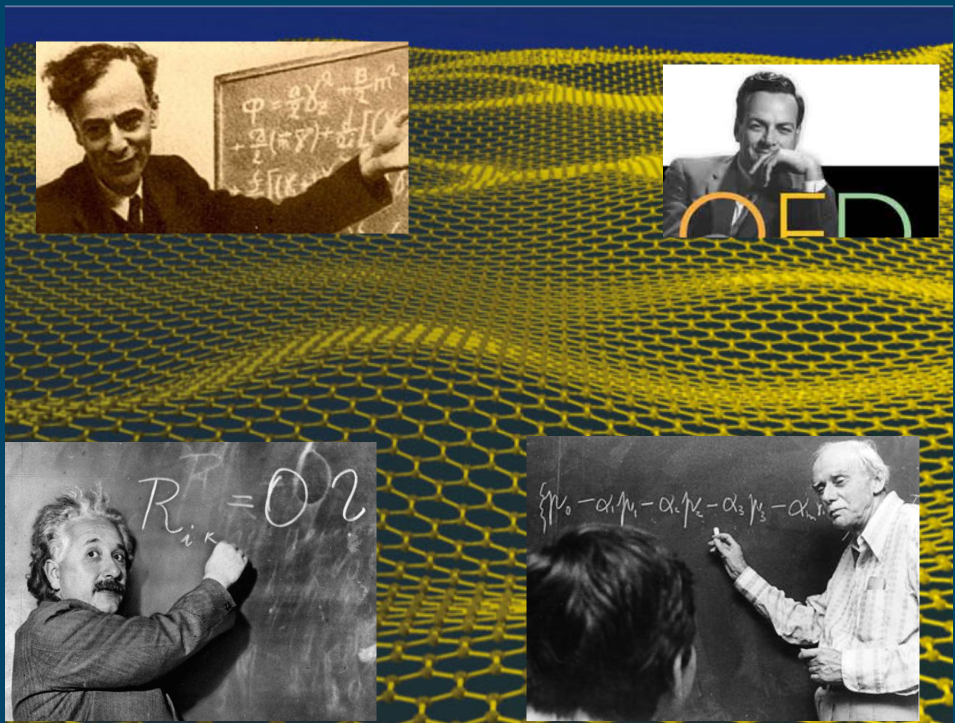
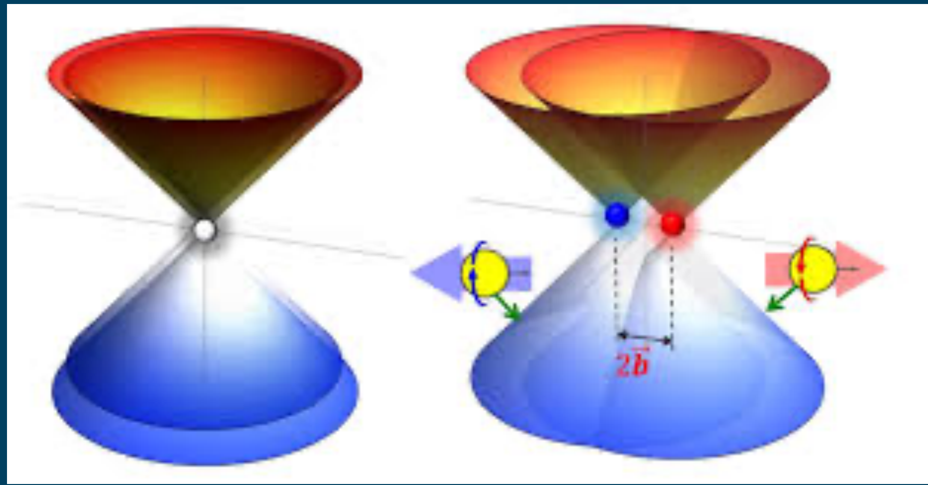
Conformal anomaly in condensed matter systems

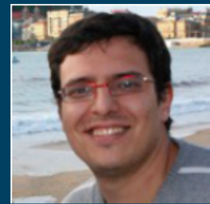


María A. H. Vozmediano
Instituto de Ciencia de Materiales de Madrid, CSIC

(3+1)D



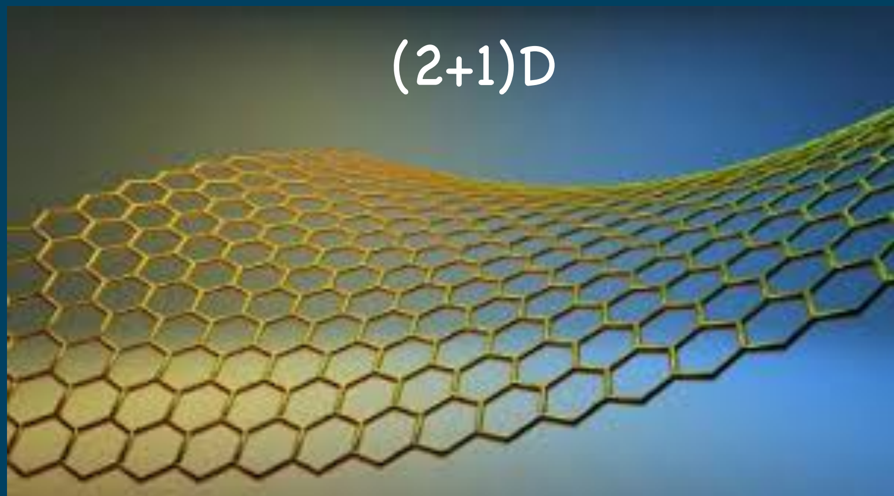
V. Arjona

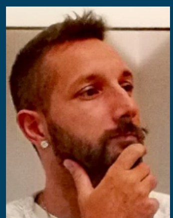


A. Cortijo

M. Chernodub

(2+1)D



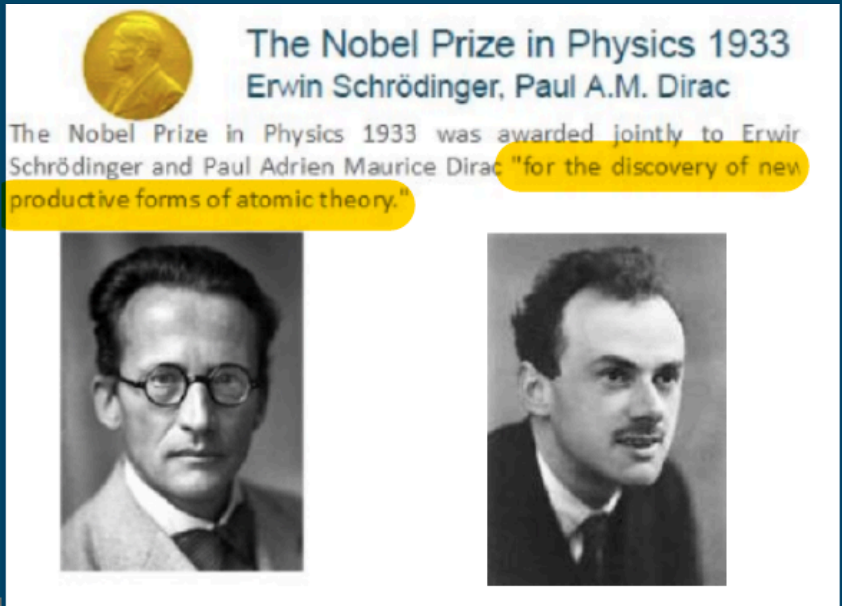


M. Baggioli

K. Landsteiner

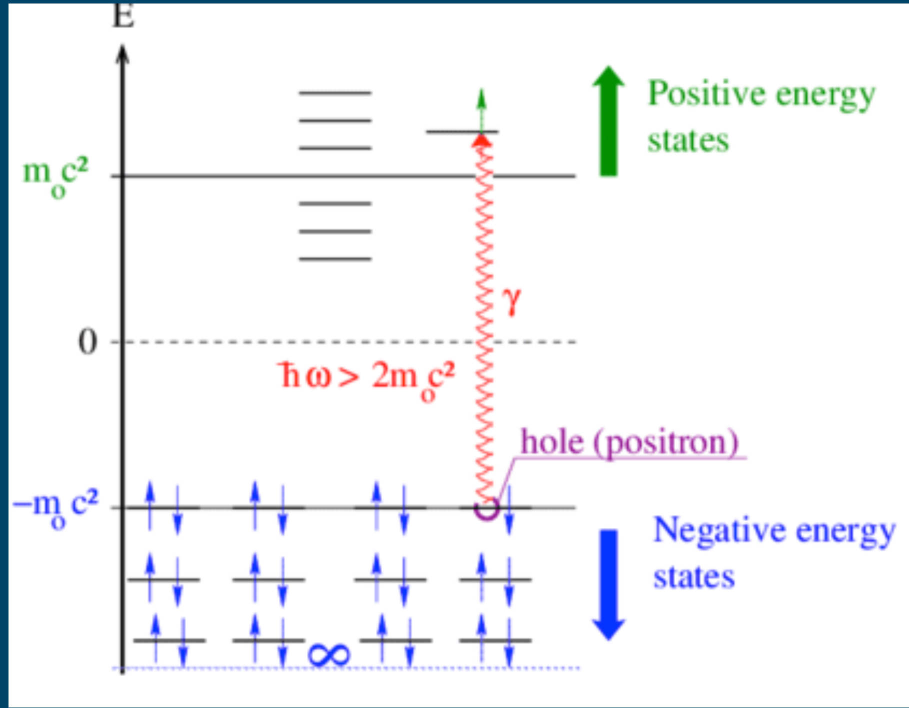
1. Dirac physics: HEP vs CM

QFT fermions



$$H(t)|\psi(t)\rangle = i\hbar\frac{\partial}{\partial t}|\psi(t)\rangle$$

$$(i\gamma\cdot\partial - m)\psi = 0$$



$$E = \sqrt{(mc^2)^2 + (cp)^2}$$

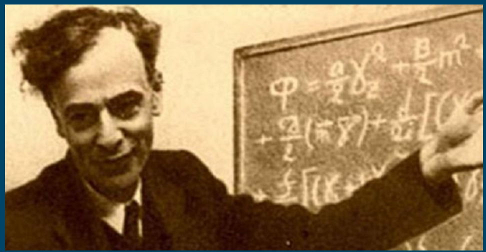
The Dirac equation has negative energy solutions:

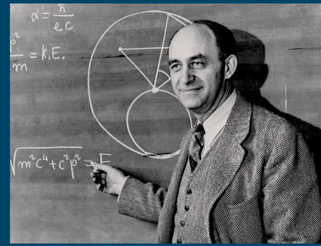
Vacuum: the infinite, filled, Dirac sea.

Particles: quantum excitations of the Dirac sea

The standard model of condensed matter

The Landau Fermi liquid for metals*





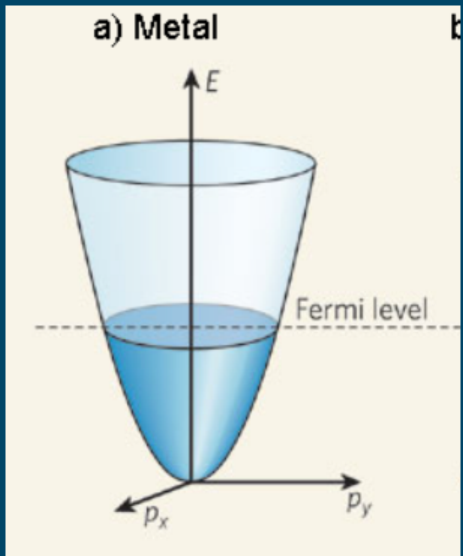
Vacuum: the Fermi sea

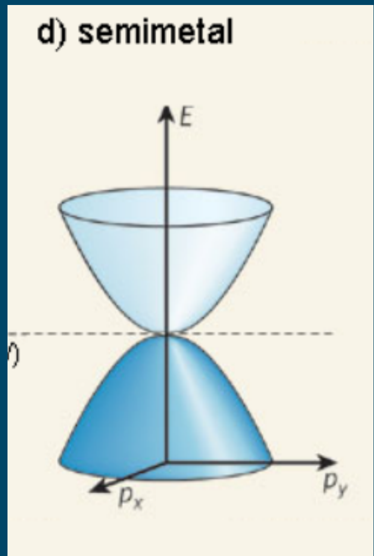
$$\varepsilon(k) = k^2 / 2m^*$$

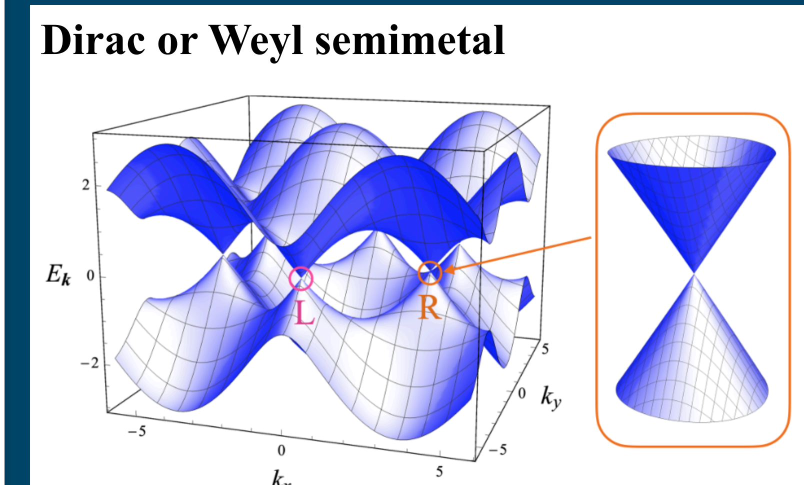
$$\varepsilon(k) = k^2 / 2m^*$$
$$m^* = \partial^2 \varepsilon(k) / \partial k^2 \big|_{kF}$$

$$\varepsilon(k) = \pm v |k|$$

?



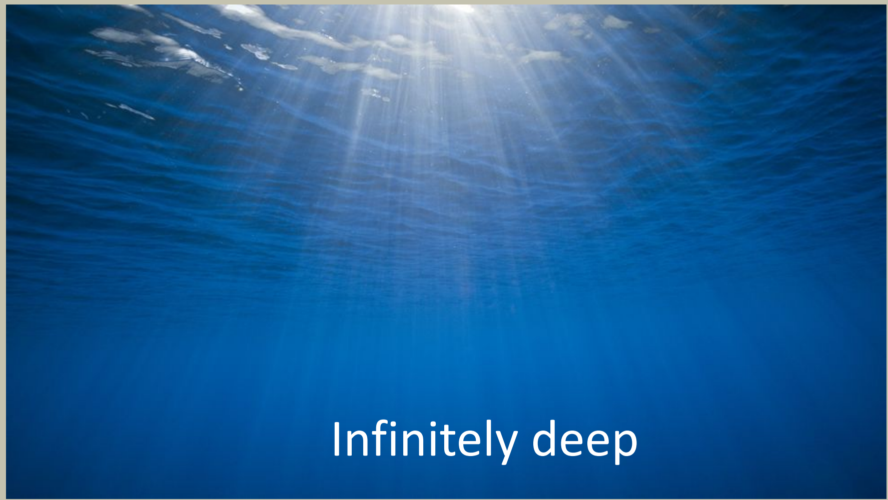




Quasiparticles: quantum excitations of the (filled) Fermi sea.

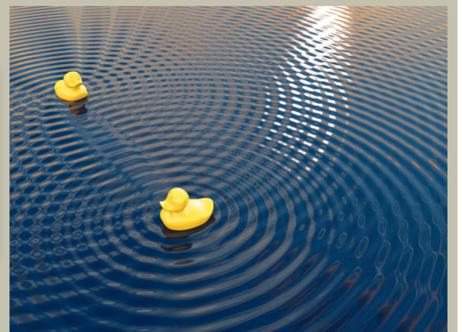
*HEP ref.: J. Polchinski, Effective Field Theory and the Fermi Surface, hep-th/9210046

Particles and quasiparticles



Dirac and Fermi seas

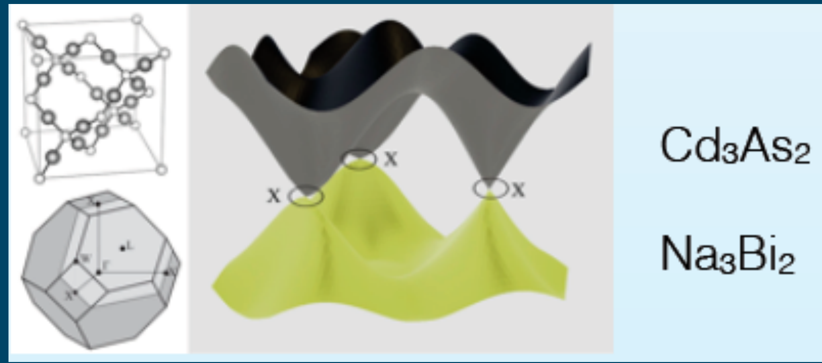
Particles / Quasiparticles { are elementary excitations of the { Dirac / Fermi } seas



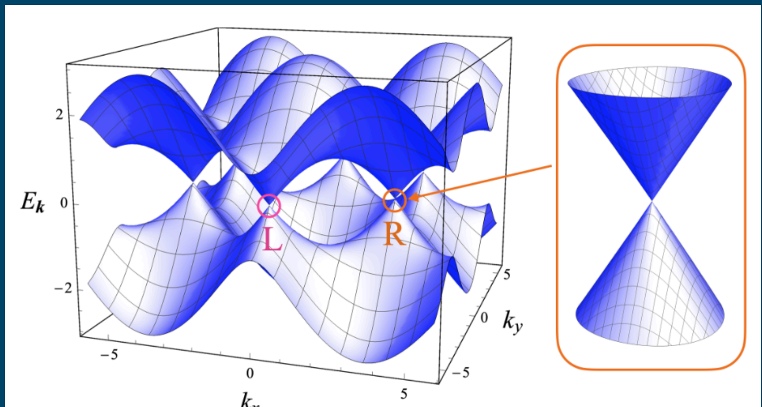
Same equations, same solutions!

Dirac matter

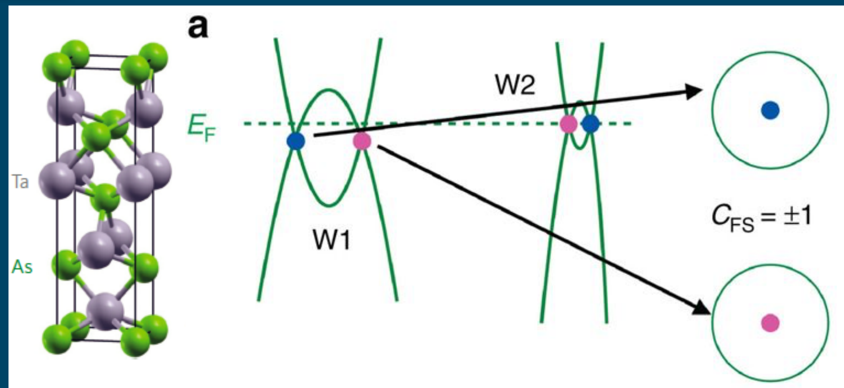
Effective low energy description
around band crossings in crystals.



$$\varepsilon(k) = \pm v|k|$$



(2+1)D: graphene



(3+1)D: Dirac and Weyl semimetals

QPs are charged **massless** fermions

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi} i \not{D} \psi \,,$$

Relativistic QFT at work

2. Conformal anomaly (3+1)

Warning

Scale vs conformal symmetry

For a rigid scale transformation $\sigma = \text{const.}$

$$\int d^d x \sqrt{g} T^\mu{}_\mu = 0$$

Implies the existence of a "virial" current D_μ

$$T^\mu{}_\mu = \nabla_\mu D^\mu$$

The current *must not* have anomalous dimension, but I believe

$$\langle T^\mu{}_\mu - \nabla_\mu D^\mu \rangle = \text{beta terms} + \text{anomaly}$$

Omar Zanusso

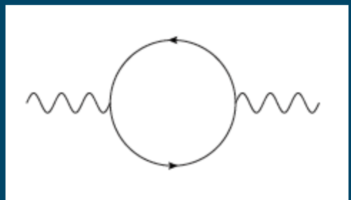
I will call this scale, or conformal anomaly

3D: QED conformal anomaly

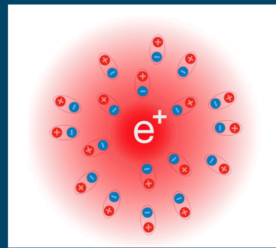
$$\mathscr{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \bar{\psi} i \gamma^{\mu} D_{\mu} \psi$$

Massless QED is scale invariant:
no masses, dimensionless coupling

After quantizing, the running coupling constant induces a scale anomaly



$$\beta_e = \frac{\mathrm{d}e(E)}{\mathrm{d}\ln E},$$



$$\langle T^{\alpha}_{\ \alpha}(x)\rangle = \frac{\beta_e(e)}{2e} F^{\mu\nu}(x) F_{\mu\nu}(x),$$

Anomaly induced transport

$$g_{\mu\nu}(x) = e^{2\tau(x)}\eta_{\mu\nu}, \quad \rightarrow \quad S_{eff} = \int d^4x\, \tau(x) T^{\mu}_{\mu}(x) \quad \rightarrow \quad J^{\mu}(x) = -\frac{2\beta_e(e)}{e} F^{\mu\nu}(x)\partial_{\nu}\tau(x).$$

B

E

Scale magnetic effect

$$\vec{J}(x) = -\frac{2\beta_e(e)}{e} \vec{B}(x) \times \vec{\nabla}\tau(x).$$

Scale electric effect

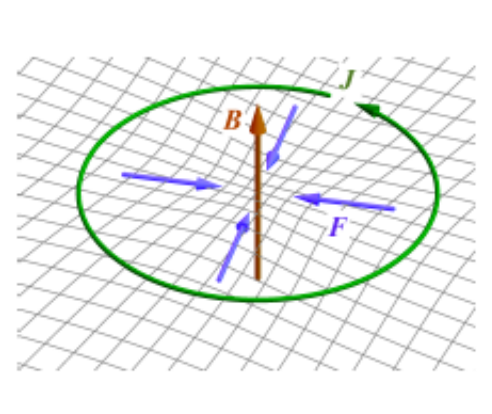
$$\vec{J}_{\mathrm{SEE}} = \sigma(x)\vec{E}(x),$$

$$\sigma(t, \vec{x}) = -\frac{2\beta_e(e)}{e}\frac{\partial \tau(t,\vec{x})}{\partial t}.$$

M. N. Chernodub, Anomalous Transport due to the Conformal Anomaly, Phys. Rev. Lett. 117, 141601 (2016).

3D Condensed matter

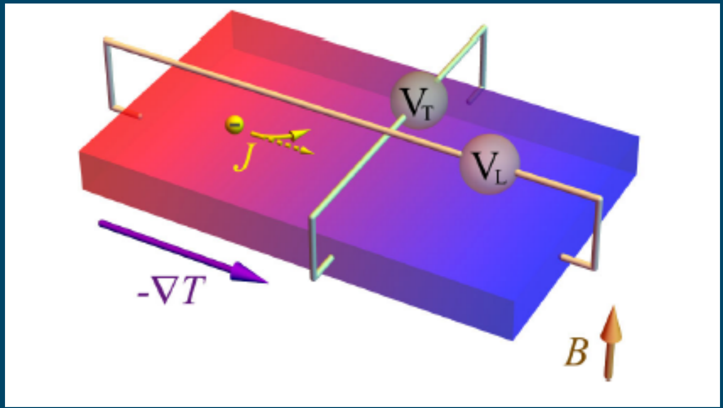
Scale magnetic effect



$$J = -\frac{2\beta(e)}{e}\nabla\tau(x)\times B(x).$$

$$\nabla\phi \leftrightarrow \frac{\nabla T}{T}$$

Nernst effect



$$J = \frac{e^2 v_F}{18\pi^2 T\hbar} B\times\nabla T\,.$$

Experimental signatures

The thermoelectric coefficient α should.

- Extrapolate to a non zero value as T->0 and μ=0.
- Linear in B.
- Proportional to v_F (larger in cleaner samples).

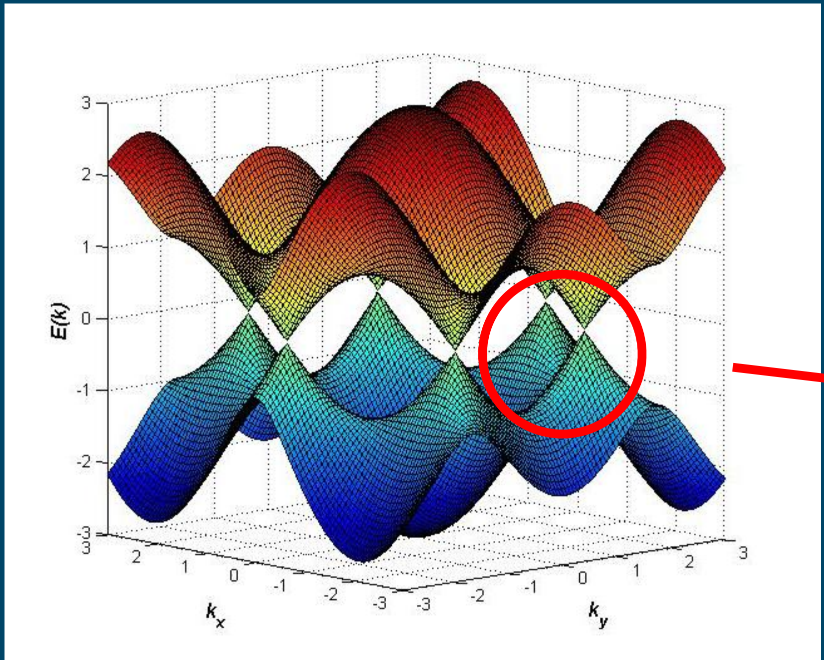
$$J = \sigma E + \alpha\nabla T,$$

$$\alpha \sim v_F B/T$$

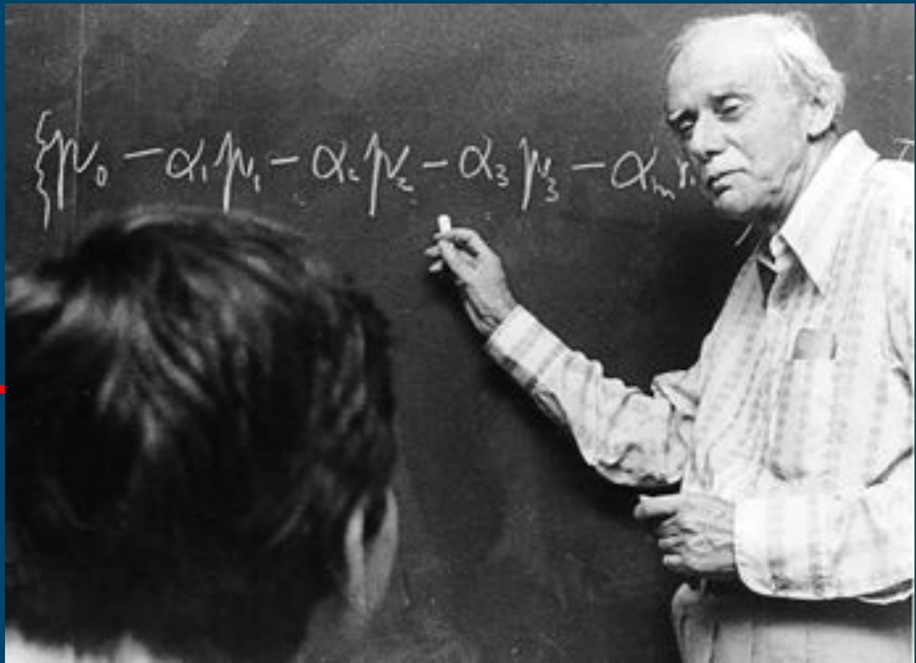
Anomaly based: Chernodub, Cortijo, MV, **PRL120**, 206601 (2018)
Kubo calculation: Arjona, Chernodub , MV, PRB**99** , 235123 (2019)
Effect of tilt: Ballestad, Cortijo, MV, Qaiumzadeh, PRB**107** , 014410 (2023)

3. Graphene: QED(2+1)?

Summary of graphene features



TB Dispersion relation



Low energy expansion around a Fermi point

$$H_K^{eff} = v_F \vec{\sigma}.\vec{p}$$

$$v_F = \frac{3}{2} t \, . \, a \sim \frac{c}{300}$$

E ≤ 1–1.5 eV

Free action: (2+1) Dirac

$$\mathscr{L} = \int dt d^2 x \bar{\Psi} \gamma^\mu \partial_\mu \Psi \quad , \quad \gamma^\mu = (\gamma^0, v_F \gamma^i)$$

Coulomb interaction

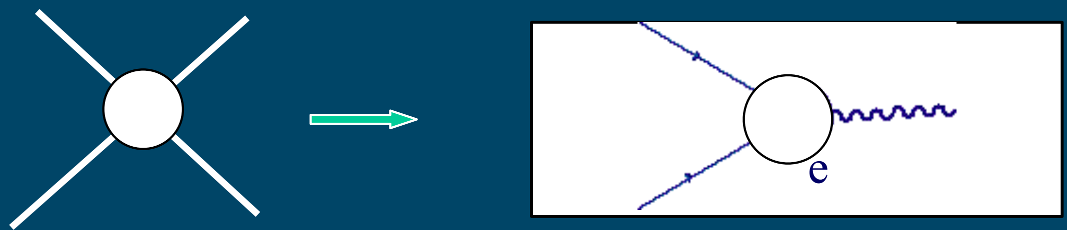
J. González, F. Guinea, MV
Nucl. Phys. **B424**, 593 (1994)

$$\mathcal{H} = v_F \int d^2\mathbf{r}\, \bar{\psi}(\mathbf{r})\, \gamma^i \partial_i \psi(\mathbf{r}),$$

$$\mathcal{H}_{\rm int} = e^2 \int d^2\mathbf{r}\, d^2\mathbf{r}' \frac{\psi^+(\mathbf{r})\psi(\mathbf{r})\psi^+(\mathbf{r}')\psi(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}.$$

$$\alpha \equiv \frac{e^2}{4\pi v_F}$$

Substitute the four fermi non-local interaction by a local term



$$\Gamma_0^\mu(k_1, k_2, k_3) = ie\gamma^\mu$$

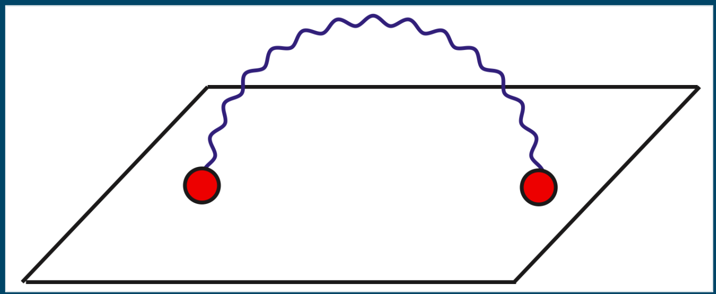
$$H_{int} = e \int d^2 r \bar{\Psi}(\mathbf{r})\gamma^0 A_0 \Psi(\mathbf{r})$$

$$L = \int d^2 r \bar{\Psi}(\mathbf{r}, t)\gamma^\mu(\partial_\mu - ieA_\mu)\Psi(\mathbf{r}, t)$$

Non-relativistic QED (2+1)?

$$J^\mu \sim (\bar{\Psi}\gamma^0\Psi, v\bar{\Psi}\vec{\gamma}\Psi)$$

Graphene: brain reduced QED



Charges confined to a plane.
Photons live in D= 3+1

Effective 2D photon propagator

$$V(\mathbf{k}) = \frac{1}{|\mathbf{k}|}$$

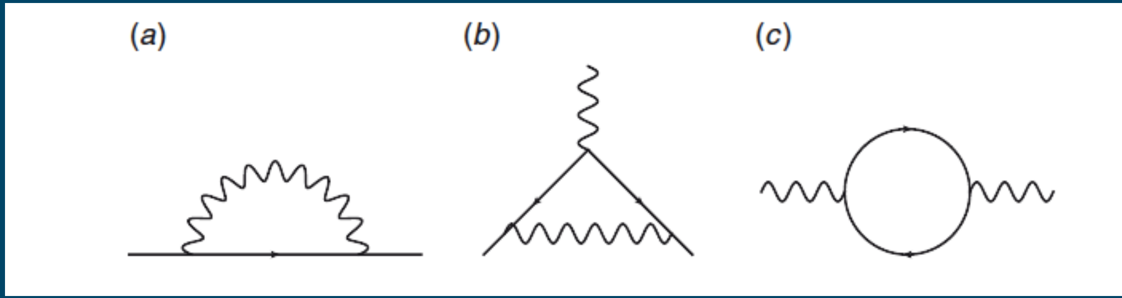
$$\begin{aligned}\Pi^0_{\mu\nu} &= \langle T A_\mu(t, \boldsymbol{r}) A_\nu(t', \boldsymbol{r}') \rangle \\ &\approx -i\delta_{\mu\nu} \int \frac{\mathrm{d}^2k \, \mathrm{d}\omega}{(2\pi)^3} \int \frac{\mathrm{d}k_z}{2\pi} \frac{\exp[i\boldsymbol{k}\cdot(\boldsymbol{r}-\boldsymbol{r}')] \exp[-i\omega(t-t')]}{\boldsymbol{k}^2 + k_z^2 - i\epsilon} \\ &= -i\delta_{\mu\nu}\delta(t-t')\frac{1}{2}\int \frac{\mathrm{d}^2k}{(2\pi)^2} \frac{1}{|\boldsymbol{k}|} \exp[i\boldsymbol{k}\cdot(\boldsymbol{r}-\boldsymbol{r}')].\end{aligned}$$

Effective **dimensionless** coupling constant

$$\alpha \equiv \frac{e^2}{4\pi\epsilon v_F} = \alpha_{QED}\frac{c}{v_F}\frac{\epsilon_0}{\epsilon}$$

Unlike QED(2+1) (superrenormalizable) this theory is renormalizable.
And scale invariant!

RG Analysis. Results



Diverges

Finite

$$\beta_v = -\frac{e^2}{16\pi v_F}$$

$$\beta_e = 0$$

$$\Pi_{\mu\nu}(k) = \frac{1}{2\pi}\frac{e^2}{v^2}\left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)|\mathbf{k}|$$

The Fermi velocity grows to the infrared

$\alpha \to 0$ Free infrared quantum critical point

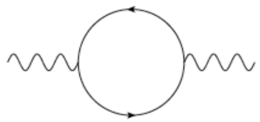
$$v_F(E) = v_F^0\left(1 + \frac{\alpha}{4}\log\frac{E_0}{E}\right)$$

The electric charge is not renormalized
(non-perturbative result)

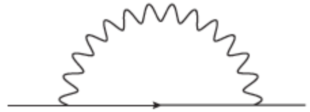
J. González, F. Guinea, MAHV
Nucl. Phys. **B424**, 593 (94)
Phys. Rev. Lett.**77**, 3589 (96)
Phys. Rev. **B59**, R2474 (99)

RG Analysis. Results

Retarded interaction: v/c finite



$$\Pi_{\mu\nu} = \frac{1}{2\pi}\frac{e^2}{v^2}\frac{k^2}{\sqrt{v_F^2 k^2 - \omega^2}}\left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)$$ finite



$$\Sigma(\omega, \vec{k}) = Z_\Psi(\omega, \vec{k})[\omega\gamma^0 - Z_v(\omega, \vec{k}) v \vec{\gamma}\cdot\vec{k}]$$

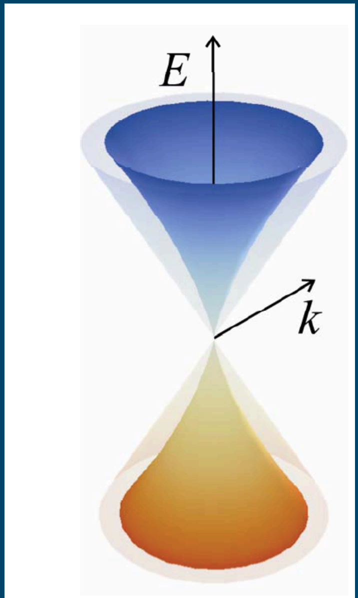
$$\gamma = \partial \log \frac{Z_\Psi}{\partial l} \quad , \quad G(\omega, \vec{k}) \sim_{\omega \to 0} \frac{1}{\omega^{1-\eta}} \quad , \quad \eta = \frac{e^2}{12\pi^2}$$

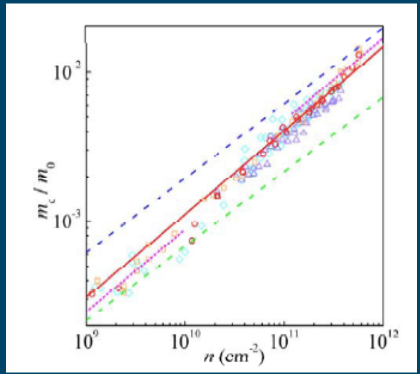
$$\beta_v = \frac{1}{v}\frac{1 - 2v^2 + 4v^4}{(1-v^2)^{3/2}} \arccos v + \frac{1 - 4v^2}{1 - v^2} = 0.$$ -> v=1

- New zero of the beta function: v_F-> c
- Non trivial infrared fixed point with $g=\alpha_{QED}$
- Wave function renormalization (anomalous dimension)

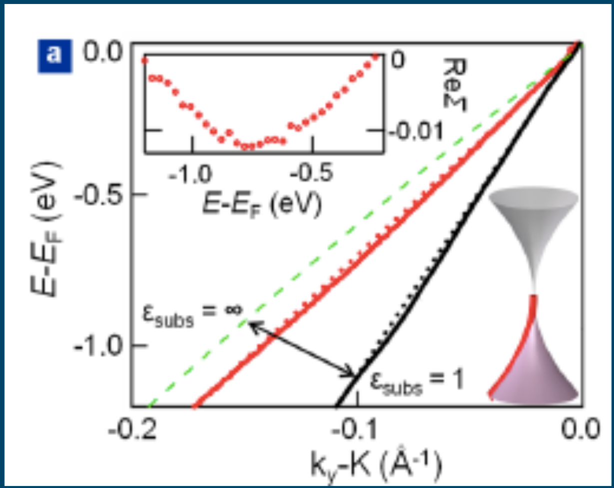
J. González, F. Guinea, MAHV
Nucl. Phys. B '94

Experimental confirmation of $v_F(E)$





From cyclotron mass- Suspended. Clean. (Elias et al Nat. Phys. 2011)



From ARPES. Epitaxial (Lanzara's group PNAS 2011)

Energy dependent!

Coulomb interactions make it grow at lower energies.
Disorder does the opposite.
If you see it constant as decreasing energies it intrinsically grows.

(Also space dependent in deformed samples)

A reasonable question: will $v_F(E)$ generate a scale anomaly?

(With observable consequences?)

4. Conformal anomaly in graphene: thermodynamics and hydro effects

Hydro aspects of Dirac matter

$\tau_{ee} >> \tau_{any}$ Fermi liquids in ultrapure crystals

JHEP March 24, 2015
Negative magnetoresistivity in chiral fluids and holography Karl Landsteiner, Yan Liu and Ya-Wen Sun

Negative local resistance caused by viscous electron backflow in graphene SCIENCE 4 MARCH 2016

D. A. Bandurin,[1] I. Torre,[2] R. Krishna Kumar,[1,3] M. Ben Shalom,[1,4] A. Tomadin,[5] A. Principi,[6] G. H. Auton,[4] E. Khestanova,[1,4] K. S. Novoselov,[4] I. V. Grigorieva,[1] L. A. Ponomarenko,[1,3] A. K. Geim,[1*] M. Polini[7*]

Evidence for hydrodynamic electron flow in $PdCoO_2$ SCIENCE 4 MARCH 2016

Philip J. W. Moll,[1,2,3] Pallavi Kushwaha,[3] Nabhanila Nandi,[3] Burkhard Schmidt,[3] Andrew P. Mackenzie[3,4*]

ELECTRON TRANSPORT

Observation of the Dirac fluid and the breakdown of the Wiedemann-Franz law in graphene SCIENCE 4 MARCH 2016

Jesse Crossno,[1,2] Jing K. Shi,[1] Ke Wang,[1] Xiaomeng Liu,[1] Achim Harzheim,[1] Andrew Lucas,[1] Subir Sachdev,[1,3] Philip Kim,[1,2*] Takashi Taniguchi,[4] Kenji Watanabe,[4] Thomas A. Ohki,[5] Kin Chung Fong[5*]

ARTICLE NATURE COMMUNICATIONS | (2018)
DOI: 10.1038/s41467-018-06688-y OPEN

Thermal and electrical signatures of a hydrodynamic electron fluid in tungsten diphosphide

J. Gooth[1,2], F. Menges[1,4], N. Kumar[2], V. Süß[2], C. Shekhar[2], Y. Sun[2], U. Drechsler[1], R. Zierold[3], C. Felser[2] & B. Gotsmann[1]

PNAS

Hydrodynamic theory of thermoelectric transport and negative magnetoresistance in Weyl semimetals

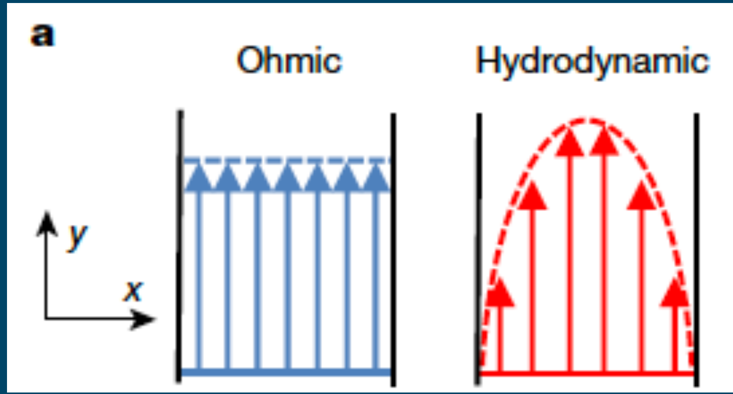
Andrew Lucas[a,1], Richard A. Davison[a,1], and Subir Sachdev[a,b,1] August 23, 2016

Graphene: more recent hydro exp.

Visualizing Poiseuille flow of hydrodynamic electrons

Sulpizio, Geim et al Nature | Vol 576 | 5 December 2019

Scanning carbon nanotube single-electron transistor to image the Hall voltage of electronic flow through channels of high-mobility graphene.



Article

Imaging viscous flow of the Dirac fluid in graphene

Nature | Vol 583 | 23 July 2020 | 537

Probing viscous electronic transport via magnetic field imaging

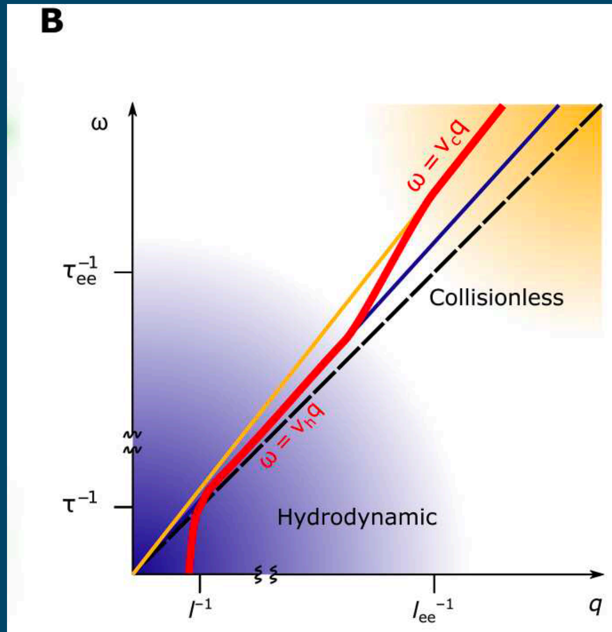
Kim, Jacobi et al

Observation of hydrodynamic plasmons and energy waves in graphene

THz absorption spectra. Zhao et al.

Nature | Vol 614 | 23 February 2023

Experimental signatures of the transition from acoustic plasmon to electronic sound in graphene



Barcons Ruiz *et al.*, *Sci. Adv.* **9**, eadi0415 (2023)

Driving viscous hydrodynamics in bulk electron flow in graphene using micromagnets

Jack N. Engdahl, Aydın Cem Keser, Thomas Schmidt, and Oleg P. Sushkov

Phys. Rev. B **109**, 195402 – Published 2 May 2024

Key expressions

General for massless QED

- The action $$S_0 = \int dt\, d^2\vec{x}\, \bar{\Psi}\, i\, (\gamma^t \partial_t + v_F\, \vec{\gamma} \cdot \vec{\nabla})\Psi$$
- The stress tensor $$T^{\mu\nu} = \frac{i}{2}\bar{\Psi}\left(\gamma^\mu \nabla^\nu + \gamma^\nu \nabla^\mu\right)\Psi - \eta^{\mu\nu}\bar{\Psi}\, i\, (\gamma^t \partial_t + v_F \vec{\gamma} \cdot \vec{\nabla})\Psi$$
- The energy density $$\epsilon \equiv T^{00} = -\, v_F\, \bar{\Psi} i \vec{\gamma} \cdot \vec{\nabla}\Psi$$
- The trace $$T^\alpha_{\ \alpha} = -\, 2\bar{\psi} i \gamma^\mu \nabla_\mu \psi \equiv 0$$ (traceless using eq. of motion)

The (new) conformal anomaly

Under a scale transformation $\quad g_{\mu\nu} = e^{2\tau}\eta_{\mu\nu} \quad , \quad \delta g_{\mu\nu} = 2\tau\eta_{\mu\nu}$

$$S \to S_\tau = S + \tau \int dt d^2\vec{x}\, T^\mu_\mu(x) + O(\tau^2)$$

$$v_F \to v_F + \tau\beta_v$$

$$\frac{\partial S_\psi}{\partial \tau} = < \int dt\, d^2\vec{x}\, T^\mu_{\ \mu} >$$

$$T^\mu_\mu = \beta_v < \bar{\Psi} i \vec{\gamma} \vec{\nabla} \Psi > = -\frac{\beta_v}{v_F} < \epsilon >$$

Consequences of conformal anomaly on hydro

- Conformal equation of state:

$$< T^{\mu}_{\mu} > = E - 2P = 0\ ,\ \ E = 2P$$

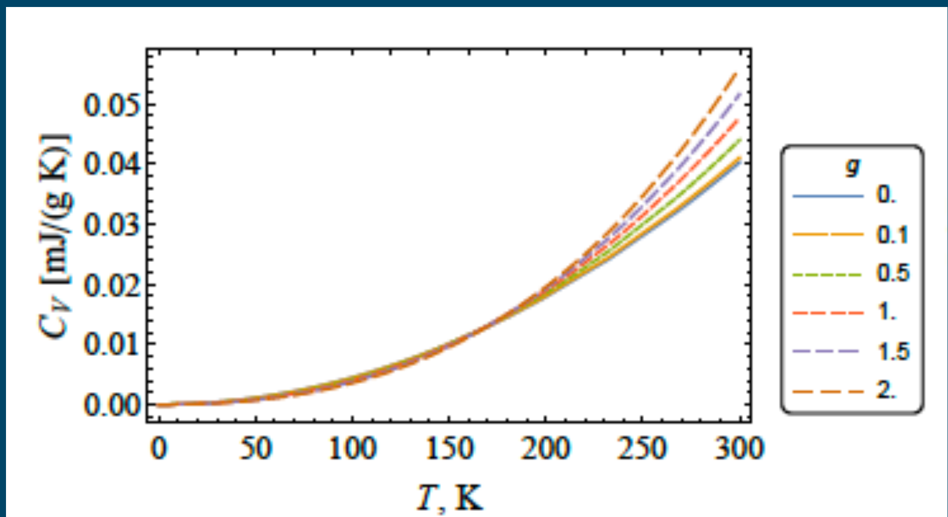
- Conformal anomaly:

$$< T^{\mu}_{\mu} > = -\frac{\beta_v}{v_F} < E >$$

- Modified EOS:

$$\boxed{\left(1 + \frac{\beta_v}{v_F}\right) E = 2P}$$

Specific heat*



* First noticed by Vafek PRL07.

Measurable

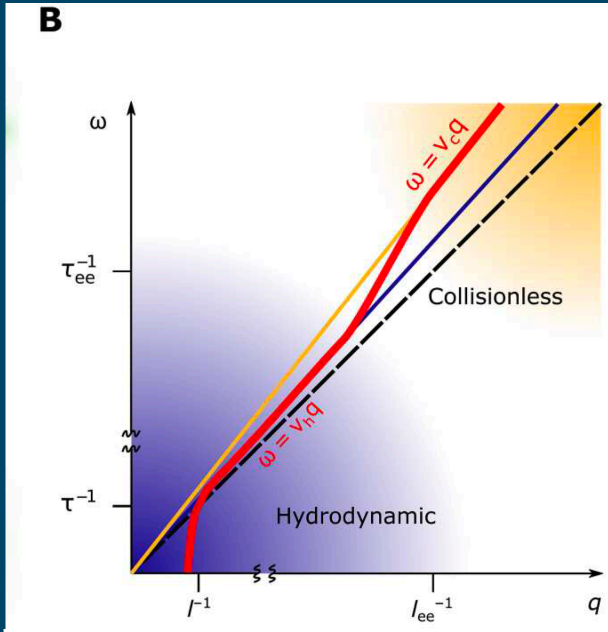
- Sound wave velocity

EOS of relativistic fluid $(c \to v_F)$

$$\frac{\partial P}{\partial E} = \frac{v_s^2}{v_F^2} \qquad \left(1 + \frac{\beta_v}{v_F}\right) E = 2P$$

$$v_s(T) = \frac{v_F(T)}{\sqrt{2}} \left(1 - \frac{\beta_v}{v_F(T)}\right)^{-1/2}$$

Measurable?



Barcons Ruiz *et al.*, *Sci. Adv.* **9**, eadi0415 (2023)

Bulk viscosity

Kubo:

$$\zeta = \lim_{\omega \to 0} \frac{1}{4\omega} \int_0^\infty dt \int d^2x\, e^{i\omega t} \langle [T^\mu_\mu(x), T^\nu_\nu(0)] \rangle.$$

Graphene:

$$\zeta = \frac{\beta_v^2}{4} \lim_{\omega \to 0} \frac{i}{\omega} \lim_{\boldsymbol{k} \to 0} G^R_{TT}(\omega, \boldsymbol{k}).$$

$$G^R_{TT}(\omega, \boldsymbol{k}) = \frac{1}{i v_F^2} \int_0^\infty dt\, e^{i\omega t} \int d^2x\, e^{-i\boldsymbol{k}\boldsymbol{x}} \langle [\bar\Psi(x) i\gamma^t \partial_t \Psi(x), \bar\Psi(0) i\gamma^t \partial_t \Psi(0)] \rangle,$$

Measurable?

Main effect: acoustic dispersion (attenuation)

Temperature dependence of bulk viscosity in water using acoustic spectroscopy

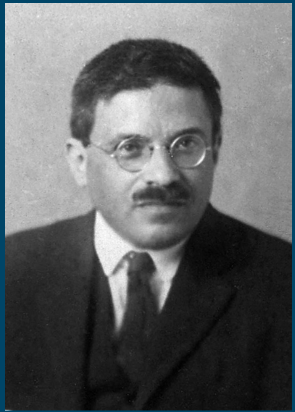
To cite this article: M J Holmes *et al* 2011 *J. Phys.: Conf. Ser.* **269** 012011

Summary and conclusion

- Novel CM systems modeled with massless Dirac -> scale invariant
- Coulomb interactions { 3D -> standard QED; 2D -> brane QED }
- Novel scale anomaly in (2+1) from renormalization of Fermi velocity
- Thermodynamics and hydro effects
- Experimentally accessible?

Condensed matter merges HEP (again)

The adventure of our science of physics is a perpetual attempt to recognize that the different aspects of nature are really different aspects of the same thing.



Tolman-Ehrenfest effect

Precursors

APRIL 15, 1930 *PHYSICAL REVIEW* *VOLUME 35*

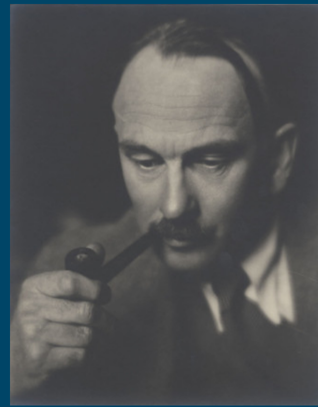
ON THE WEIGHT OF HEAT AND THERMAL EQUILIBRIUM IN GENERAL RELATIVITY

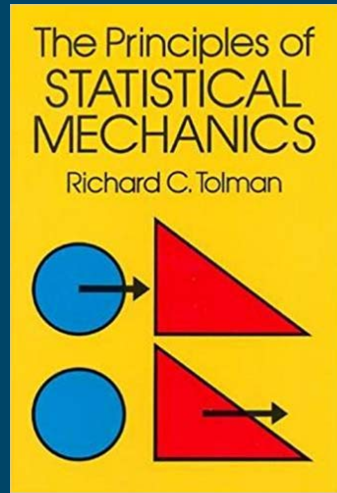
By Richard C. Tolman

DECEMBER 15, 1930 *PHYSICAL REVIEW* *VOLUME 36*

TEMPERATURE EQUILIBRIUM IN A STATIC GRAVITATIONAL FIELD

By Richard C. Tolman and Paul Ehrenfest





The problem: thermodynamic equilibrium in gravitational fields.

heat has weight

$$\frac{1}{T}\nabla T = -\frac{1}{c^2}\nabla\Phi,$$

Variations of gravitational potentials induce variations of temperature

PHYSICAL REVIEW VOLUME 135, NUMBER 6A 14 SEPTEMBER 1964

Theory of Thermal Transport Coefficients*

How can we get Kubo formulas for thermal transport coefficients?

Just as the space- and time-varying external electric potential produced electric currents and density variations, so a varying gravitational field will produce, in principle,[7] energy flows and temperature fluctuations.

Gravitational potential Φ as a local source of thermal (energy) currents

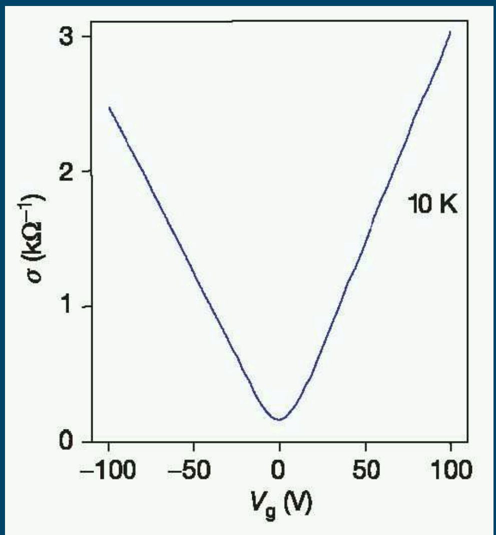
$$\frac{1}{T}\nabla T = -\frac{1}{c^2}\nabla\Phi,$$

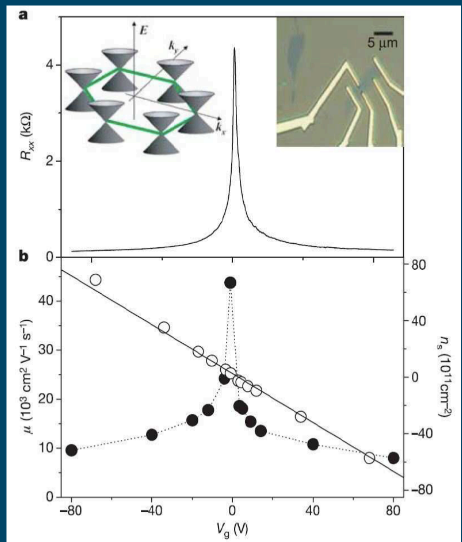
7 calling these interesting references to my attention.) Although the effect is very small, in practice we are only interested in questions of principle, and an arbitrarily small effect is just as good as a large one. In fact, if the gravitational field didn't exist, one could invent one for the purposes of this paper.

Digression: A puzzling question still remaining in graphene

Early experiments explained with single particle picture.
Continuum model accounts for most (almost all) low energy features.

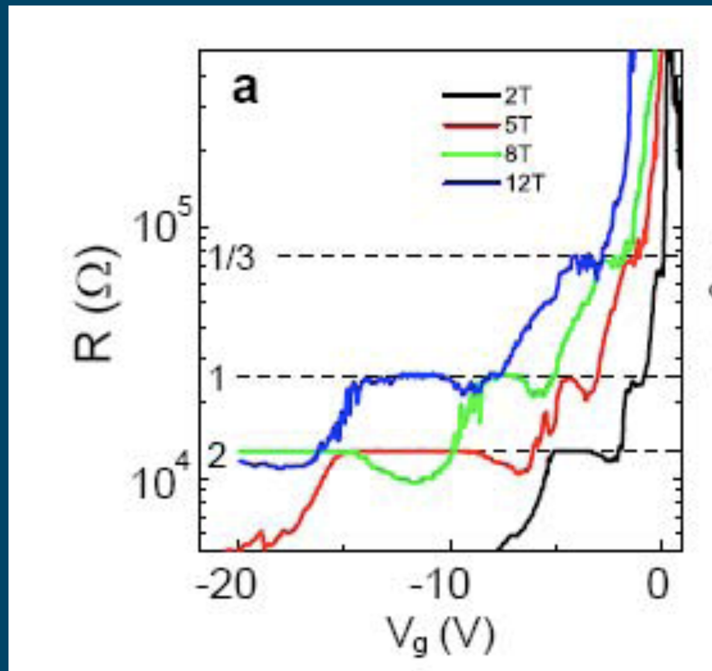
Fractional Quantum Hall Effect





Novoselov et al, Nature 2005

$$\alpha \equiv \frac{e^2}{\epsilon v_F} = \alpha_{QED} \frac{c}{v_F} \frac{\epsilon_0}{\epsilon} \sim 300 \alpha_{QED} \frac{\epsilon_0}{\epsilon}$$

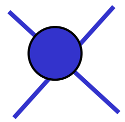


E. Andrei´s andP. Kim's groups
Nature 09

Coulomb interactions (I)

Nucl. Phys. **B424**, 593 (94)
Phys. Rev. Lett.**77**, 3589 (96)
Phys. Rev. **B59**, R2474 (99)

$$H = \frac{3}{2} ta \int d^2 r \bar{\psi}(\mathbf{r})\, \gamma . \nabla \psi(\mathbf{r}) + \frac{e^2}{2} \int d^2 r_1 d^2 r_2 \frac{\bar{\psi}(r_1)\sigma_3 \psi(r_1)\, \bar{\psi}(r_2)\sigma_3 \psi(r_1)}{4\pi \left| r_1 - r_2 \right|}$$



Substitute the four fermions non-local interaction by a local term $L_j = j^\mu A_\mu$,

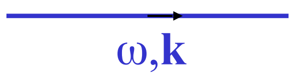
$$L = \frac{3}{2} v_F \int d^2 r \, \mathrm{d}t \, \bar{\psi}(\mathbf{r},\mathrm{t}) \gamma^\mu (\partial_\mu - ieA_\mu) \psi(\mathbf{r},t)$$

Non-relativistic QED (2+1)

$$j^\mu \sim (\bar{\psi}\gamma^0\psi \,,\, \mathrm{v}\bar{\psi}\vec{\gamma}\psi)$$

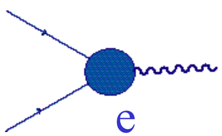
Feymann diagrams building blocks:

$$L = L[\Psi, A, v_F, \alpha]$$



$$G^0(\omega, \mathbf{k}) = i \frac{-\gamma_0 \omega + \mathrm{v}\gamma . \mathbf{k}}{-\omega^2 + \mathrm{v}^2 \mathbf{k}^2 - i\varepsilon}$$

$$\Pi^0_{\mu\nu} = \langle T A_\mu(t, \mathbf{r}) A_\nu(t', \mathbf{r}') \rangle = -i \delta_{\mu\nu} \int \frac{d^4 k}{(2\pi)^4} \frac{e^{i\mathbf{k}(\mathbf{r}-\mathbf{r}') - i\omega(t-t')}}{-\omega^2 + \mathbf{k}^2 - i\varepsilon}$$

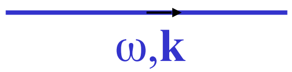


$$\Gamma^0{}_\mu(k_1, k_2, k_3) = ie$$

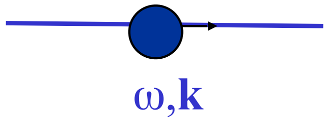
Electron self-energy

- **Electron self-energy**

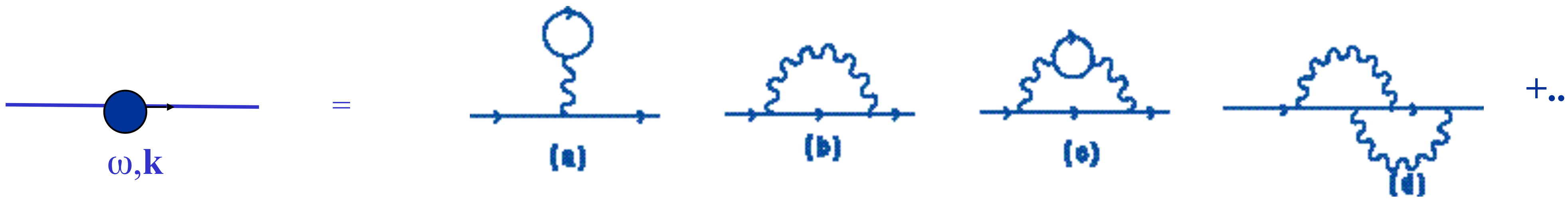
$$\frac{1}{G} = \frac{1}{G^0} - \Sigma$$



$$\Sigma_0(\omega, k) = \omega - v_F |k|$$



$$\Sigma(\omega, k) = Z_\psi(\omega, k)[\omega - Z_v(\omega, k)|k|]$$

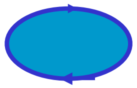


- (a) Density of states $n(\omega) = \mathrm{Im} \int d^2k \ \mathrm{tr}\left[\mathrm{G}(\omega, \mathbf{k})\sigma_3\right]$
- (b) Renormalization of the Fermi velocity (or hopping parameter).
- (c),(d) Wave function renormalization
  - Quasiparticle lifetime. $\tau^{-1} \sim \lim_{\omega \to 0} \mathrm{Im}\ \Sigma(\omega, \mathbf{0})$
  - Anomalous exponent: η $G(\omega) \underset{\omega \to 0}{\sim} \frac{1}{\omega^{1-\eta}}$

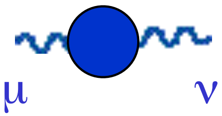
  (affects the interlayer hopping)

Photon self-energy and vertex

- **Photon self-energy: Π (ω,k)**



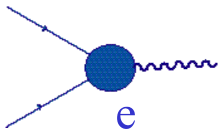
Its real part renormalizes the interaction.
Im χ(ω,**k**) gives the density of electron-hole pairs.
Its real poles give the plasmon spectra.



$$\Pi_{\mu\nu}(k) = \frac{1}{2\pi}\frac{e^2}{v^2}\frac{k^2}{\sqrt{v_F^2 k^2 - \omega^2}}\left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)$$

Finite at the one loop level

- **Vertex corrections**:

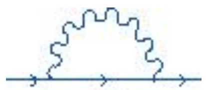


Renormalizes the electric charge.
Related to photon self-energy by gauge invariance.

$$Z_A^{1/2} = Z_e^{-1}$$

The electric charge is not renormalized. Coulomb unscreened.
The result stays at a non-perturbative level if gauge inv. is maintained.

RG Analysis. Results

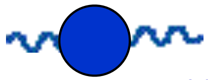


(b)

$$Z_v = 1 - \frac{1}{16\pi}\frac{e^2}{v}\log\left(\frac{\Lambda}{\omega}\right)$$

• The Fermi velocity grows in the infrared (v→c).

Quantum critical point

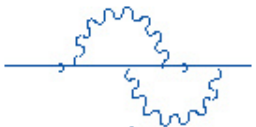


μ ν

$$\Pi_{\mu\nu}(k) = \frac{1}{2\pi}\frac{e^2}{v^2}\left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)|\mathbf{k}|$$

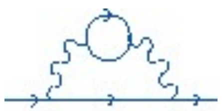
• The electric charge is not renormalized (non-perturbative result).

The effective coupling constant e^2/v_F renormalizes to zero. Free IR fixed point.



(d)

$$Z_\psi(k_F\omega) \sim 1 - \frac{1}{16\pi^2}\frac{e^4}{v^2}\log\left(\frac{\Lambda}{\omega}\right)$$



(c)

• The density of states acquires anomalous exponents η.
• The quasiparticle lifetime grows linearly with the energy.

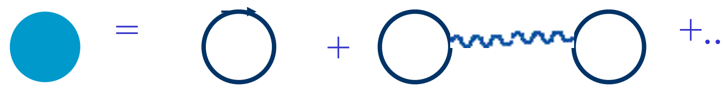
A non-perturbative fixed point

Phys. Rev. **B59**, R2474 (99)

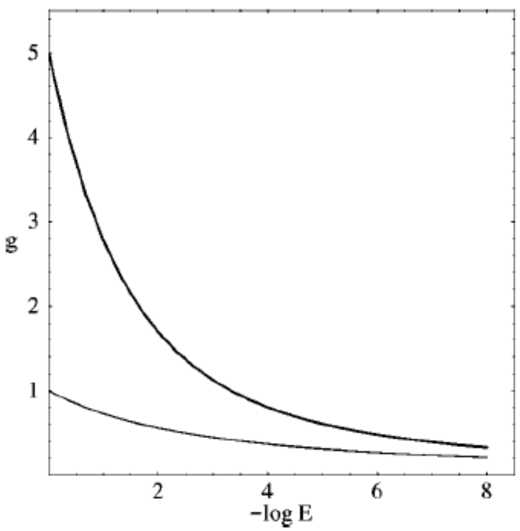
$$H=-iv_F\int d^2r\,\Psi^+(\mathbf{r})\boldsymbol{\sigma}\cdot\boldsymbol{\nabla}\Psi(\mathbf{r})$$
$$+e\int d^2r\,\Psi^+(\mathbf{r})\Psi(\mathbf{r})\phi(\mathbf{r}),$$

Scalar potential

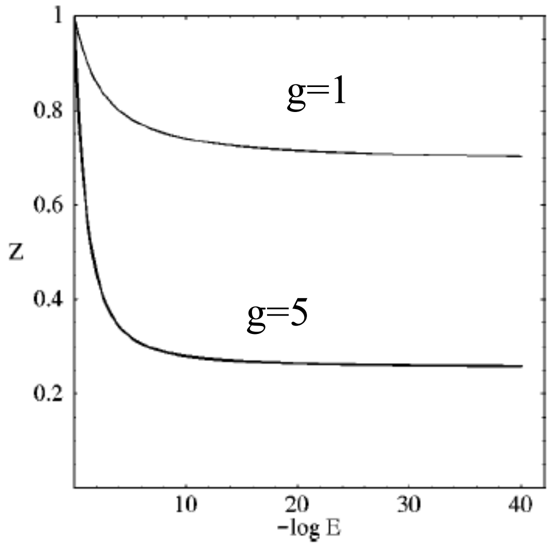
RPA:



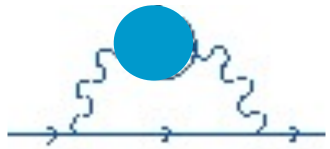
$$\Pi_{eff}(\omega,k)=\frac{-i}{2|k|+\frac{e^2}{8}\frac{k^2}{\sqrt{v_F^2k^2-\omega^2}}}$$



Flow of the coupling constant



Wave function renormalization



$$E_c\frac{d}{dE_c}g(E_c)=\frac{8}{\pi^2}\left(g+\frac{\arccos g}{\sqrt{1-g^2}}\right)-\frac{4}{\pi}.$$

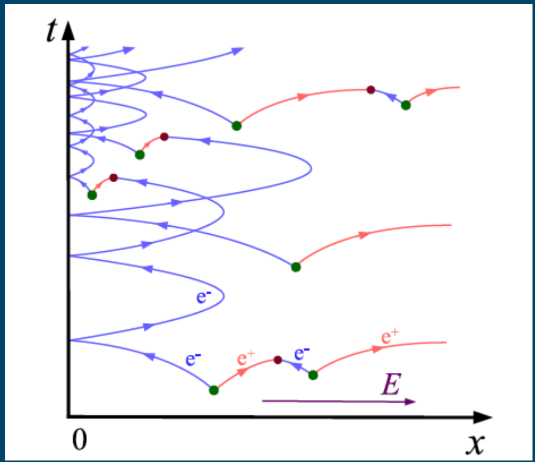
$$\mathrm{Re}\,\Sigma\sim g^2\omega\log(\omega)$$

$$\mathrm{Im}\,g\approx\omega\log^2\omega$$

There is no phase different from that of the perturbative regime, and the strong-coupling regime is connected to it through RG transformations.

3D Condensed matter II

Scale electric effect at the boundary



$$\frac{E(x)}{E(0)} \propto e^{-x/\lambda},$$

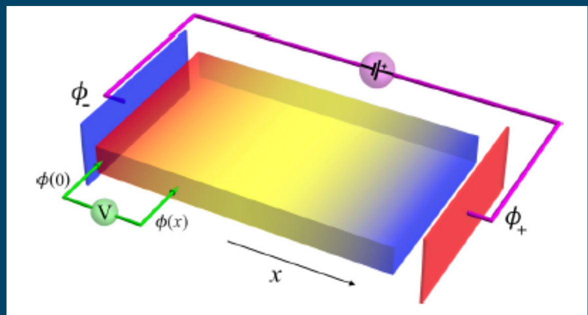
Thomas-Fermi **screening**

Conformal electromagnetic edge effects[1]

$$J^\mu(x) = -\frac{2c\beta_e}{e\hbar}\frac{F^{\mu\nu}(x)n_\nu}{x}, \quad \xrightarrow{\mu=0} \quad \rho = -\frac{2v_F\beta_e}{e\hbar c^2}\frac{\vec{n}\cdot\vec{E}}{x}.$$

It generates a "**conformal screening**"[2]

$$\frac{E(x)}{E(0)} \propto x^{-\nu}, \quad \nu = \frac{2\beta_e}{ec\hbar\varepsilon_0} = \frac{e^2}{6\pi^2\hbar v_F\varepsilon\varepsilon_0}$$

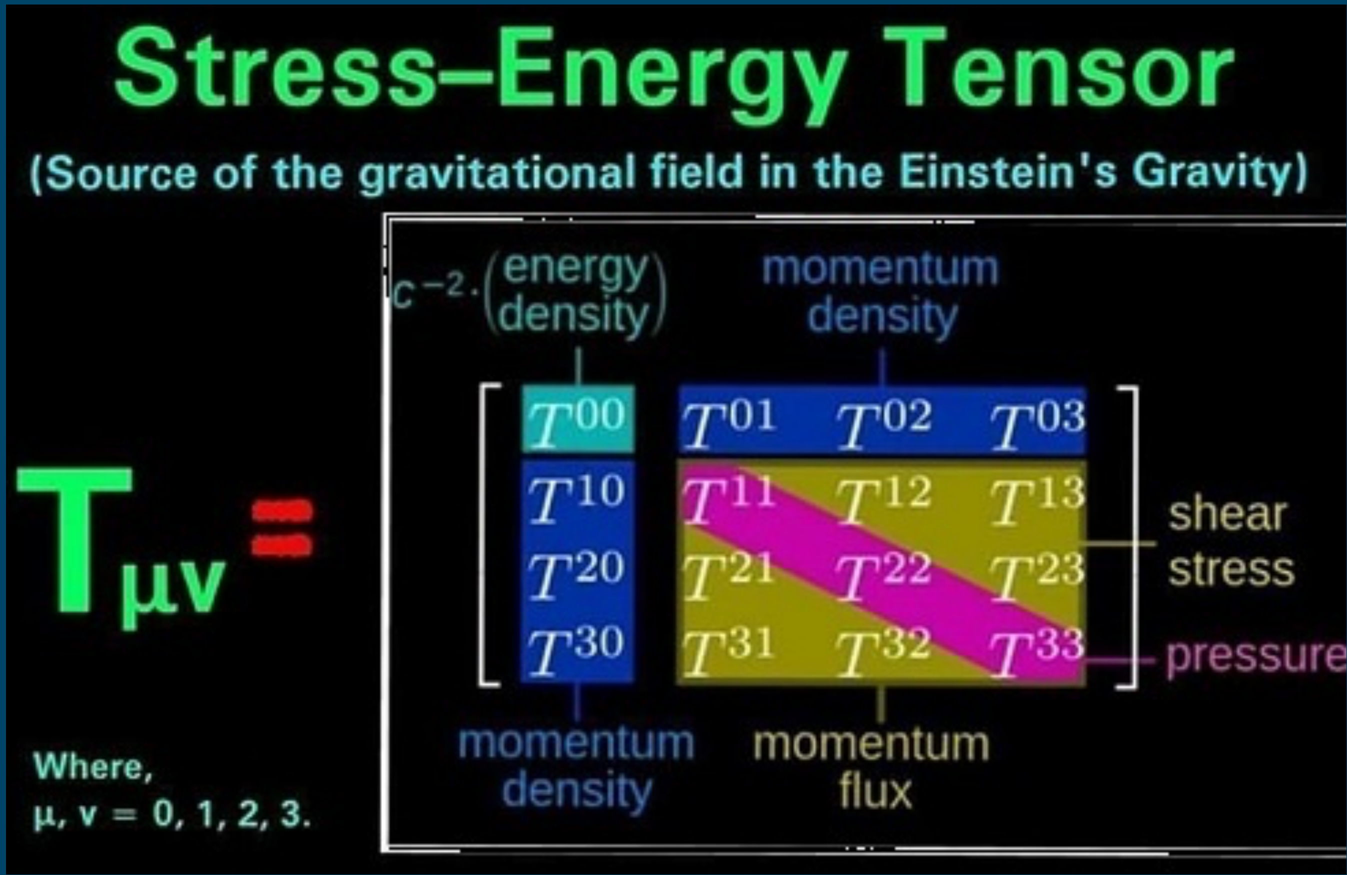


Measuring the screening exponent?

1.D. M. McAvity and H. Osborn, A DeWitt expansion of the heat kernel for manifolds with a boundary, Class. Quantum Gravity 8, 603 (1991).

2. M. Chernodub, MV, Direct measurement of a beta function and an indirect check of the Schwinger effect near the boundary in Dirac semimetals, PRR1, 032002(R) (2019).

Conformal anomaly in graphene



$$\frac{\partial S_\psi}{\partial \tau} = \beta_v \int \mathrm{d}t\,\mathrm{d}^2\boldsymbol{x}\,\langle \bar{\Psi} i \boldsymbol{\gamma} \boldsymbol{D} \Psi \rangle, \tag{29}$$

so finally we obtain – via Eq. (27) – the following equation for the conformal anomaly:

$$\langle T^{\mu}_{\ \mu} \rangle = \beta_v \langle \Psi i \boldsymbol{\gamma} \boldsymbol{\nabla} \Psi \rangle. \tag{30}$$

Repeating the derivation with a coordinate-dependent factor $\tau = \tau(x)$, using a variation instead of the differential operator in Eq. (29), we also get a relation for a matrix element with arbitrary insertions of T^{μ}_{μ}'s:

$$\langle T^{\mu}_{\ \mu}(x_1) \ldots T^{\mu}_{\ \mu}(x_n) \rangle = \beta_v^n \langle \Psi(x_1) i \boldsymbol{\gamma} \boldsymbol{\nabla} \Psi(x_1) \ldots \Psi(x_n) i \boldsymbol{\gamma} \boldsymbol{\nabla} \Psi(x_n) \rangle. \tag{31}$$

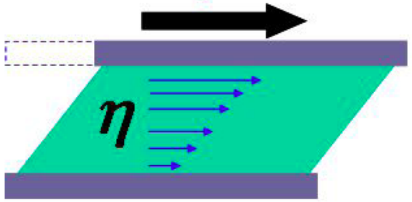
This formula implies the validity of the local relation:

$$T^{\mu}_{\ \mu}(x) = \beta_v \bar{\Psi}(x) i \boldsymbol{\gamma} \boldsymbol{\nabla} \Psi(x). \tag{32}$$

Viscosity

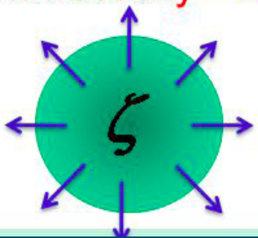
Fluids: (characterized by velocity field u)

Shear viscosity – measures the resistance to flow



act against the buildup of flow anisotropy

Bulk viscosity – measures the resistance to expansion



act against the buildup of radial flow

Elasticity:
Response of the stress tensor to time dependent strain

$$\tau_{ij} = \eta_{ijkl}\dot{u}_{kl}$$

Hydro:
Response of the stress tensor to gradients of the fluid velocity:

Coefficient of phenomenological equations

$$\langle \tau_{ij} \rangle = P\delta_{ij} - \kappa^{-1}\frac{\delta V}{V}\delta_{ij} - \eta_{ijk\ell}\frac{\partial v_\ell}{\partial x_k}$$

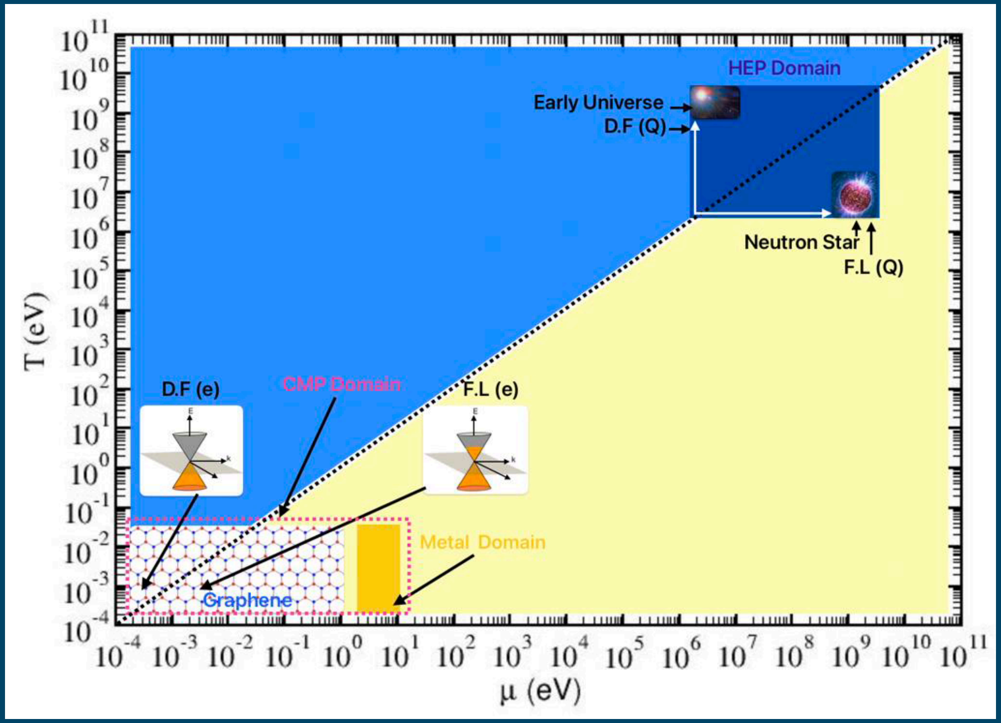
Rotational invariance
2D

Bulk — Shear

$$\eta_{ijkl} = \zeta(\omega)\delta_{ij}\delta_{kl} + \eta_{sh}(\omega)(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk} - \delta_{ij}\delta_{kl})$$

An interesting point of view

Relating hydro and thermodynamics



Graphene is neither Relativistic nor Non-Relativistic case: Thermodynamics Aspects, T. Zaw et al, arXiv:2307.05395

(No v renormalization but fun reading)

Measuring the bulk viscosity

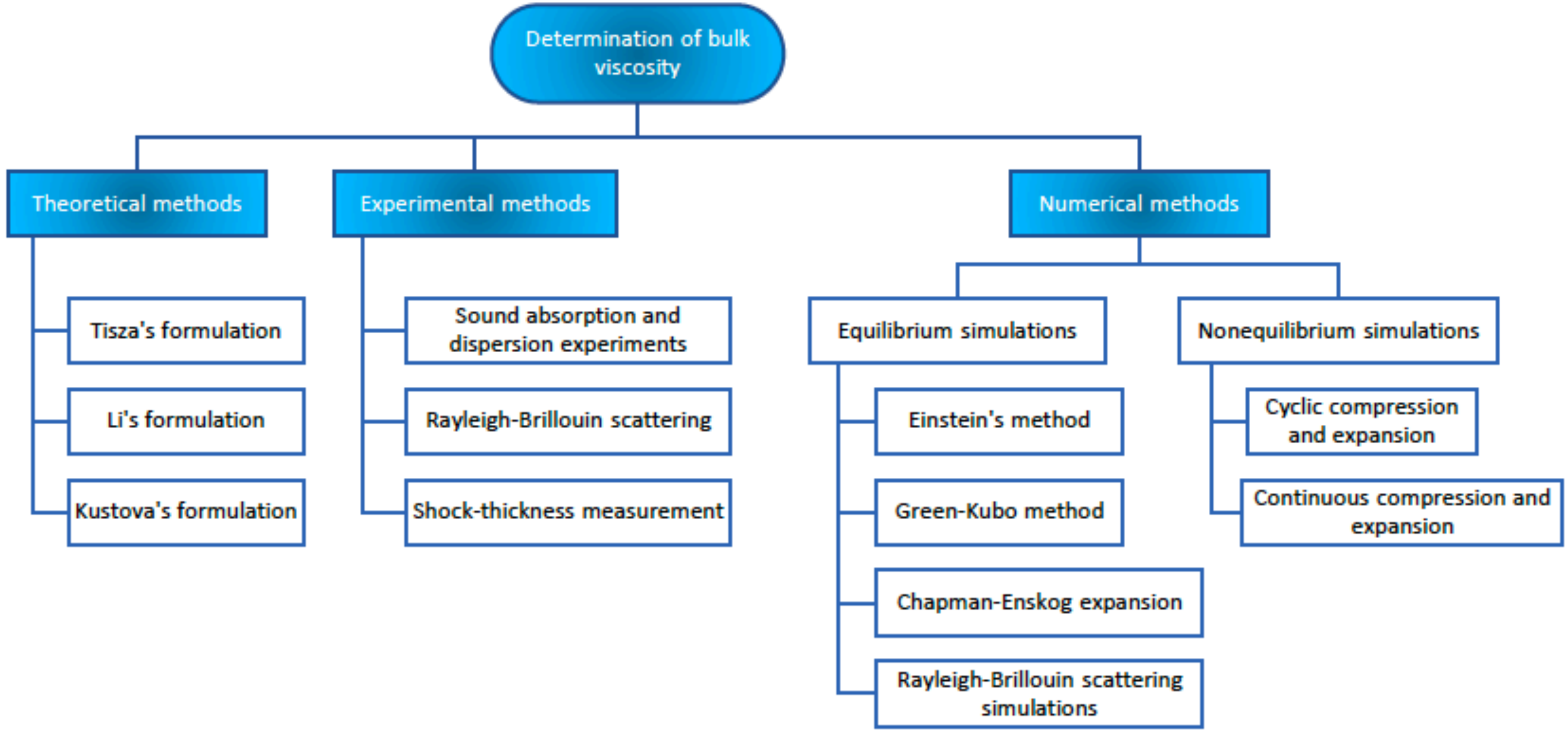


Figure 3: A survey of available methods for estimation of bulk viscosity.

A brief introduction to bulk viscosity of fluids
Bhanuday Sharma, Rakesh Kumar arXiv:2303.08400