HIGHER-DIMENSIONAL ORIGIN OF EXTENDED BLACK HOLE THERMODYNAMICS

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Goal: Use **braneworld holography** to

- ⋆ Show that dynamical Λ is natural
- ⋆ Elucidate the origin of the extended BH thermodynamics

Outline:

1. **Extended BH thermodynamics**
   - Smarr relation
   - Thermodynamic Volume
   - Isoperimetric inequalities
2. **Chemical phenomena**: Phase transitions, VdW, triple points
3. **Braneworld holography**: review
   - Quantum black holes on the brane
   - Extended thermodynamics of quantum black holes

EXTENDED BLACK HOLE THERMODYNAMICS

Schwarzschild black hole in AF:

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \frac{dr^2}{1 - \frac{2M}{r}} + r^2 d\Omega^2$$

- Important quantities:

1. Asymptotic mass (total energy)
2. BH horizon radius $r_h = 2M$

Surface area: $A = 4\pi r_h^2$ never decreases

3. Surface gravity κ

- Observation $\Rightarrow$ Black hole mechanics

$$\delta M = \kappa \delta A \overset{\text{Bekenstein}}{\Longleftrightarrow} \delta E = T \delta S$$

- Hawking (1974) When quantum effects are taken into account, black holes radiate away particles

First law of black hole thermodynamics:

$$\boxed{\delta M = T\delta S + \Omega\delta J + \Phi\delta Q}$$

Smarr-Gibbs-Duhem relation (when $\Lambda = 0$) :

$$\boxed{M = 2TS + 2\Omega J + \Phi Q}$$

Energy	E	$\longleftrightarrow$	M	BH mass
Temperature	T	$\longleftrightarrow$	$\frac{\hbar\kappa}{2\pi}$	Surface gravity
Entropy	S	$\longleftrightarrow$	$\frac{A}{4G\hbar}$	Horizon Area

$$\boxed{\delta E = T\delta S + \begin{array}{c}\text{work}\\\text{terms}\end{array} \quad\longleftrightarrow\quad \delta M = \frac{\kappa}{8\pi G}\delta A + \Omega\delta J + \Phi\delta Q}$$

1st law and Smarr relation are connected by a scaling argument

Euler's theorem:

$$f(\alpha^p x,\ \alpha^q y) = \alpha^r f(x,y) \Longrightarrow r f(x,y) = p\left(\frac{\partial f}{\partial x}\right)x + q\left(\frac{\partial f}{\partial y}\right)y$$

Smarr relation for $\Lambda \neq 0$:

Mass of the black hole in AdS: $M = M(A, \Lambda)$

$\Rightarrow$ since $[\Lambda] = L^{-2}$, $\quad [A] = L^2$, $\quad [M] = L$, follows

$$M = 2A\left(\frac{\partial M}{\partial A}\right) - 2\Lambda\left(\frac{\partial M}{\partial \Lambda}\right)$$

It can be generalized in generic spacetime dimensions

Where is the PδV term?

- PδV is commonplace in everyday thermodynamics
  $\Rightarrow$ but there is no obvious notion of P or V associated with a black hole

Where is the $P\delta V$ term?

- $P\delta V$ is commonplace in everyday thermodynamics
  $\Rightarrow$ but there is no obvious notion of P or V associated with a black hole
- In the last few years **a new perspective has emerged that incorporates these P and V into black hole thermodynamics** [Dolan, Creighton, Mann, Kastor, Traschen, Padmanabhan, Cvetic, Gibbons, Kubiznak, Pope, Gregory, Fischler, Nguyen, Johnson, Karch, Ortin,....]
- The idea: pressure can be associated with a negative Λ
  $\Rightarrow$ a form of energy whose (positive) pressure is equal in magnitude to its (negative) energy density

Where is the $P\delta V$ term?

BRIEF HISTORY OF THE DYNAMICAL Λ: An old tale

- Study of the dynamics of the gravitational field when $\Lambda < 0$. Regard Λ as a constant of the motion (”hair” in BH as M or Q) rather than as a fundamental parameter.
  [Henneaux, Teitelboim, Asymptotically Anti-de Sitter Spaces, Commun. Math. Phys. 98, 391-424 (1985)]
- Examine the consequences of this new role of Λ for the laws governing the evolution of event horizons in BHs and cosmology
  [Teitelboim, The Cosmological constant as a thermodynamic black hole parameter, PLB 158 (1985)]

Where is the $P\delta V$ term?

1. Consider an asymptotically AdS black hole spacetime
2. Identify the cosmological constant with a thermodynamic pressure
$$P = -\frac{\Lambda}{8\pi} = \frac{3}{8\pi L^2}$$
3. Allow this to be a (thermo)dynamical quantity

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It brings our understanding of certain properties of AdS black holes more closely in parallel with well-known results in the AF case.

[Kastor, Ray and Traschen, CQG 26 (2009) 195011, arXiv:0904.2765]

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- Consistency between 1^{st} law and Smarr formula in AdS
- Concept of the thermodynamic Volume (conjugate quantity) $\rightarrow$ Isoperimetric inequalities conjecture for AdS black hole
- Study of black hole phase transitions

Enthalpy	H	$\longleftrightarrow$	M	BH mass
Temperature	T	$\longleftrightarrow$	$\frac{\hbar\kappa}{2\pi}$	Surface gravity
Entropy	S	$\longleftrightarrow$	$\frac{A}{4G\hbar}$	Horizon Area
Pressure	P	$\longleftrightarrow$	$-\frac{\Lambda}{8\pi}$	Cosmological Constant

$$\delta H = T\delta S + V\delta P + \cdots \longleftrightarrow \delta M = \frac{\kappa}{8\pi G}\delta A + V\delta P + \cdots$$

The Smarr Relation, in general dimensions, reads:

$$\frac{d-3}{d-2} M = T S + \sum_i \Omega^i J^i + \frac{d-3}{d-2}\Phi Q - \frac{2}{d-2} PV$$

THERMODYNAMIC VOLUME

Extended phase space thermodynamics enables one to define a new "intrinsic" quantity — **thermodynamic volume** — associated with the BH horizon.

$$V \equiv \left(\frac{\partial M}{\partial P}\right)_{S,Q,J,..}$$

- Often gives a finite result in the limit to AF ($P \to 0$)
- Smoothly connected to its AdS counterpart: a way for defining a thermodynamic volume of asymptotically flat black holes.

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Geometrical derivation: Surface integral of a two-form potential from the Killing field $\propto$ a finite, **effective volume** for the region outside the AdS black hole horizon [Kastor, Ray, Traschen, Enthalpy and the Mechanics of AdS Black Holes, Class. Quant. Grav. 26:195011 (2009)]

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- Often gives a finite result in the limit to AF ($P \to 0$)
- ‘Smoothly connected’ to its AdS counterpart: a way for defining a thermodynamic volume of asymptotically flat black holes.

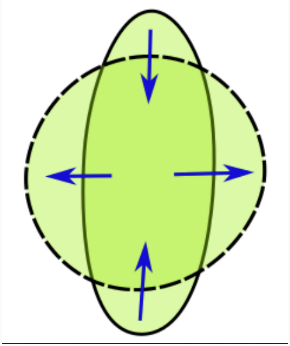
Different proposal: The **thermodynamic volume** should be replaced with a more general notion of **gravitational tension** that describes the extra energy associated with the presence of gravitational fields surrounding a black hole. [Armas, Obers and Sanchioni, Gravitational Tension, Spacetime Pressure and Black Hole Volume, JHEP 09 (2016) 124, arXiv:1512.09106]

Euclidean space: The isoperimetric inequality for volume V bounded by a surface of area A

$$\mathcal{R} = \left[\frac{V}{\mathcal{V}_0}\right]^{1/(d-1)} \left(\frac{\mathcal{A}_0}{A}\right)^{\frac{1}{d-2}} \leq 1$$

$\mathcal{A}_0$ and $\mathcal{V}_0$: area and volume of the unit (d–2)-sphere

The isoperimetric inequality states that a sphere has the smallest surface area per given V.



Conjecture for the thermodynamic volume:

Reverse Isoperimetric Inequality (RII): [Cvetic, Gibbons, Kubiznak and Pope, "Black Hole Enthalpy and an Entropy Inequality for the Thermodynamic Volume", PRD 84 024037 (2011)]

$$\boxed{\mathcal{R} = \left[\frac{V}{\mathcal{V}_0}\right]^{1/(D-1)} \left(\frac{\mathcal{A}_0}{A}\right)^{\frac{1}{D-2}} \geq 1}$$

- ⋆ This inequality is the "inverse" of the isoperimetric inequality
- ⋆ **At fixed thermodynamic volume, the Schwarzschild-AdS black hole has maximum entropy (saturate the RII)**

$$A_{Sol}(V) \leq A_{Schw}(V)$$

- Violated only in two cases: Charged BTZ & rotating AdS black hole solutions with non-compact event horizons (supergravity)

$$\boxed{\mathcal{R} = \left[\frac{(D-1)\,V}{\omega_{D-2}}\right]^{\frac{1}{D-1}} \left(\frac{\omega_{D-2}}{A}\right)^{\frac{1}{D-2}} \geq 1}$$

⋆ The original RII holds as an equality for certain solutions (Schwarzschild-AdS, Reissner-Nordstrom-AdS, ...)

$$A_{Sol}(V) \leq A_{Schw}(V)$$

- **Does there exist a generalization that holds with angular momentum?**
- Can the original RII be one member of a hierarchy of inequalities that relate the thermodynamic volume and entropy ?

NEW CONJECTURE: STRONG VARIANT (RRII)

[Amo, **Frassino**, Hennigar, Phys.Rev.Lett.131(2023)24, 241401arXiv : 2307.03011]

$$\boxed{A_{Sol}(V, J, M) \leq A_{Kerr}(V, J, M)}$$

For fixed values of (M, J, V) the Kerr-AdS black hole (if it exists) has maximum entropy.

- Any deformation of the solution, e.g. through the incorporation of additional charges or matter fields, leads to a decrease in the black hole entropy.
- In the limit $J_i \to 0$, the Kerr-AdS area reduces to the Schwarzschild-AdS area and the previous RII is recovered

CHEMICAL PHENOMENA

- Let´s consider charged (and rotating) AdS black holes in a canonical (fixed Q or J) ensamble
- The corresponding thermodynamic potential is **Gibbs free energy**

$$G = M - TS = G(P, T, J, Q)$$

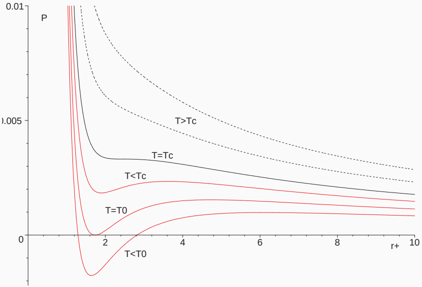
  An equilibrium state corresponds to the global minimum of G
- **Local thermodynamic stability:** positivity of the specific heat

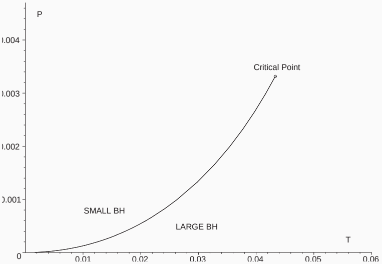
$$C_P = C_{P,J,Q} = T\left(\frac{\partial S}{\partial T}\right)_{P,J,Q}$$

- **Phase diagrams:** P-T diagrams
- **Critical points:** calculate critical exponents

NOVEL CHEMICAL-TYPE PHASE BEHAVIOUR

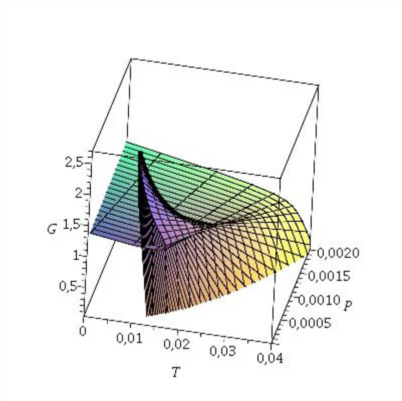
Van der Waals system in charged AdS-Schwarzschild 4D BHs

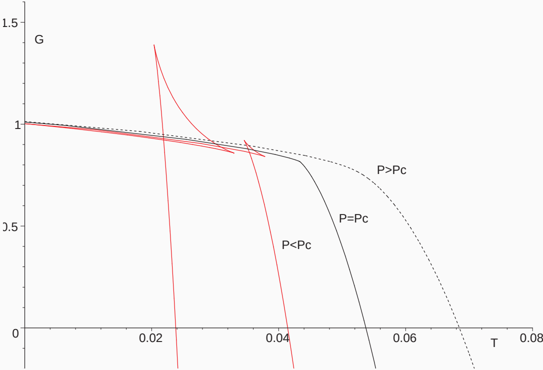




- The VdW analogy for a charged black hole was discovered in: [Chamblin, Emparan, Johnson and Myers, “Charged AdS Black Holes and Catastrophic Holography ”, PRD 60 (1999) 064018]
- Properties of these transitions in the ’extend phase space’ have been worked out in: [Kubiznak and Mann, “P-V criticality of charged AdS black holes”, JHEP 1207 (2012) 033]
- In this approach, HP phase transition $\rightarrow$ solid-liquid phase transition.

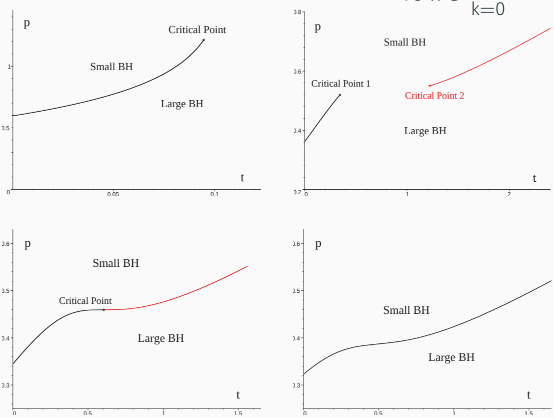
GIBBS FREE ENERGY FOR CHARGED ADS BHS

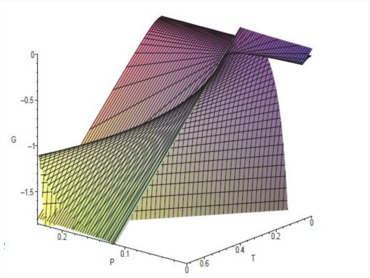




Lovelock higher curvature gravity:

$$\mathcal{L} = \frac{1}{16\,\pi G} \sum_{k=0}^{k_{max}} \hat{\alpha}_{(k)} \mathcal{L}^{(k)}$$



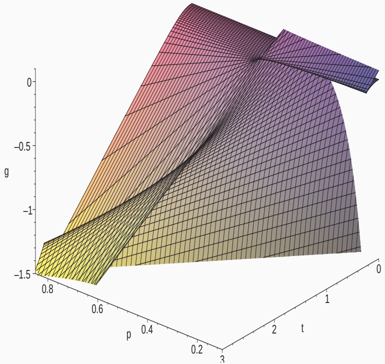


$k_{max} = 3, d = 7$

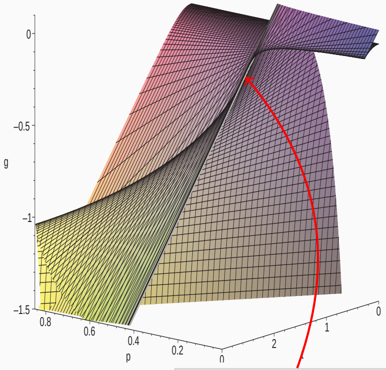
Interesting critical exponents

$$\alpha = 1, 1.65, \sqrt{3}, 1.85$$

$\alpha = 1.65$



$\alpha = \sqrt{3}$

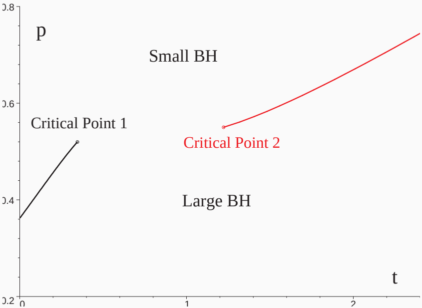


Two critical points merge together

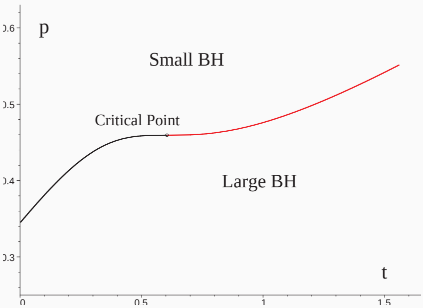
In the Gibbs free energy: two swallowtails emerge,
giving rise to two first-order phase transitions between small and large black holes. [**Frassino**, Kubiznak, Mann, Simovic, JHEP 1409 (2014)]

3rd order Lovelock $q = 0, d = 7, \kappa = -1, \alpha = \alpha_2/\sqrt{\alpha_3}$

$\alpha = 1.65$



$\alpha = \sqrt{3}$



- The system can be solved analytically (corresponding BH has $M = 0$)
- The special isolated critical point is characterized by non-standard critical exponents

$$\tilde{\alpha} = 0, \qquad \tilde{\beta} = 1, \qquad \tilde{\gamma} = 2, \qquad \tilde{\delta} = 3$$

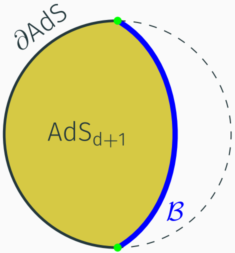
VARIABLE Λ : BRANEWORLD HOLOGRAPHY

[**Frassino**, Pedraza, Svesko, Visser, Phys. Rev. Lett. 130 (2023), 161501 (arXiv:2212.14055)]

As it happened with the initial BH thermodynamics – started from an analogy – now we go beyond the analogy and find a stronger indication of the extended thermodynamics:

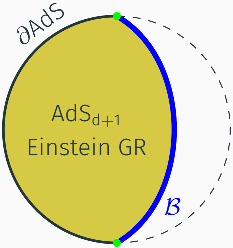
- Holographic braneworlds are used to present a higher-dimensional origin of extended BH thermodynamics.
- In this framework, classical, asymptotically AdS black holes map to quantum BHs in one dimension less, with a conformal matter sector that backreacts on th brane geometry.
- Varying the brane tension alone leads to a dynamical cosmological constant on thebrane, and, correspondingly, a variable pressure attributed to the brane black hole.
- Thus, standard thermodynamics in the bulk (including a work term coming from the brane) induces extended thermodynamics on the brane, exactly, to all orders in the backreaction.

Basic idea of BRANEWORLD gravity: recover gravity localized on a lower dimensional surface of a higher dimensional bulk spacetime



One application of considering warped geometries with branes is that it provides a generalization of AdS/CFT to spaces with boundaries. [de Haro, Skenderis, Solodukhin, "Gravity in warped compactifications and the holographic stress tensor´´, Classical and Quantum Gravity, 2001]

AdS/CFT: Gravity theory in a $(d+1)$-dim spacetime with a d-dim asymptotic boundary = CFT in a d-dim spacetime



Cutting the bulk with a brane

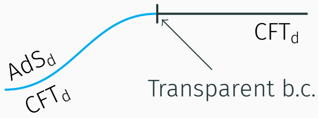
- The CFT is cutoff in the UV and one gets dynamics on the brane

Holographic interpretation (AdS/CFT):

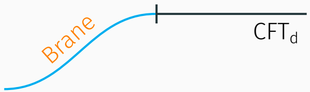
- **Classical bulk** governed by GR + brane $\mathcal{B}$:
  $I_{\text{bulk}} = I_{\text{EH}} + I_{\text{brane}}$ with: $I_{\text{brane}} = -\tau \int_{\mathcal{B}} d^d z \sqrt{-h}$
- **Brane:** Integrate out bulk from ∂AdS to $\mathcal{B}$: $I_{\text{eff}}^{\mathcal{B}} = I_{\text{grav}}[\mathcal{B}] + I_{\text{CFT}}^{\text{cut-off}}[\mathcal{B}]$

1 Start with a CFT_{d-1} and then couple it with a CFT_d

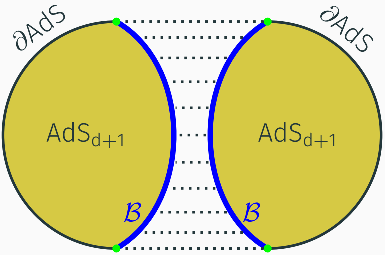
2 Brane perspective: Replace the original CFT_{d-1} by a d-dimensional bulk – then the CFT_d can leak to the dynamical bulk
$\Rightarrow$ EFT of gravity coupled to an holographic CFT_d and a 'bath' that is connected to the bulk with transparent b.c.



3 Gravitational description in terms of the classical EE with a brane source, which can be solved classically



Karch-Randall braneworld [Karch, Randall '00]



- As in RS one glues on the other side of the brane an identical spacetime
- Israel junction conditions fix the location of the brane [Emparan, Johnson, Myers, '99], [Balasubramanian, Kraus, '99]
- 'Massive' gravity due to massive graviton localized on the brane [Karch, Randall, '00] (when the tension is very small and the brane is very close to the boundary and the graviton is almost massless.)

- Holographically induced scales

$$G_d = \frac{(d-2)}{2L_{d+1}} G_{d+1}, \qquad \frac{1}{L_d^2} = \frac{2}{L_{d+1}^2}\left(1 - \frac{4\pi G_{d+1} L_{d+1}}{d-1}\,\tau\right)$$

- Natural to tune $\tau \Rightarrow$ tuning position of the brane
- Brane tension controls effective $\Lambda_{\mathcal{B}}$ on the brane

$$\delta\tau = \frac{\delta\Lambda_{\mathcal{B}}}{8\pi G_d}$$

$\Rightarrow$ Classical BHs thermodynamics in the bulk, plus work done by the brane would **naturally** induce **extended** thermodynamics of (quantum) BHs on the brane

- It is possible to construct exact 4D solutions describing localized black holes bound to a brane
- To obtain these solutions, notice that a black hole on a brane in AdS is **accelerating**

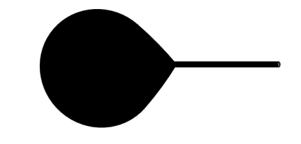
> There is a solution to Einstein's equation that describes accelerating black holes: **the C-metric**

- ⋆ This solution can be extended to include the cosmological constant Λ
- ⋆ Mechanism to accelerate these black holes:
  cosmic strings pulling on the black holes
  $\rightarrow$ usually contains conical singularities along the axis from the BH to ∞

AdS$_4$ C-metric with Karch-Randal brane

$$ds^2 = \frac{\ell^2}{(\ell + xr)}\left[-H(r)dt^2 + H^{-1}(r)dr^2 + r^2\left(G^{-1}(x)dx^2 + G(x)d\phi^2\right)\right]$$

Accelerating due to cosmic string, acceleration $\tau = 1/(2\pi G_4 \ell)$



Brane:

- Umbilic surface at $x = 0$: $K_{ij} = \ell^{-1} h_{ij}$
- Brane at $x = 0$, where Israel-junction conditions are satisfied

AdS$_4$ C-metric with Karch-Randal brane

$$ds^2 = \frac{\ell^2}{(\ell + xr)}\left[-H(r)dt^2 + H^{-1}(r)dr^2 + r^2\left(G^{-1}(x)dx^2 + G(x)d\phi^2\right)\right]$$

$$H(r) = \kappa + \frac{r^2}{\ell_3^2} - \boxed{\frac{\mu\ell}{r}} \qquad G(x) = 1 - \kappa x^2 - \boxed{\mu x^3}$$

- Parameters:
  - $\kappa = \pm 1, 0 \Rightarrow$ slicing on the brane;
  - $\mu \geq 0$ mass parameter: quantum corrections on the brane
  - ℓ is acceleration & brane position $\tau = 1/(2\pi G_4 \ell)$
  - ℓ_3: related to the brane cosmological constant:

$$\frac{1}{\ell_4^2} = \frac{1}{\ell^2} + \frac{1}{\ell_3^2}$$

- Given the symmetries $\Rightarrow$ brane $x = 0$ [Emparan, Horowitz, Myers '99]

To introduce a brane into the spacetime, we need a surface whose extrinsic curvature is proportional to the intrinsic metric.

- The metric on the brane is obtained by selecting $x = 0$

Brane geometry:

$$ds^2 = -f(r)dt^2 + f^{-1}(r)dr^2 + r^2 d\phi^2, \qquad f(r) = \frac{r^2}{\ell_3^2} + \kappa - \frac{\mu\ell}{r}$$

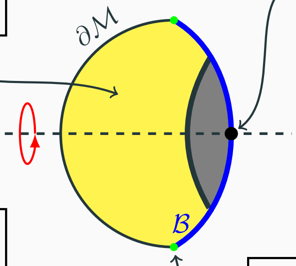
⋆ Classical limit: $\mu = 0$
$\Rightarrow \kappa = -1$ (BTZ), $\kappa = +1$ (Global or Conical AdS_3)
⋆ Quantum effects: $\mu \neq 0$
$\Rightarrow \kappa = -1$ (qBTZ) different properties of the horizon & curvature singularity

• Recover $2 + 1$ gravity at large distances along the brane
• Deviations from $2 + 1$ gravity arise at order $1/r$, reflecting the 4D nature of the black hole.

QBTZ

The bulk spacetime is a solution of the vacuum EE with negative Λ in 4D

black hole in the bulk is intersecting the brane



3D $\Lambda_{\mathcal{B}}$ obtained from 4D Λ & brane tension

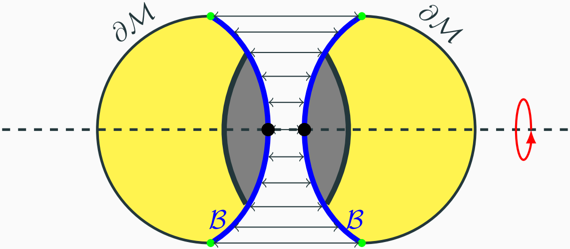
$$\frac{1}{\ell_4^2} = \frac{1}{\ell^2} + \frac{1}{\ell_3^2}$$

Transparent boundary conditions for the CFT btw the boundary and the brane

- Classical dynamics of AdS_{d+1} bulk encodes quantum dynamics of brane [Emparan, Fabbri, Kaloper '02]

Classical GR $\Leftrightarrow$ Semi-classical gravity

- Classical BHs localized on braneworld $\leftrightarrow$ quantum BHs



- Study semi-classical backreaction to all orders

$$G_{\mu\nu}(g_{\alpha\beta}) = 8\pi G_N \langle T_{\mu\nu} \rangle$$

Quantum BTZ black hole: [Emparan, **Frassino**, Way, '20]

- The stress tensor for the CFT_3 in this state is the stress tensor of a thermal CFT in equilibrium with the BH
- It has the generic structure of the renormalized stress tensor of conformal fields in the presence of the BTZ black hole

$$\langle T_\mu^\nu \rangle = \frac{\ell}{16\pi G_3} \frac{F(M)}{r^3} \mathrm{diag}\{1, 1, -2\} + \ldots$$

- Strength of backreaction controlled by ℓ
- Comparison with other calculations:
  1. Free conformal scalar in BTZ
  2. Holographic w/out backreaction ($\ell \to 0$)

BRANEWORLD THERMODYNAMICS

Bulk BH thermodynamics doubles as thermodynamics of qBTZ:

$$\begin{aligned} T &= T_{\text{qBTZ}} \\ S_{\text{gen}} &\equiv S_{\text{Wald}} + S_{\text{outside}} \end{aligned}$$

$$S_{\text{gen}} = \frac{A_{d+1}}{4G_{D+1}} \quad \text{(Bulk entropy)}, \quad S_{\text{Wald}} = \frac{A_d}{4G_d} + \ldots \quad \text{(Brane entropy)}$$

First law of quantum black holes [Emparan, **Frassino**, Way, '20]

$$dM = T dS_{\text{gen}}$$

- Interplay btw 3D (M) and 4D (S, T, Ω, J)
- S_{Wald} alone will not satisfy these relations!
- In $\ell \to 0$ limit, recover thermodynamics of BTZ
- Holds exactly to all orders in the backreaction and higher-curvature corrections

- **Treat tension as variable**, like fluid surface tension
  [**Frassino**, Pedraza, Svesko, Visser, Phys. Rev. Lett. 130 (2023), 161501 (arXiv:2212.14055)]
- Brane performs work on the bulk BH system

Bulk first law:

$$dM = TdS + A_\tau d\tau$$

$$A_{tau} \equiv \left(\frac{\partial M}{\partial \tau}\right)_S$$ – "regularized brane area"

Bulk Smarr law:

$$M = 2TS - 2P_4V_4 - \tau A_\tau , \qquad P_4 = -\frac{\Lambda_4}{8\pi G_4}$$

$\Rightarrow$ variable τ induces extended thermodynamics!

$$\delta\tau = \frac{\delta\Lambda_3}{8\pi G_3} = -\delta P_3$$

- Extended first law of qBTZ:

$$dM = TdS_{gen} + V_3 dP_3$$

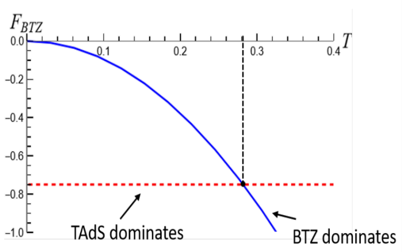
- Smarr law for qBTZ:

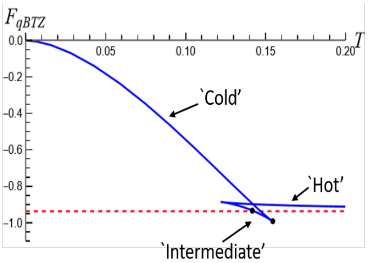
$$0 = TS_{gen} - 2P_3V_3 + \mu_3 c_3$$

  ⋆ 3D extended thermodynamics for charged and rotating BTZ [**Frassino**, Mann, Mureika, '22]

REENTRANT PHASE TRANSITIONS OF QUANTUM BLACK HOLES

Backreaction of quantum fields on black hole geometries can trigger new thermal phase transitions





$$F_{qBTZ} = M - TS_{gen}$$

- Large backreaction ⇒ 'reentrant' phase transitions
  As T increases, $T_A dS \longrightarrow^{1st} qBTZ \longrightarrow^{0th} TAdS$
- Intermediate BH always thermodynamically stable, $C_{P_3} > 0$

[Frassino, Pedraza, Svesko, Visser, Phys. Rev. D (2024) (arXiv:2310.12220)]

The double-holographic dictionary allows many new perspectives:

- E.g., on the evaporation problem of the BH for example – with the emergence for example of the so-called **islands**

In our setup:

- Single holography: AdS_4/CFT_3
  - Dual CFT_3 on bdry with $c_3 = L_4^2/G_4 \sim \ell$
- Double holography: AdS_3/CFT_2
  - Dual CFT_2 with $c_2 = 3L_3/2G_3$

$$dE = TdS_2 - P_2 dV_2 + \mu_2 dc_2$$

[**Frassino**, Pedraza, Svesko, Visser, Phys. Rev. Lett. 130 (2023), 161501 (arXiv:2212.14055)]

CONCLUSIONS

Summary:

- Bulk holographic AdS_{d+1} coupled to a brane with dynamical gravity
- **classical** BHs $\leftrightarrow$ **semi-classical** BHs
- Brane tension controls effective $\Lambda_{\mathcal{B}}$ on the brane $\delta\tau = \frac{\delta\Lambda_{\mathcal{B}}}{8\pi G_{\mathcal{B}}}$
- Mechanical work due to the brane induces extended themodynamics
- New thermal phase structure of black holes corrected due to semi-classical backreaction

Future directions:

- Other quantum black holes (rotating, dS)
- Quantum Penrose Inequalities and quantum RII
  [In preparation: 2406.XXXXX]
- Cosmic Censorship [**Frassino**, Rocha, Sanna 2405.04597 + in preparation]
- Stringy origins? [Karch, Sun, Uhlemann, '21]

THANK YOU