheta^2 + \sin^2\vartheta\,d\varphi^2) \qquad (\star) \\
N &= \left(1 - \frac{r_{\rm ex}}{r}\right)^2 \left(1 - \frac{\Lambda}{3}\left[r^2 + 2rr_{\rm ex} + 3r_{\rm ex}^2\right]\right)
\end{aligned}
$$

78% of the magnetic charge is inside the horizon and 22% is outside in the hair. Far away from the horizon the geometry approaches RN described by ($\star$) with

$$
N = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} + \mathcal{O}(1/r^3)
$$

where (!!!)

$$
M < |Q|
$$

Weak gravity

The hair carries 22% of the total charge, $Q_{\rm h} = 0.22 \times Q$, but hair mass $M_{\rm h}$ is very small due to the negative Zeeman energy of the condensate interacting with the magnetic field of the black hole, which shifts the W-mass as

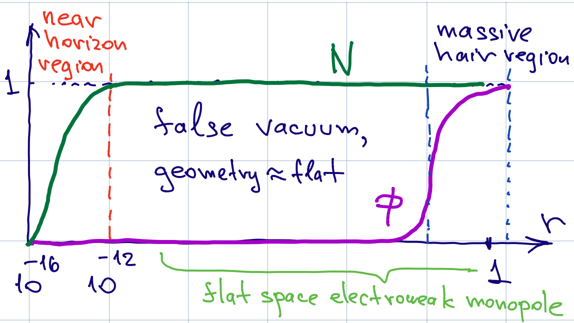
$$m_{\rm w}^2 \to m_{\rm w}^2 - |{\rm B}| \approx 0$$

As a result, the mass-to-charge ratio for the hair is very small, $M_{\rm h}/|Q_{\rm h}| \sim \sqrt{\kappa} \ll 1$. This can be viewed as a manifestation of the weak gravity conjecture. The condensate is magnetically repelled by the black hole stronger than attracted gravitationally, but it cannot fly away because it has to follow the Yukawa law. Since the hair mass is small, one has (if $Q \ll Q_\star$)

$$M = M_{\rm H} + M_{\rm h} \approx M_{\rm H} = \frac{r_{\rm ex}}{2} + \frac{g^2 Q^2}{2 r_{\rm ex}} \approx g|Q| = 0.88\,|Q| < |Q|$$

Hairy black hole is less energetic than RN for which $M \geq Q$.

Extrapolating toward $\kappa \sim 10^{-33}$ if $Q \ll Q_\star$



$$ds^2 \approx -N\,dt^2 + \frac{dr^2}{N} + r^2\left(d\Theta^2 + \sin^2\Theta\, d\varphi^2\right)$$

near-horizon region is parametrically small as compared to the hair region. The hair lives in flat geometry and does not see the black hole, but the latter renders the mass finite by putting the cutoff at the horizon.

Hairy black holes as magnetic monopoles

- Most popular magnetic monopoles of t'Hooft-Polyakov are not described by the Standard Model. They are described by GUT theories, which may or may not exist, hence the existence of these monopoles is not obvious.
- Standard Model definitely exists and admits solutions describing electroweak monopoles, but in flat space their energy diverges because $B \sim n/(2r^2)$. This divergence might be cured by renormalization, but so far nobody has proven that renormalization works for magnetic charges.
- Gravity converts monopoles to hairy black holes and renders their mass finite:

$$M \approx 5.1\,|n|\,M_{\rm Pl}$$

Therefore, it is possible that these black holes are the only magnetic monopoles which may exist in Nature.

Increasing the charge

- When the charge $Q \propto n$ grows, the sizes of the vacuum bubble and of the hairy region scale as $\sqrt{|Q|}$. The hair grows longer till macroscopic size of order 1 cm for very large Q. The horizon size grows even faster, $r_{\rm ex} \propto |Q|$, and the black hole absorbs the bubble.
- The hair mass $M_{\rm h}$ grows faster than the horizon mass $M_{\rm H}$ and the ratio $M/|Q|$ increases, always remaining smaller than one.
- The horizon value of the hypermagnetic field $B \sim 1/|Q|$ decreases and when $B < m_{\rm h}^2$, the horizon value of the Higgs deviates from zero and the system enters phase II.

Extremal hairy solutions in phase II

- In phase I, for $Q < Q_\star$, the horizon spherical – the oblateness is zero, $\delta = 0$. In phase II, for $Q > Q_\star$, symmetry changes and horizon squashes, $\delta > 0$. Near the transition point one has (with $s \approx 10.8$ if $\kappa = 10^{-2}$)

$$\delta \propto (|Q| - Q_\star)^s$$

  This looks like a second order phase transition.

- The fraction of the hair charge which was constant in phase I, $P_{\rm h}/P = 0.22$, starts decreasing. The black hole is getting less hairy, the ratio $M/|Q|$ grows, the geometry approaches extreme RN.

- The solution merges with extreme RN when the horizon size overtakes the hair size, for

$$Q_{\rm max} = 2.15\, Q_\star = \frac{1.29}{2g\sqrt{\Lambda}}$$

  No hairy solutions for $Q > Q_{\rm max}$.

Existence diagram

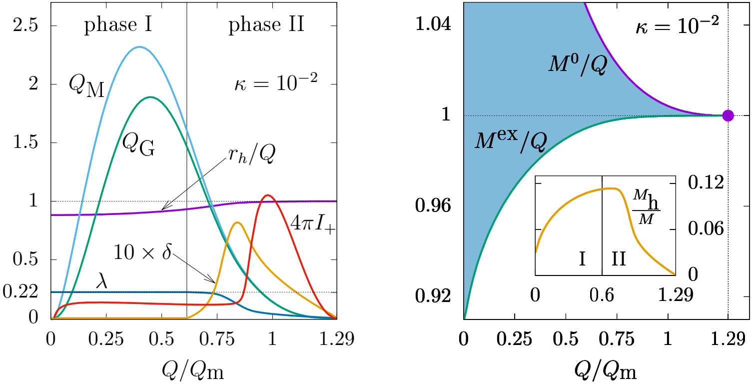


Figure: The parameters of extremal solutions (left) and the existence diagram for hairy solutions (right) for $\kappa = 10^{-2}$; $Q_m = 1/(g\sqrt{\Lambda})$.

Planetary mass

The black hole is maximally hairy around the phase transition point when the fraction of the hair mass M_{h}/M is maximal. Then

$$|n| \approx 1.5 \times 10^{32}, \qquad r_h \approx 3 \text{ cm},$$

the black hole mass has a planetary value,

$$M \approx 2 \times 10^{25} \quad \text{kg}$$

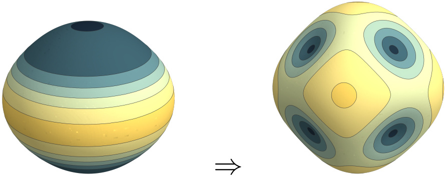
of which $\approx 11\%$ is contained in the hair condensate.

Stability

According to Maldacena, the corona greatly enhances the Hawking evaporation rate. Therefore, non-extremal black holes quickly relax to the extremal state when their temperature is zero and

$$M < |Q|$$

Therefore, they cannot decay into RN black holes. However, axially symmetric black holes can further reduce their mass by splitting their hair into a hedgehog of vortices, and then they seem to become absolutely stable. The corresponding solutions have not yet been obtained.



Conclusions

- We constructed for the first time hairy black holes described by well-tested theories, GR and SM. This suggests that they may really exist in Nature. Perhaps they could have been created by fluctuations in primordial electroweak plasma.
- They provide a spectacular example of the electroweak condensation. They seem to be absolutely stable. Perhaps they are the only magnetic monopoles which may exist.
- Estimates based on the Parker bound and proton decay catalysis show that contribution of magnetic black holes into Dark Matter should be small. However, it is possible that they form neutral pairs stabilized by a scalar attraction, then the estimates may change.