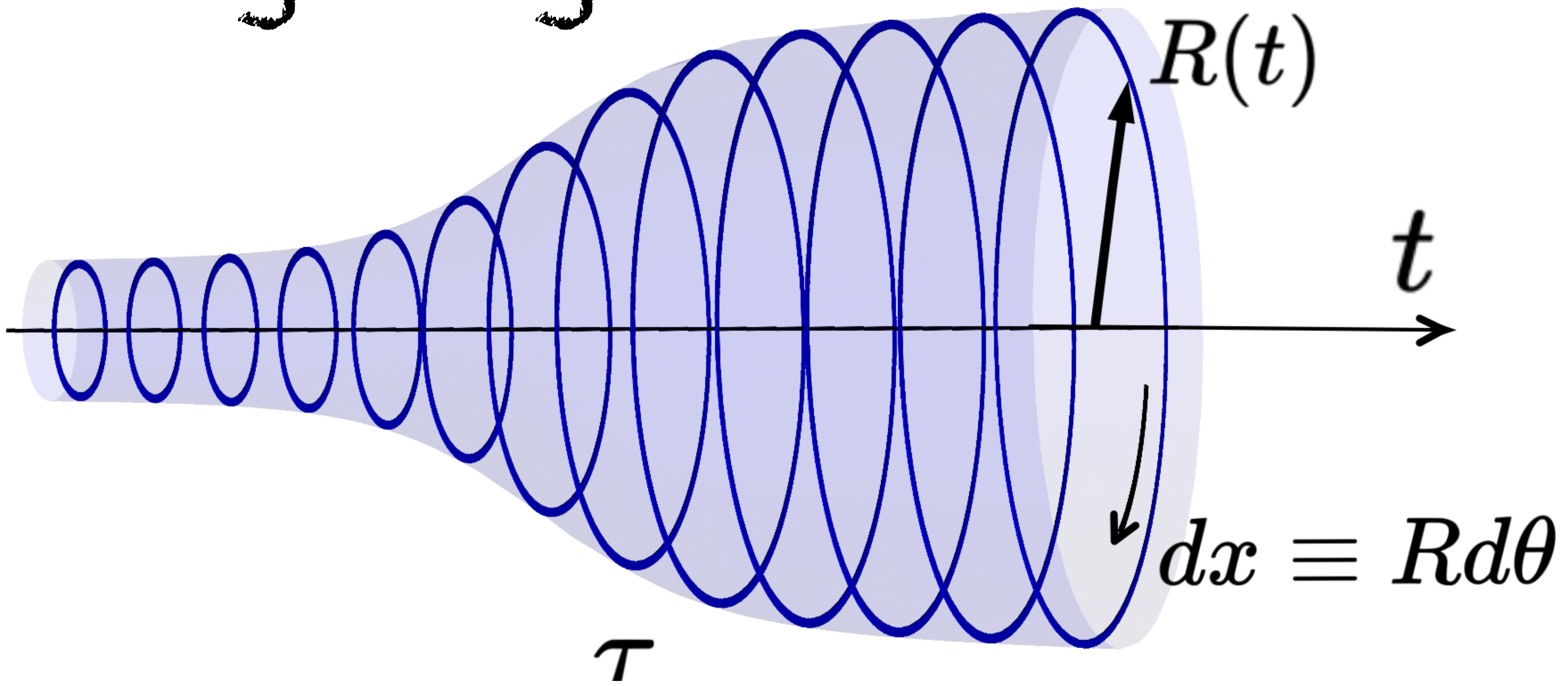
Anomalous Casimir effect in an expanding ring

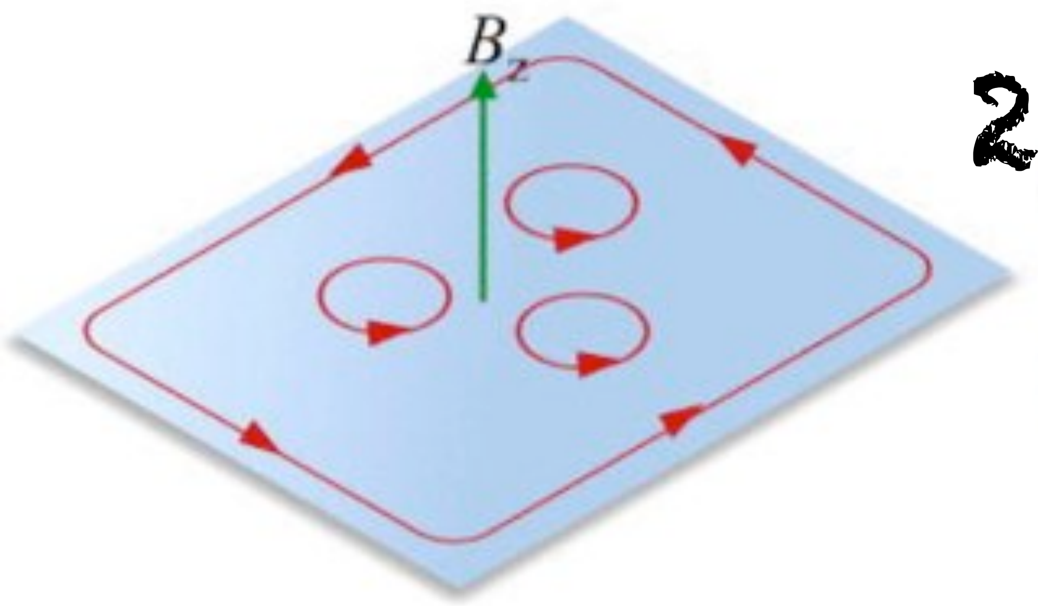


* B.Bermond, A.Grushin and, D.Carpentier
(2024), ArXiv 2402.08610

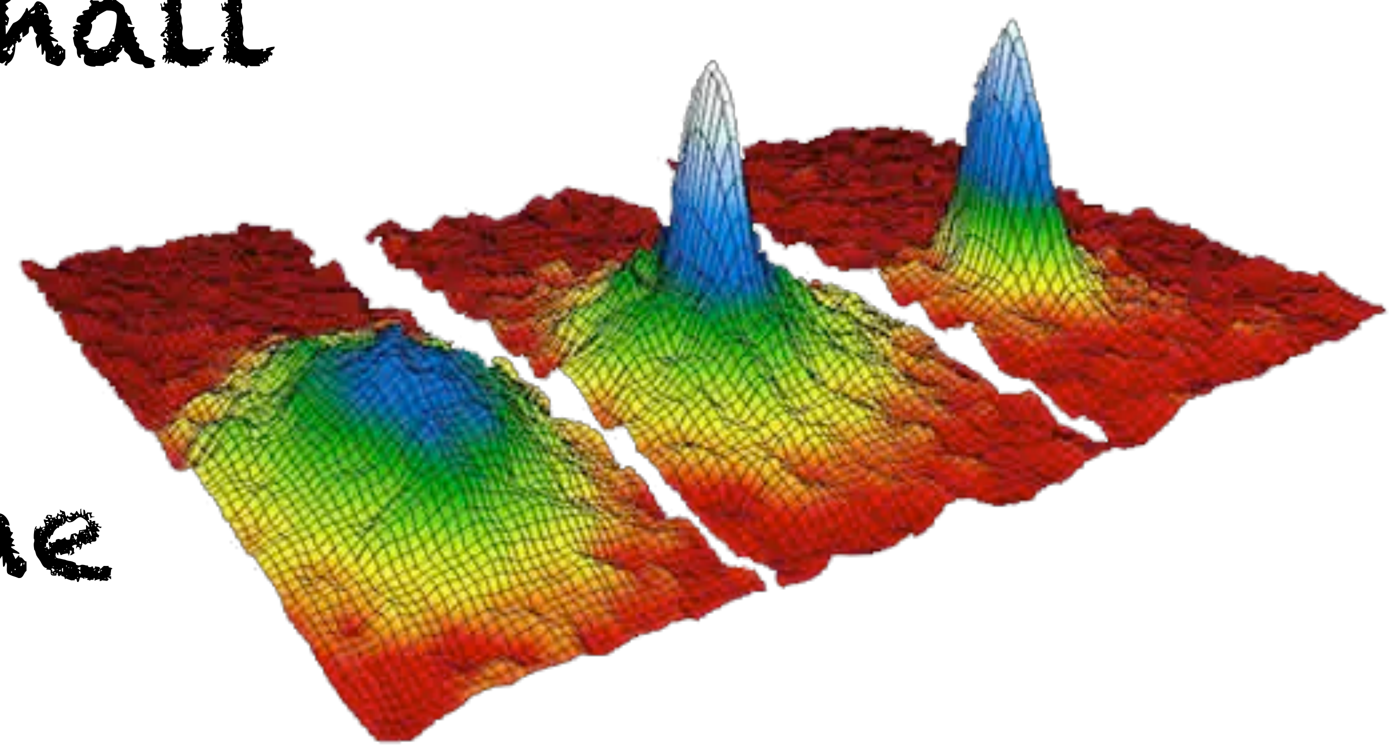
Lab PHYS ENS de LYON
ENS DE LYON

Outline

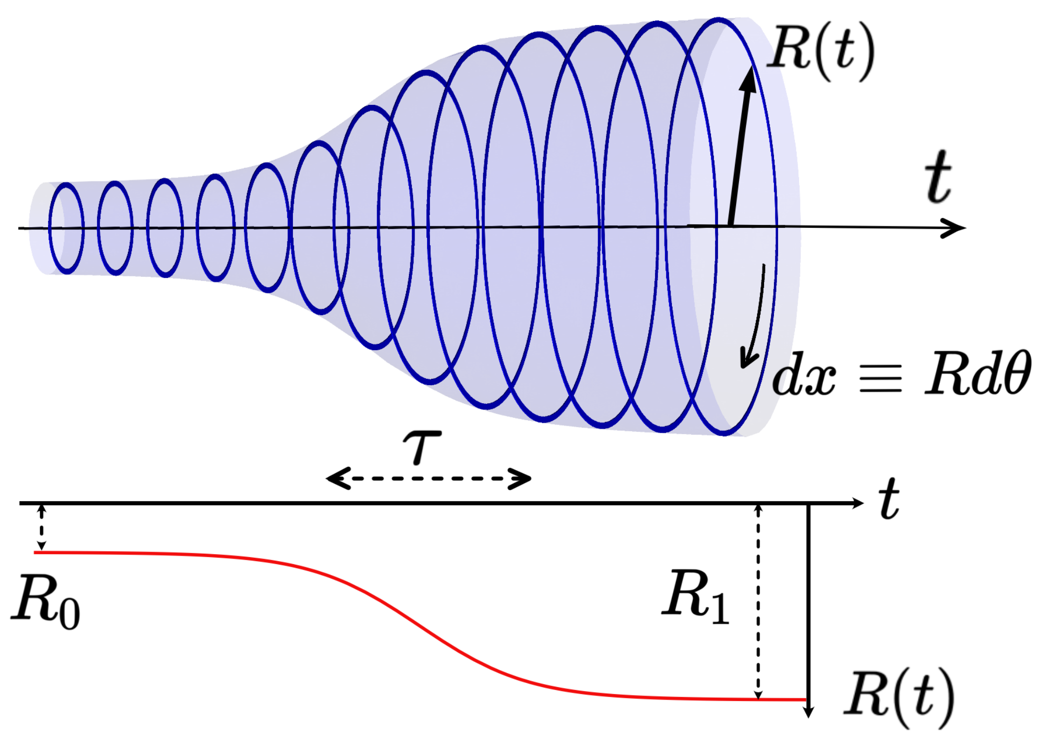
1. Out of equilibrium thermodynamics



2. Relativistic physics in condensed matter: From BEC to quantum hall edges



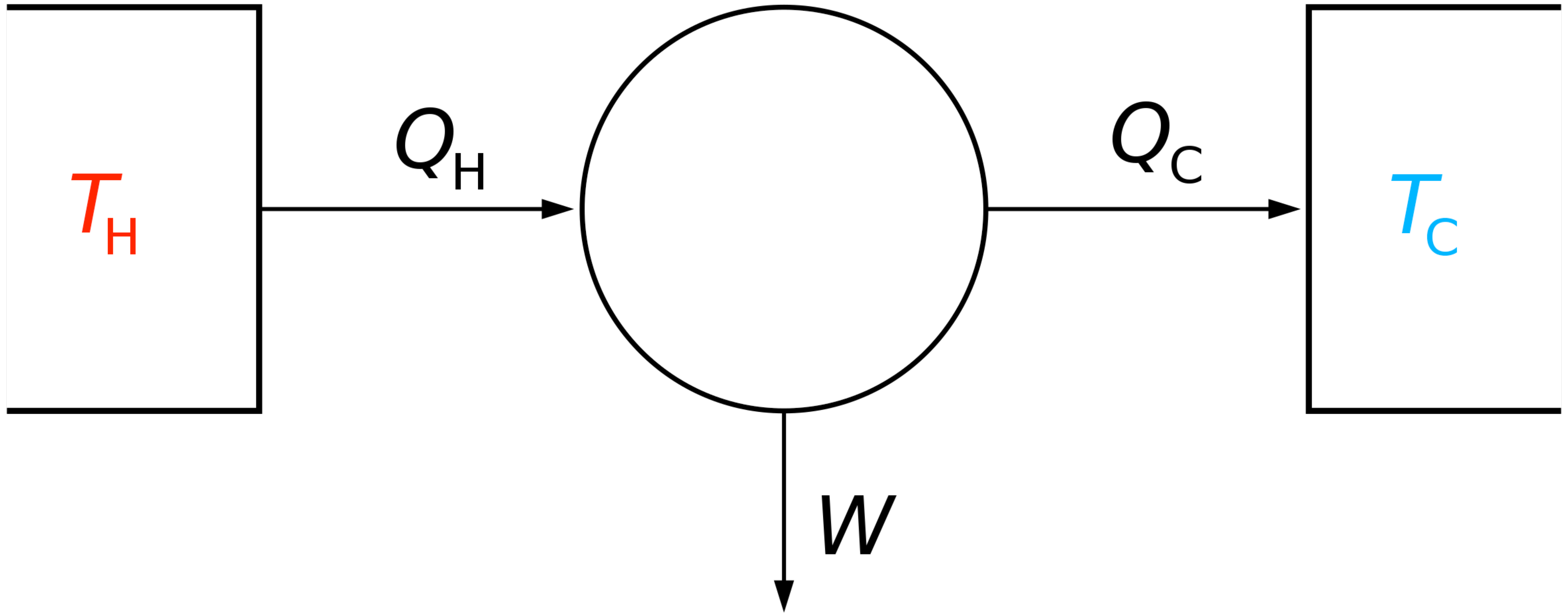
3. Gravitational anomalies and the anomalous Casimir effect



Conclusion/Outlook

1. Out of equilibrium thermodynamics

1.a. Out of equilibrium classical thermodynamics



1.b. Out of equilibrium thermodynamics in quantum physics

Non-equilibrium thermodynamics

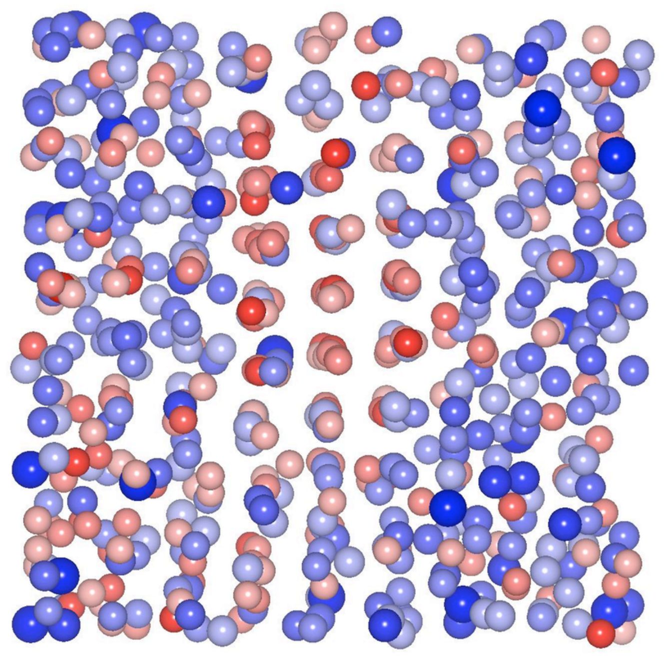
Thermodynamics based on equilibrium properties

$$\neq$$

Out of equilibrium systems

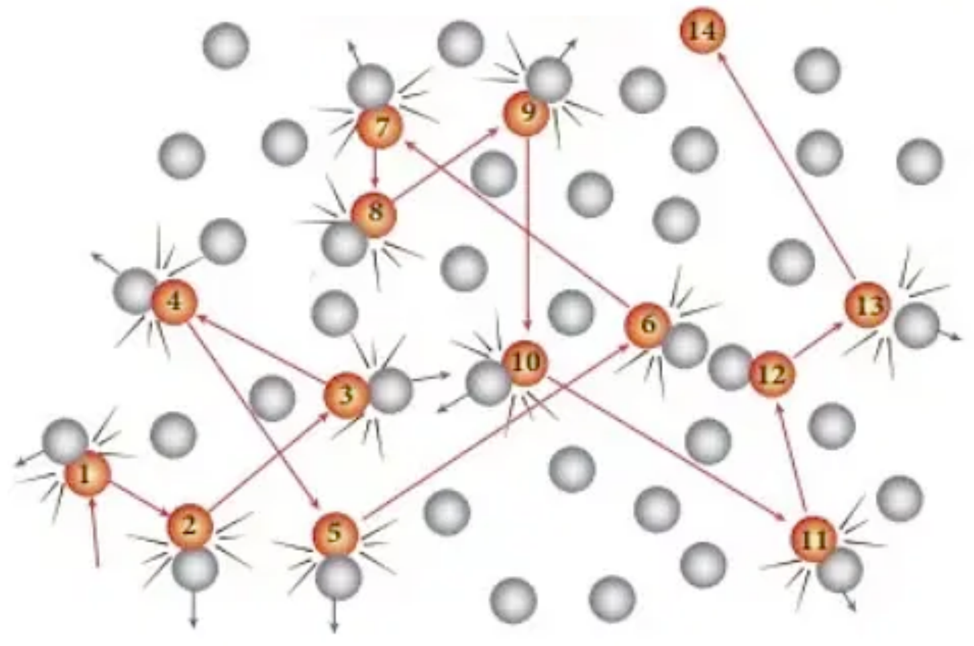
Possible reasons:

- **Extremely slow (beyond experimental reach) relaxation processes**



E.g. Critical slowing down,
Glassy dynamics.

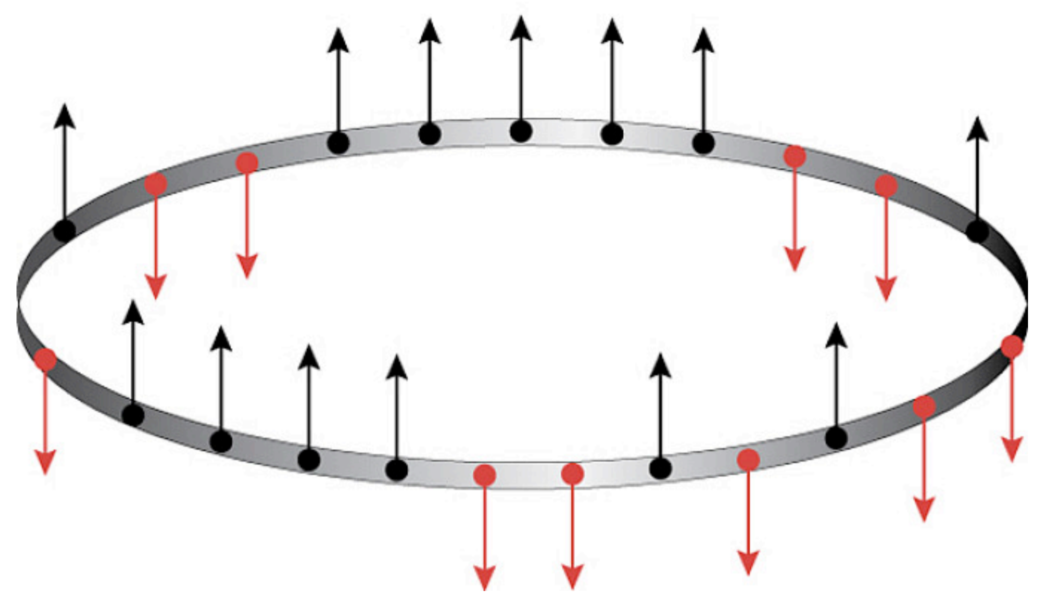
- **Driven systems (Energy injection)**



E.g. Active matter,
Quenched systems

- **Integrability**

Too many constant of motion inhibit equilibration



E.g. 1D Bosonic gases

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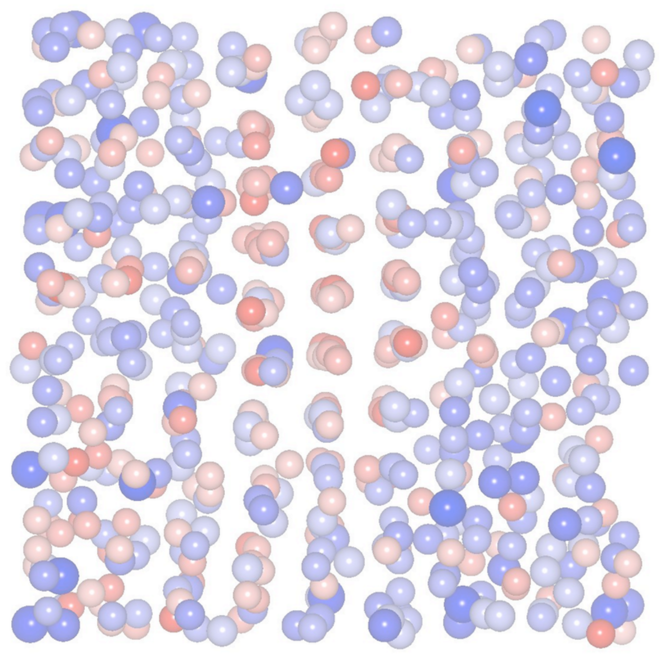
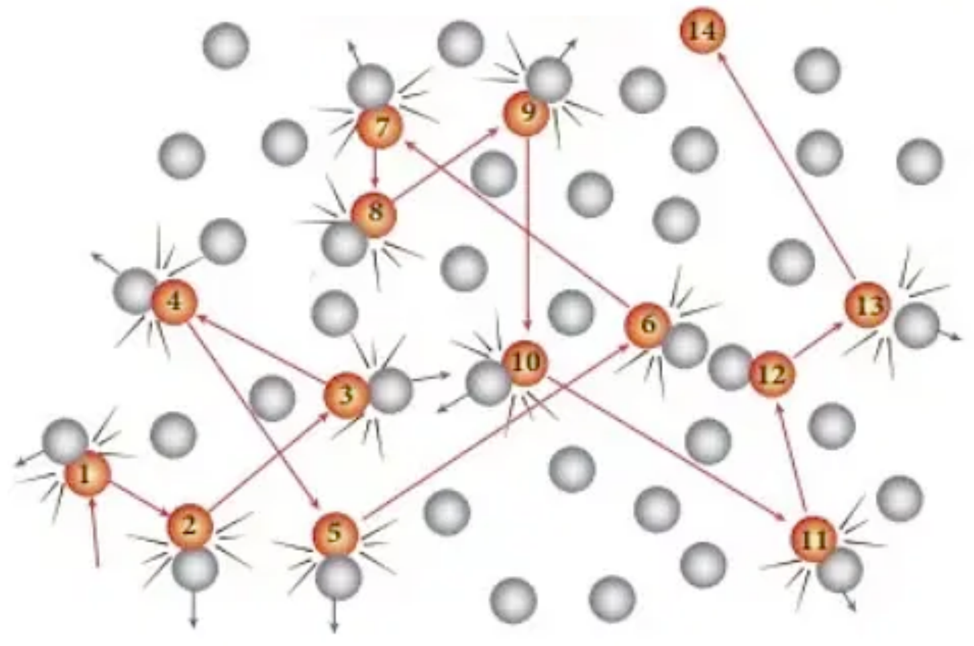


E.g. Critical slowing down, Glassy dynamics.

Our focus

- **Driven systems (Energy injection)** in 1+1 dimensions



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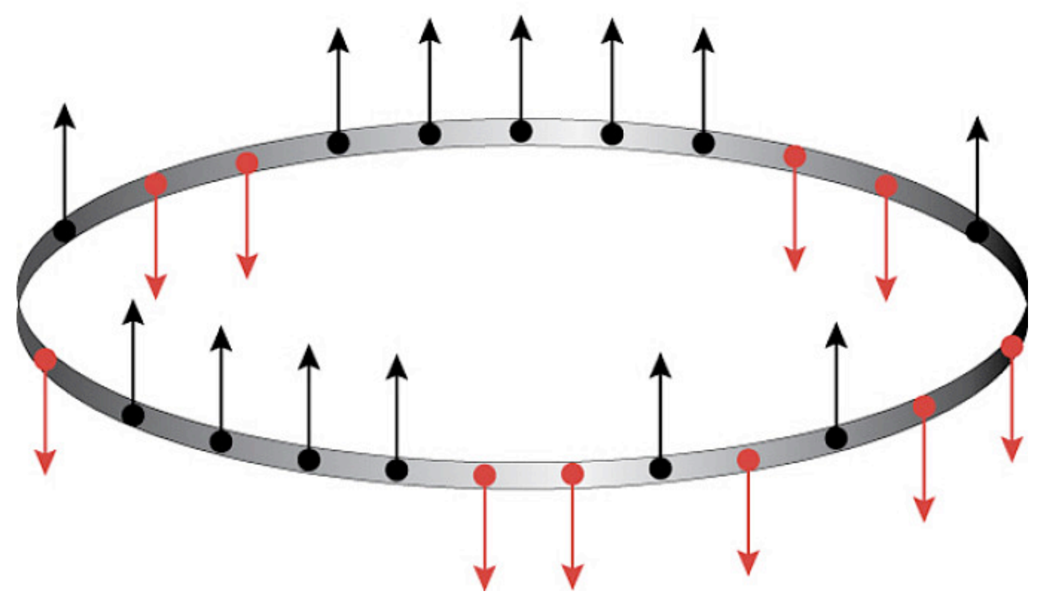
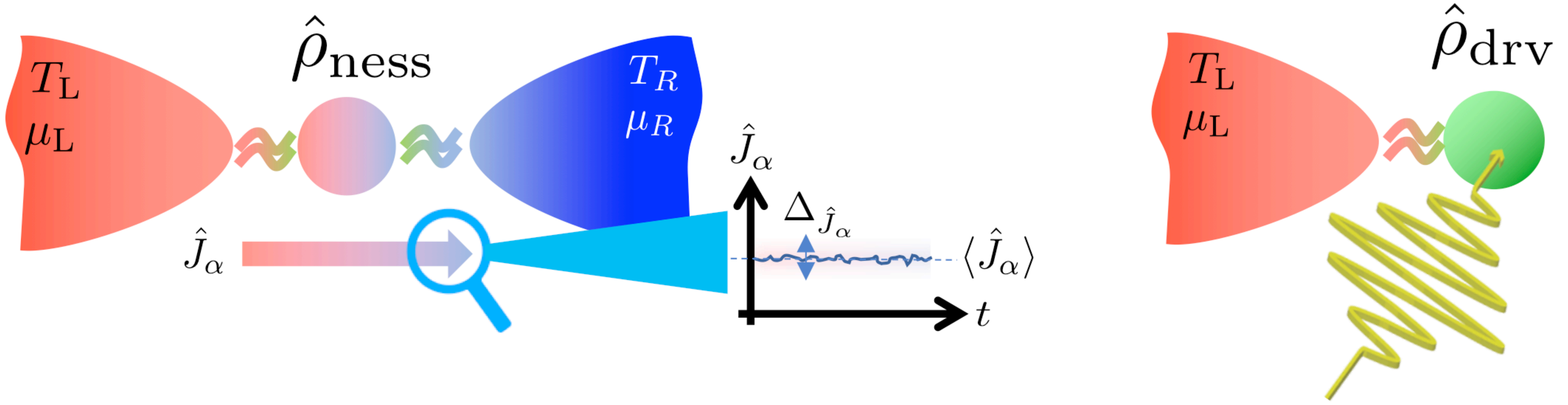


E.g. 1D Bosonic gases

1. Out of equilibrium thermodynamics

1.a. Out of equilibrium classical thermodynamics

1.b. Out of equilibrium thermodynamics in quantum physics



Out of equilibrium quantum physics

Driven quantum systems

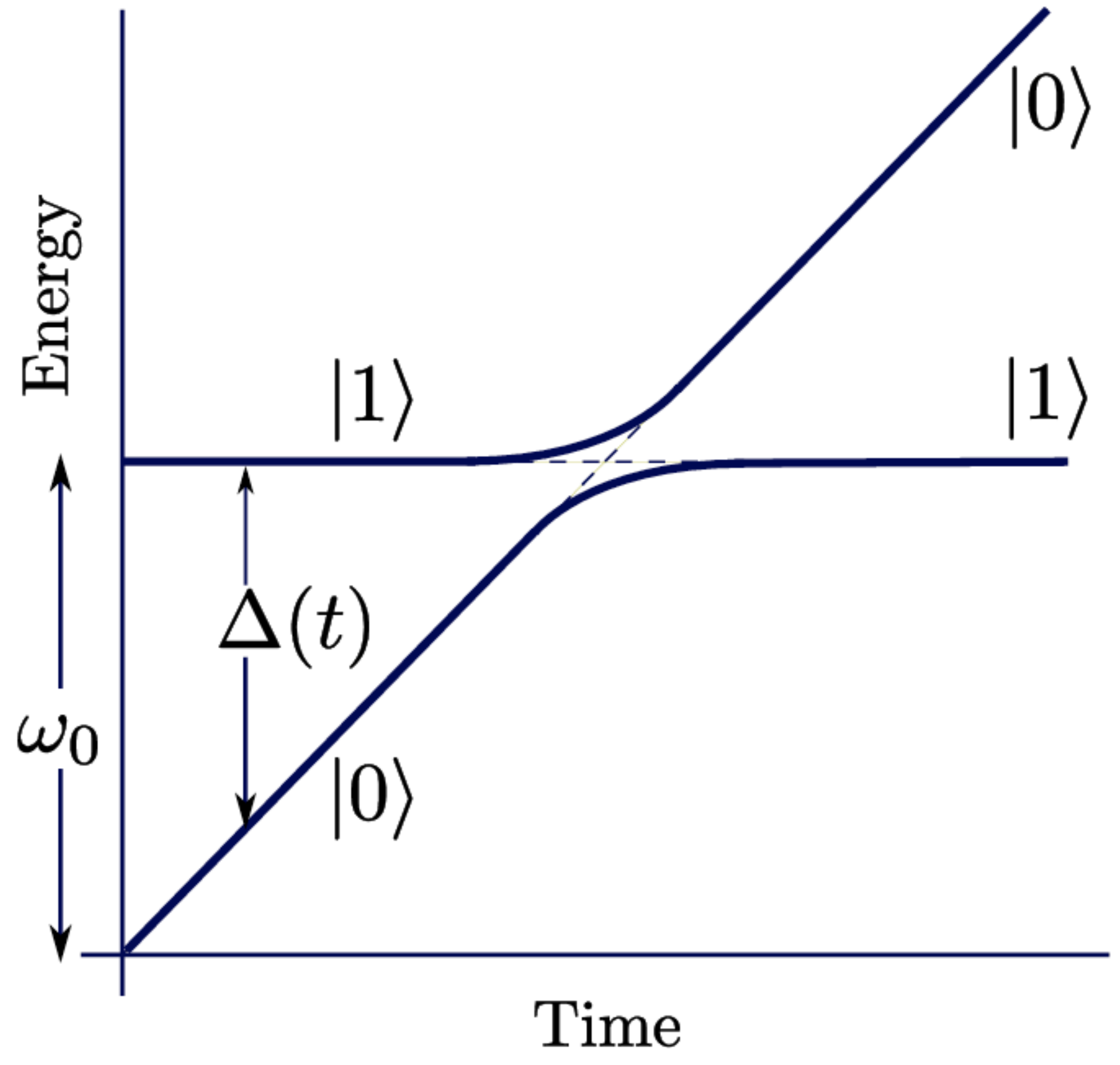
Smooth evolution of system parameters over time

E.g. Hamiltonian parameters, external parameters(temperature, environment), Floquet systems

- **Questions:**

- **Does the system remains in its ground state?**

Adiabatic evolution? Landau Zener transition?



L. Landau (1932)
C. Zener (1932)
E. Stueckelberg (1932)
E. Majorana (1932)

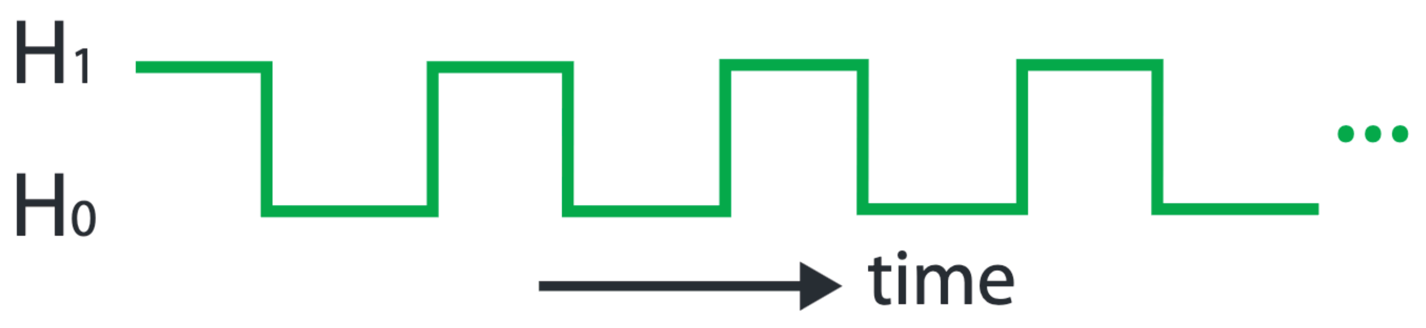
- **Floquet theory**

Periodic in time Hamiltonian

$$\hat{H}(t) = \hat{H}(t+T)$$

In thermodynamics: Thermalization? Steady states? Thermodynamic?

T. Mori (2022)



Out of equilibrium quantum physics (In 1+1 dimensions)

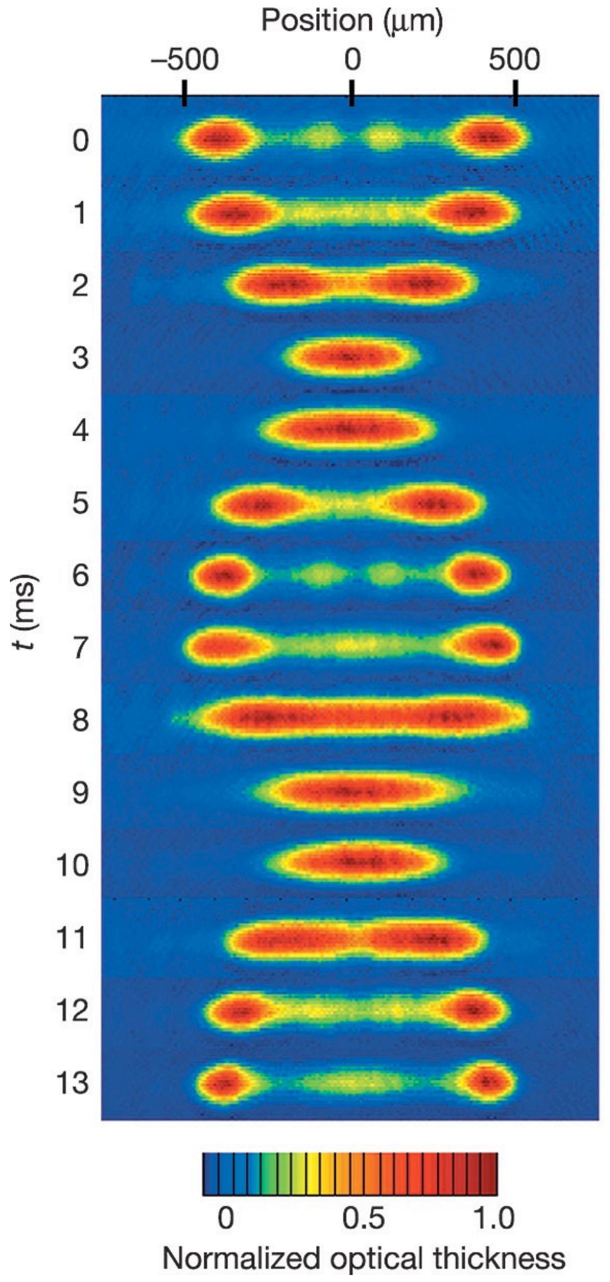
Quantum quenches

Sudden change of system parameters

E.g. Hamiltonian parameters, external parameters(temperature, environment)

Questions:

- **Will the system eventually thermalize?**



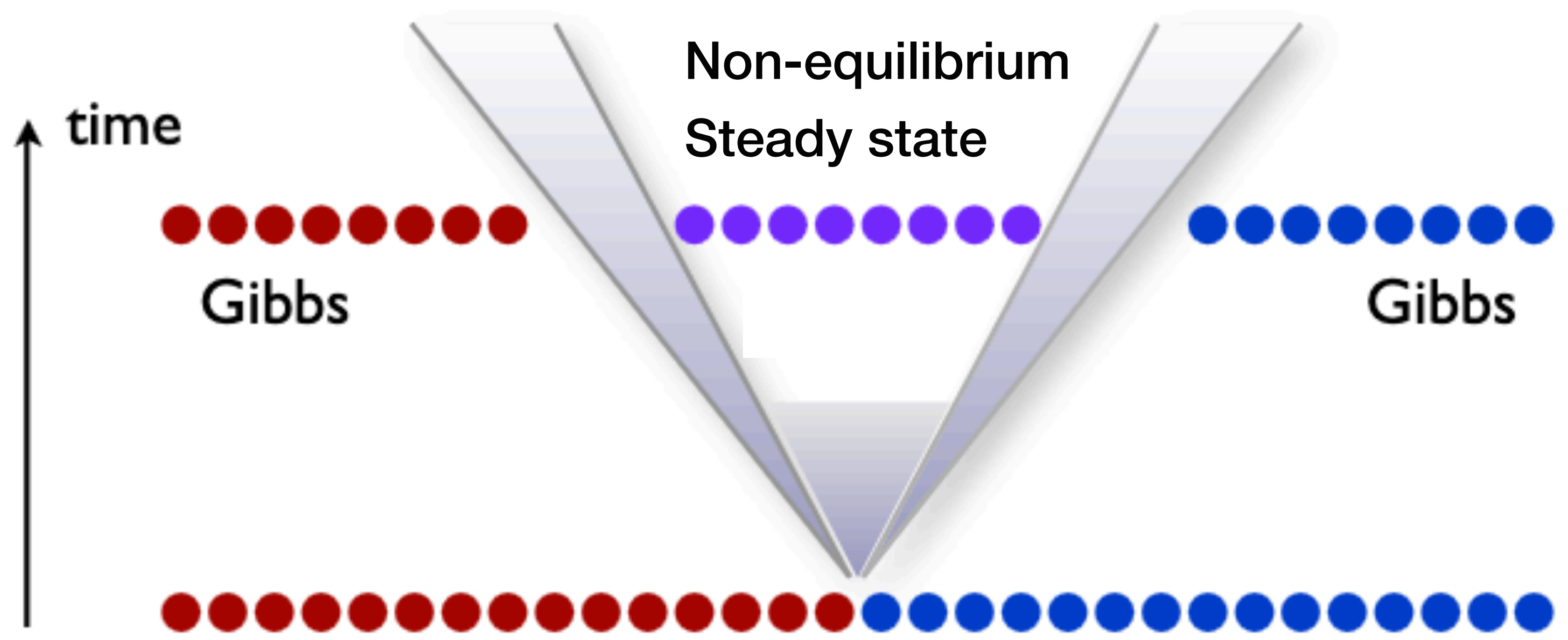
- A quantum Newton cradle

T Kinoshita Et. Al. (2006)

1D Bose Einstein condensate set out of equilibrium by a laser pulse

No sign of thermalization

- **What does the asymptotic steady state looks like? Boltzmann? Others?**



B. Doyon & D. Bernard (2016)

System at two different temperature are put into contact

Energy density? Energy currents?

Out of equilibrium quantum physics (In 1+1 dimensions)

Quantum quenches

Sudden change of system parameters

E.g. Hamiltonian parameters, external parameters(temperature, environment)

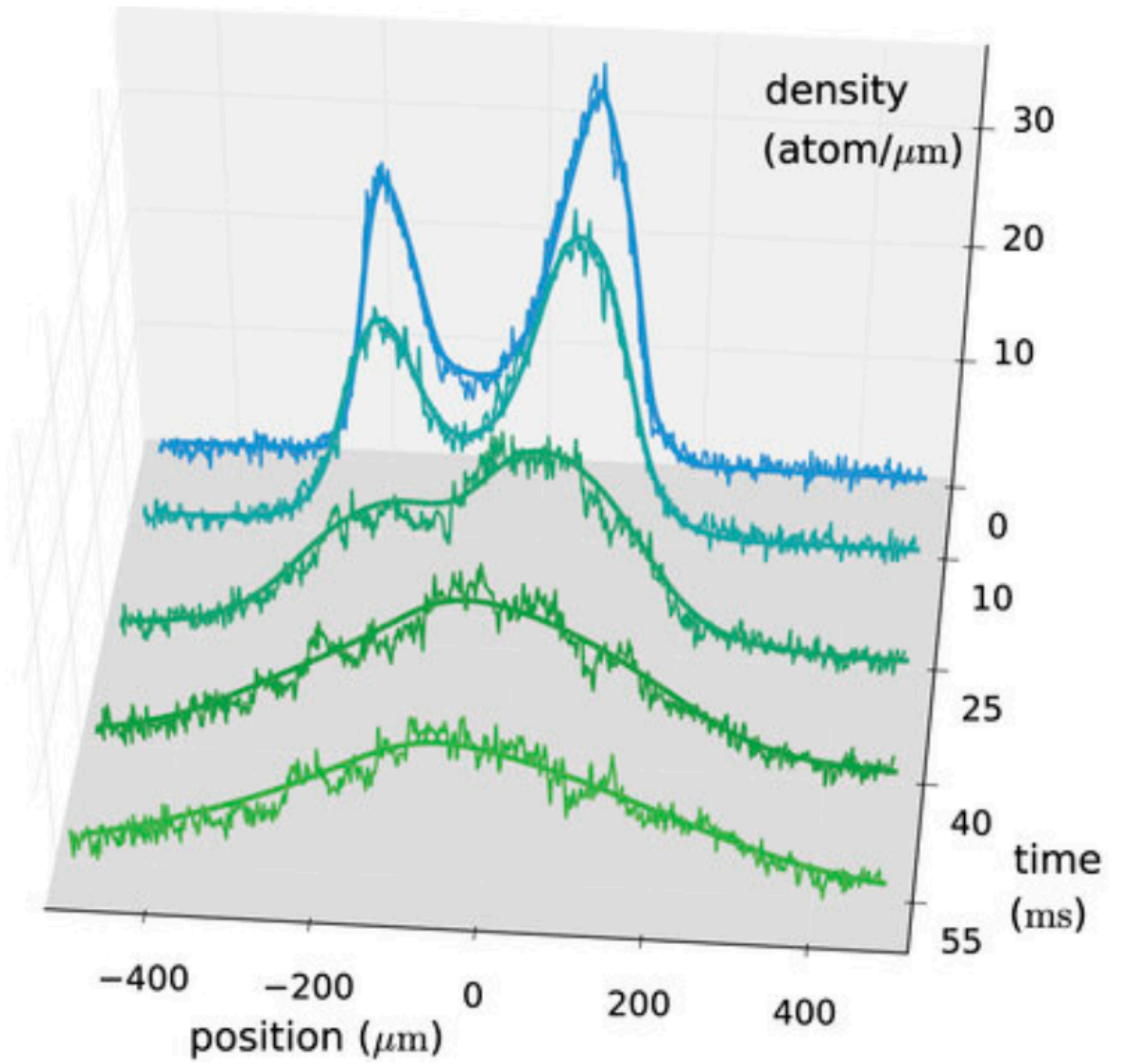
- **Strategies: Generalized hydrodynamics**

  Fast local equilibration allow one to define local thermodynamics quantities

  Local conservation equations defines time evolution of these local quantities

  B. Doyon & D. Bernard (2016)

  Example of follow up questions: How to include defects?



M. Schemmer et. Al. (2019)

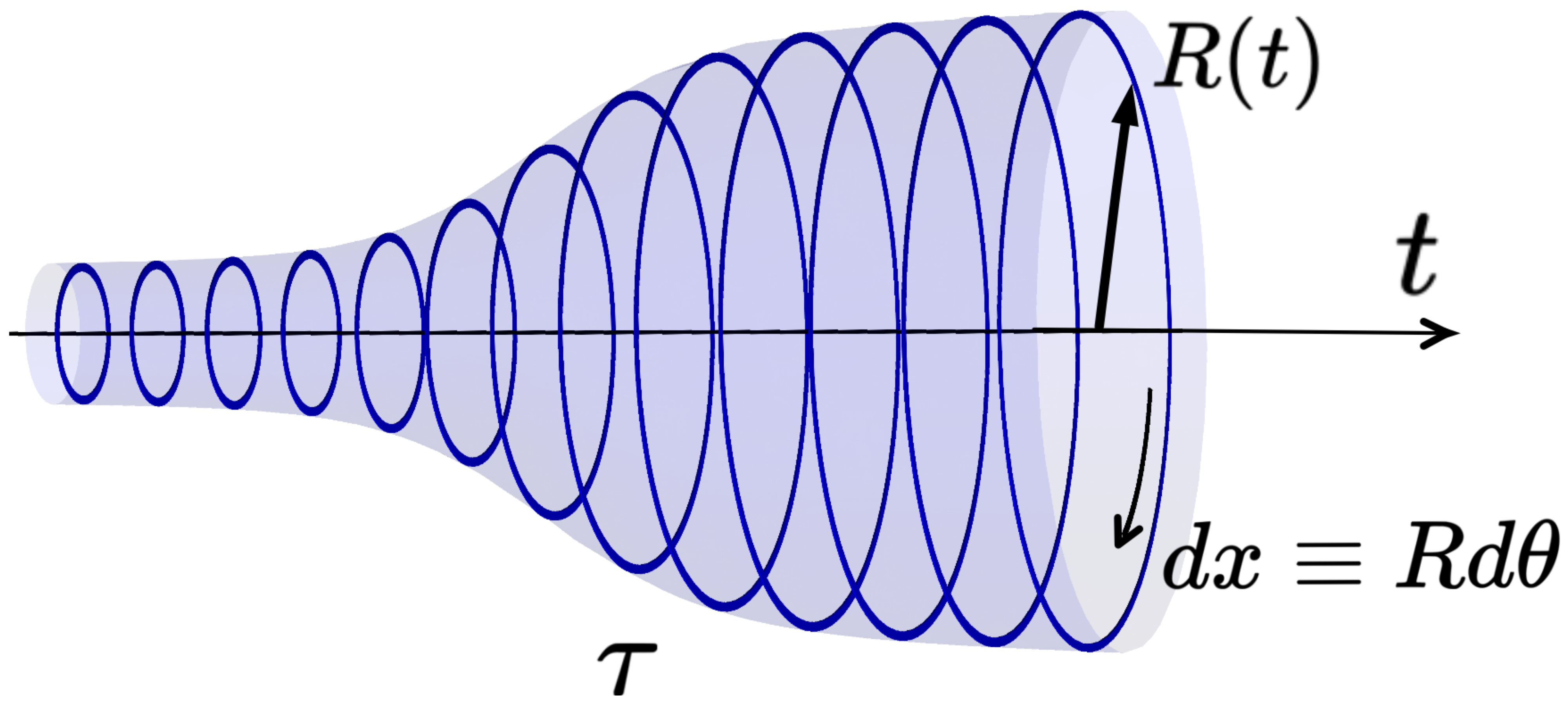
- **Result compared with exact computation done with Bethe ansatz/ exact diagonalisation**

  For transverse field Ising model, Luttinger liquids …

Out of equilibrium quantum physics (In 1+1 dimensions)

Non-equilibbrium quantum systems

- This talk:

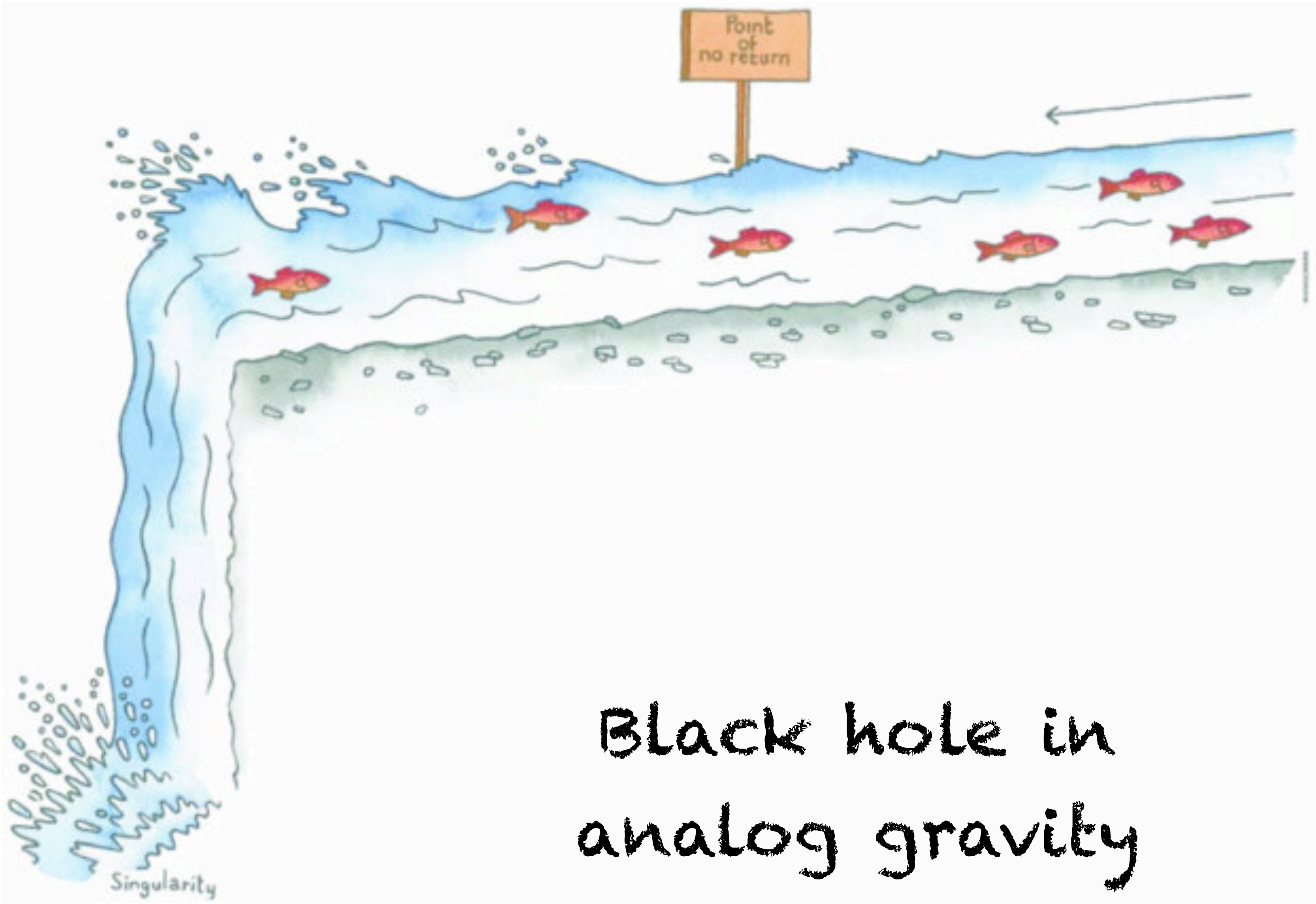


- Within generalized hydrodynamics
- 1+1D out-of-equilibrium systems: driven by a time-dependent geometry

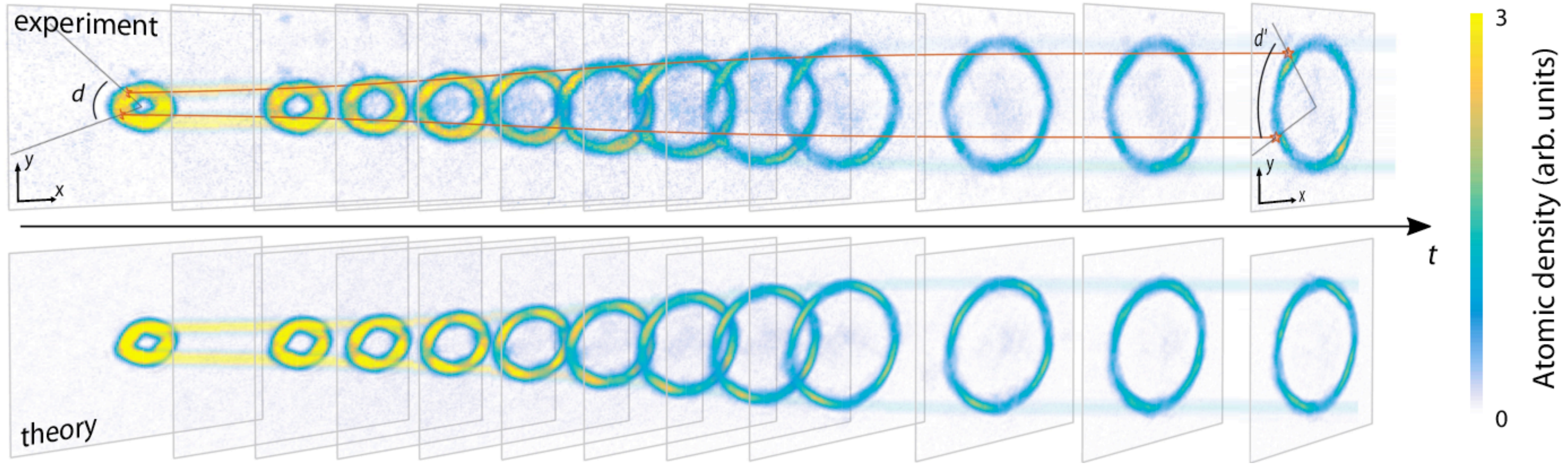
How do gravitational anomalies modifies the thermodynamic properties of such a non-equilibrium system?

2. Relativistic physics in condensed matter: From BEC to quantum hall edges

2.a. Curved spacetimes in the laboratory: Analog gravity in Bose-Einstein condensates



Black hole in analog gravity

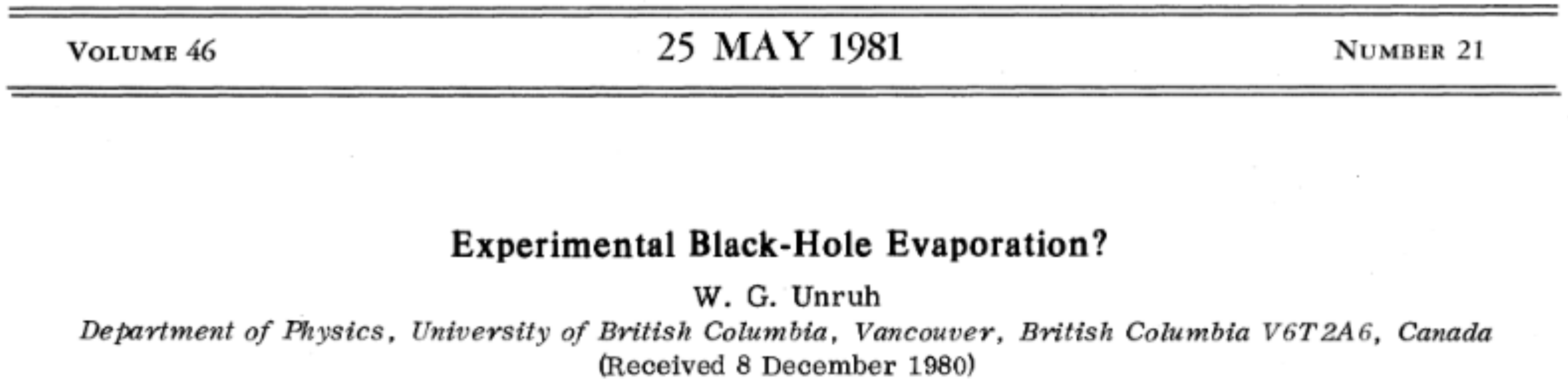


Analog expanding universe in a Bose Einstein Condensate

2.b. Chiral fields and curved spacetimes in quantum Hall edges

Analog gravity

W.G. Unruh



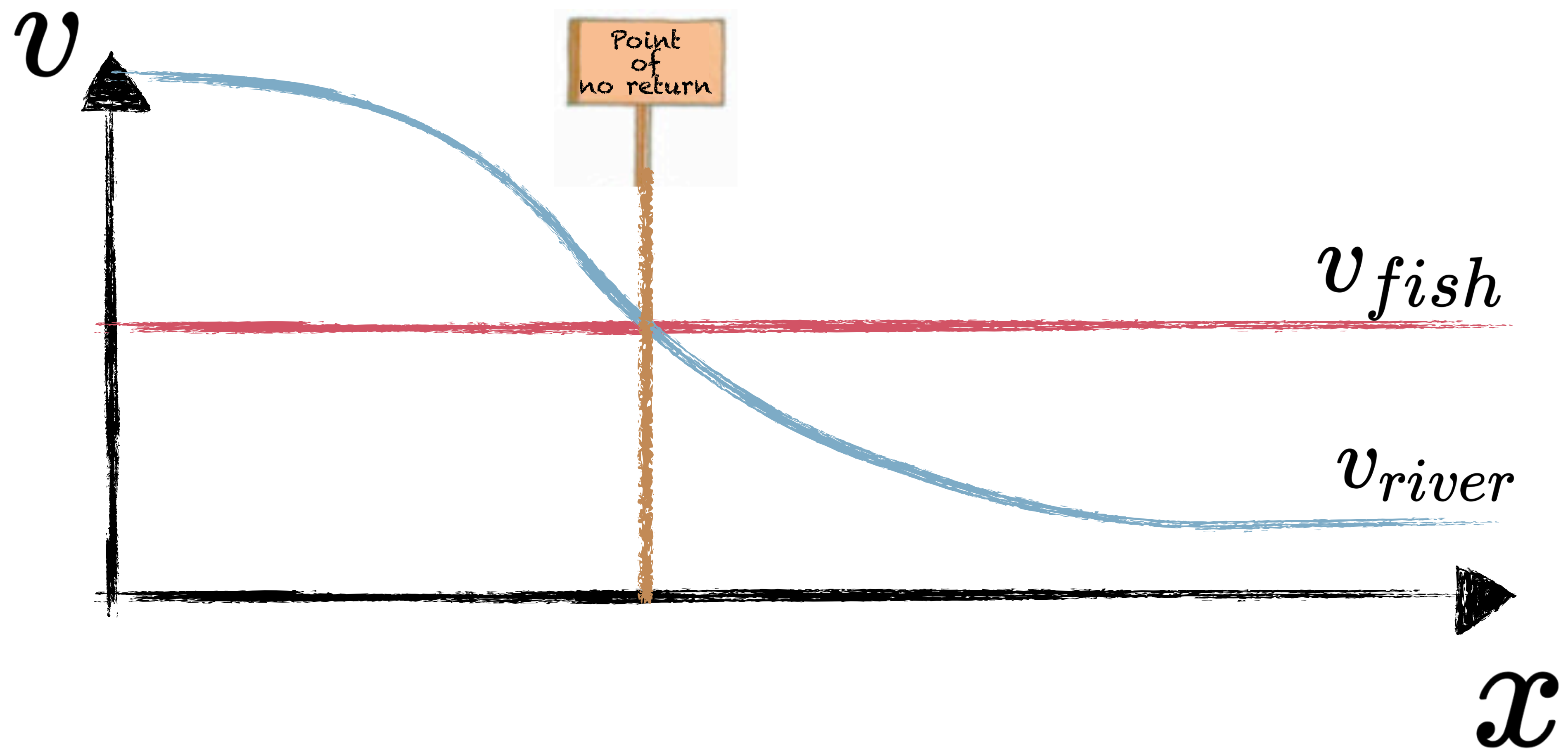
VOLUME 46 25 MAY 1981 NUMBER 21

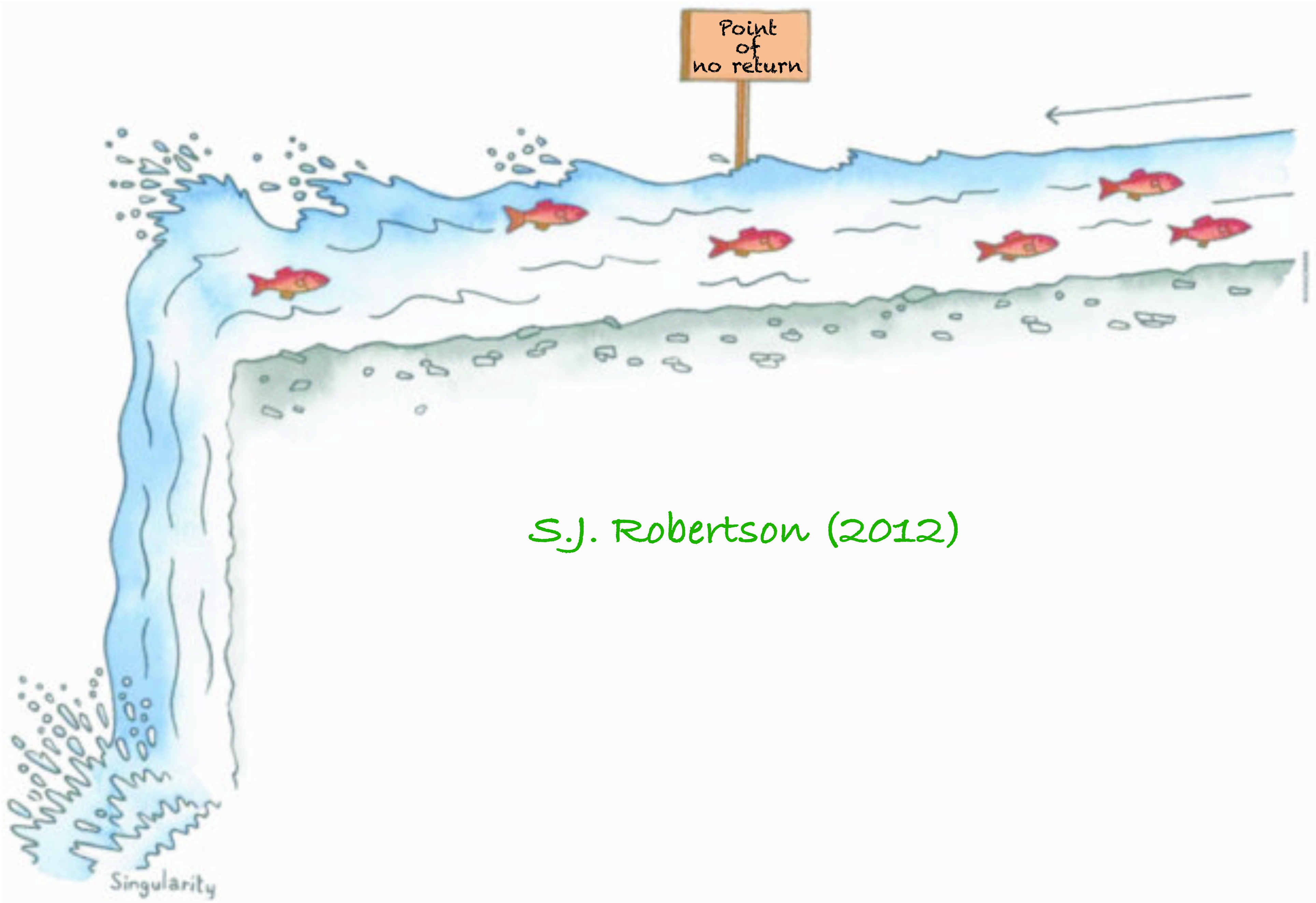
Experimental Black-Hole Evaporation?

W. G. Unruh
Department of Physics, University of British Columbia, Vancouver, British Columbia V6T 2A6, Canada
(Received 8 December 1980)

General idea:

Black holes physics can be reproduced in classical hydrodynamics





Analog gravity

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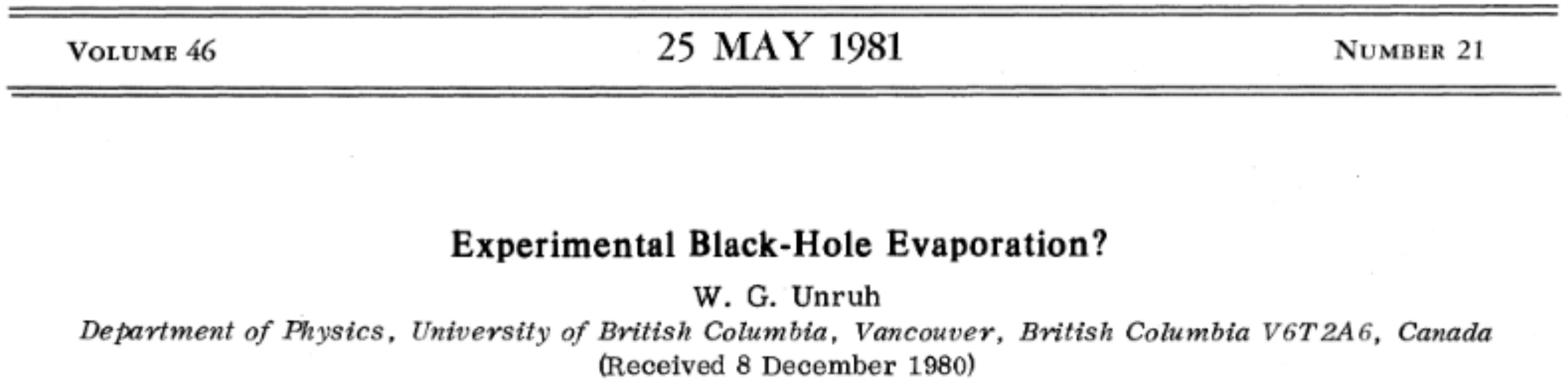

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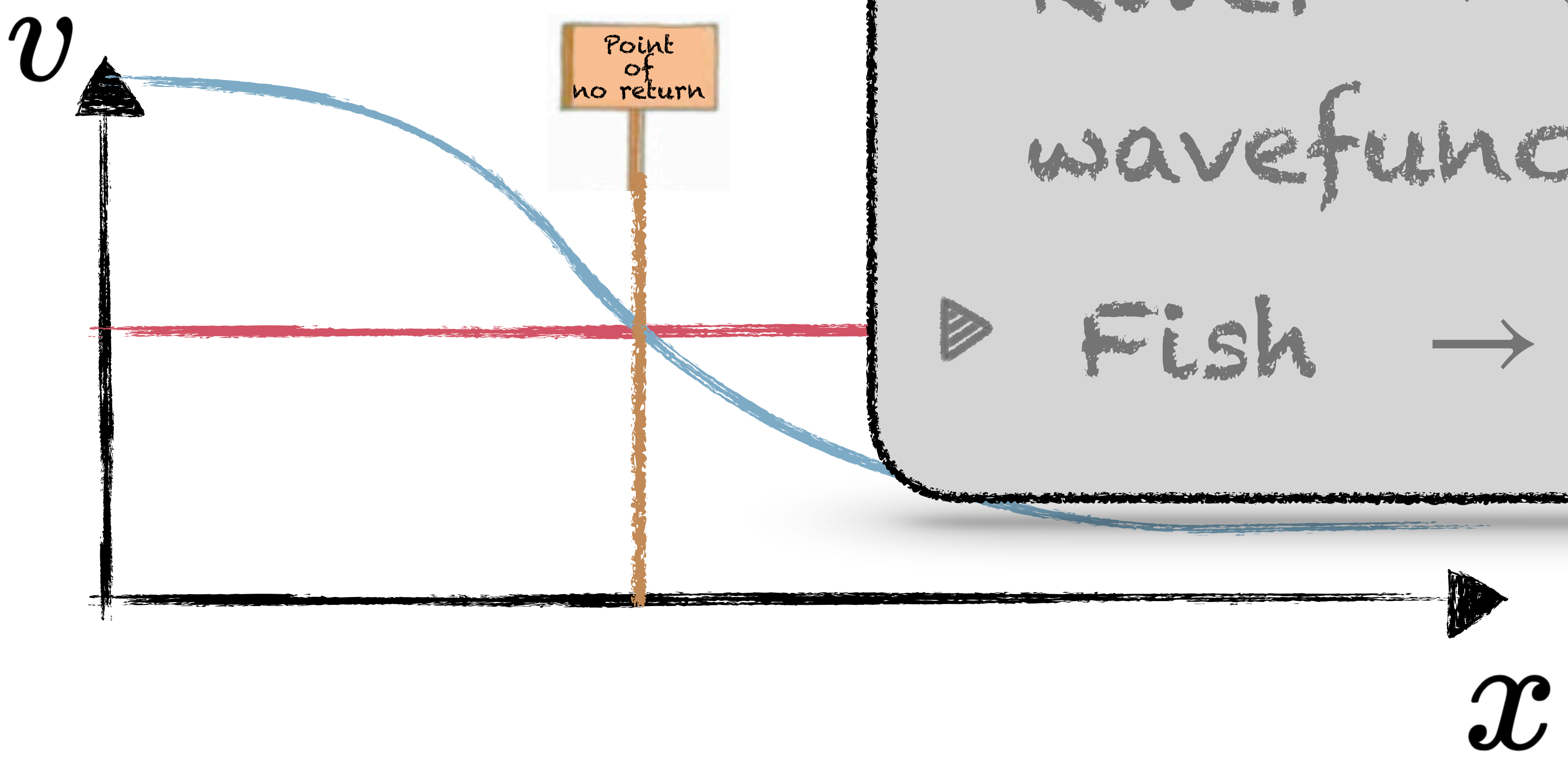

- River → Bose-Einstein wavefunction
- Fish → Sound waves

N interacting bosons: Bose Einstein condensation

Second quantized Hamiltonian

$$\mathscr{H} = \int d\vec{x}\, \hat{\Psi}^\dagger(\vec{x},t)\left[-\frac{\hbar^2}{2m}\vec{\nabla}^2 + \boxed{V_{ext}(\vec{x})}\right]\hat{\Psi}(\vec{x},t) + \frac{1}{2}\int d\vec{x}\, d\vec{y}\, \hat{\Psi}^\dagger(\vec{x},t)\hat{\Psi}^\dagger(\vec{y},t)\boxed{V\left(\vec{x}-\vec{y}\right)}\,\hat{\Psi}(\vec{y},t)\hat{\Psi}(\vec{x},t)$$

Potential

Particle-particle interactions

$$V\left(\vec{x}\right) = \kappa\delta\left(\vec{x}\right)$$

N interacting bosons: Bose Einstein condensation

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Potential

Particle-particle interactions

$$V\left(\vec{x}\right) = \kappa\delta\left(\vec{x}\right)$$

Mean field solution (Gross-Pitaevskii equation)

$$\hbar\partial_t\psi = \left(-\frac{\hbar^2}{2m}\vec{\nabla}^2 + V_{ext}(\vec{x}) + \kappa\psi^\dagger\psi\right)\boxed{\psi}$$

Wave function of the condensate

$$\psi\left(\vec{x},t\right) = \left\langle \hat{\Psi}\left(\vec{x},t\right)\right\rangle$$

N interacting bosons: Bose Einstein condensation

Mean field solution (Gross-Pitaevskii equation)

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Wave function of the condensate

$$\psi\left(\vec{x}, t\right) = \left\langle \hat{\Psi}\left(\vec{x}, t\right) \right\rangle$$

- Assuming $\psi = \sqrt{n} e^{i\frac{\theta}{\hbar}}$

$$\begin{cases} \partial_t n + \frac{1}{m} \vec{\nabla} \cdot \left(n \vec{\nabla} \theta \right) = 0\,, \\ \partial_t \theta + \frac{1}{2m} \left(\vec{\nabla} \theta \right) \cdot \left(\vec{\nabla} \theta \right) + V_{ext} + \kappa n - \boxed{\frac{\hbar^2}{2m} \frac{\vec{\nabla}^2 \sqrt{n}}{\sqrt{n}}} = 0\,. \end{cases}$$

Quantum potential

V_q

In classical hydrodynamics

$$\vec{v} = \vec{\nabla} \phi$$

$$\begin{cases} \partial_t \rho - \vec{\nabla} \cdot \left(\rho \vec{\nabla} \phi \right) = 0\,, \\ \partial_t \phi - h(p) - \frac{1}{2} \left(\vec{\nabla} \phi \right) \cdot \left(\vec{\nabla} \phi \right) = 0\,. \end{cases}$$

N interacting bosons: Bose Einstein condensation

Relativistic waves in a curved spacetimes

- Setting $\begin{cases} n &= n_0 + \delta n\,, \\ \theta &= \theta_0 + \delta\theta\,, \end{cases}$ assuming $\frac{k}{2\pi} < \boxed{\xi^{-1}}$

Healing length $\xi = \frac{\hbar}{\sqrt{m\kappa n_0}}$

Sound wave

$$\partial_t \delta n + \frac{1}{m}\vec{\nabla}\cdot\left(\delta n\,\vec{\nabla}\theta_0 + n_0\,\vec{\nabla}\delta\theta\right) = 0\,,$$

$$\partial_t \delta\theta + \frac{1}{m}\left(\vec{\nabla}\delta\theta\right)\cdot\left(\vec{\nabla}\theta_0\right) + \kappa\delta n = 0$$

Background flow

$$\partial_t n_0 + \frac{1}{m}\vec{\nabla}\cdot\left(n_0\,\vec{\nabla}\theta_0\right) = 0\,,$$

$$\partial_t \theta_0 + \frac{1}{2m}\left(\vec{\nabla}\theta_0\right)\cdot\left(\vec{\nabla}\theta_0\right) = V_q - V_{ext} - \kappa n_0\,.$$

$$\partial_\mu\left[f^{\mu\nu}\partial_\nu\left(\delta\theta\right)\right] = 0$$

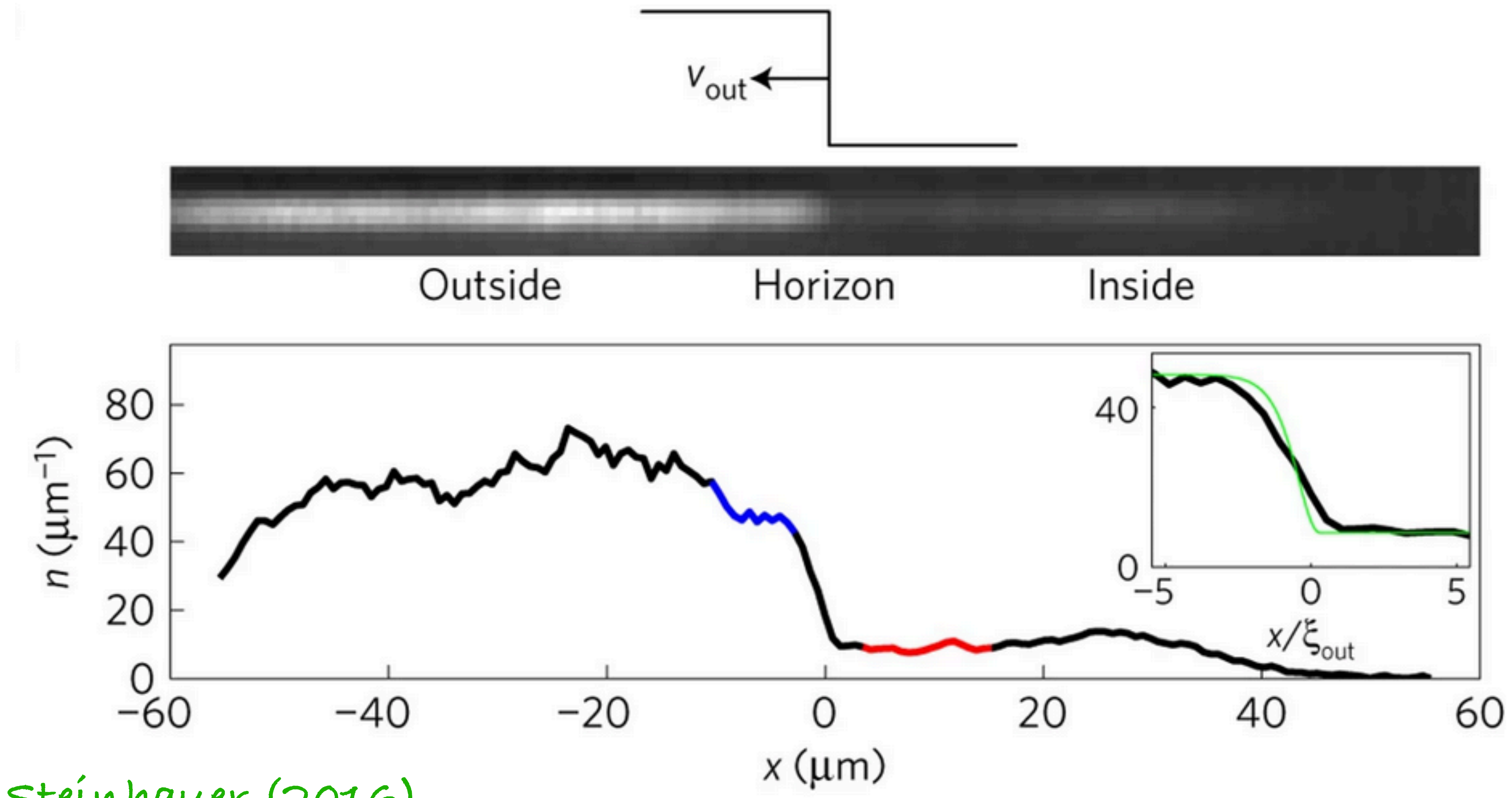
with,

$$f^{\mu\nu} = \sqrt{det\left(g_{\rho\sigma}\right)}\,g^{\mu\nu} = \frac{n_0}{m\,\boxed{c_s^2}}\begin{pmatrix} 1 & -m\partial_i\theta_0 \\ -m\partial_i\theta_0 & -c_s^2\delta^{ij} + m^2\partial_i\theta_0\partial_j\theta_0 \end{pmatrix}$$

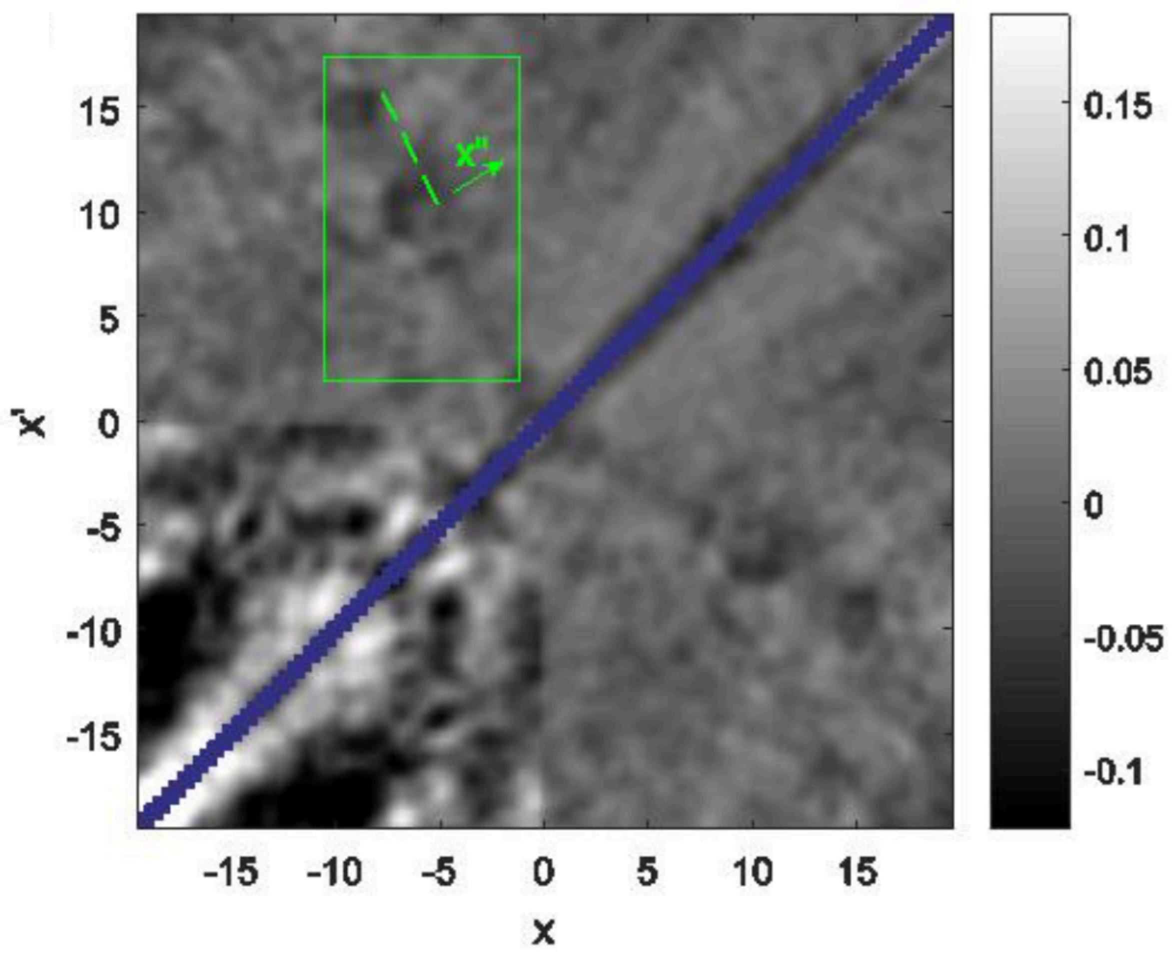
Speed of sound $c_s^2 = \frac{\kappa n_0}{m}$

Analog gravity

Ex: Hawking radiation in Bose Einstein condensates



J. Steinhauer, (2016)



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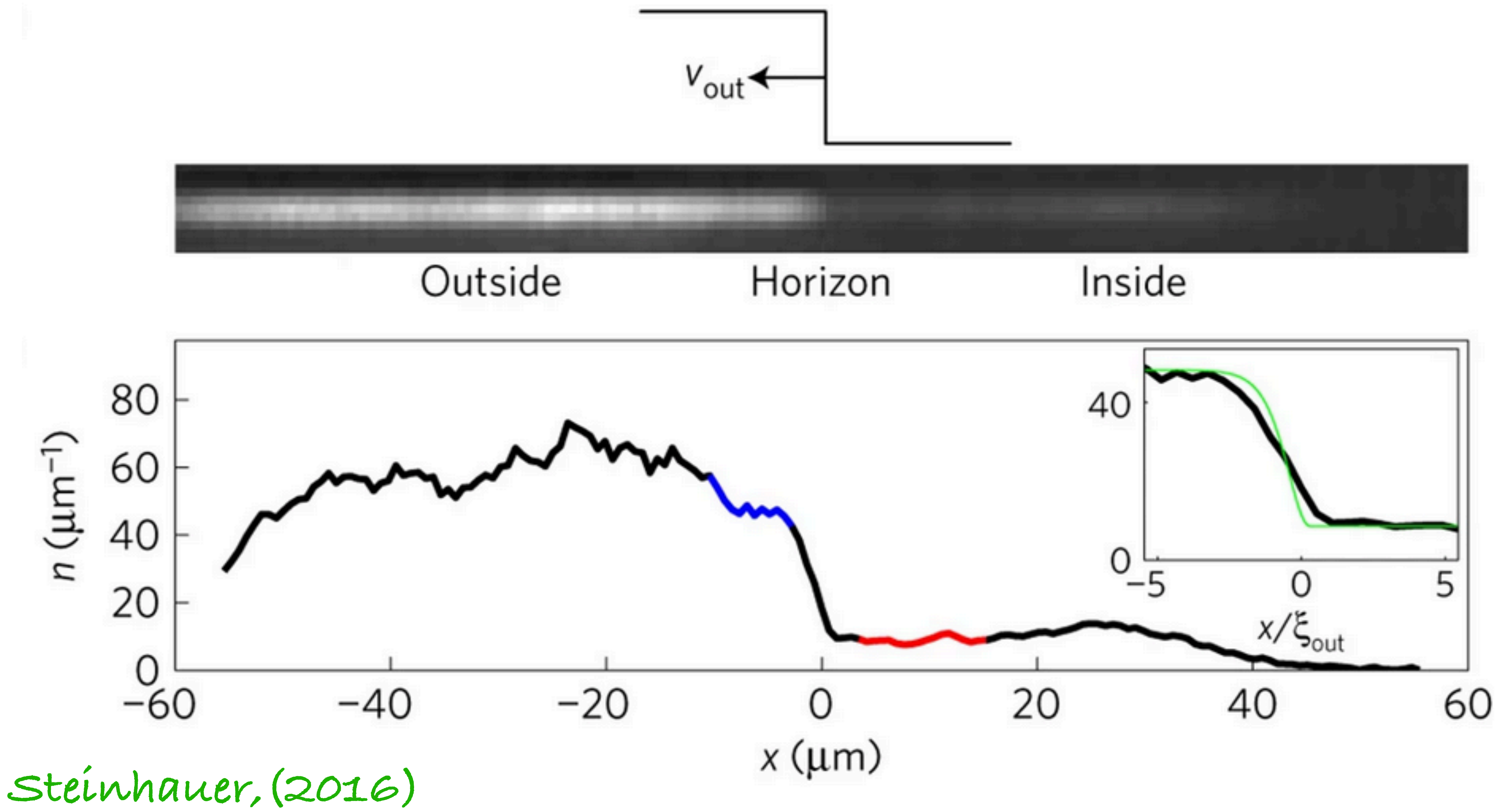

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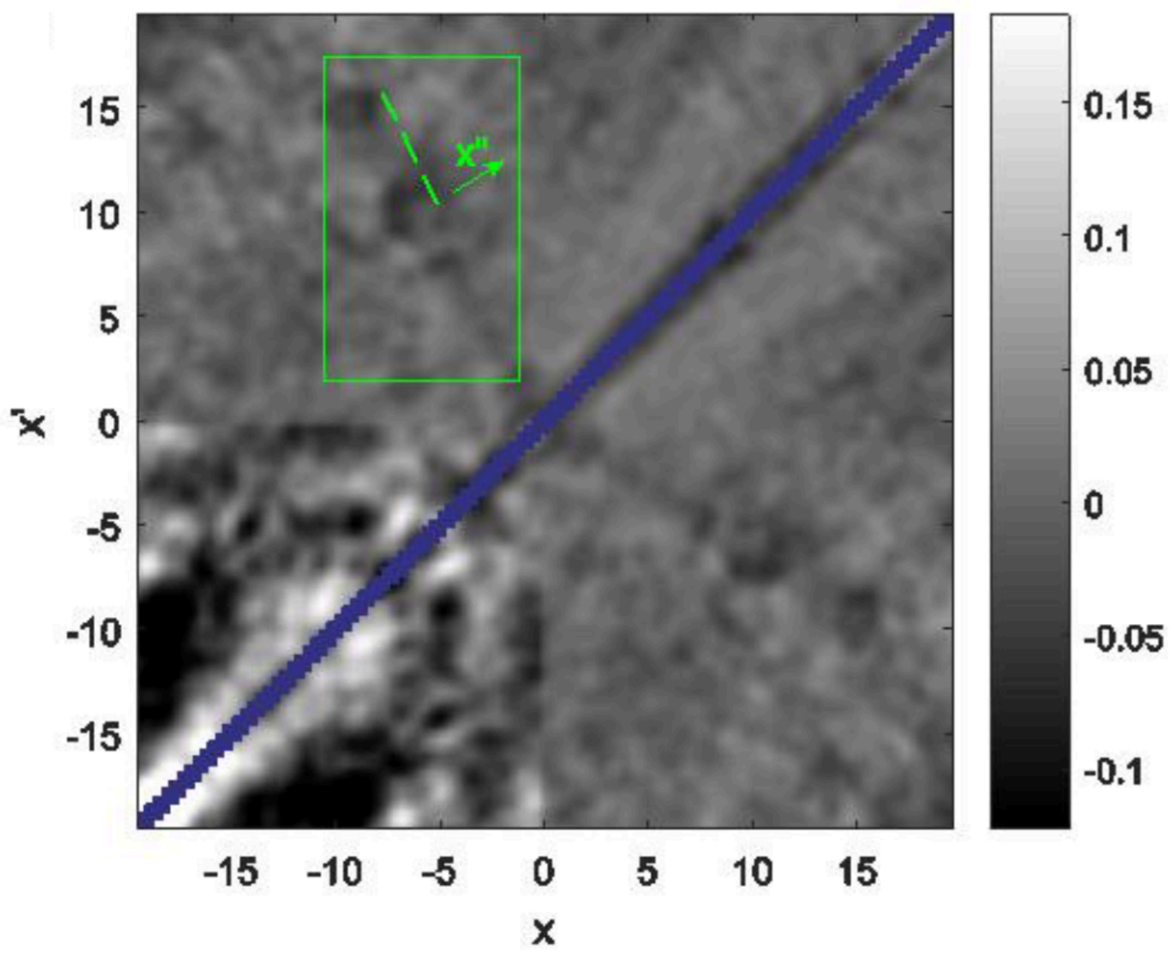
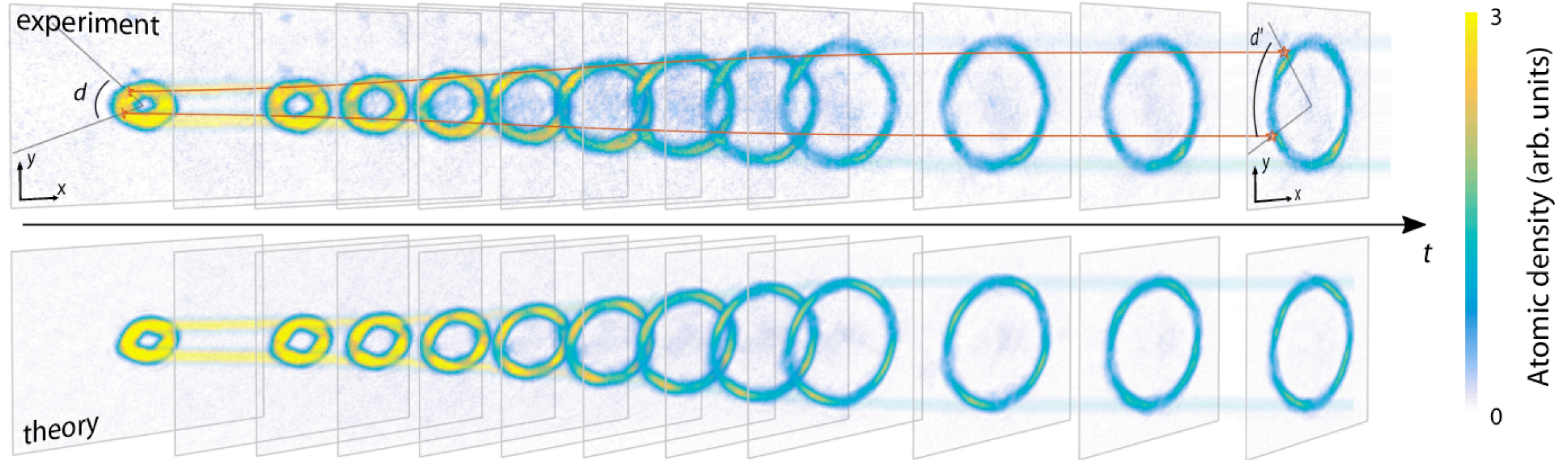


J. Steinhauer et al, (2019)

Ex: Analog de Sitter universe in Bose-Einstein condensates

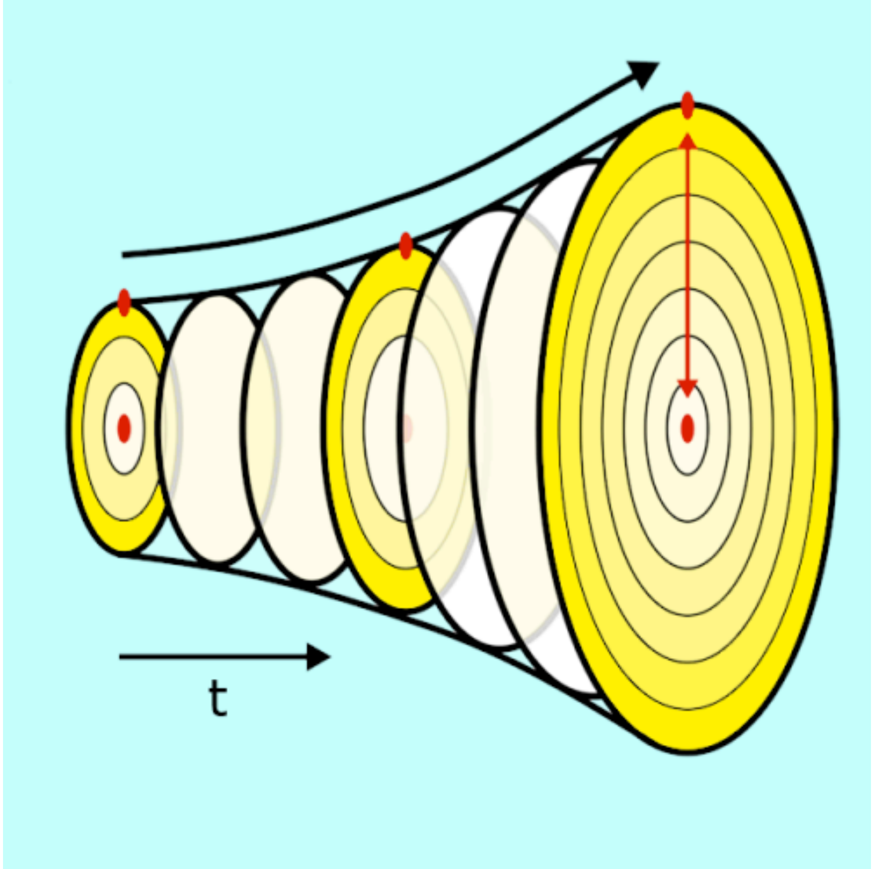
S.Weinfurtner(2004)

1+1D



S.Eckel et. Al. (2021)

2+1D



C. Viermann et al. (2022)

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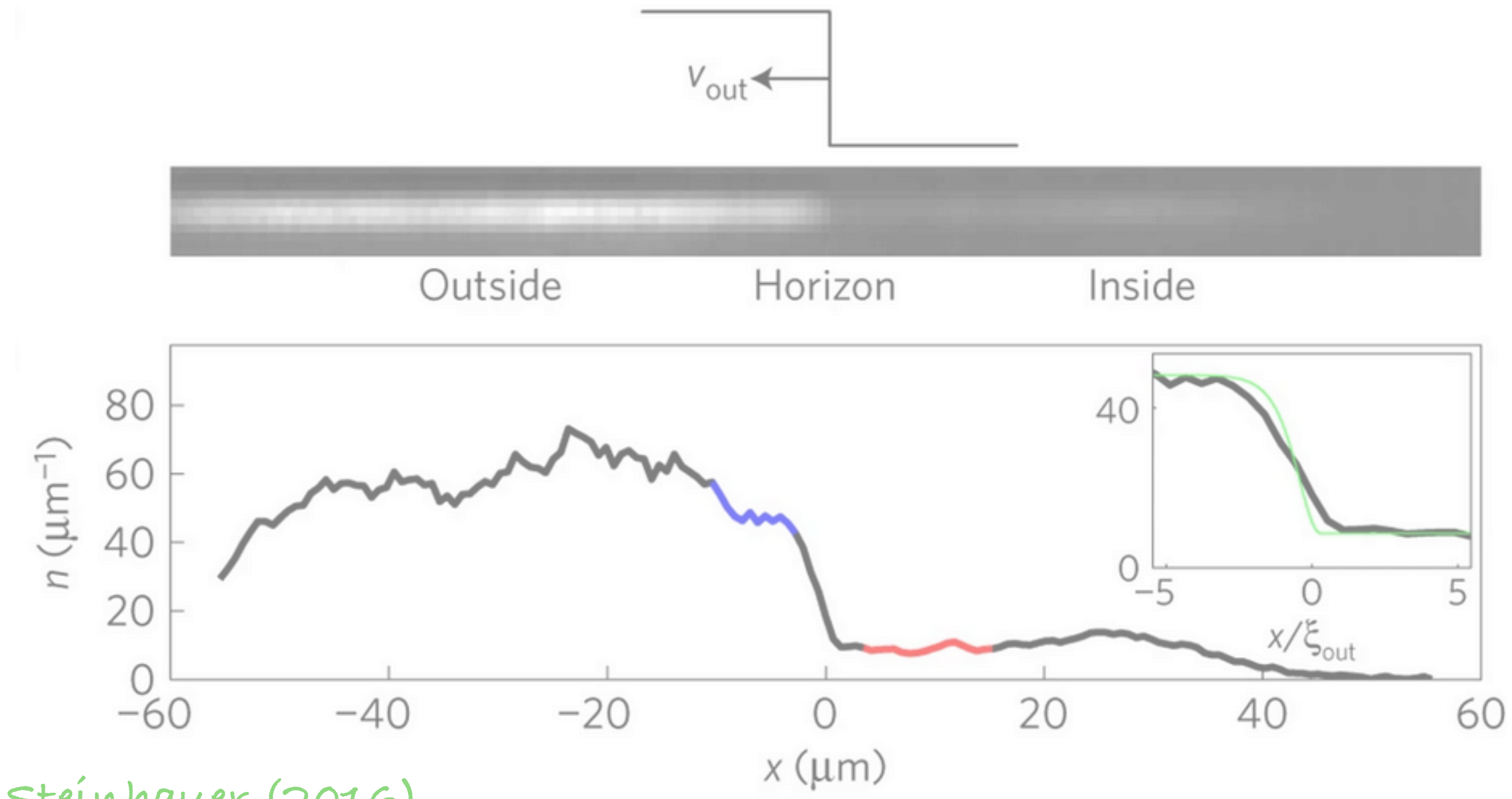

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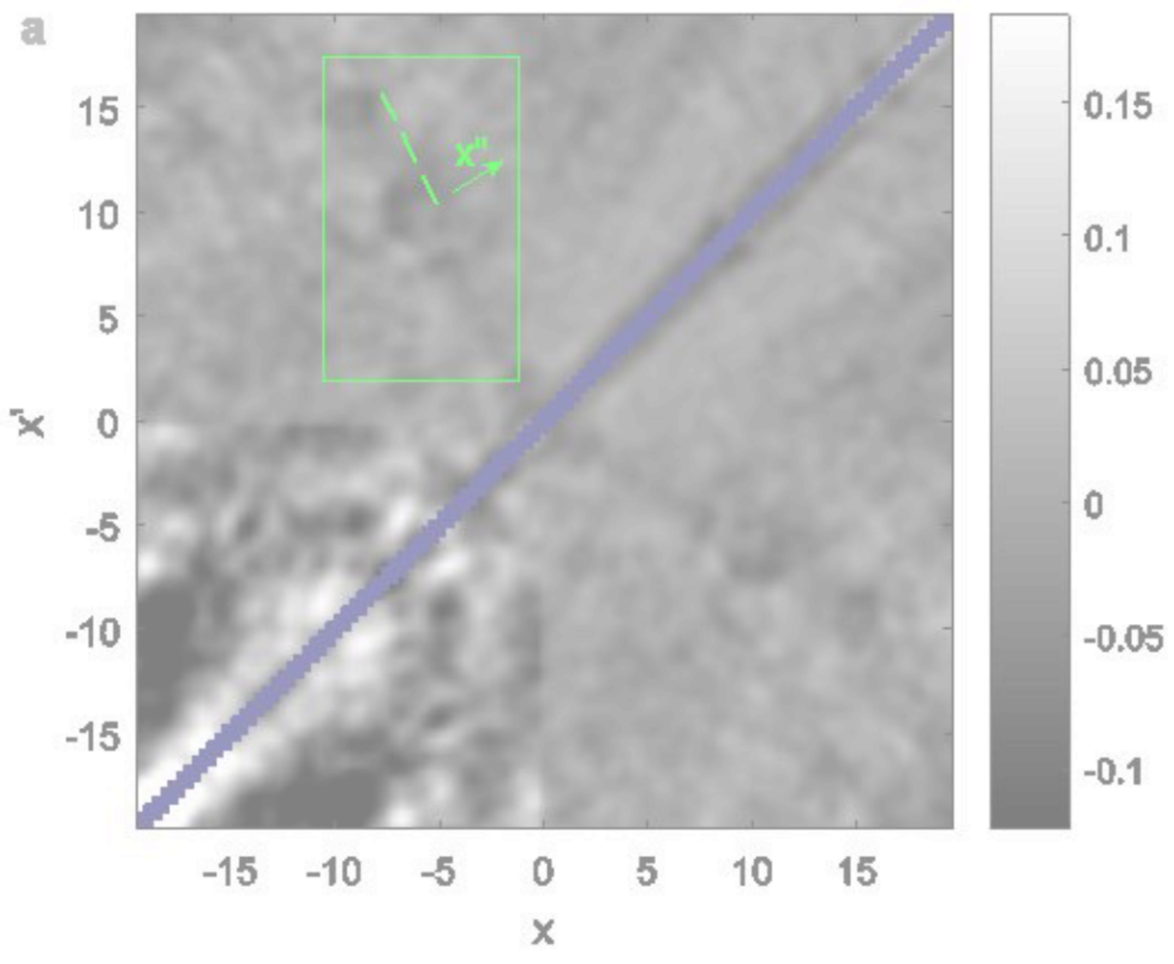

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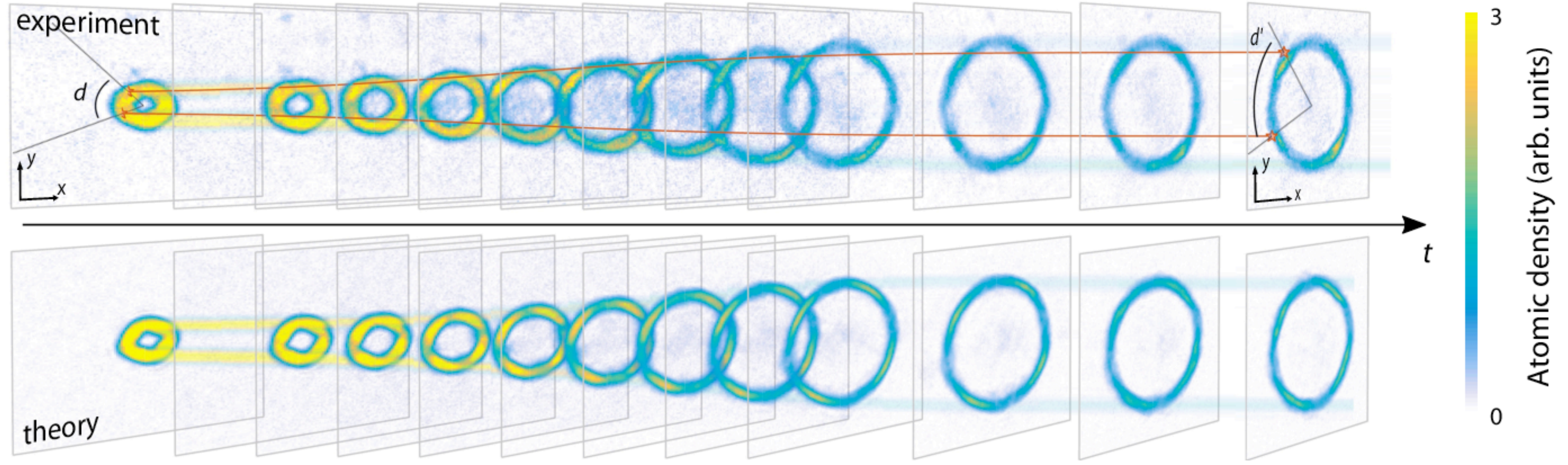

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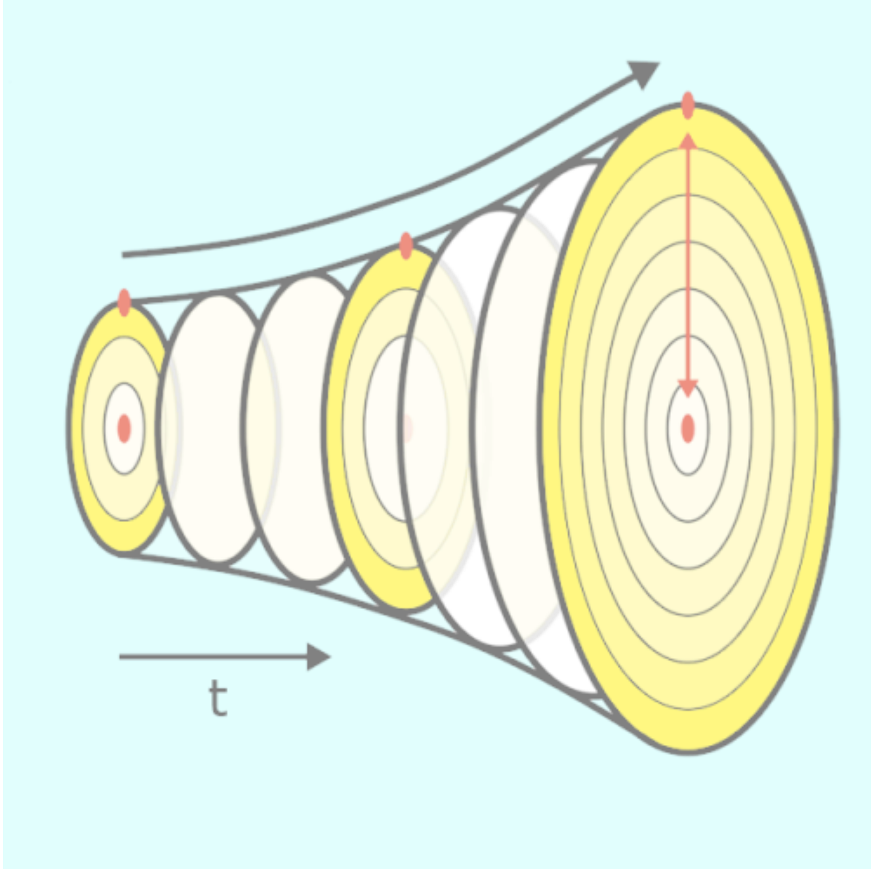

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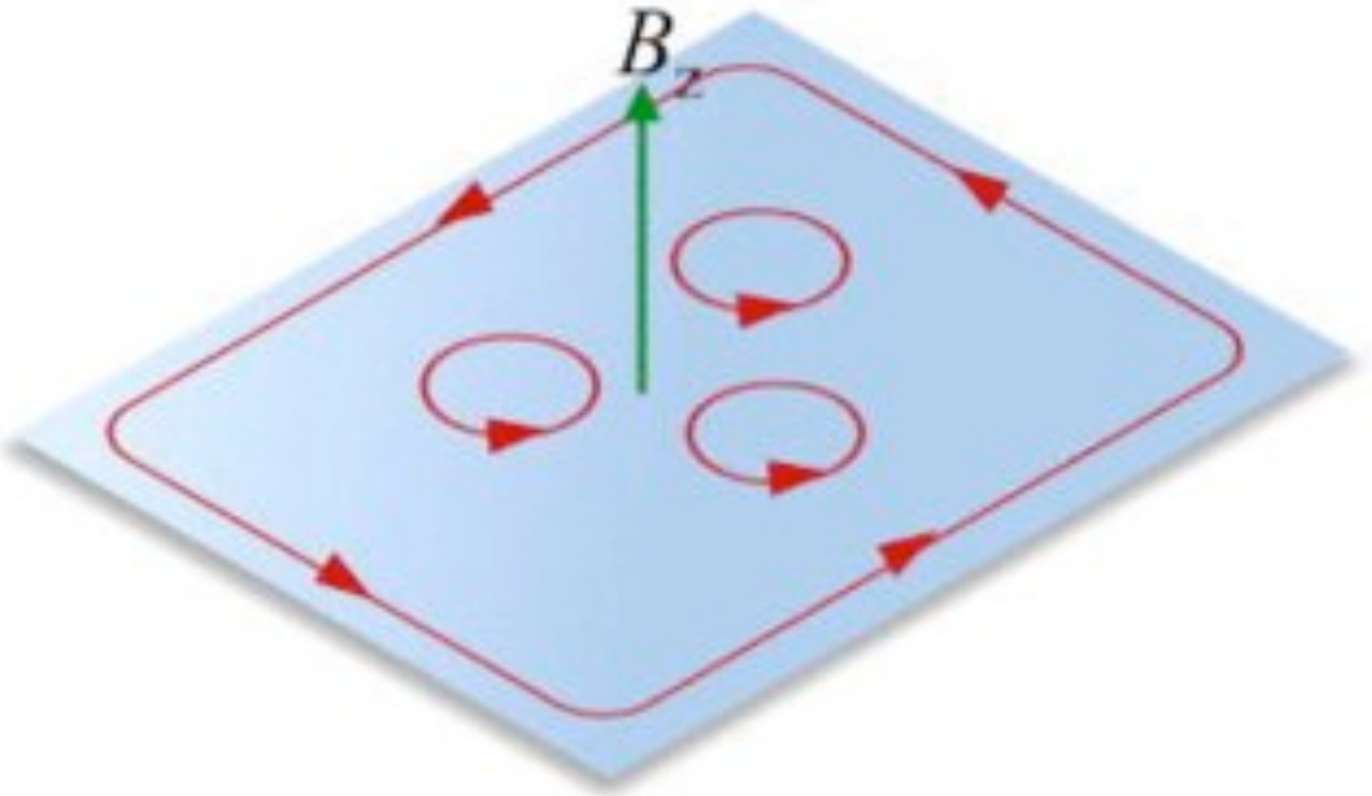
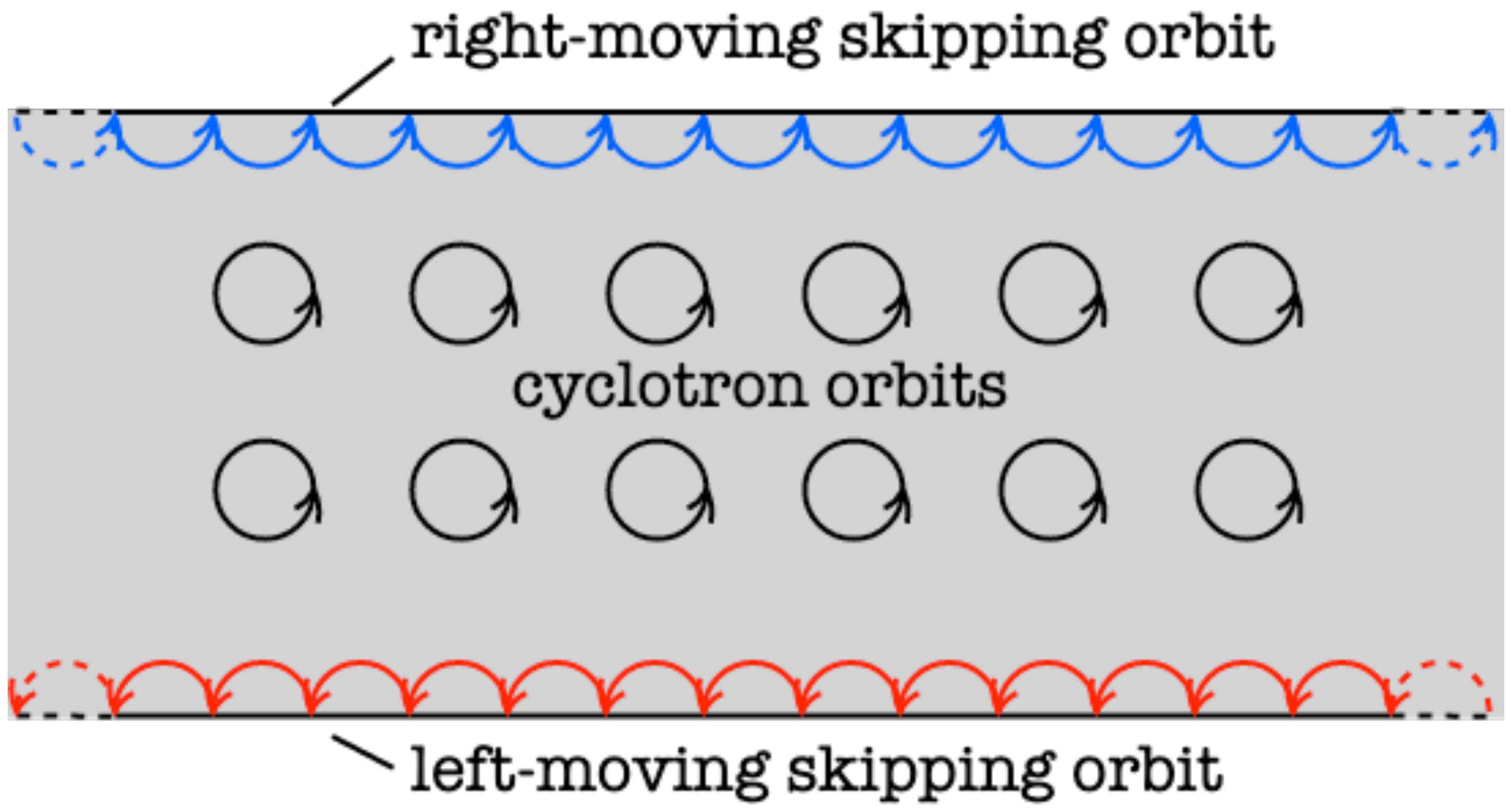
2. Relativistic physics in condensed matter: From BEC to quantum hall edges

2.a. Curved spacetimes in the laboratory:
Analog gravity in Bose-Einstein condensates

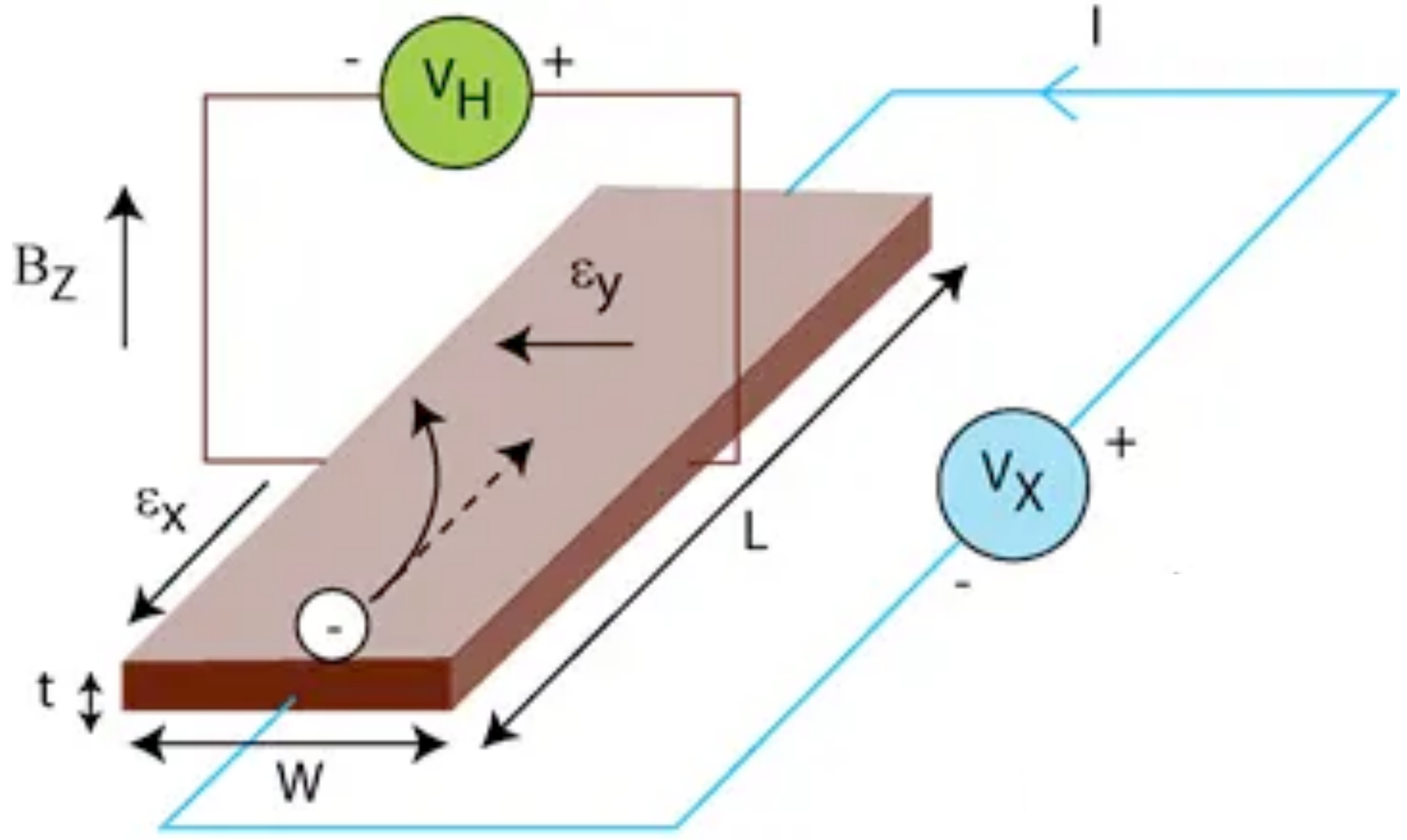
2.b. Chiral fields and curved spacetimes in quantum Hall edges

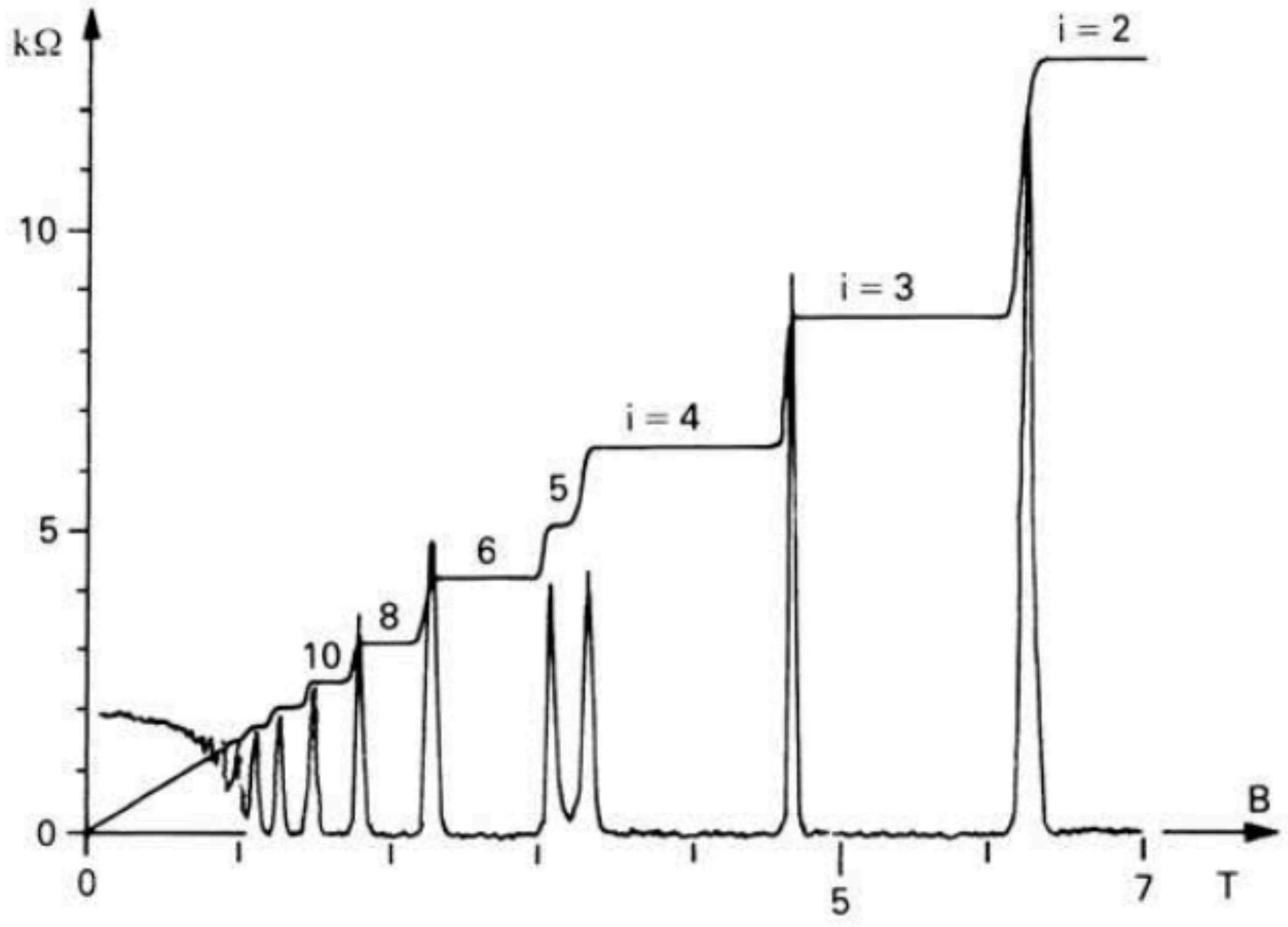




Integer quantum Hall effect: Historics

Integer quantum Hall effect





$$\rho_{xy} = \frac{e^2}{h}\frac{1}{\nu}, \nu \in \mathbb{Z}$$

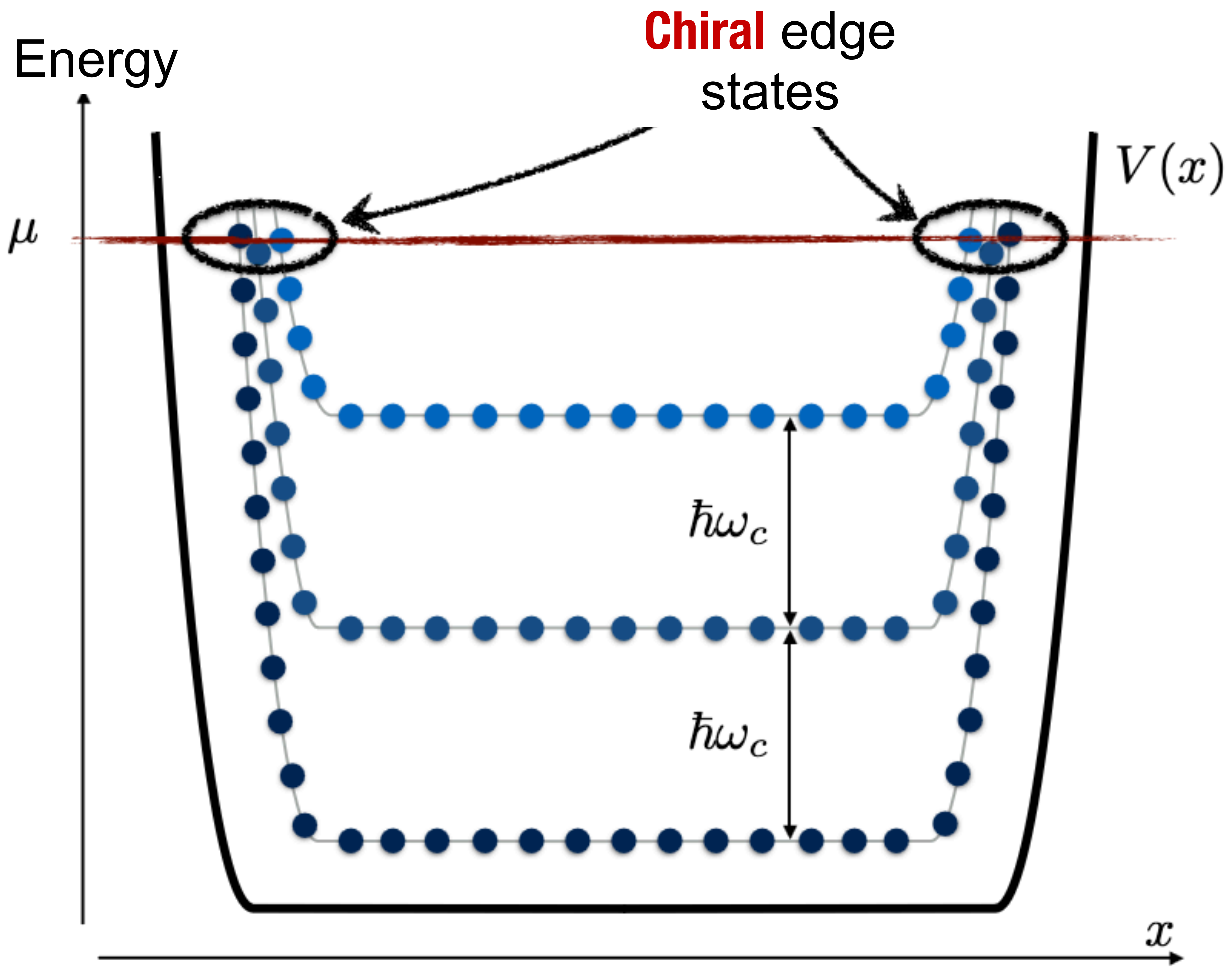
K. Von Klitzing

K. Von Klitzing (1980)
Nobel Prize 1985

Integer quantum Hall effect:

A theory on the edges

- In the presence of B, levels split into quantize non dispersive, Landau levels
- Deformation of the level by scalar potentials/confinement



Linear dispersion on the edges with

$$\vec{v} = \overrightarrow{\nabla} V \wedge \overrightarrow{B}$$

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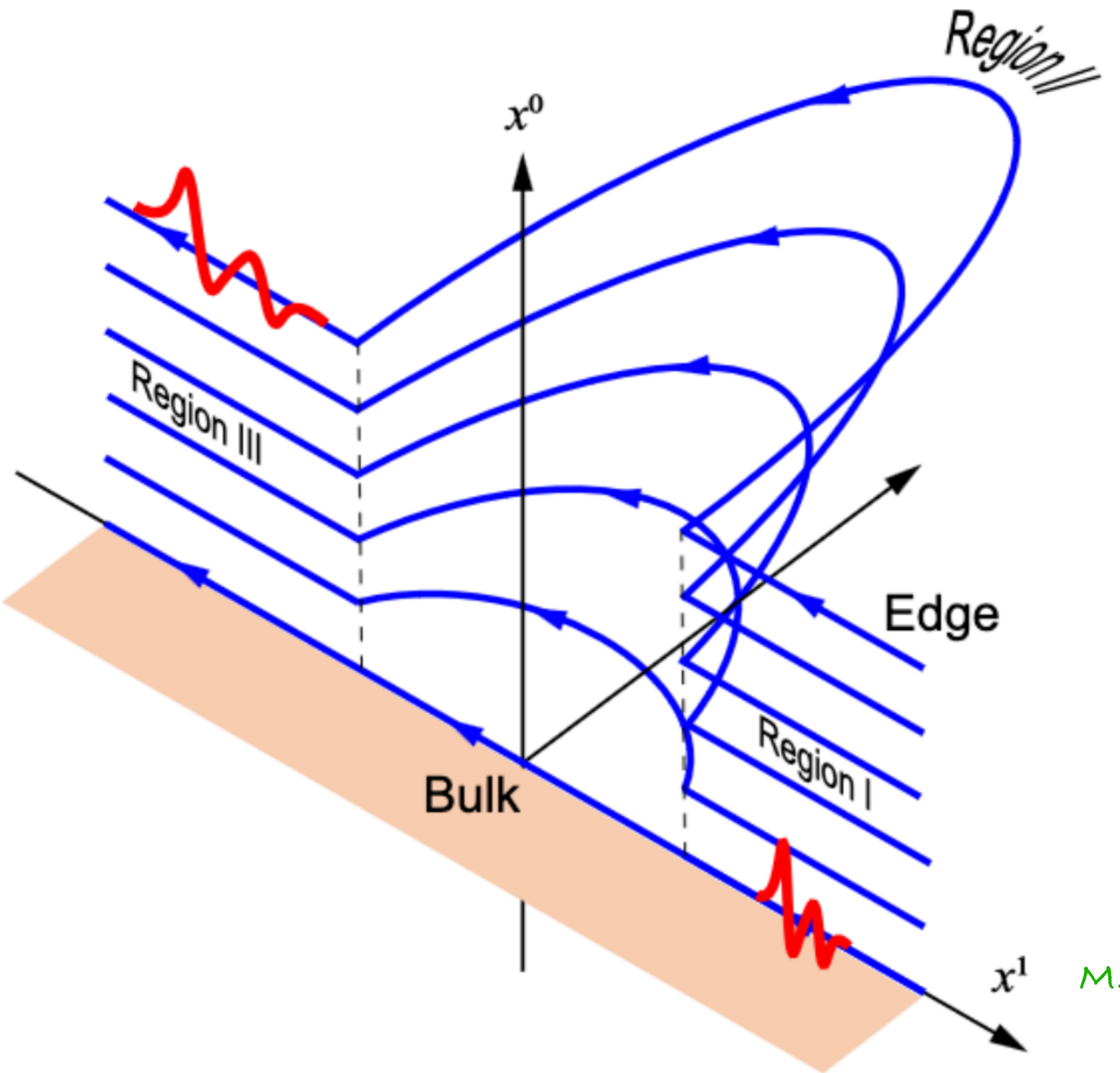

M. Hotta & Y. Nambu (2022)

Linear dispersion on the edges with

$$\vec{v} = \vec{\nabla} V \wedge \vec{B}$$

Possibility to shape space-time by modifying the potential V in space and time

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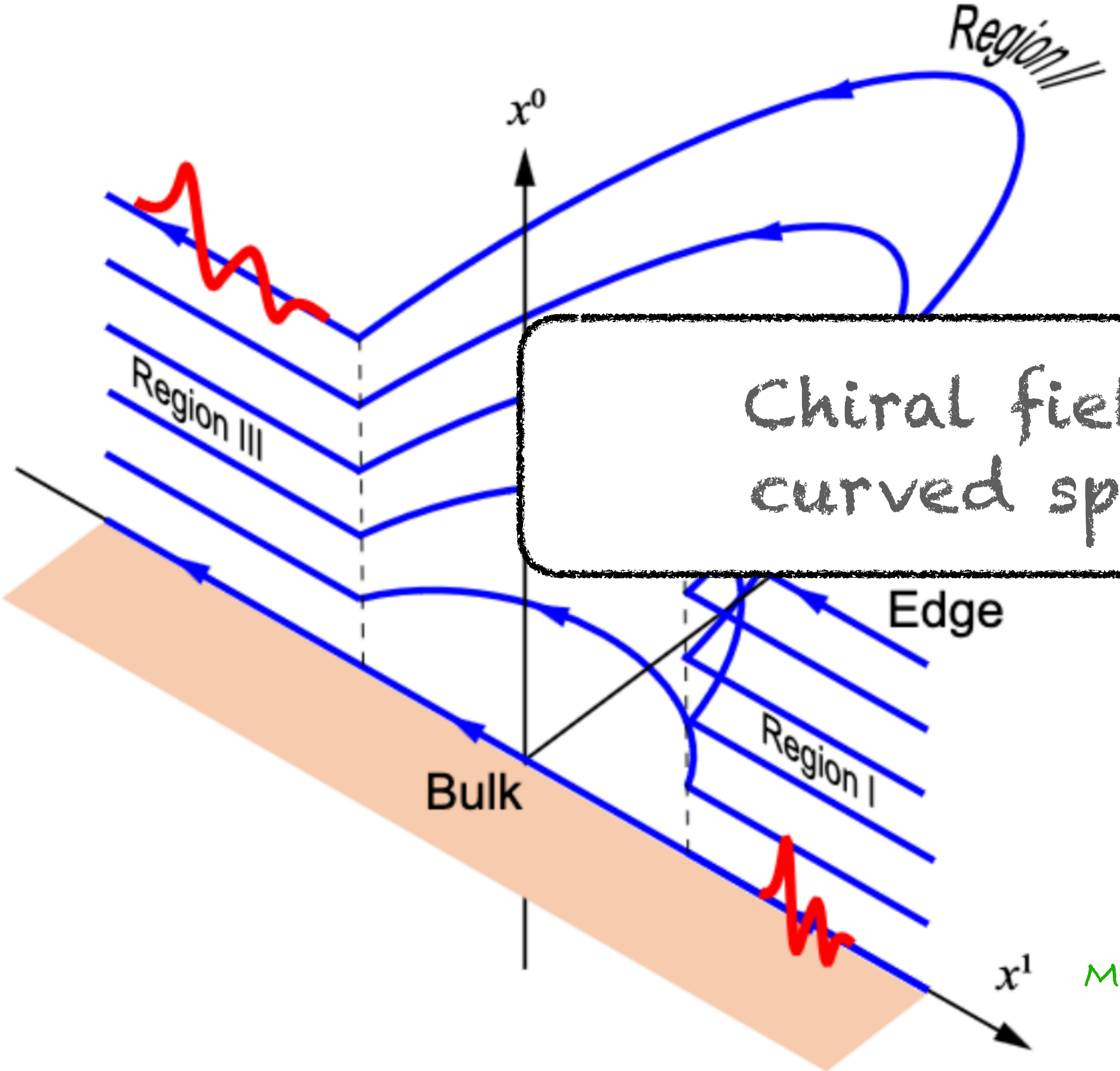


M. Hotta & Y. Nambu (2022)

Chiral fields in a curved spacetime

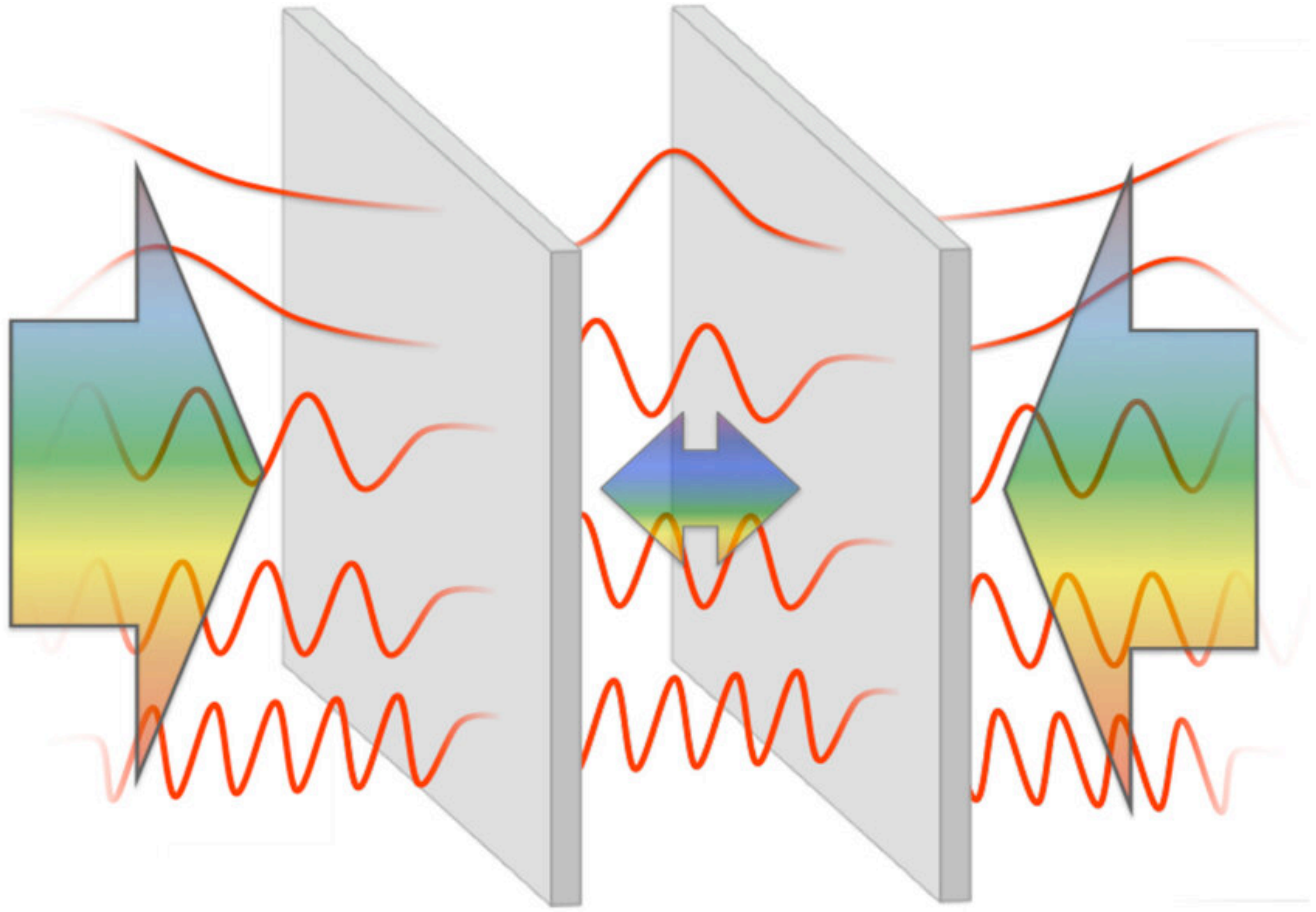
Linear dispersion on the edges with

$\vec{B}$

...ty to shape space-time by modifying the potential V in space and time

3. Gravitational anomalies and the anomalous Casimir effect

3.a. The Casimir effect, or how confinement modify vacuum properties of a system



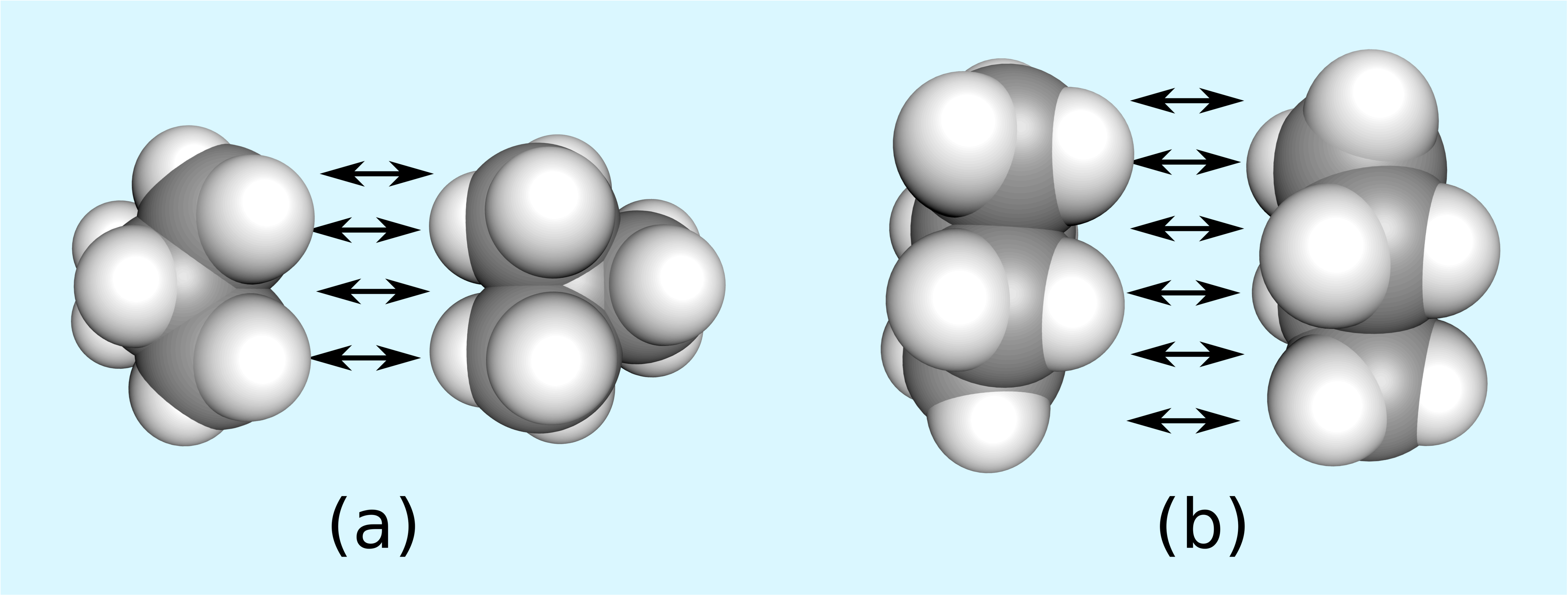
3.b. Casimir effect in expanding ring: First predictions and Curved space time analog

3.c. Another geometric effect: gravitational anomalies and and the anomalous Casimir effect

3.d. Extension to velocity modulated systems

The Casimir effect

- H. Casimir used quantum mechanics to investigate Wan der Walls forces between polarizable molecules in 1948

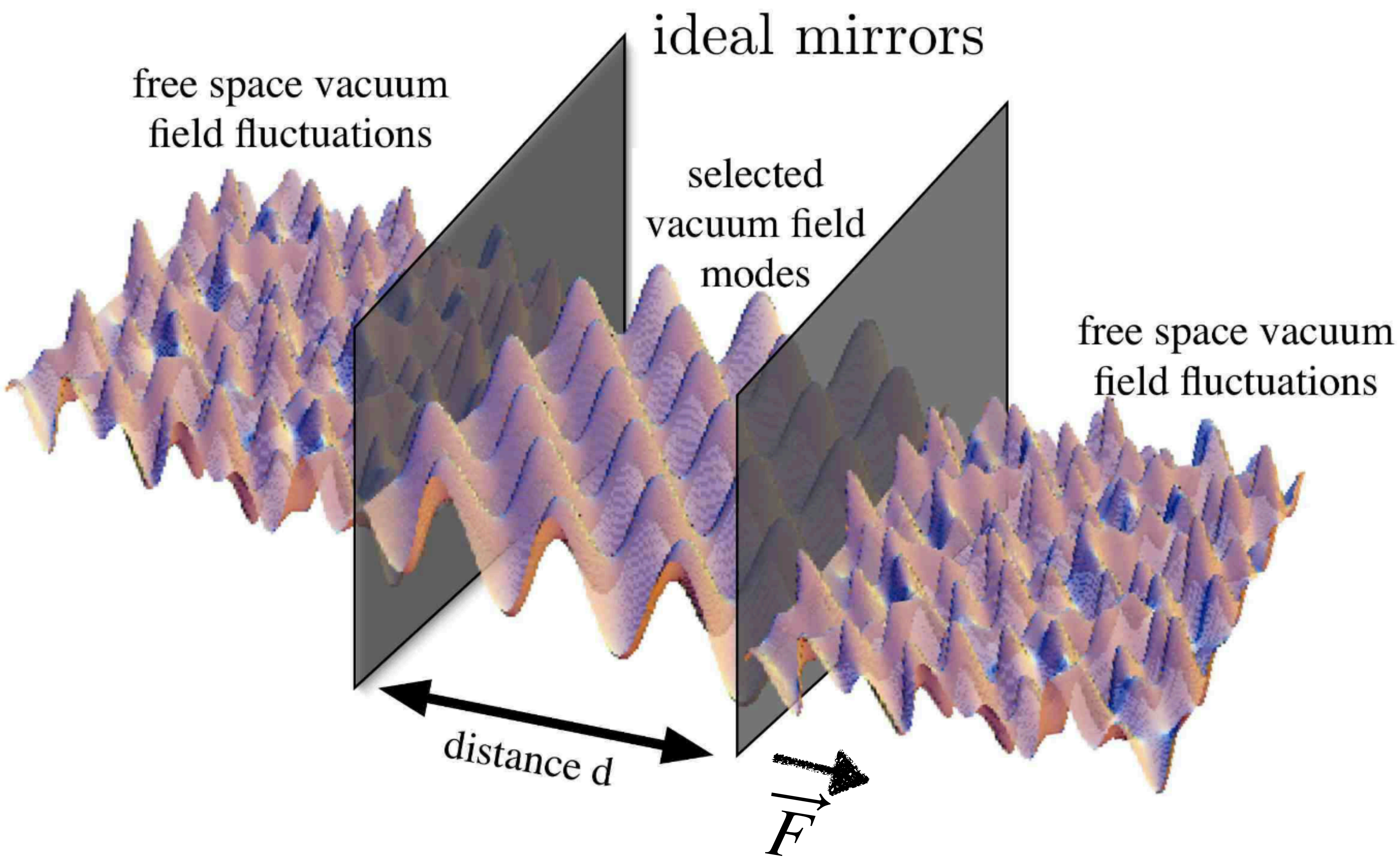


H. Casimir

The Casimir effect

- Attractive force between two uncharged metal plates when they are very close (a few nm)

- Can be interpreted as a modification of a quantum vacuum properties in the presence of geometrical constraints (here confinement)



H. Casimir

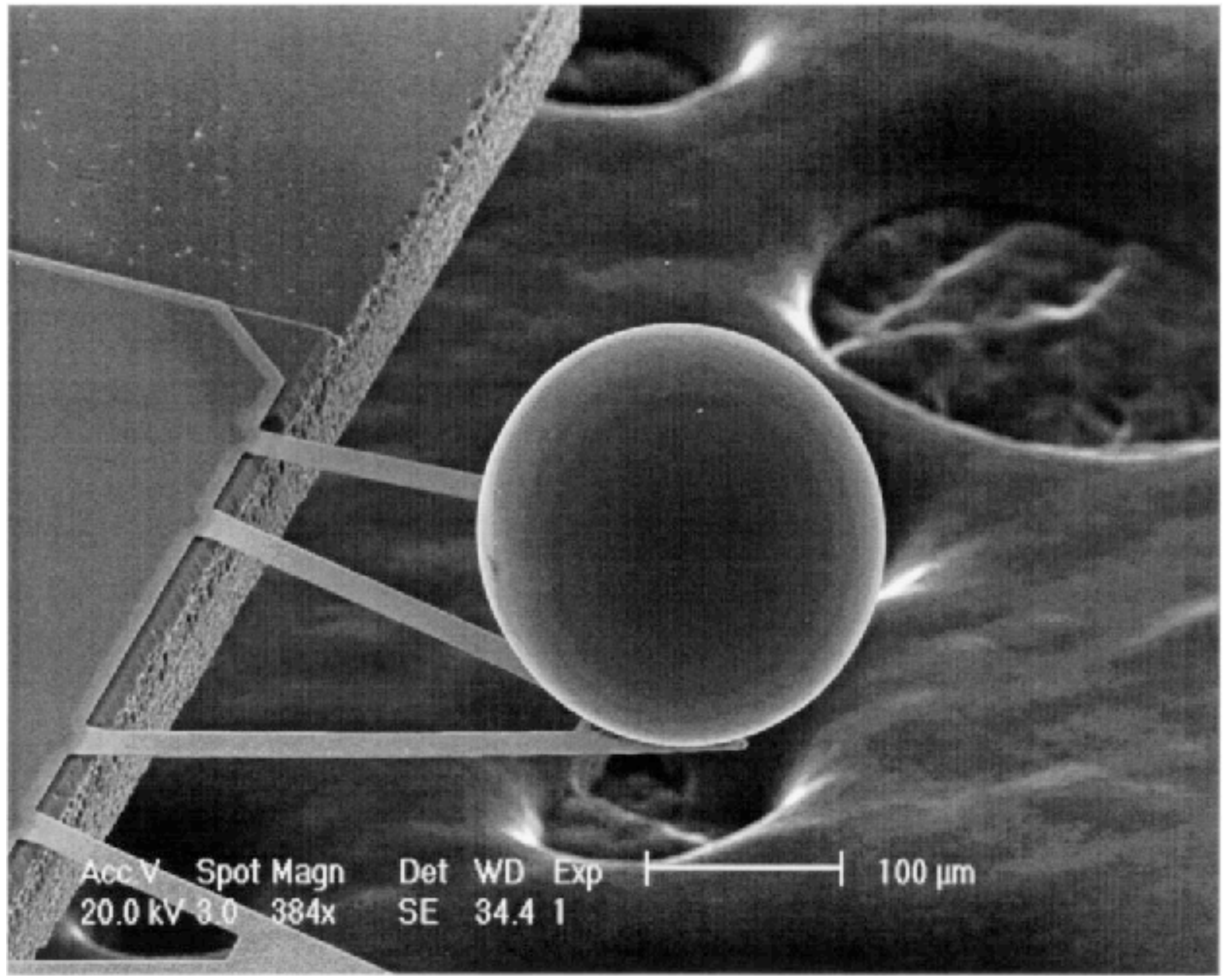
$$F = -\frac{\hbar c \pi^2 A}{240 d^4}$$

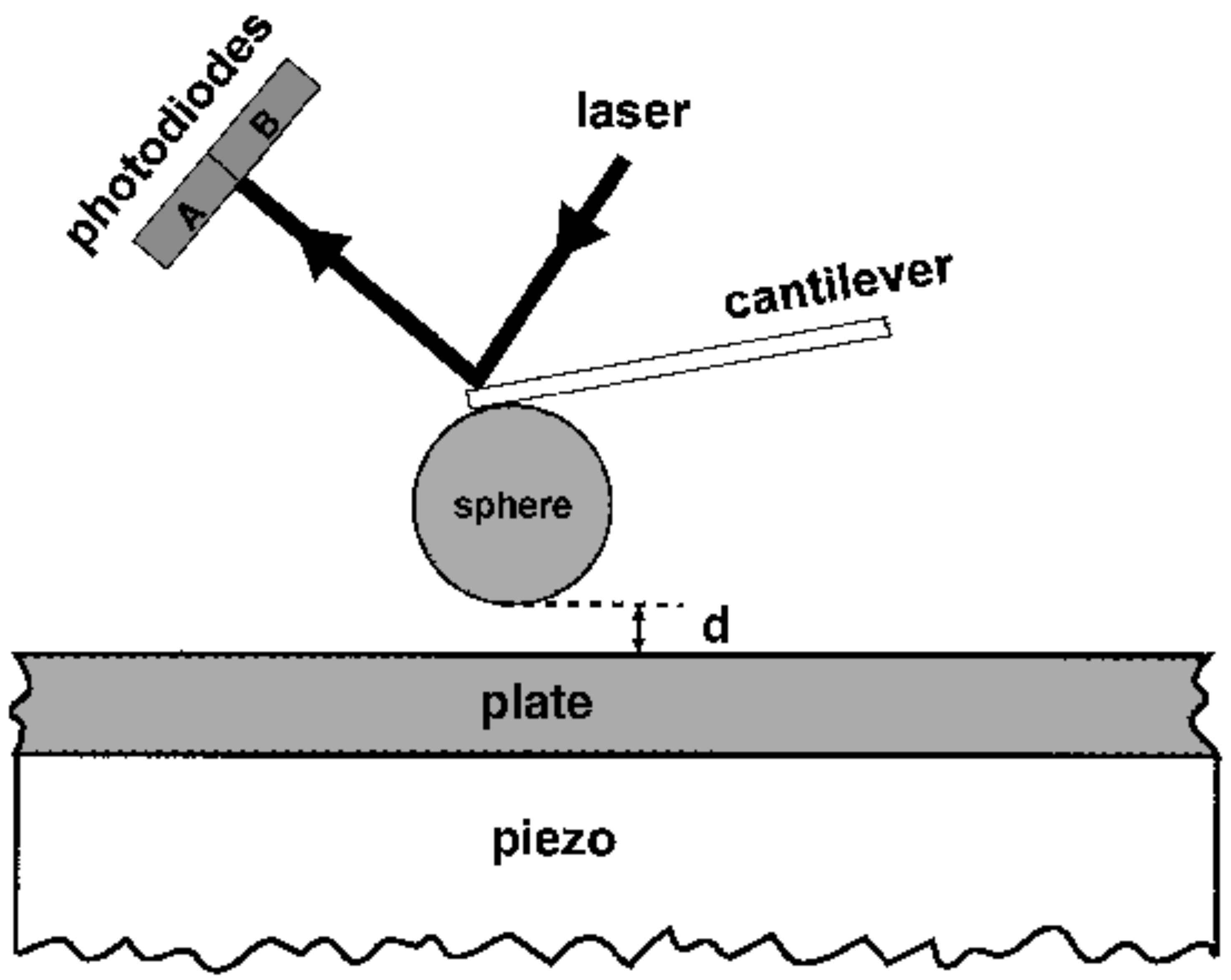
Casimir (1948)

The Casimir effect: Experimental verifications?

- Hard to test experimentally within H.Casimir setup

- First experimental evidence by S.K.Lamoureux at Yale in 1997, and reproduced by Mohideen and Roy in 1999, using atomic force microscopy





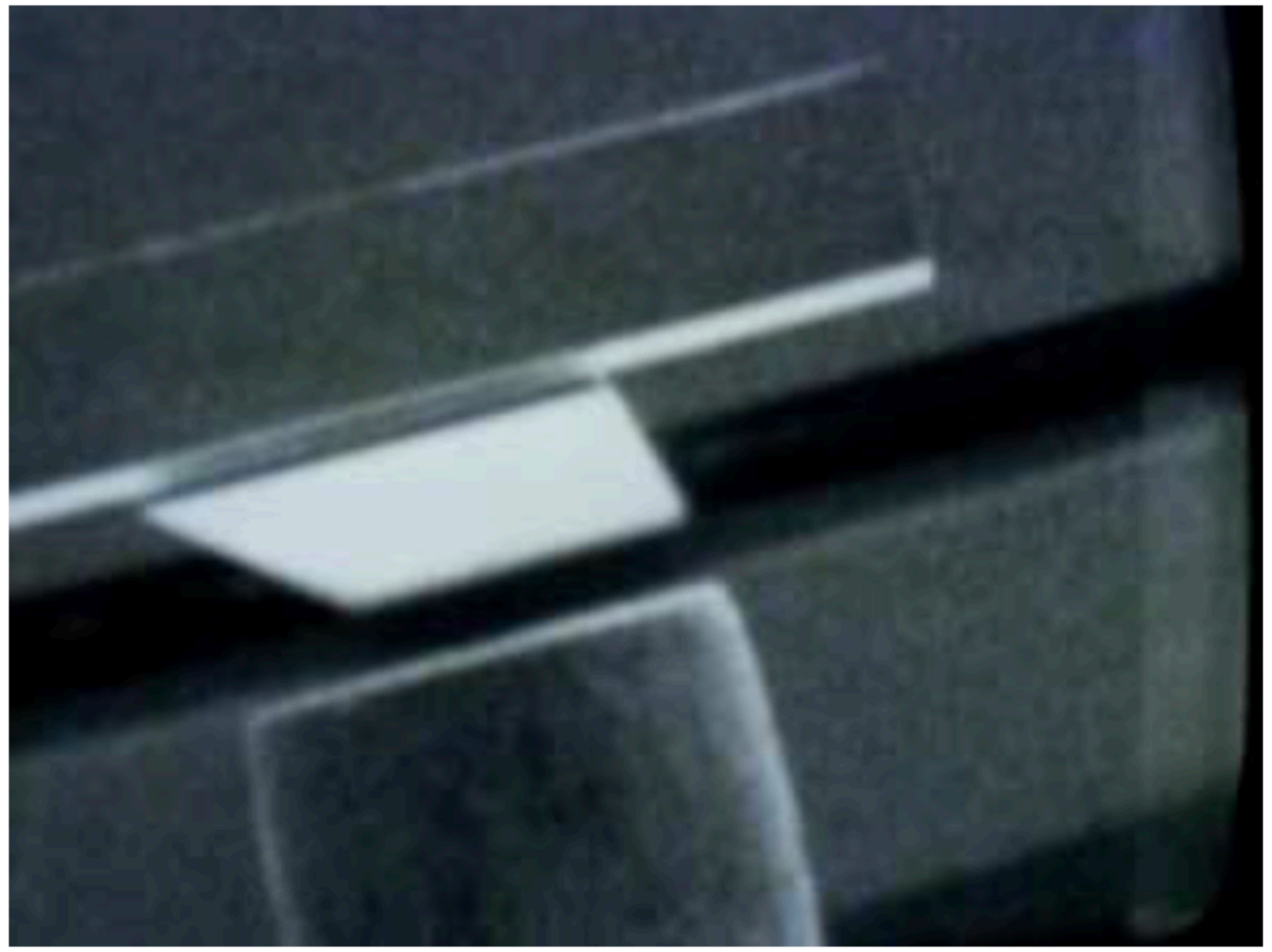
Mohideen & Roy (1999)

The Casimir effect: Experimental verifications?

- Hard to test experimentally within H.Casimir setup

- First experimental evidence by S.K.Lamoureux at Yale in 1997, and reproduced by Mohideen and Roy in 1999, using atomic force microscopy

- Finally verified between metallic plates in 2002 by Bressi et al



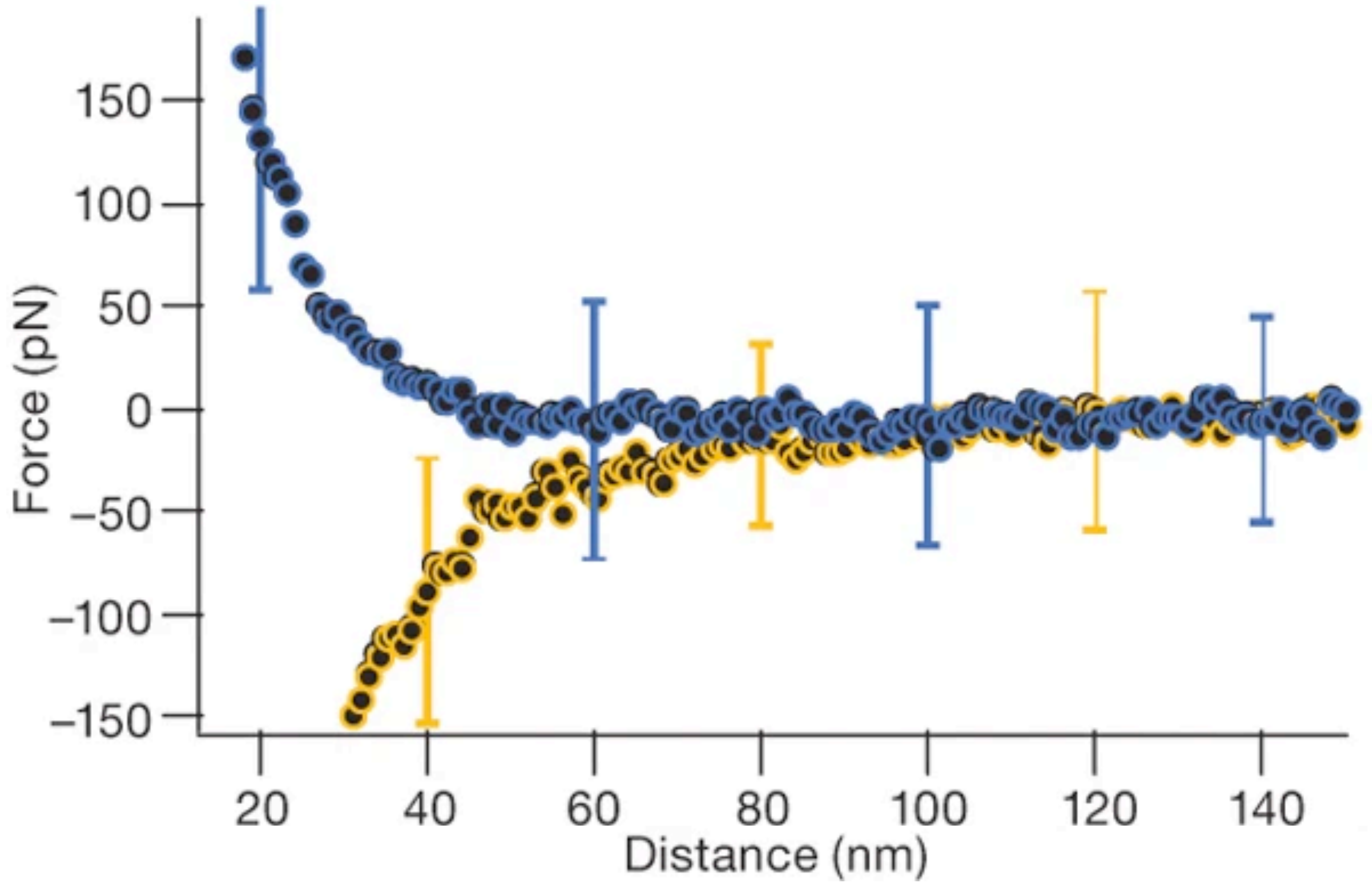
Bressi et al (2002)

The Casimir effect: Modern experiments

Other type of materials

Effect of dielectric response, repulsive Casimir force

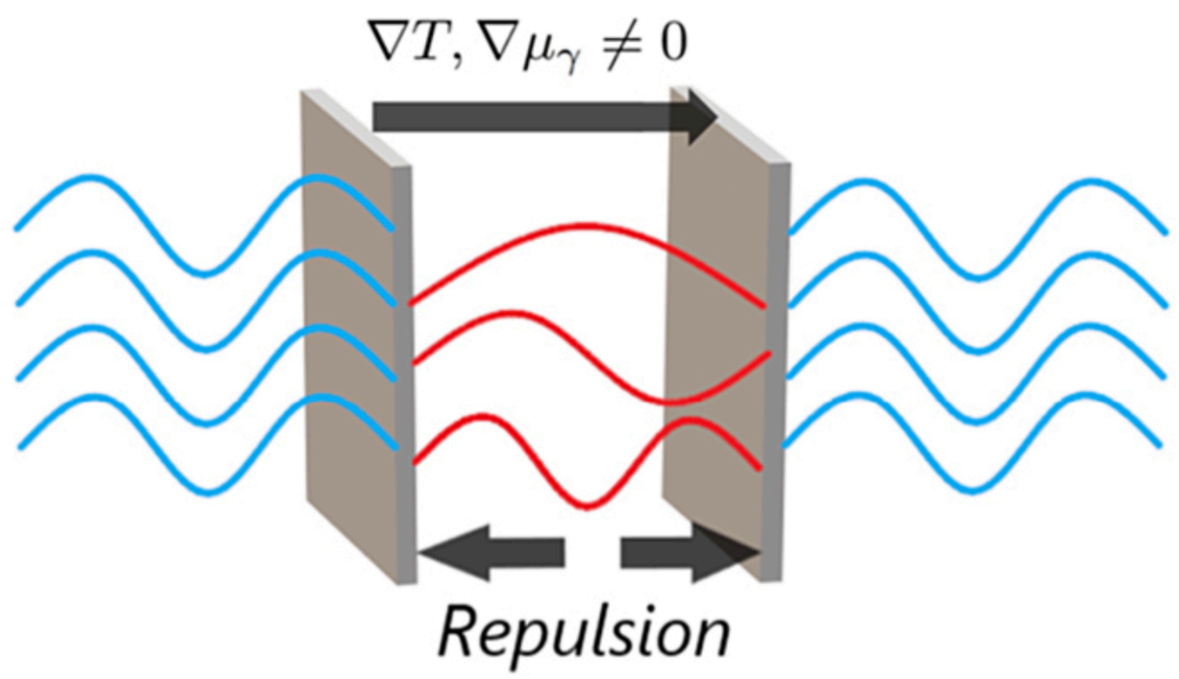
J.Munday & F. Capasso (2009)



Systems beyond static equilibrium

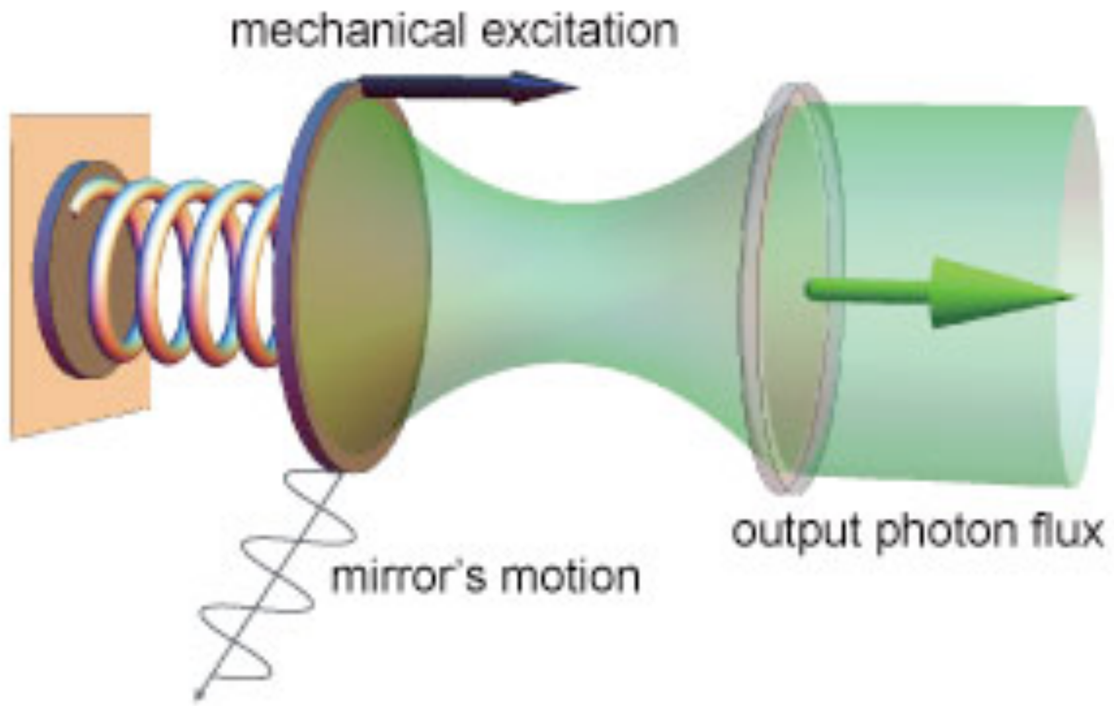
Effect of finite chemical potential and temperature gradients

C. Henkel et al (2002)
K. Chen & S. Fan (2016)



Generation of photon from vacuum: **Dynamical Casimir effect**

G. Moore (1970)



The Casimir effect: effect of dimensions

Generalizing Casimir arguments in a d+1 dimensional cavity of size L

$$\varepsilon = d\,.\,P \propto L^{-d-1}$$

J. Ambjørn & S. Wolfram (1983)

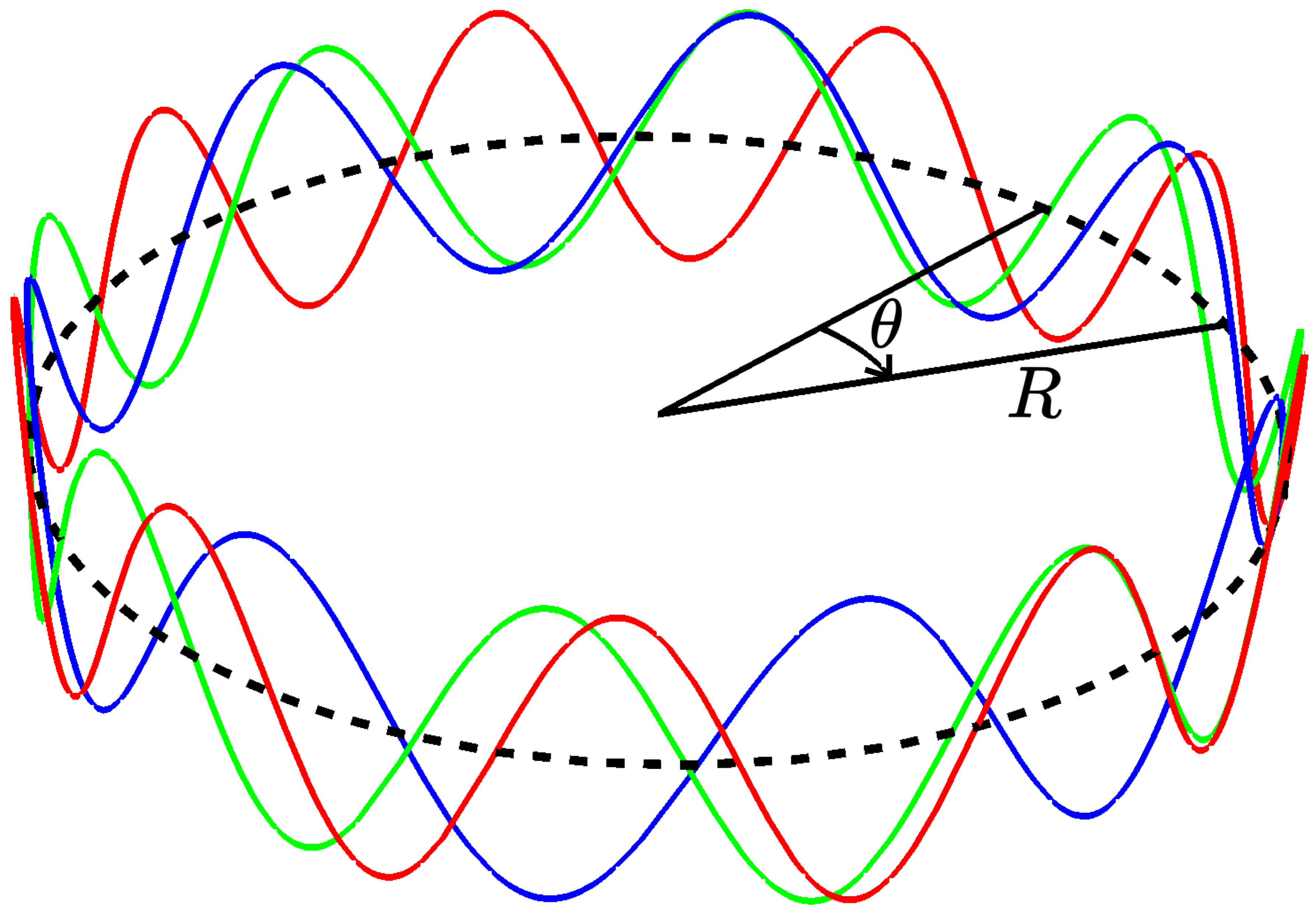
The Casimir effect: effect of dimensions

Generalizing Casimir arguments in a d+1 dimensional cavity of size L

$$\varepsilon = d\,.\,P \propto L^{-d-1}$$

J. Ambjørn & S. Wolfram (1983)

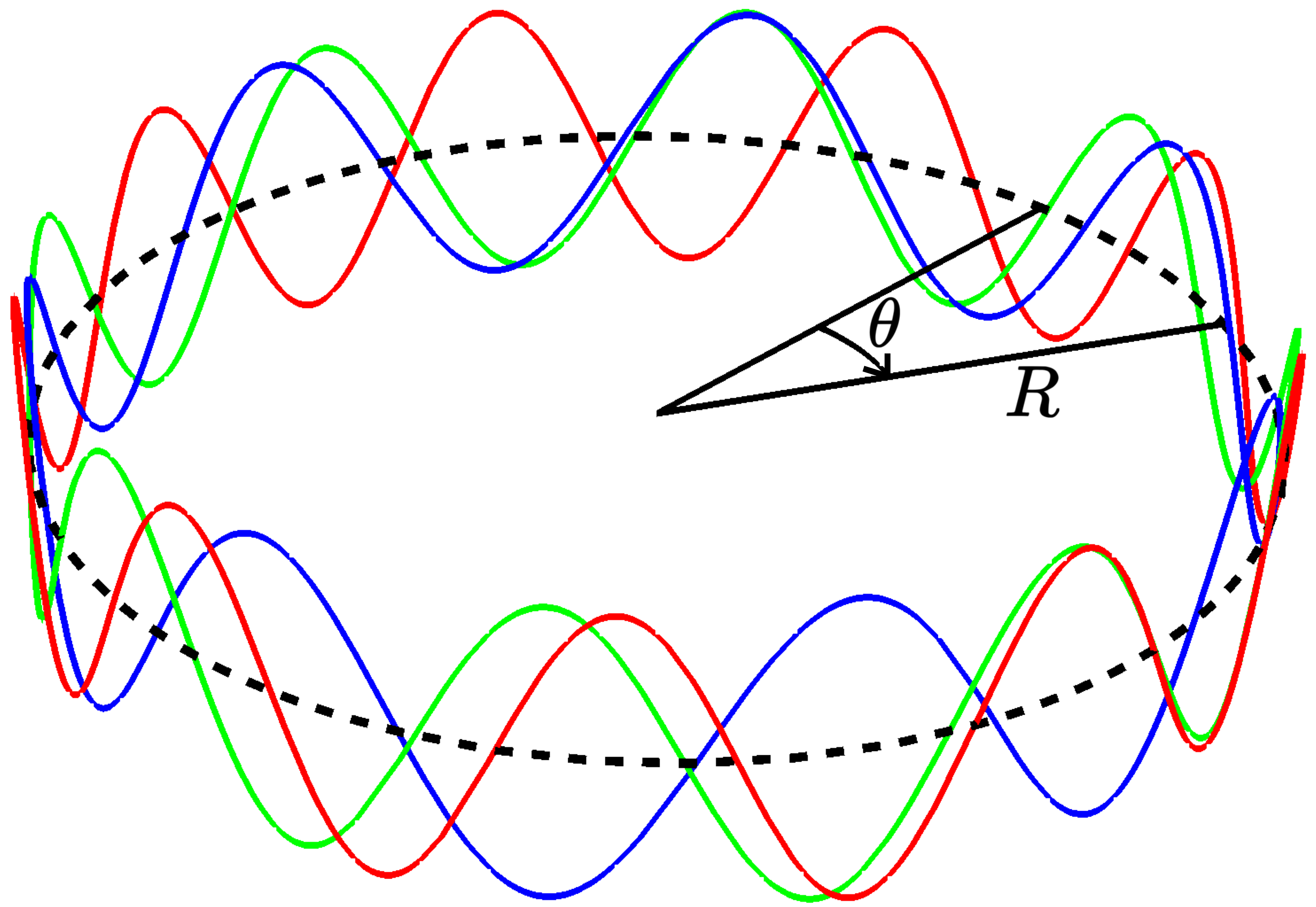
In particular, for a 1+1 dimensional ring



On a d=1 ring: $\varepsilon = P = -\underbrace{\frac{1}{2}\frac{\hbar c_s}{24\pi R^2}}_{\text{Casimir energy density: } \varepsilon_{\mathscr{C}}}$

The Casimir effect: effect of dimensions

In particular, for a 1+1 dimensional ring



On a d=1 ring: $\varepsilon = P = -\underbrace{\frac{1}{2}\frac{\hbar c_s}{24\pi R^2}}$

Casimir energy density: $\varepsilon_{\mathscr{C}}$

Generalization to interacting 1+1 dimensional theories

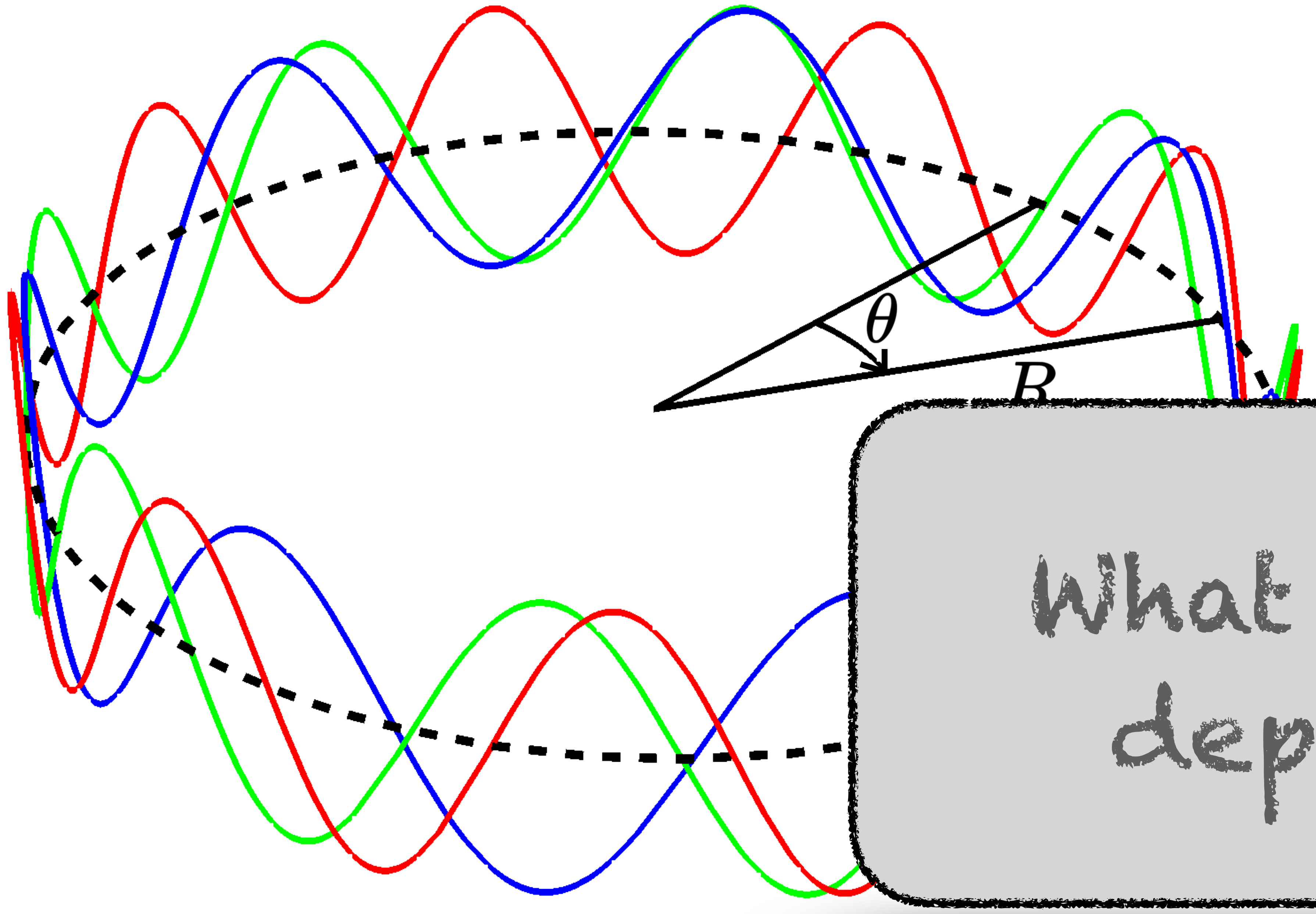
$$\varepsilon = P = -\mathscr{C}_W \frac{\hbar c_s}{48\pi R^2}$$

$$J_\varepsilon = -\mathscr{C}_g \frac{\hbar c_s}{48\pi R^2}$$

$\frac{\mathscr{C}_+ + \mathscr{C}_-}{2}$ Central charges $\frac{\mathscr{C}_+ - \mathscr{C}_-}{2}$

The Casimir effect: effect of dimensions

In particular, for a 1+1 dimensional ring



$$= P = -\underbrace{\frac{1}{2}\frac{\hbar c_s}{24\pi R^2}}$$

Casimir energy density: $\varepsilon_{\mathscr{C}}$

What If the radius R depends on time?

Generalization to interacting 1+1 dimensional theories

$$\varepsilon = P = -\mathscr{C}_W \frac{\hbar c_s}{48\pi R^2}$$

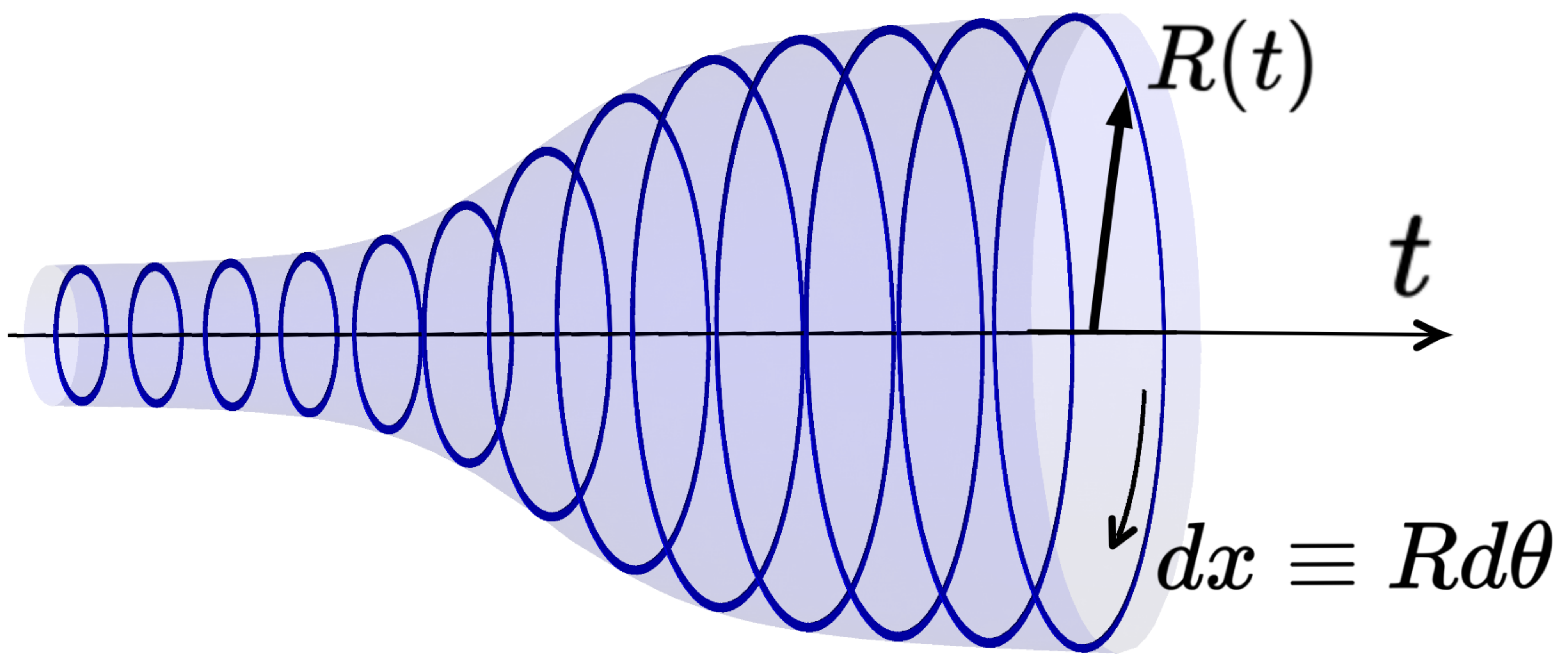
$$J_\varepsilon = -\mathscr{C}_g \frac{\hbar c_s}{48\pi R^2}$$

$\frac{\mathscr{C}_+ + \mathscr{C}_-}{2}$ **Central charges** $\frac{\mathscr{C}_+ - \mathscr{C}_-}{2}$

3. Gravitational anomalies and the anomalous Casimir effect

3.a. The Casimir effect, or how confinement modify vacuum properties of a system

3.b. Casimir effect in expanding ring: First predictions and Curved space time analog

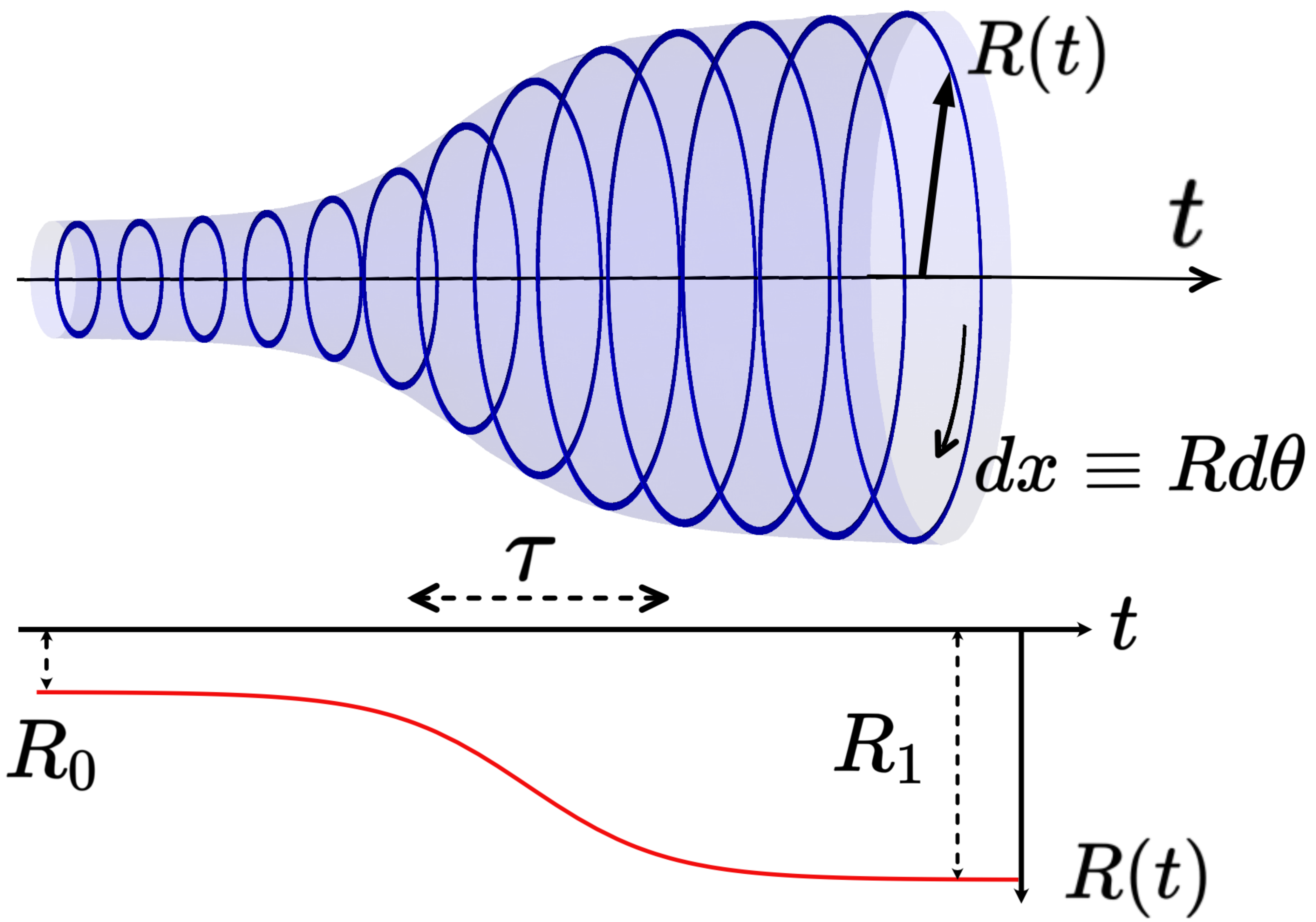


3.c. Another geometric effect: gravitational anomalies and and the anomalous Casimir effect

3.d. Extension to velocity modulated systems

1+1D Casimir effect in expanding cavities

What if the radius R depends on time?



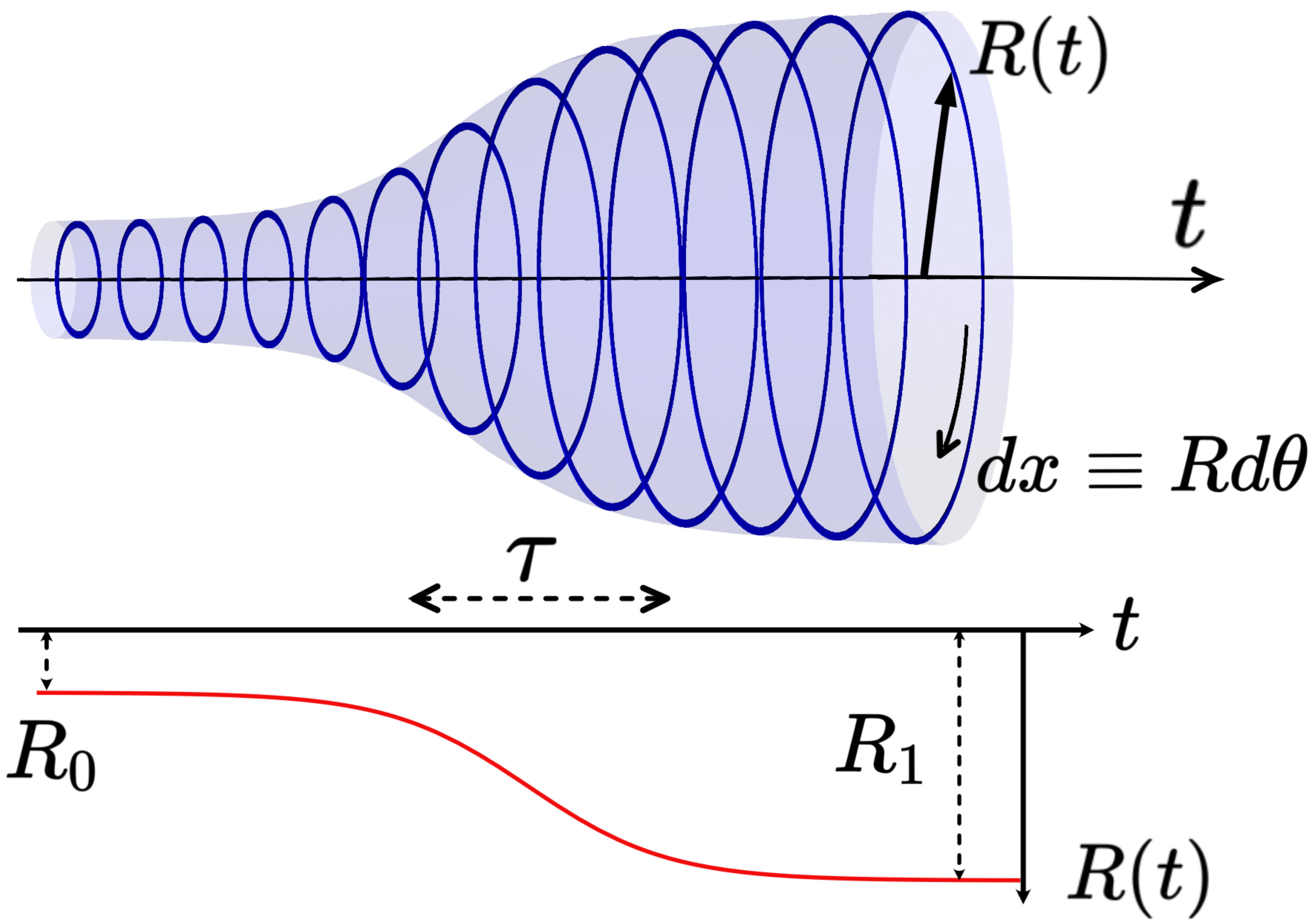
Instantaneous Casimir energy density: $\varepsilon_{\mathscr{C}}(t)$

$$\varepsilon_{\mathscr{C}}(t) = \frac{\hbar c_s}{24\pi R(t)^2}$$

$$\Rightarrow \varepsilon = p = -\frac{\mathscr{C}_w}{2}\varepsilon_{\mathscr{C}}(t)$$

1+1D Casimir effect in expanding cavities

What if the radius R depends on time?



Instantaneous Casimir energy density: $\varepsilon_{\mathscr{C}}(t)$

ε

BUT

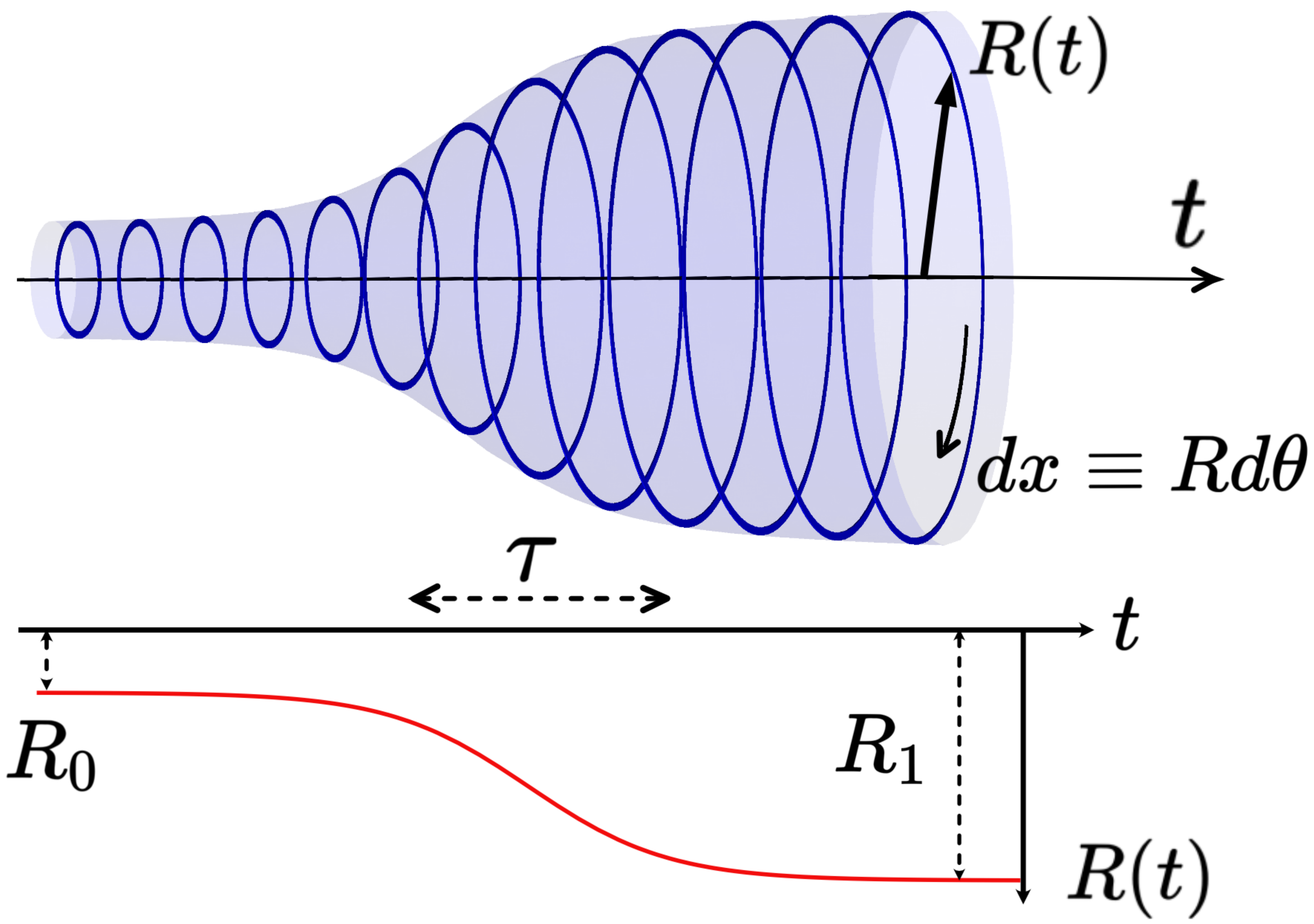
New length scale

$$\frac{1}{c_s}\frac{\partial_t R}{R}$$

$$\Rightarrow \varepsilon = p = -\frac{\mathscr{C}_w}{2}\varepsilon_{\mathscr{C}}(t)$$

1+1D Casimir effect in expanding cavities

What if the radius R depends on time?

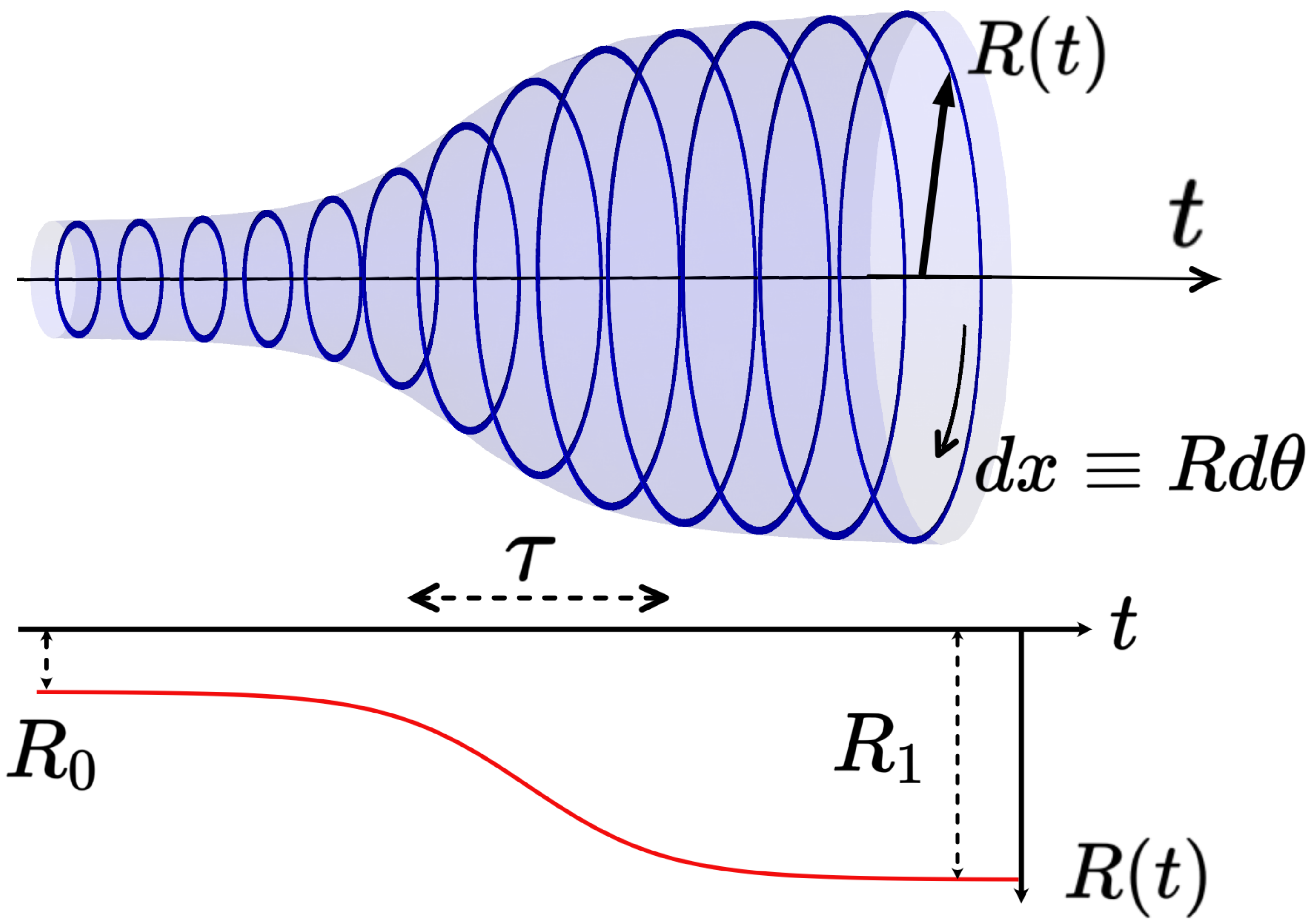


Curved spacetime description

$$ds^2 = c_s^2 dt^2 - R(t)^2 d\theta^2$$

1+1D Casimir effect in expanding cavities

What if the radius R depends on time?



Curved spacetime description

$$ds^2 = c_s^2 dt^2 - R(t)^2 d\theta^2$$

Similar to the Friedman-Lemaitre-Robertson-Walker metric

$$ds^2 = c_s^2 dt^2 - a(t)^2 dx^2$$

with $dx = R_0 d\theta$ and $a(t) = \dfrac{R(t)}{R_0}$

A. Friedman (1922)
G. Lemaître (1927)
H. Robertson (1929)
A. Walker (1935)

Symmetries in curved spacetime

Metric tensor

$$g_{\mu\nu} = \begin{pmatrix} f_1(t) & 0 \\ 0 & -f_2(t) \end{pmatrix}$$

Invariance of a theory in curved spacetime

$\Leftrightarrow$

Properties of the momentum-energy tensor

Momentum-energy tensor

Energy density

Density of energy current

$$\mathcal{T}^{\mu}{}_{\nu} = \begin{pmatrix} \varepsilon & \sqrt{f_1/f_2}\frac{1}{c_s}J_\varepsilon \\ -\sqrt{f_2/f_1}\,c_s\Pi & -p \end{pmatrix}$$

Momentum density

(radiative) Pressure

Symmetries in curved spacetime

Metric tensor

$$g_{\mu\nu} = \begin{pmatrix} f_1(t) & 0 \\ 0 & -f_2(t) \end{pmatrix}$$

Momentum-energy tensor

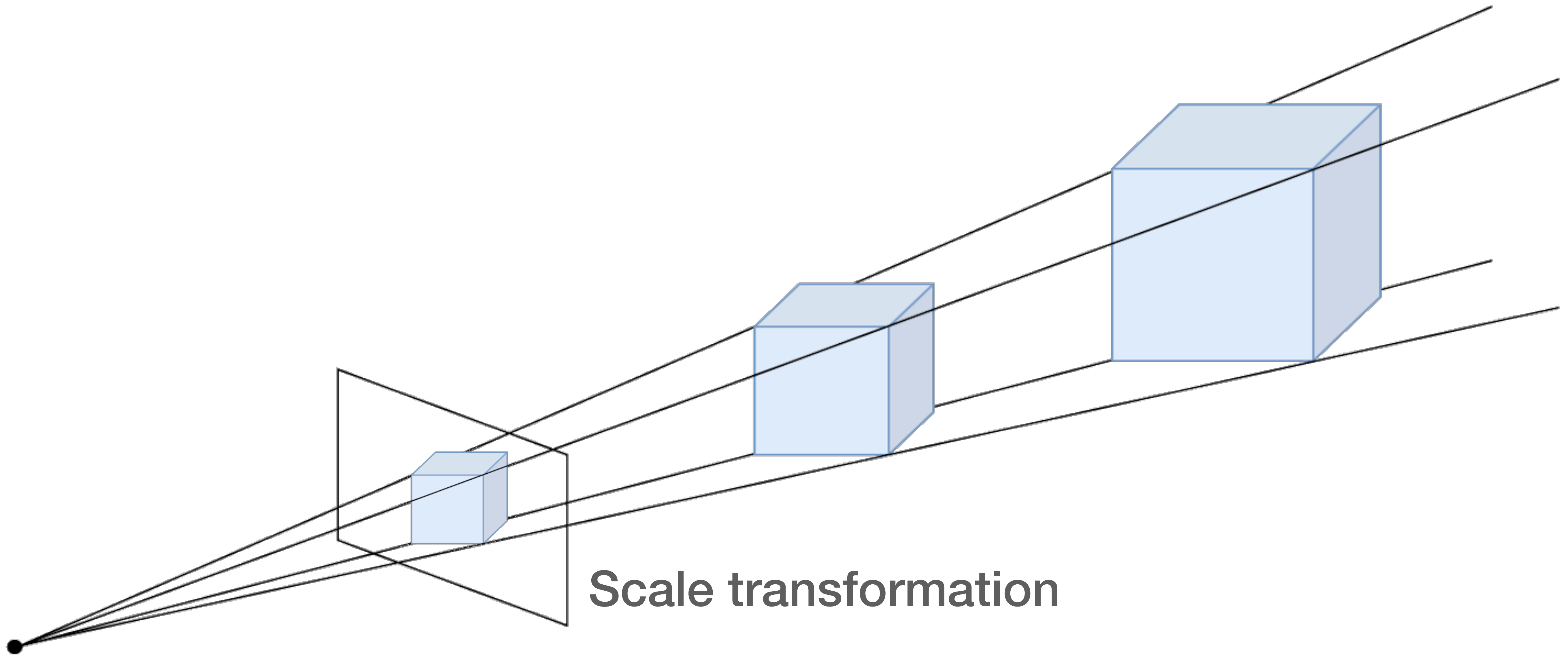
$$\mathcal{T}^{\mu}{}_{\nu} = \begin{pmatrix} \varepsilon & \sqrt{f_1/f_2}\frac{1}{c_s}J_\varepsilon \\ -\sqrt{f_2/f_1}c_s\Pi & -p \end{pmatrix}$$

Invariance of a theory in curved spacetime

$\Leftrightarrow$

Properties of the momentum-energy tensor

- Conformal/Weyl invariance $\mathcal{T}^{\mu}_{\mu} = 0$ $\leftrightarrow$ $\varepsilon = p$



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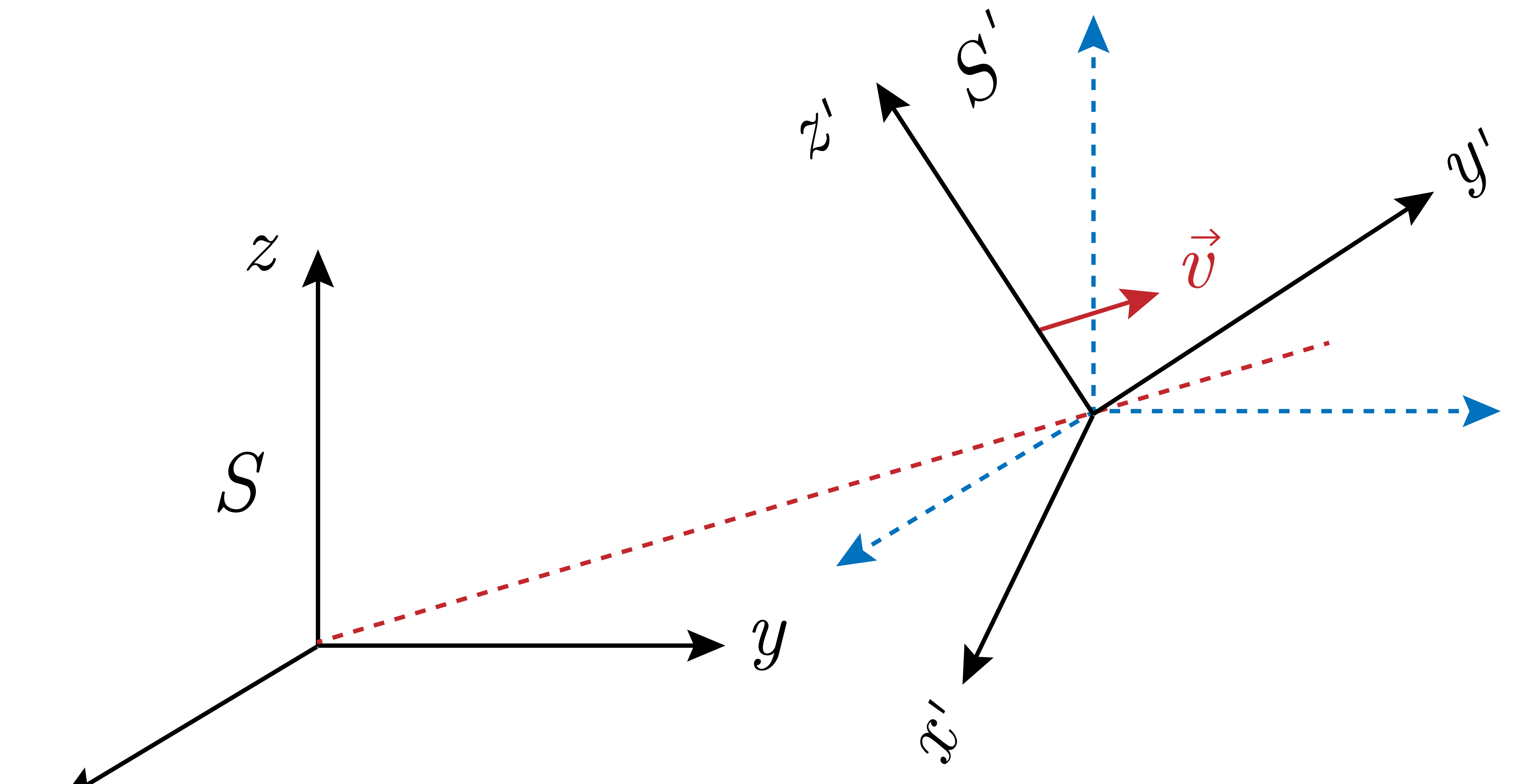
Properties of the momentum-energy tensor

Momentum-energy tensor

$$\mathcal{T}^{\mu}{}_{\nu} = \begin{pmatrix} \varepsilon & \sqrt{f_1/f_2}\frac{1}{c_s}J_\varepsilon \\ -\sqrt{f_2/f_1}c_s\Pi & -p \end{pmatrix}$$

- Lorentz invariance $\mathcal{T}^{\mu\nu} - \mathcal{T}^{\nu\mu} = 0$ $\longleftrightarrow$ $v_F \Pi = v_F^{-1} J_\varepsilon$



Lorentz transformation= Boost+Rotation

Symmetries in curved spacetime

Metric tensor

$$g_{\mu\nu} = \begin{pmatrix} f_1(t) & 0 \\ 0 & -f_2(t) \end{pmatrix}$$

Momentum-energy tensor

$$\mathcal{T}^{\mu}{}_{\nu} = \begin{pmatrix} \varepsilon & \sqrt{f_1/f_2}\,\frac{1}{c_s} J_\varepsilon \\ -\sqrt{f_2/f_1}\, c_s\, \Pi & -p \end{pmatrix}$$

Invariance of a theory in curved spacetime

$\Leftrightarrow$

Properties of the momentum-energy tensor

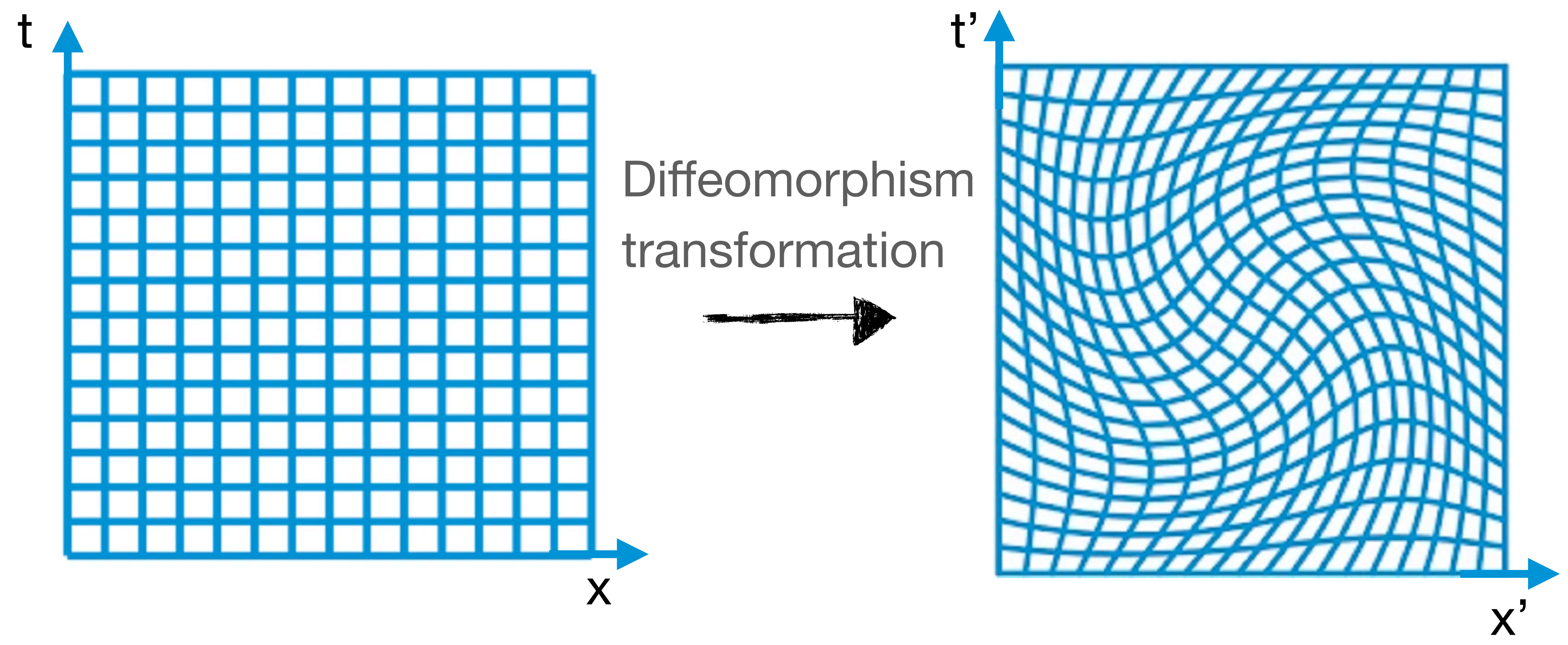
- Diffeomorphism invariance $\quad \nabla_\mu \mathcal{T}^{\mu\nu} = 0$

$\longleftrightarrow$

Energy conservation

$$\begin{cases} \partial_t \left(f_2\, \varepsilon\right) + \partial_x \left(\sqrt{f_1 f_2}\, J_\varepsilon\right) = 0 \\ -\partial_t \left(f_2\, \Pi\right) + \partial_x \left(\sqrt{f_1 f_2}\, p\right) = 0 \end{cases}$$

Momentum conservation



Symmetries in curved spacetime

Metric tensor

$$g_{\mu\nu} = \begin{pmatrix} f_1(t) & 0 \\ 0 & -f_2(t) \end{pmatrix}$$

- Conformal/Weyl invariance $\mathcal{T}^{\mu}_{\ \mu} = 0 \quad \longleftrightarrow \quad \varepsilon = p$
- Lorentz invariance $\mathcal{T}^{\mu\nu} - \mathcal{T}^{\nu\mu} = 0 \quad \longleftrightarrow \quad v_F\,\Pi = v_F^{-1} J_\varepsilon$
- Diff. invariance $\nabla_\mu \mathcal{T}^{\mu\nu} = 0 \quad \longleftrightarrow$

Energy conservation

$$\begin{cases} \partial_t\left(f_2\,\varepsilon\right) + \partial_x\left(\sqrt{f_1 f_2}\,J_\varepsilon\right) = 0 \\ -\partial_t\left(f_2\,\Pi\right) + \partial_x\left(\sqrt{f_1 f_2}\,p\right) = 0 \end{cases}$$

Momentum conservation

Solution from a uniform initial state

$$\varepsilon = p \propto \frac{1}{f_2(t)}$$

Looking into asymptotics

Instantaneous Casimir energy

$$\varepsilon = p = -\frac{\mathscr{C}_w}{2}\frac{\hbar c_s}{24\pi R^2 f_2(t)} \equiv -\frac{\mathscr{C}_w}{2}\frac{\hbar c_s}{24\pi R^2(t)}$$

3. Gravitational anomalies and the anomalous Casimir effect

3.a. The Casimir effect, or how confinement modify vacuum properties of a system

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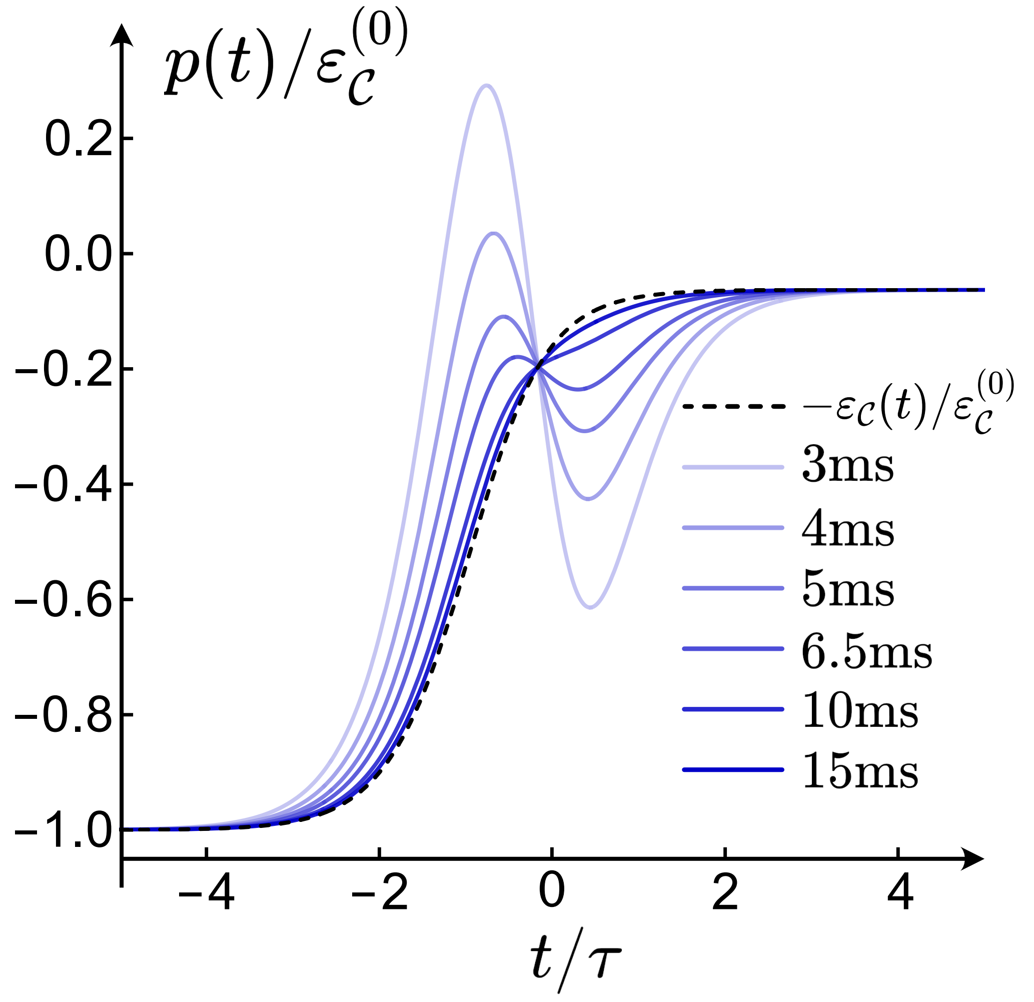
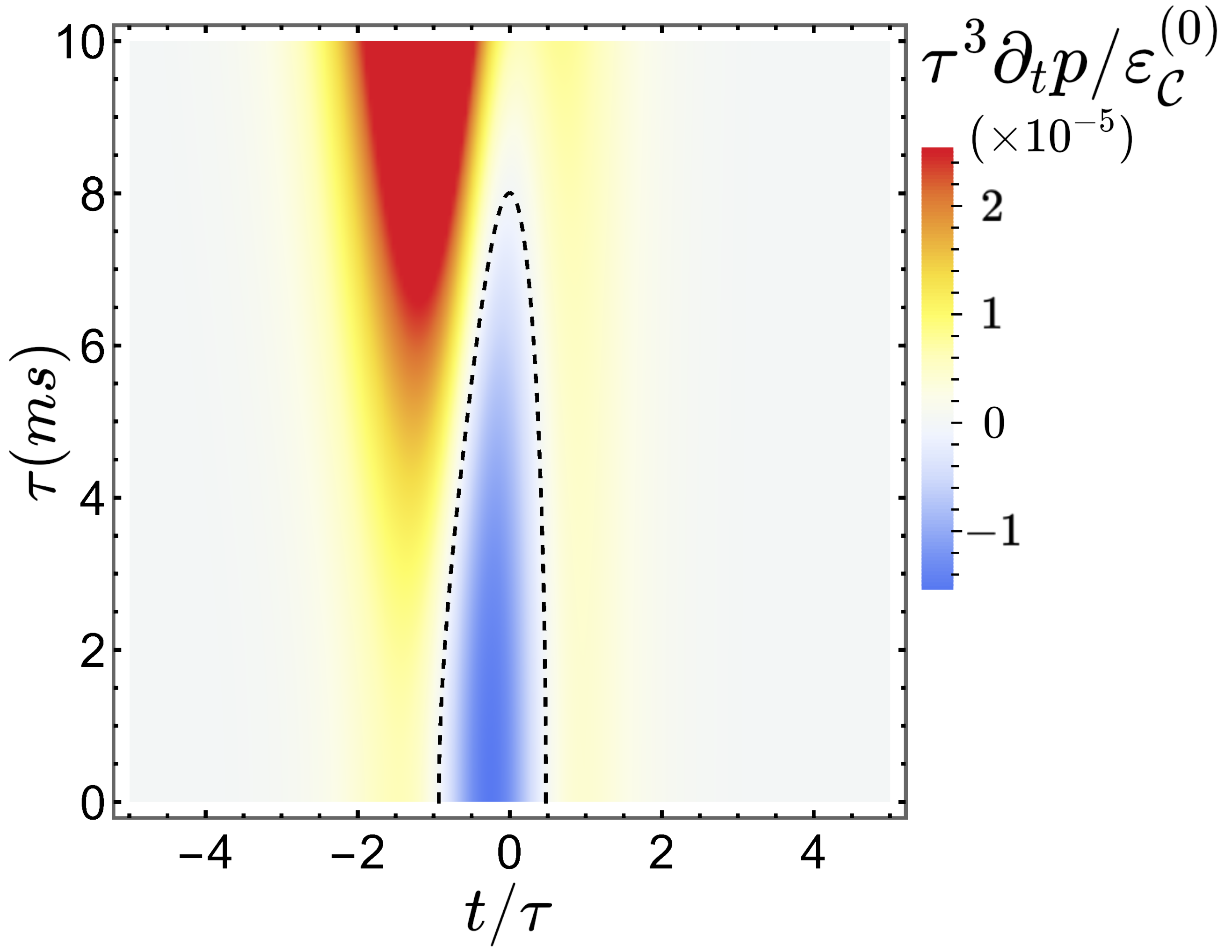




3.d. Extension to velocity modulated systems

Anomalies in physics

- Conformal/Weyl invariance
- Lorentz invariance
- Diff. invariance

Symmetries of the Hamiltonian / action

Anomaly: symmetry of the Hamiltonian, but not of the field theory
➡ Signals anomalous **quantum fluctuations**
➡ **Conservation law spoiled** by quantum fluctuations

Bertlmann, Anomalies in Quantum Field Theory (2001)

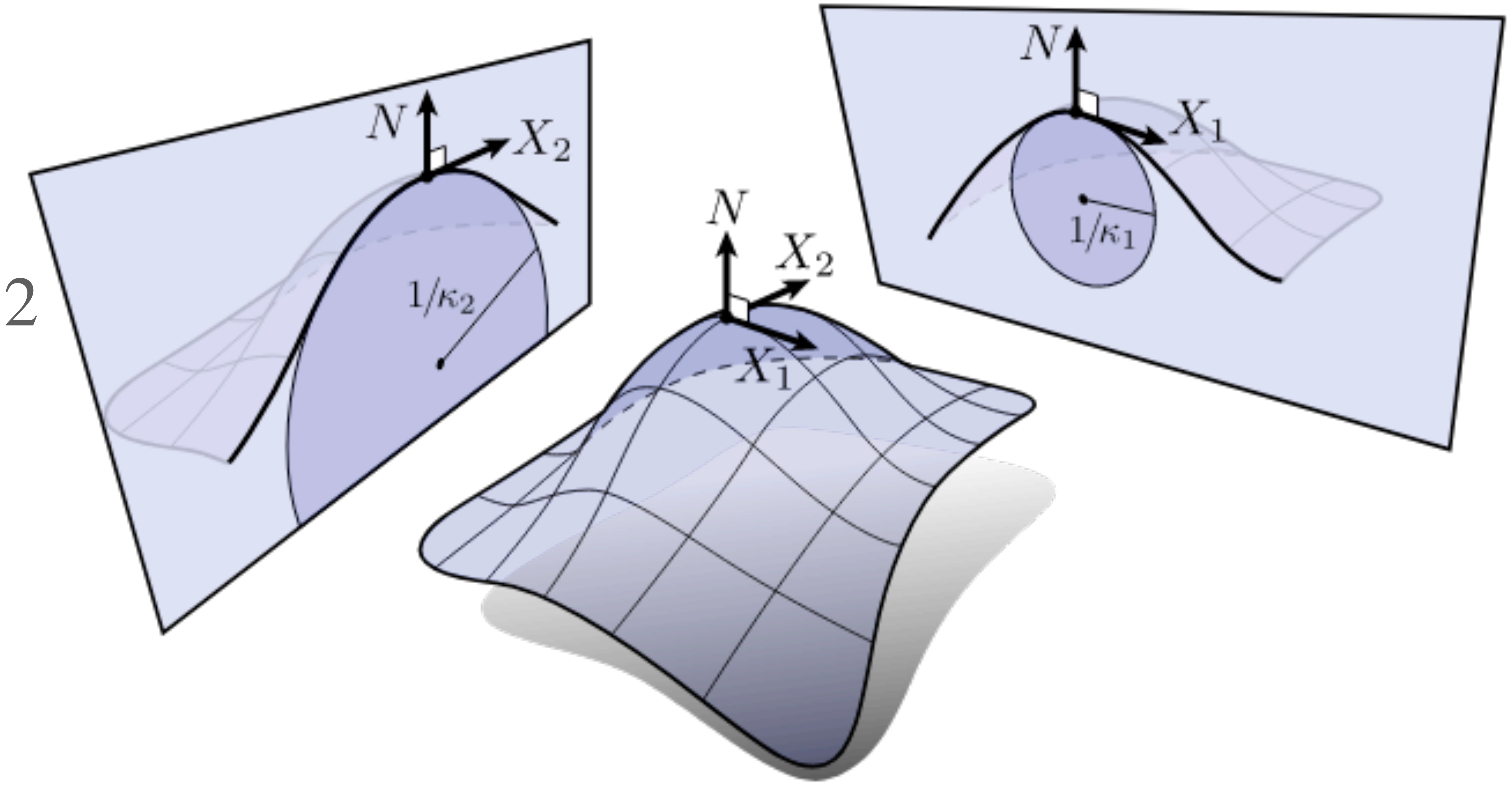
Relativistic quantum theory in a curved spacetime :

Gravitational anomalies: anomalous vacuum fluctuations induced by the curvature of spacetime

Anomalies in curved spacetimes

- Relativistic physics in **flat** spacetime
  - A single energy scale $\varepsilon_{\mathscr{C}} \propto 1/L^2$
- Relativistic physics in **curved** spacetime
  - One new energy scale $\hbar v_F \mathscr{R}$

Spacetime scalar curvature



$$\mathscr{R} = \kappa_1 \kappa_2$$

- **Question:** How does this new energy scale affect the conservation equations ?

Anomalies in curved spacetimes

- Consequences of the new energy scale $\frac{\hbar c_s}{48\pi}\mathscr{R}$ on the conservation laws

Bertlmann, Anomalies in Quantum Field Theory (2001)

Metric tensor

$$g_{\mu\nu} = \begin{pmatrix} f_1(t) & 0 \\ 0 & -f_2(t) \end{pmatrix}$$

$$\mathscr{R} = \frac{1}{c_s^2}\left(-\frac{\partial_t^2 f_2}{f_1 f_2} + \frac{1}{2}\frac{\partial_t f_2}{f_1 f_2}\left[\frac{\partial_t f_1}{f_1} + \frac{\partial_t f_2}{f_2}\right]\right)$$

- ~~Conformal/Weyl invariance~~

⇒ Conformal/Weyl anomaly

$$\varepsilon = p + \mathscr{C}_w \frac{\hbar c_s}{48\pi}\mathscr{R}$$

$$\left(\mathscr{T}^{\mu}_{\ \mu} = \mathscr{C}_w \frac{\hbar c_s}{48\pi}\mathscr{R}\right)$$

Anomalies in curved spacetimes

Metric tensor

$$g_{\mu\nu} = \begin{pmatrix} f_1(t) & 0 \\ 0 & -f_2(t) \end{pmatrix}$$

$$\mathscr{R} = \frac{1}{c_s^2}\left(-\frac{\partial_t^2 f_2}{f_1 f_2} + \frac{1}{2}\frac{\partial_t f_2}{f_1 f_2}\left[\frac{\partial_t f_1}{f_1} + \frac{\partial_t f_2}{f_2}\right]\right)$$

- **Consequences of the new energy scale $\frac{\hbar c_s}{48\pi}\mathscr{R}$ on the conservation laws**

Bertlmann, Anomalies in Quantum Field Theory (2001)

- Lorentz invariance $c_s \Pi = c_s^{-1} J_\varepsilon$

$$\left(\mathscr{T}^{\mu\nu} - \mathscr{T}^{\nu\mu} = 0\right)$$

Anomalies in curved spacetimes

Metric tensor

$$g_{\mu\nu} = \begin{pmatrix} f_1(t) & 0 \\ 0 & -f_2(t) \end{pmatrix}$$

$$\mathscr{R} = \frac{1}{c_s^2}\left(-\frac{\partial_t^2 f_2}{f_1 f_2} + \frac{1}{2}\frac{\partial_t f_2}{f_1 f_2}\left[\frac{\partial_t f_1}{f_1} + \frac{\partial_t f_2}{f_2}\right]\right)$$

- **Consequences of the new energy scale $\frac{\hbar c_s}{48\pi}\mathscr{R}$ on the conservation laws**

Bertlmann, Anomalies in Quantum Field Theory (2001)

- ~~Diffeomorphism invariance~~

Source of momentum

⇨ Einstein anomaly

$$\begin{cases} -\partial_t\left(f_2\,\Pi\right) + \partial_x\left(\sqrt{f_1 f_2}\,p\right) = \mathscr{C}_g\dfrac{\hbar}{96\pi} f_2 \partial_t \mathscr{R} \\ \partial_t\left(f_2\,\varepsilon\right) + \partial_x\left(\sqrt{f_1 f_2}\,J_\varepsilon\right) = 0 \quad \text{(Energy conservation)} \end{cases}$$

$$\left(\nabla_\mu \mathscr{T}^{\mu\nu} = \mathscr{C}_g \frac{\hbar c_s}{96\pi}\frac{1}{\sqrt{-det(g)}}\epsilon^{\nu\mu}\nabla_\mu \mathscr{R}\right)$$

Anomalies in curved spacetimes

B.Bermond, A.Grushin and, D.Carpentier (2024), ArXiv 2402.08610

Metric tensor

$$g_{\mu\nu} = \begin{pmatrix} f_1(t) & 0 \\ 0 & -f_2(t) \end{pmatrix}$$

- Conformal/Weyl anomaly $\quad \cancel{\varepsilon = p} \quad \longrightarrow \quad \varepsilon = p + \mathscr{C}_w \frac{\hbar c_s}{48\pi}\mathscr{R}$
- Lorentz invariance $\quad c_s \Pi = c_s^{-1} J_\varepsilon \quad \longrightarrow \quad c_s \Pi = c_s^{-1} J_\varepsilon$
- Einstein anomaly

$$\cancel{-\partial_t (f_2 \Pi) + \partial_x \left(\sqrt{f_1 f_2}\, p\right) = 0}$$
$$\partial_t (f_2 \varepsilon) + \partial_x \left(\sqrt{f_1 f_2}\, J_\varepsilon\right) = 0$$

$\longrightarrow$

Source of momentum

$$-\partial_t (f_2 \Pi) + \partial_x \left(\sqrt{f_1 f_2}\, p\right) = \mathscr{C}_g \frac{\hbar}{96\pi} f_2 \partial_t \mathscr{R}$$
$$\partial_t (f_2 \varepsilon) + \partial_x \left(\sqrt{f_1 f_2}\, J_\varepsilon\right) = 0$$

Solution from a uniform initial state

$$\cancel{\mathscr{T}^\mu{}_\nu = -\frac{1}{2} \begin{pmatrix} \mathscr{C}_w & \sqrt{f_1/f_2}\frac{1}{c_s}\mathscr{C}_g \\ -\sqrt{f_2/f_1}\, c_s \mathscr{C}_g & -\mathscr{C}_w \end{pmatrix} \varepsilon_{\mathscr{C}}(t)} \longrightarrow \mathscr{T}^\mu{}_\nu = -\frac{1}{2} \begin{pmatrix} \mathscr{C}_w & \sqrt{f_1/f_2}\frac{1}{c_s}\mathscr{C}_g \\ -\sqrt{f_2/f_1}\, c_s \mathscr{C}_g & -\mathscr{C}_w \end{pmatrix} \varepsilon_{\mathscr{C}}(t) + \left(\mathscr{T}^\mu{}_\nu\right)_{An}$$

$\Rightarrow$ 2 new energy scales: $\varepsilon_{\mathscr{R}} = \frac{\hbar c_s}{48\pi}\mathscr{R} \qquad \epsilon_{\overline{\mathscr{R}}} = \frac{\hbar c_s}{48\pi}\overline{\mathscr{R}} = \frac{\hbar c_s}{48\pi}\frac{1}{f_2(t)}\int_0^t \mathscr{R}\partial_t f_2$

Anomalies in curved spacetimes

B.Bermond, A.Grushin and, D.Carpentier (2024), ArXiv 2402.08610

Metric tensor

$$g_{\mu\nu} = \begin{pmatrix} f_1(t) & 0 \\ 0 & -f_2(t) \end{pmatrix}$$

Solution from a uniform initial state

1 natural energy scale $\varepsilon_{\mathscr{C}}(t) = \dfrac{\hbar c_s}{24\pi R^2 f_2(t)}$ (Instantaneous Casimir energy density)

2 new energy scales: $\varepsilon_{\mathscr{R}} = \dfrac{\hbar c_s}{48\pi}\mathscr{R}$ $\quad \epsilon_{\overline{\mathscr{R}}} = \dfrac{\hbar c_s}{48\pi}\overline{\mathscr{R}} = \dfrac{\hbar c_s}{48\pi}\dfrac{1}{f_2(t)}\displaystyle\int_0^t \mathscr{R}\partial_t f_2$

Components of the stress energy tensor

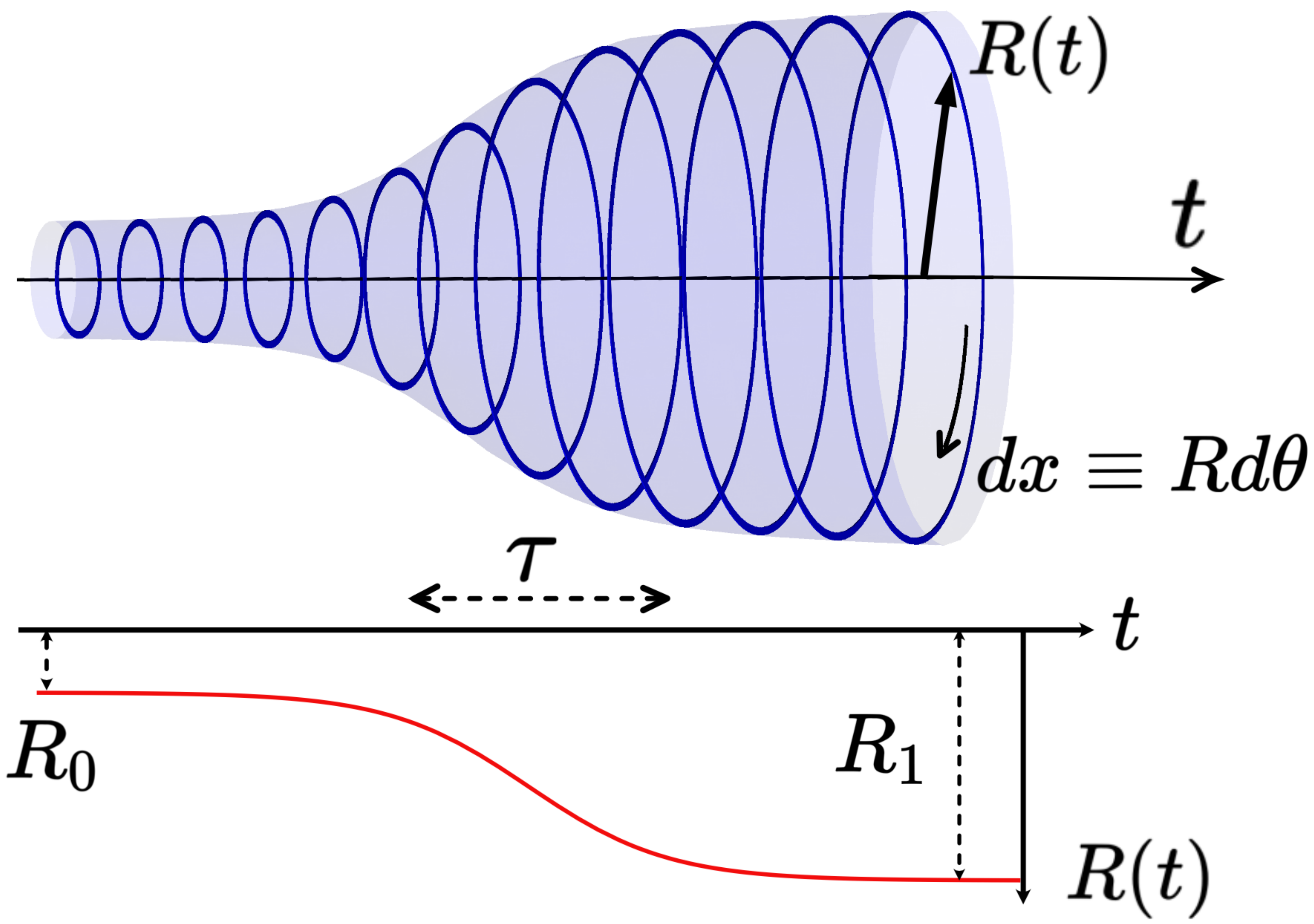
$$\varepsilon = -\cancel{\frac{\mathscr{C}_w}{2}}\varepsilon_{\mathscr{C}}(t) \longrightarrow \varepsilon = \frac{\mathscr{C}_w}{2}\left(-\varepsilon_{\mathscr{C}}(t) + \varepsilon_{\overline{\mathscr{R}}}\right)$$

$$p = -\cancel{\frac{\mathscr{C}_w}{2}}\varepsilon_{\mathscr{C}}(t) \longrightarrow p = \frac{\mathscr{C}_w}{2}\left(-\varepsilon_{\mathscr{C}}(t) + \varepsilon_{\overline{\mathscr{R}}} - 2\varepsilon_{\mathscr{R}}\right)$$

$$J_\varepsilon/c_s = \Pi c_s \cancel{=} - \frac{\mathscr{C}_g}{2}\varepsilon_{\mathscr{C}}(t) \longrightarrow J_\varepsilon/c_s = \Pi c_s = \frac{\mathscr{C}_g}{2}\left(\varepsilon_{\mathscr{C}}(t) + \varepsilon_{\overline{\mathscr{R}}} - \varepsilon_{\mathscr{R}}\right)$$

1+1D Casimir effect in expanding cavities

What if the radius R depends on time?



Curved spacetime description

$$ds^2 = c_s^2 dt^2 - R(t)^2 d\theta^2$$

In the absence of gravitational anomalies

$$\varepsilon = -\frac{\mathscr{C}_w}{2}\varepsilon_{\mathscr{C}}(t)$$

$$p = -\frac{\mathscr{C}_w}{2}\varepsilon_{\mathscr{C}}(t)$$

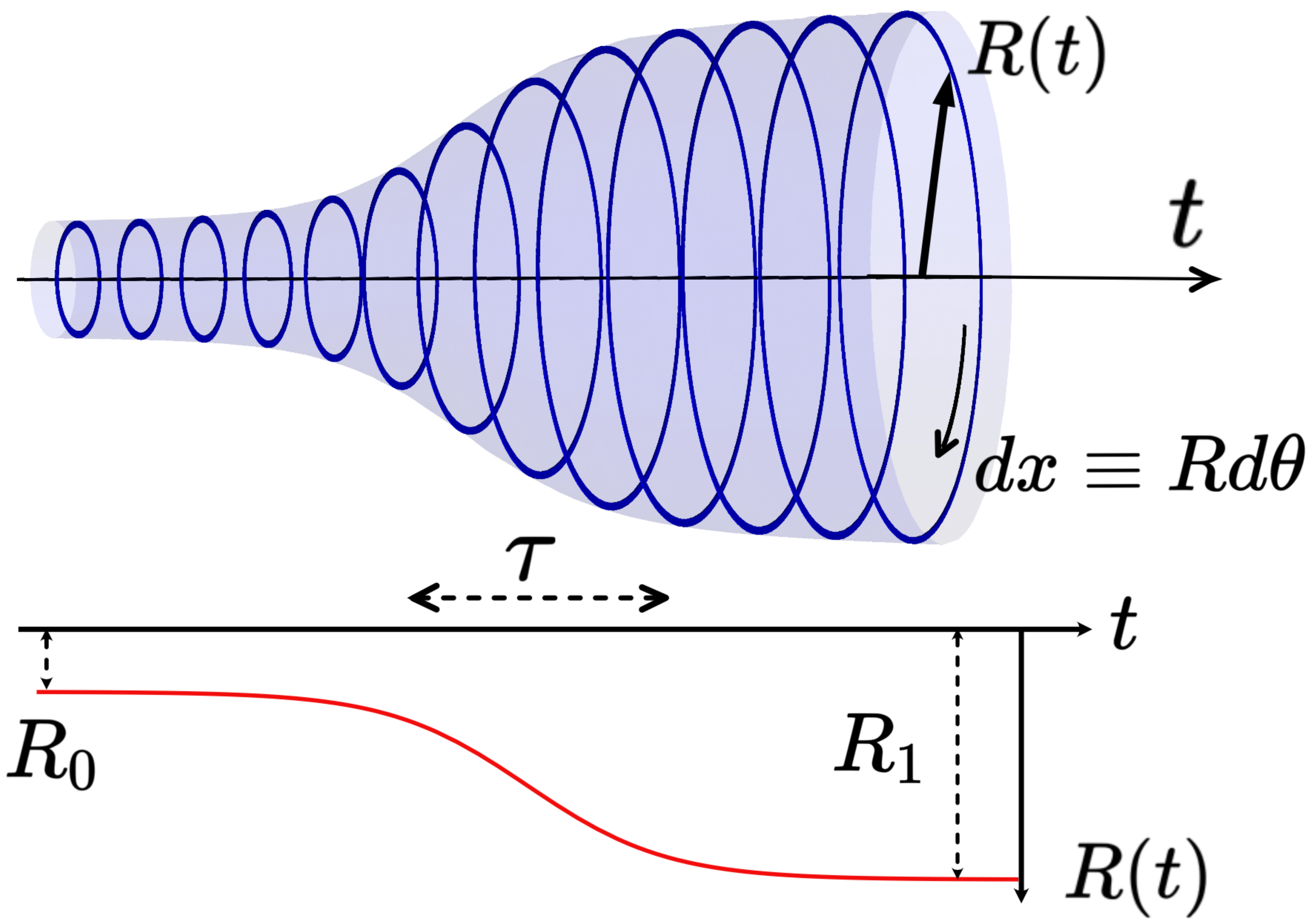
$$J_\varepsilon / c_s = \Pi c_s = -\frac{\mathscr{C}_g}{2}\varepsilon_{\mathscr{C}}(t)$$

Instantaneous Casimir energy density: $\varepsilon_{\mathscr{C}}(t)$

$$\varepsilon_{\mathscr{C}}(t) = \frac{\hbar c_s}{24\pi R(t)^2}$$

1+1D anomalous dynamical Casimir effect in expanding cavities

What if the radius R depends on time?



Curved spacetime description

$$\mathrm{d}s^2 = c_s^2\mathrm{d}t^2 - R(t)^2\mathrm{d}\theta^2 \rightarrow \mathscr{R} = -\frac{2}{c_s^2}\frac{\partial_t^2 R}{R}$$

In the presence of gravitational anomalies

$$\varepsilon = \frac{\mathscr{C}_w}{2}\left(-\varepsilon_{\mathscr{C}}(t) + \varepsilon_{\overline{\mathscr{R}}}\right)$$

$$p = \frac{\mathscr{C}_w}{2}\left(-\varepsilon_{\mathscr{C}}(t) + \varepsilon_{\overline{\mathscr{R}}} - 2\varepsilon_{\mathscr{R}}\right)$$

$$J_\varepsilon / c_s = \Pi c_s = \frac{\mathscr{C}_g}{2}\left(\varepsilon_{\mathscr{C}}(t) + \varepsilon_{\overline{\mathscr{R}}} - \varepsilon_{\mathscr{R}}\right)$$

2 new energy scales

$$\varepsilon_{\mathscr{R}} = -\frac{\hbar}{24\pi c_s}\frac{\partial_t^2 R}{R} \qquad \varepsilon_{\overline{\mathscr{R}}} = -\frac{\hbar}{24\pi c_s}\left(\frac{\partial_t R}{R}\right)^2$$

1+1D anomalous dynamical Casimir effect in expanding cavities

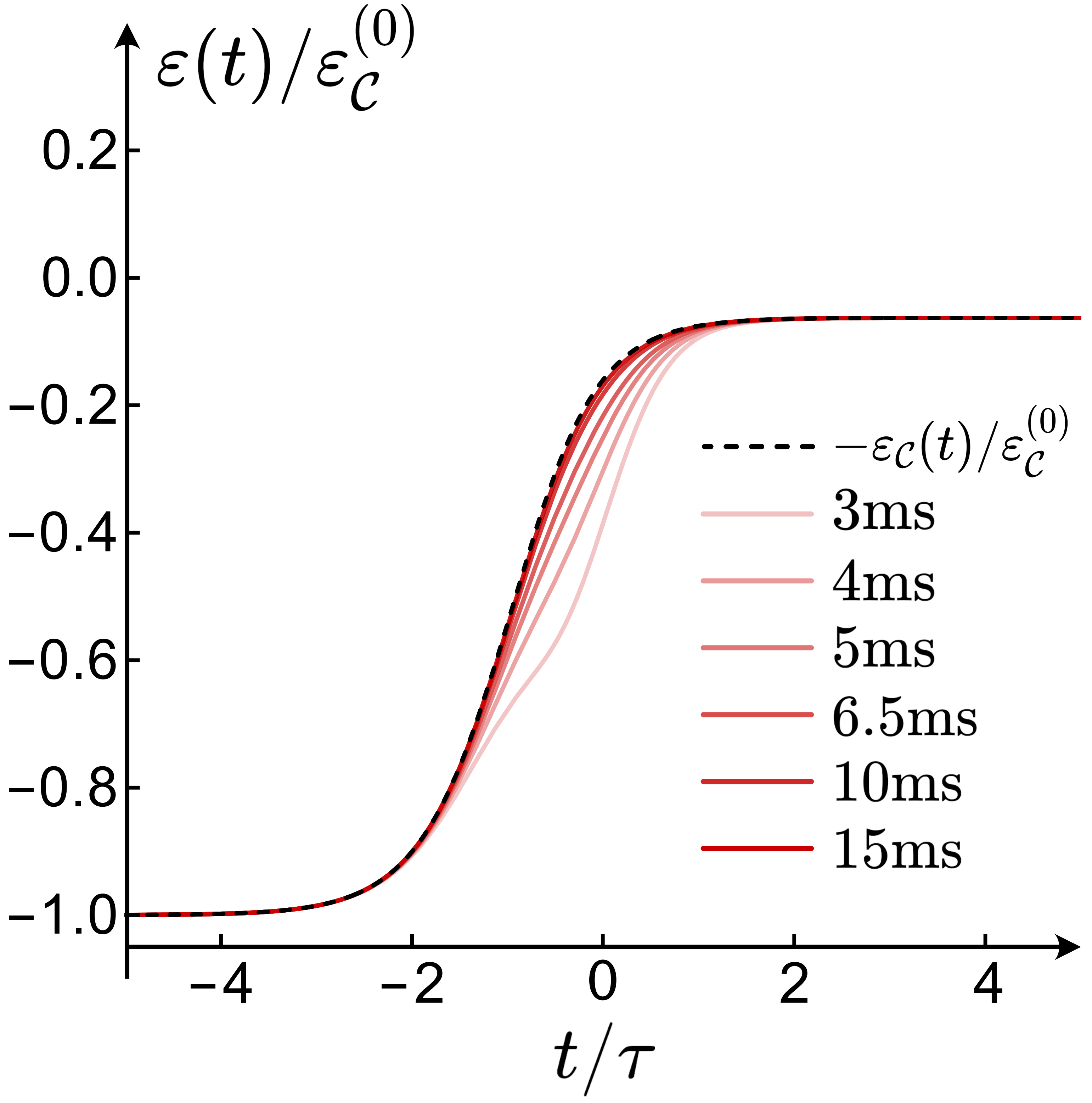
Consequences on energy:

Experimentally relevant parameters

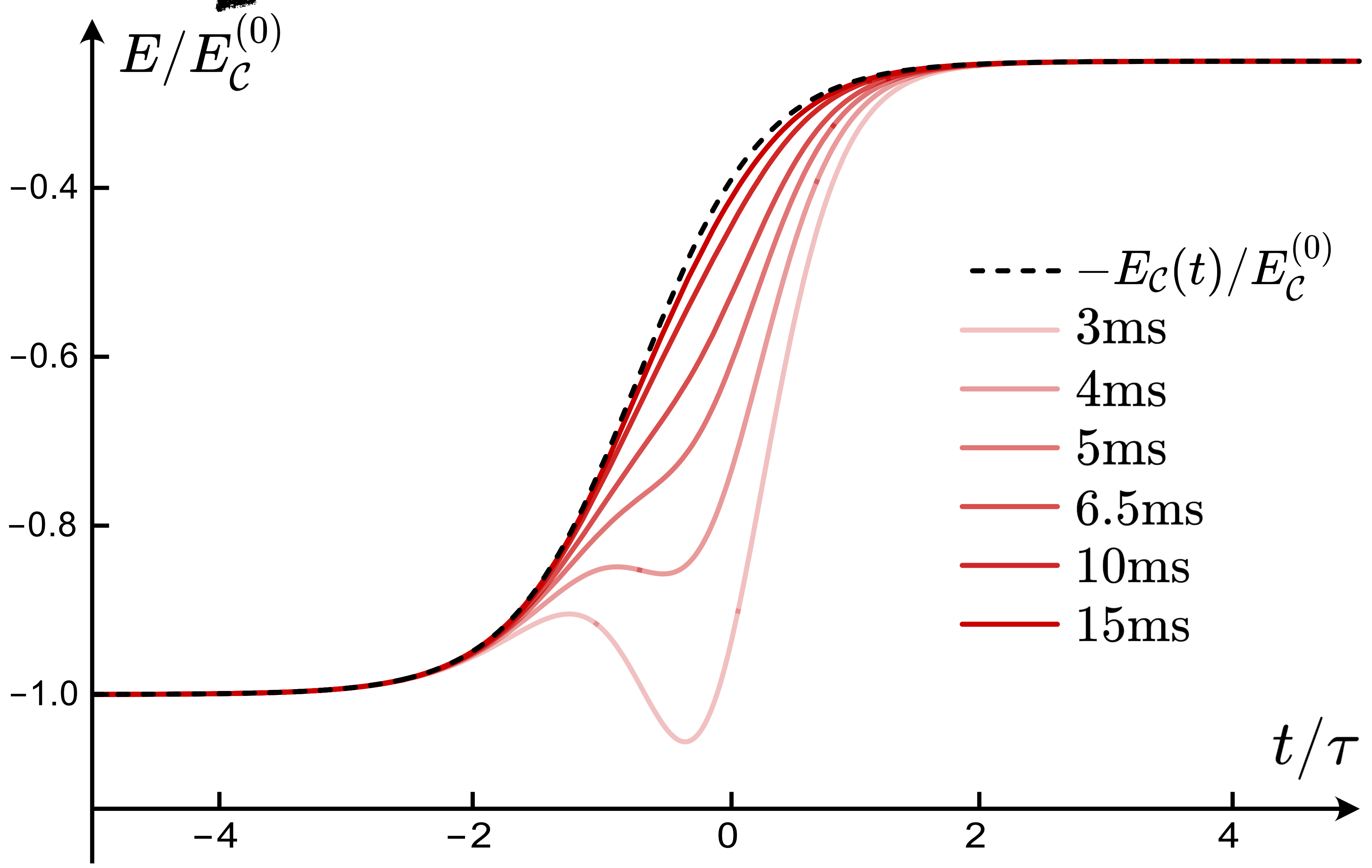
$$R_0 = 10\mu\text{m}$$
$$R_1 = 40\mu\text{m}$$
$$c_s = 4.10^{-3}\text{m.s}^{-1}$$

Typical for Bose-Einstein condensates experiments

S.Eckel et. Al. (2021)



$E(t) = 2\pi R(t)\varepsilon(t)$

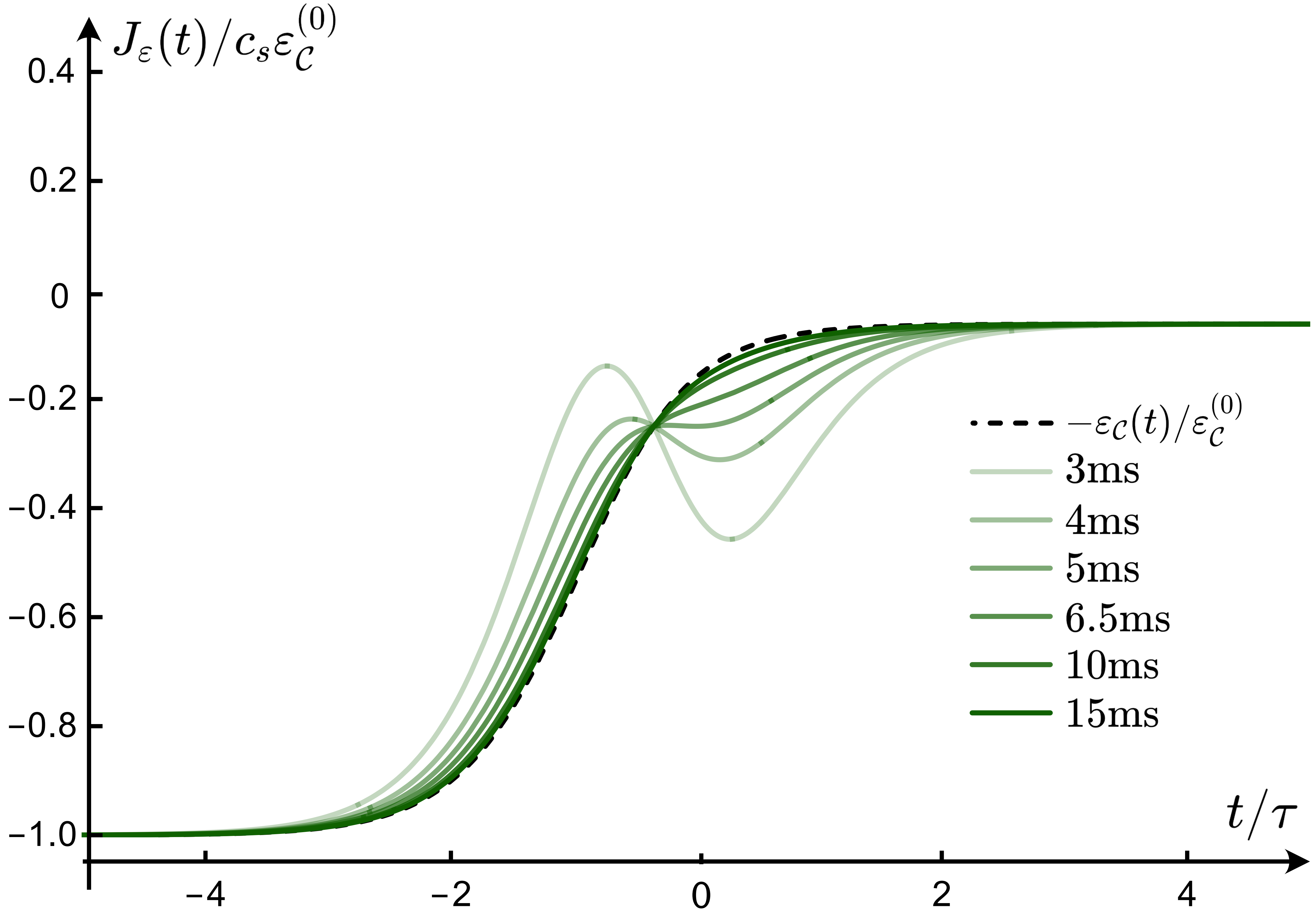


1+1D anomalous dynamical Casimir effect in expanding cavities

Consequences on energy currents for chiral systems:

Experimentally relevant parameters
$$R_0 = 10\mu\text{m} \qquad R_1 = 40\mu\text{m} \qquad c_s = 4.10^{-3}\text{m.s}^{-1}$$
Typical for Bose-Einstein condensates experiments

S.Eckel et. Al. (2021)



Energy currents in **chiral systems**

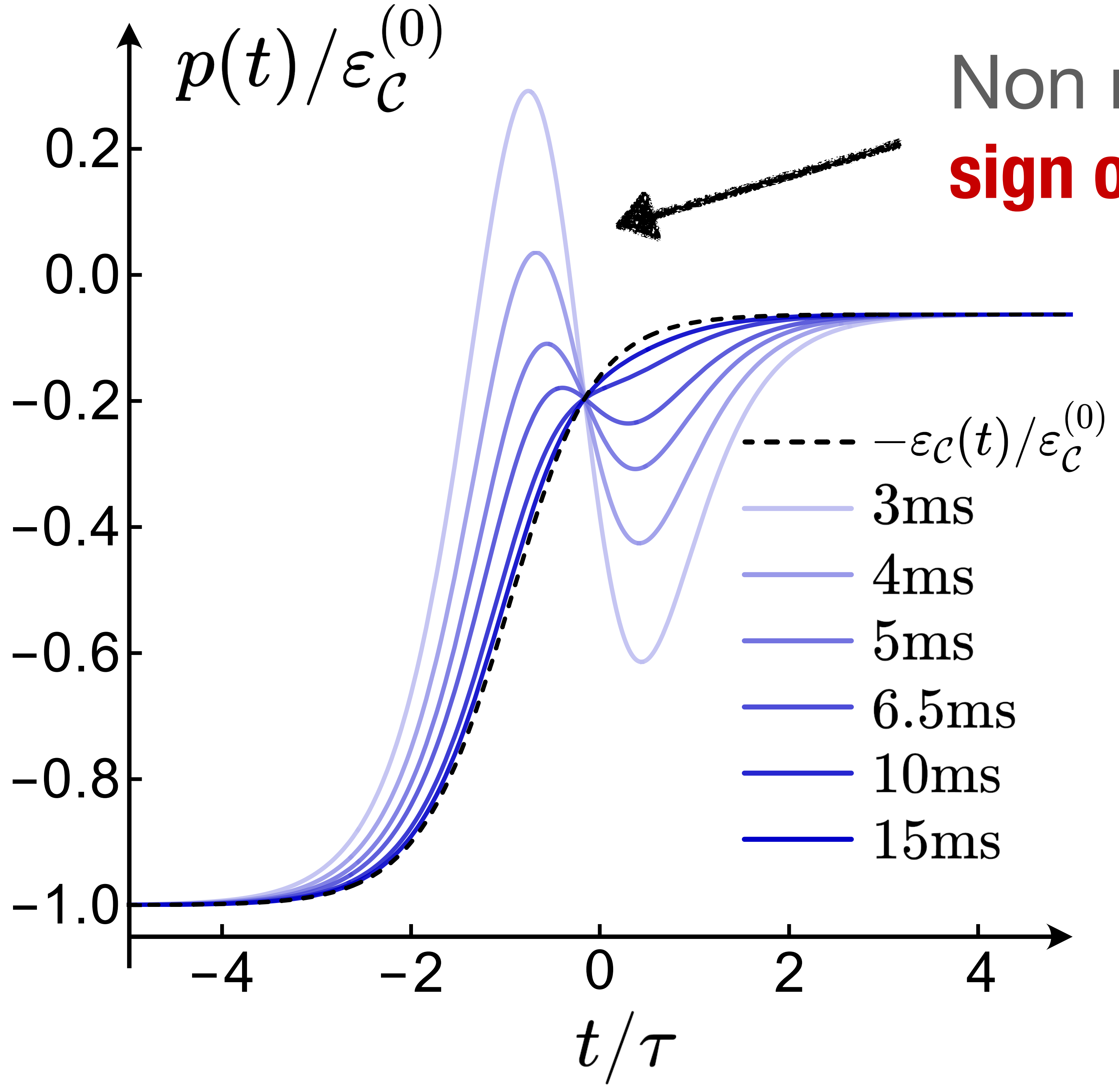
1+1D anomalous dynamical Casimir effect in expanding cavities

Consequences on pressure:

Experimentally relevant parameters

$$\left.\begin{aligned} R_0 &= 10\mu\text{m} \\ R_1 &= 40\mu\text{m} \\ c_s &= 4.10^{-3}\text{m.s}^{-1} \end{aligned}\right\}$$

Typical for Bose-Einstein condensates experiments
S.Eckel et. Al. (2021)



Non monotonous pressure
sign of an instability

1+1D anomalous dynamical Casimir effect in expanding cavities

Consequences on pressure:

Experimentally relevant parameters $\left.\begin{array}{l} R_0 = 10\mu\text{m} \\ R_1 = 40\mu\text{m} \\ c_s = 4.10^{-3}\text{m.s}^{-1} \end{array}\right\}$ Typical for Bose-Einstein condensates experiments

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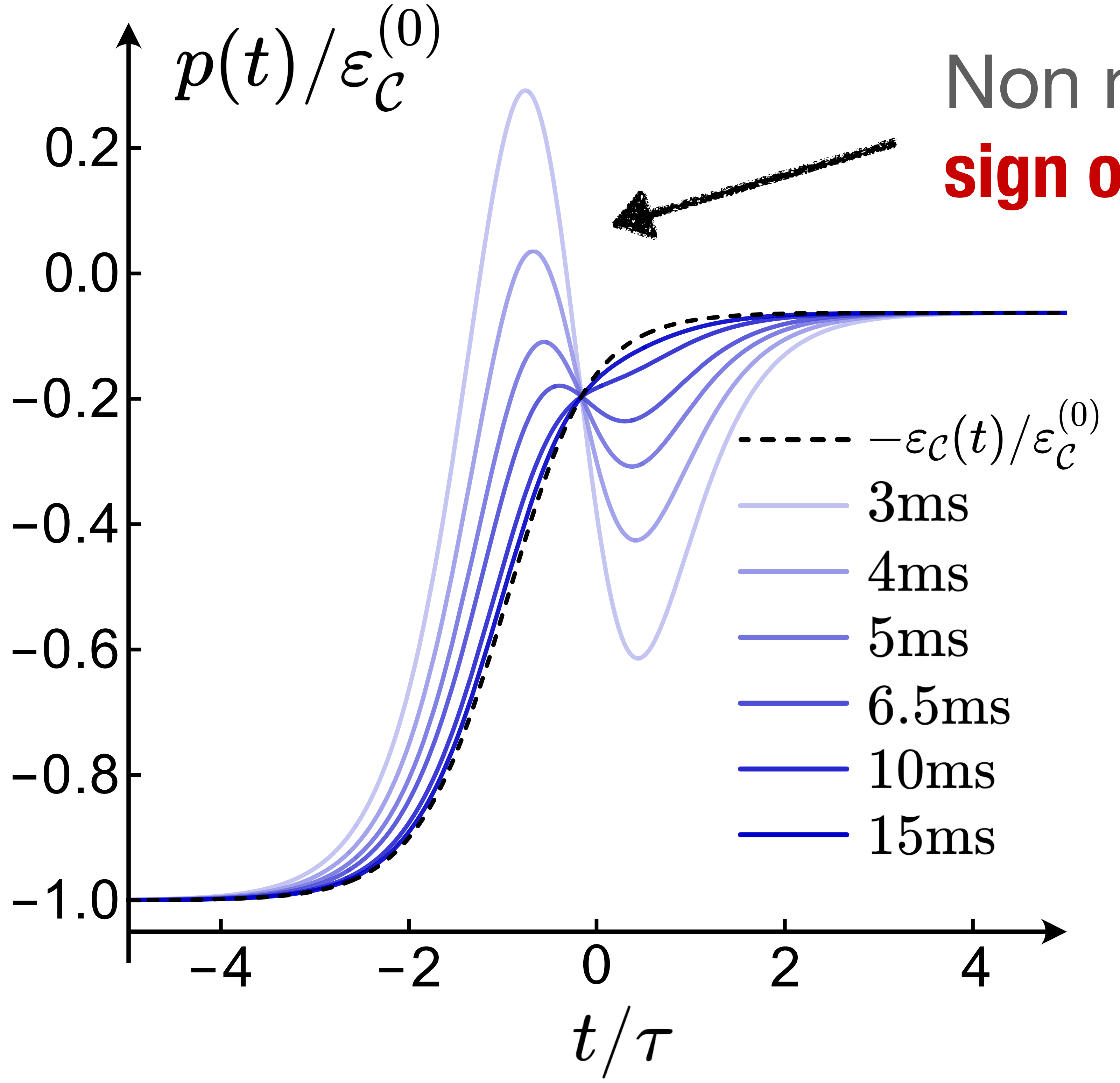
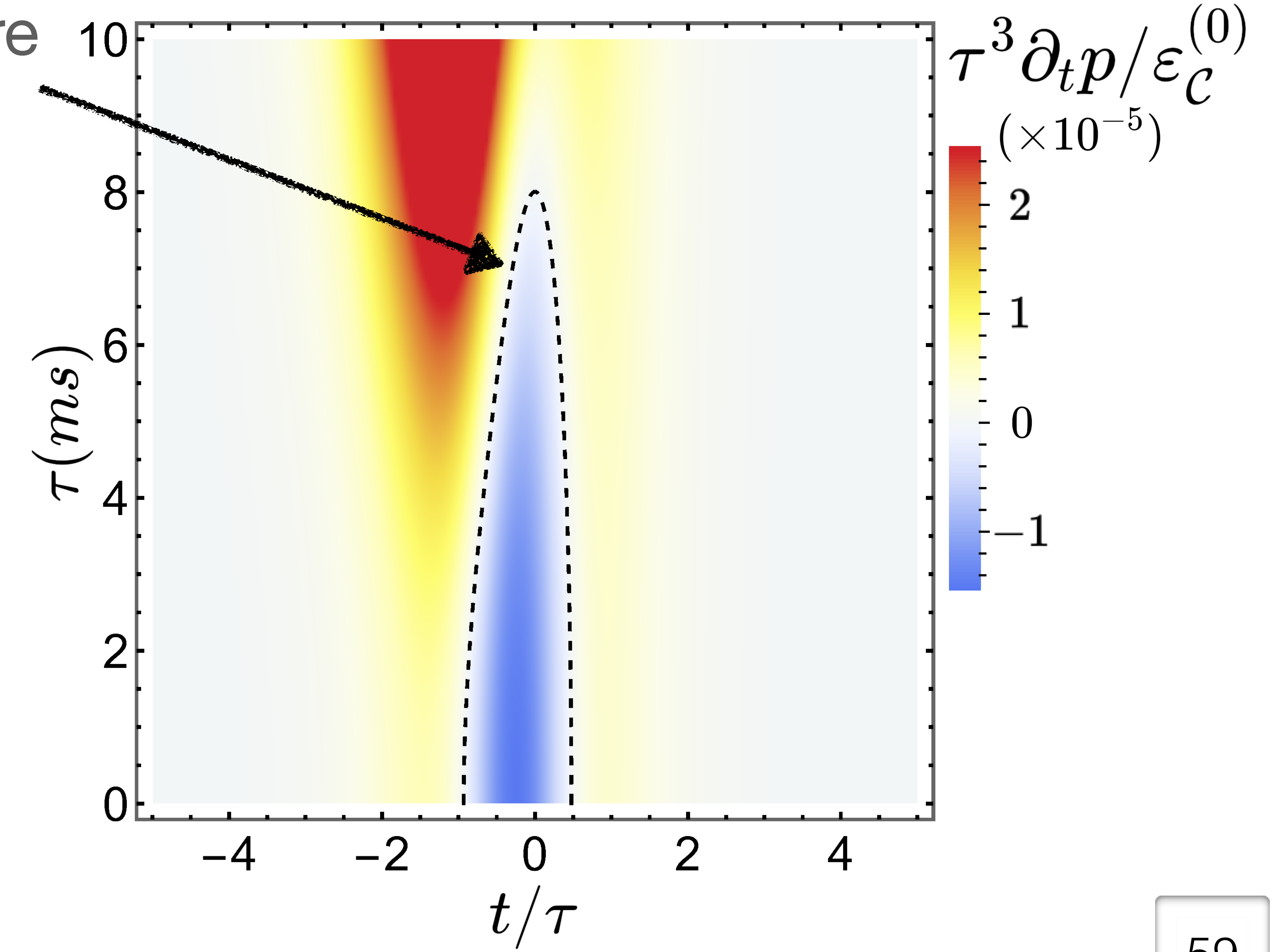


Non monotonous pressure
sign of an instability



1+1D anomalous dynamical Casimir effect in expanding cavities

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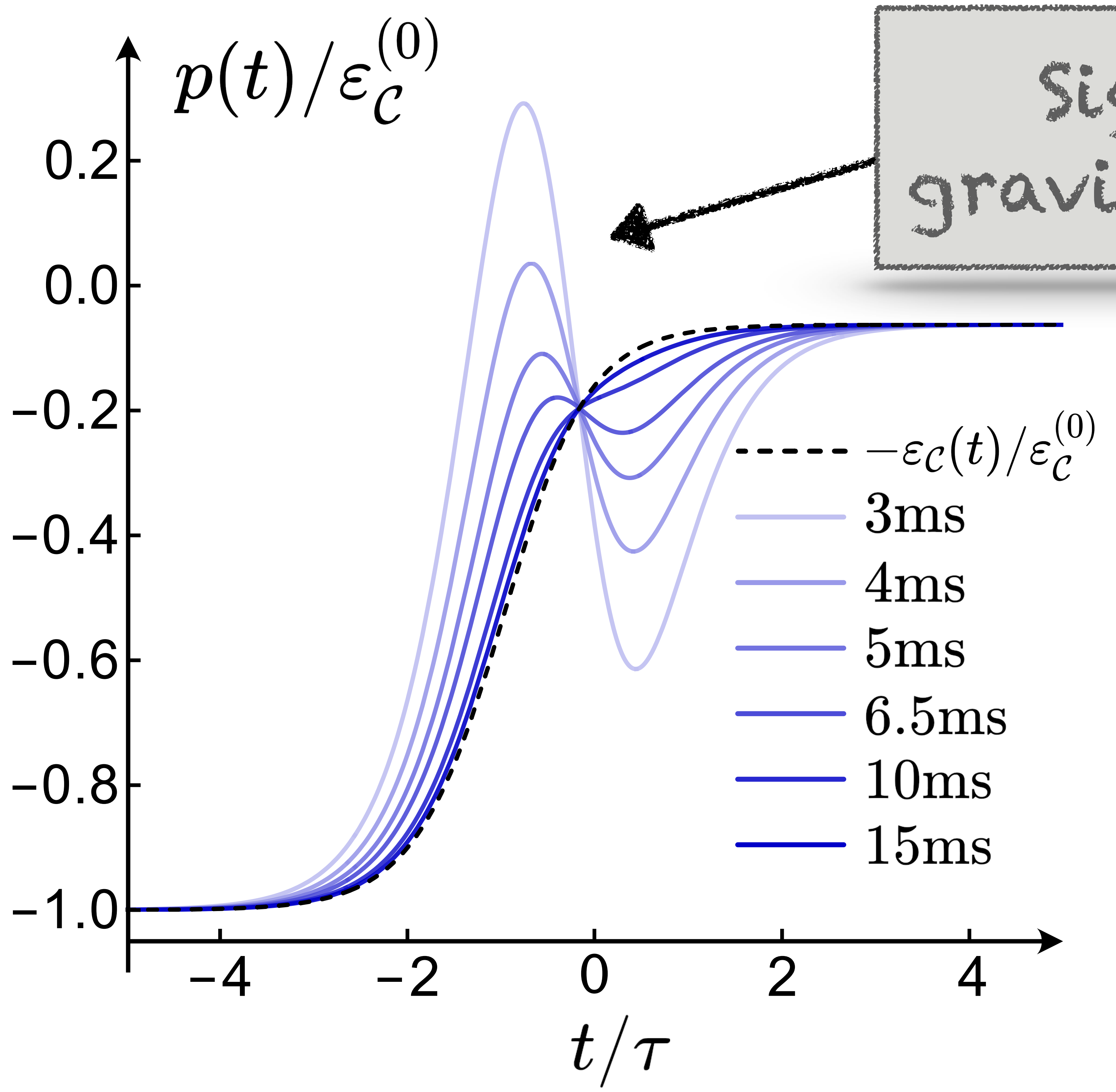
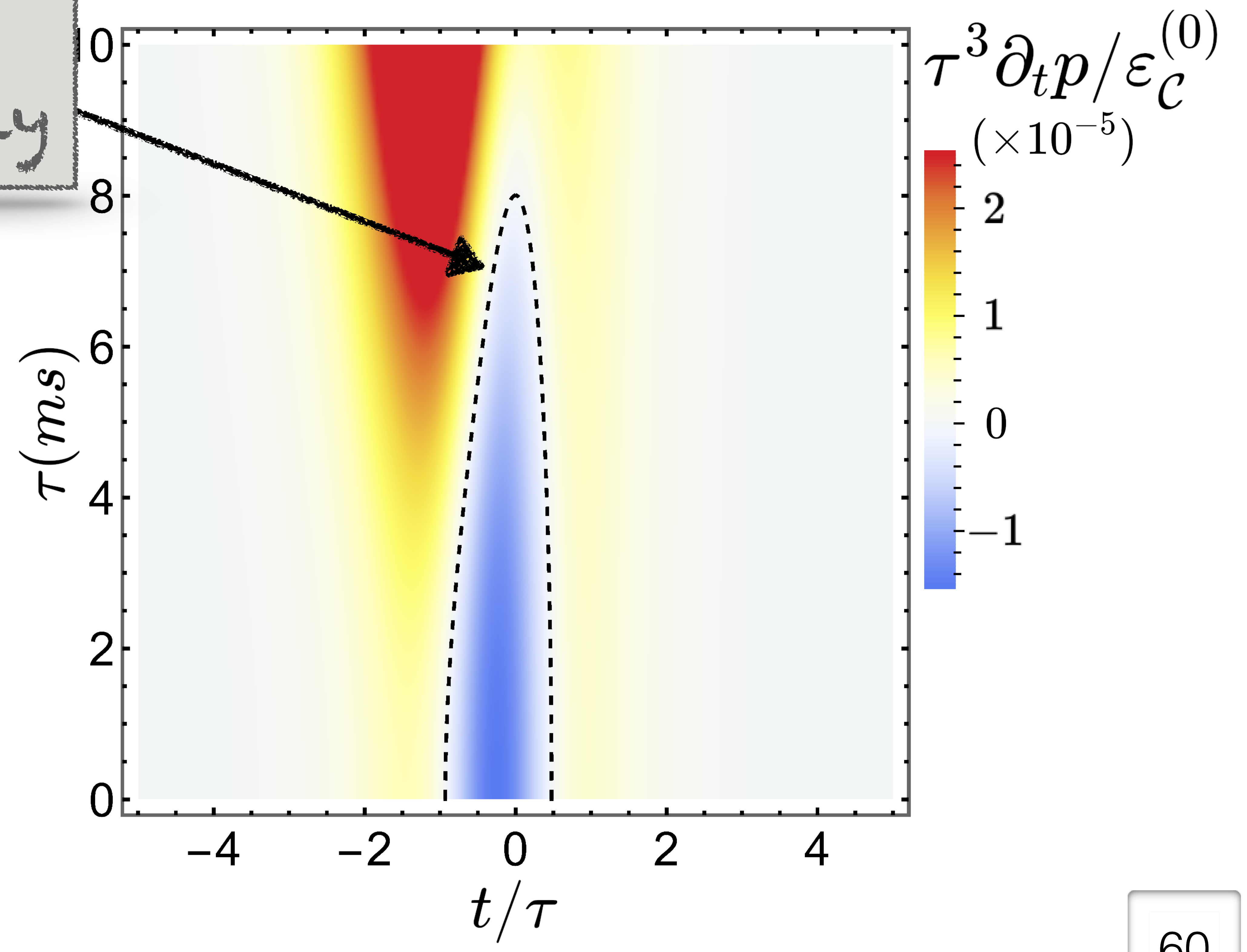
S.Eckel et. Al. (2021)



Signature of the gravitational anomaly



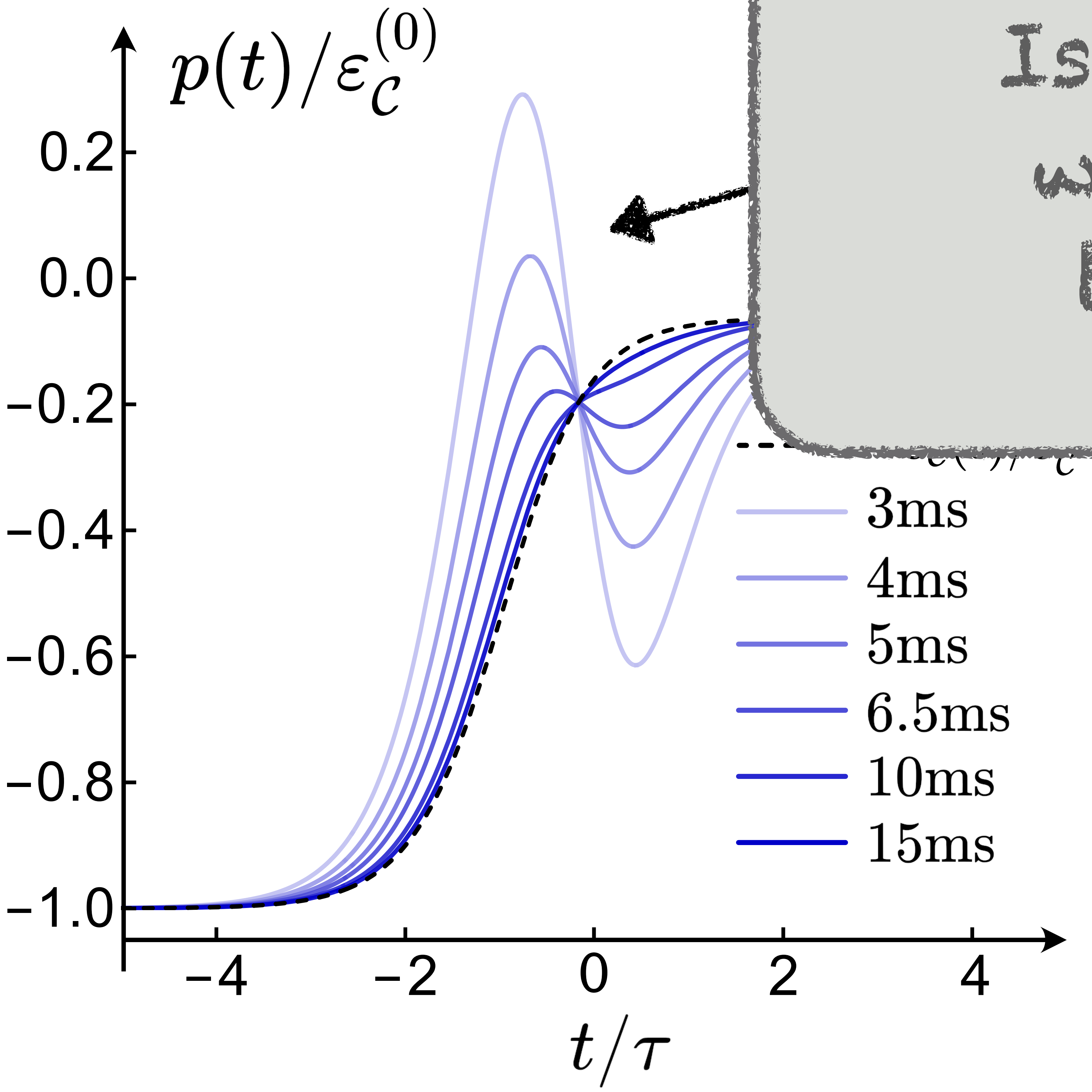
1+1D anomalous dynamical Casimir effect in expanding cavities

Consequences on pressure:

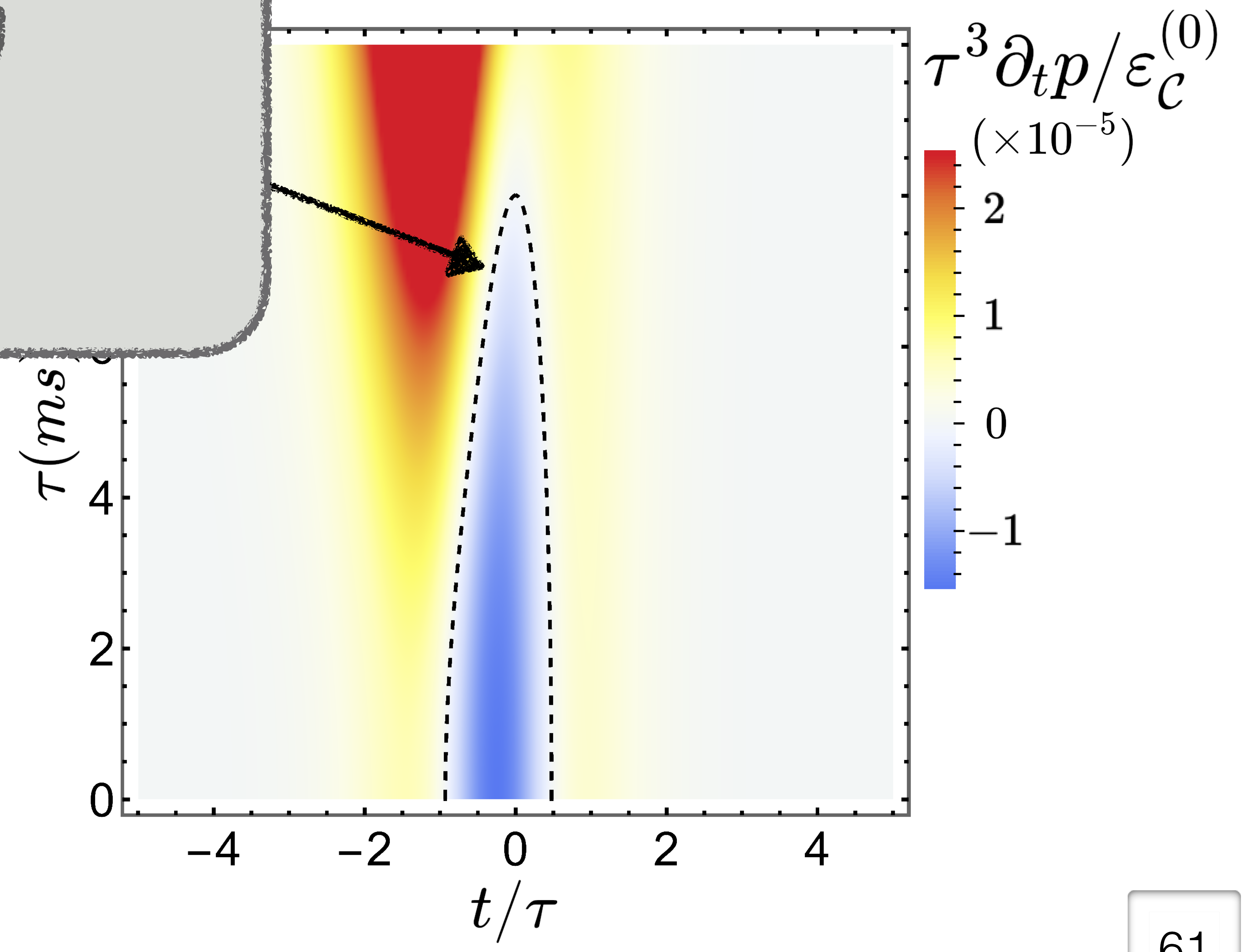
Experimentally relevant parameters $R_0 = 10\mu\mathrm{m}$, $R_1 = 40\mu\mathrm{m}$, $c = 4\,10^{-3}\mathrm{m\,s^{-1}}$

Typical for Bose-Einstein condensates experiments

S.Eckel et. Al. (2021)



Is there an easy way to recover these results?



1+1D anomalous dynamical Casimir effect in expanding cavities

An easy way to recover the results?

New length scale $\quad l^{-1} = c_s^{-1}\frac{\partial_t R}{R}$ → New energy density scale $\quad \varepsilon_l \propto l^{-2}$ → $\varepsilon = \varepsilon_{\mathscr{C}} + \varepsilon_l$

$$E = 2\pi R(t)\varepsilon = -\mathscr{C}_w\frac{\hbar c_s}{24R(t)}\left(1+\left(\frac{\partial_t R}{c_s}\right)^2\right)$$

Together with the thermodynamics identity: $\quad p = \frac{dE}{dR} \equiv \frac{\partial_t E}{\partial_t R}$

$$\Rightarrow \quad p = -\mathscr{C}_w\frac{\hbar c_s}{48\pi R^2(t)}\left(1+\left(\frac{\partial_t R}{c_s}\right)^2+\frac{R\partial_t^2 R}{c_s^2}\right)$$

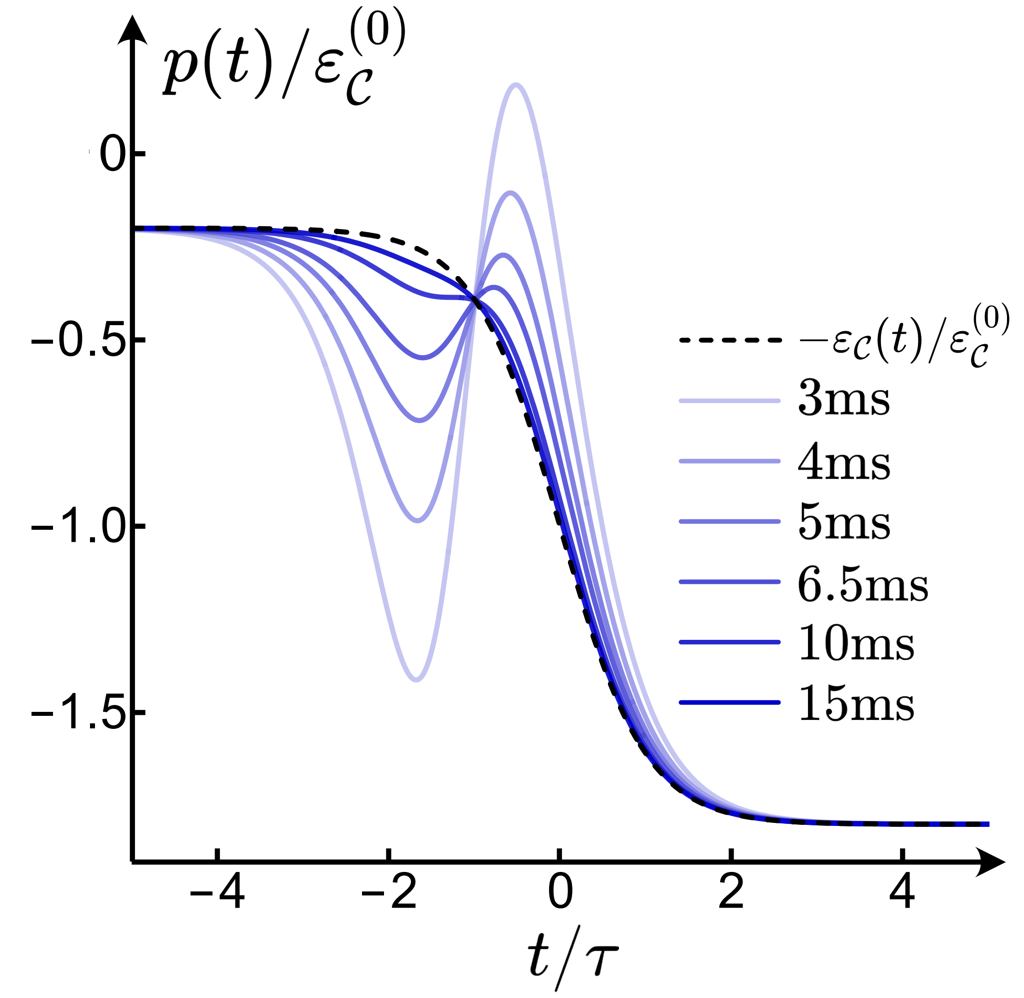
3. Gravitational anomalies and the anomalous Casimir effect

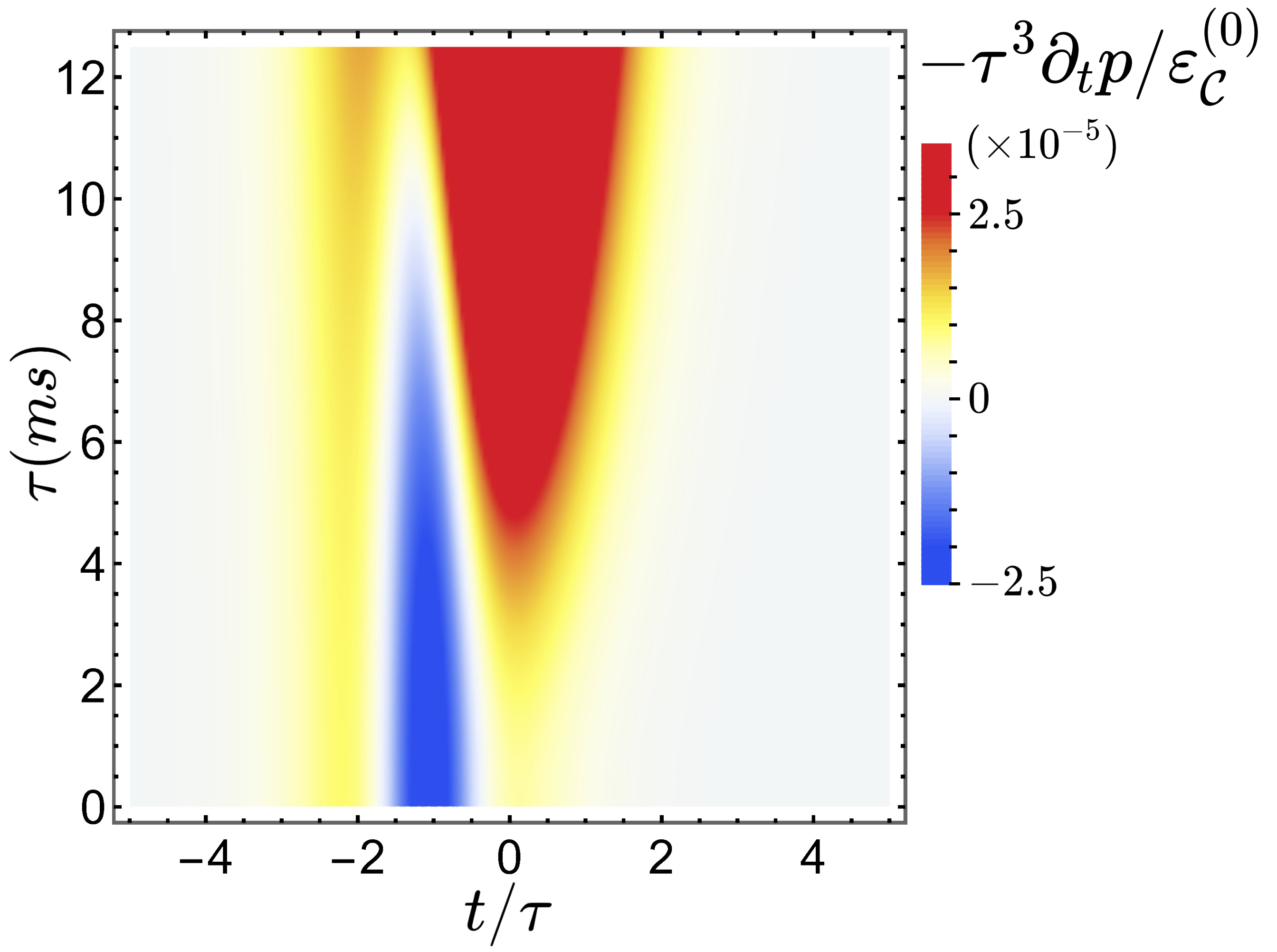
3.a. The Casimir effect, or how confinement modify vacuum properties of a system

3.b. Casimir effect in expanding ring: First predictions and Curved space time analog

3.c. Another geometric effect: gravitational anomalies and and the anomalous Casimir effect

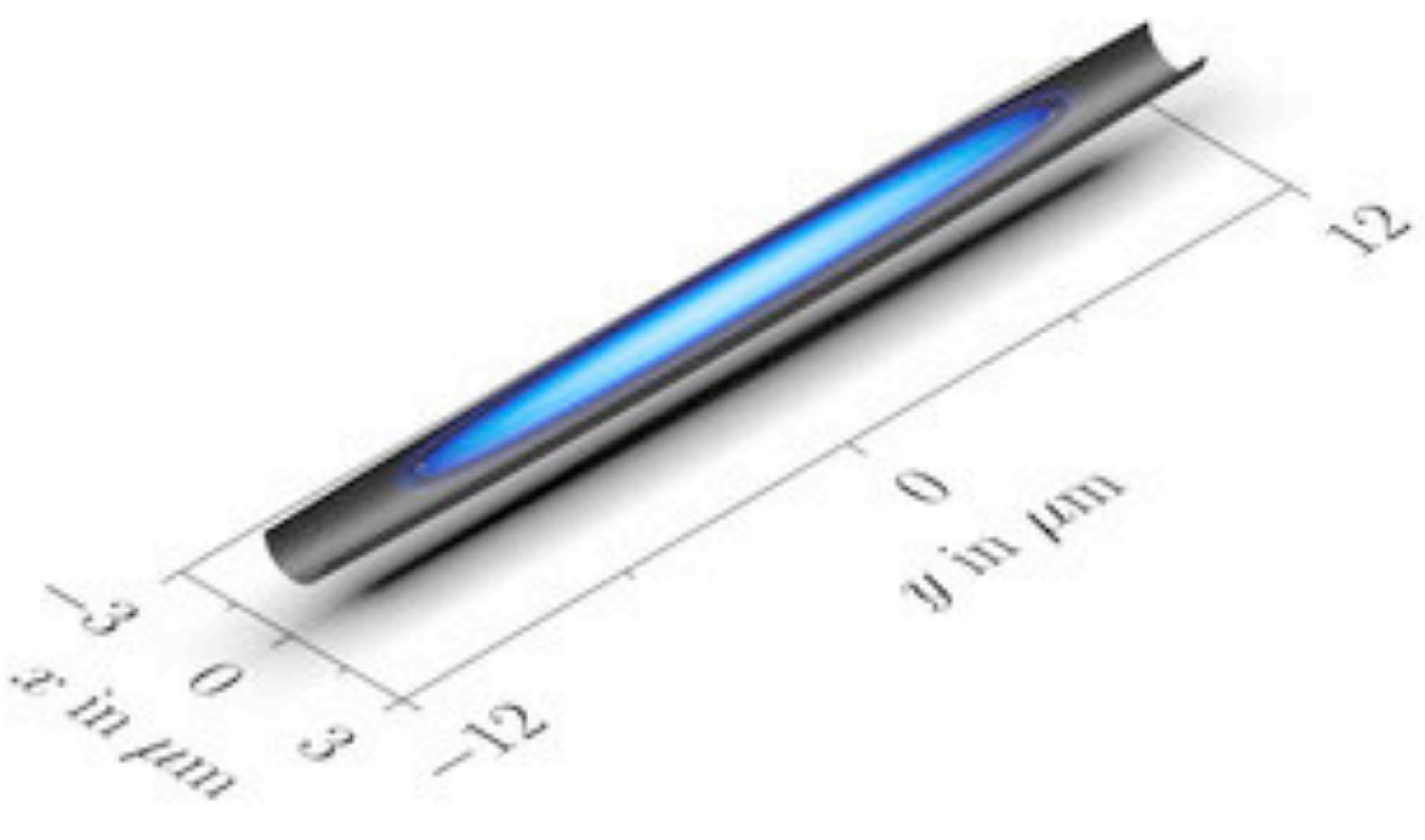
3.d. Extension to velocity modulated systems

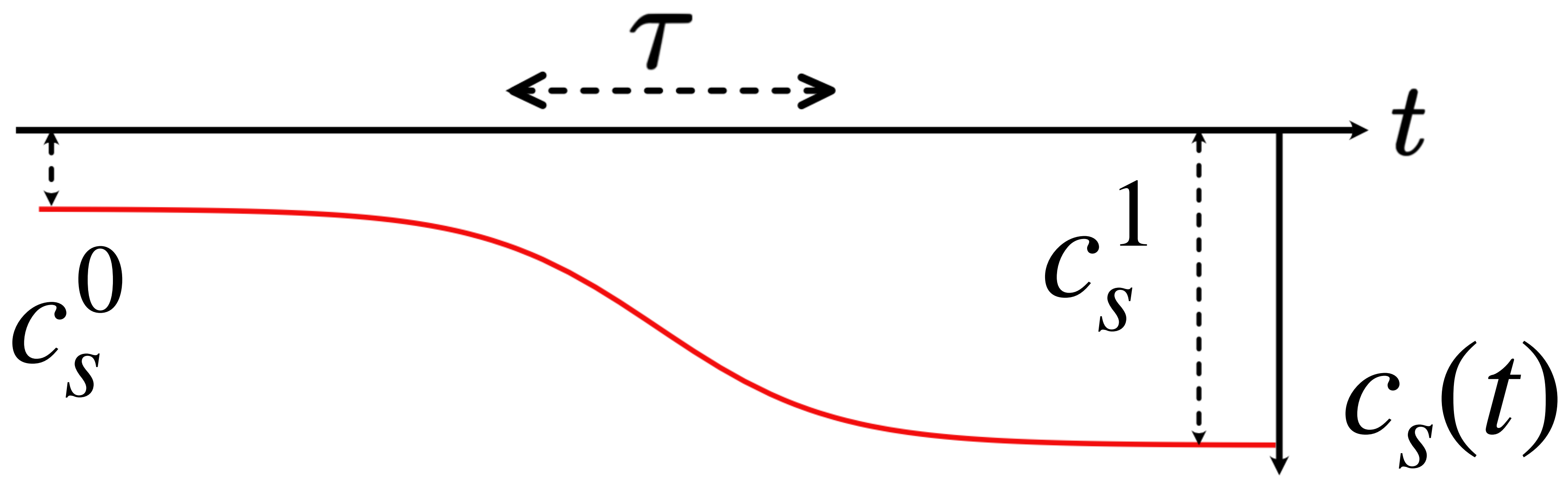




1+1D Casimir effect in velocity modulated cavities

What if the velocity c_s depends on time?





Curved spacetime description

$$\mathrm{d}s^2 = \frac{c_s(t)}{c_0} c_0^2 \mathrm{d}t^2 - \frac{c_0}{c_s(t)} \mathrm{d}x^2$$

In the absence of gravitational anomalies

$$\varepsilon = -\frac{\mathscr{C}_w}{2} \varepsilon_{\mathscr{C}}(t)$$

$$p = -\frac{\mathscr{C}_w}{2} \varepsilon_{\mathscr{C}}(t)$$

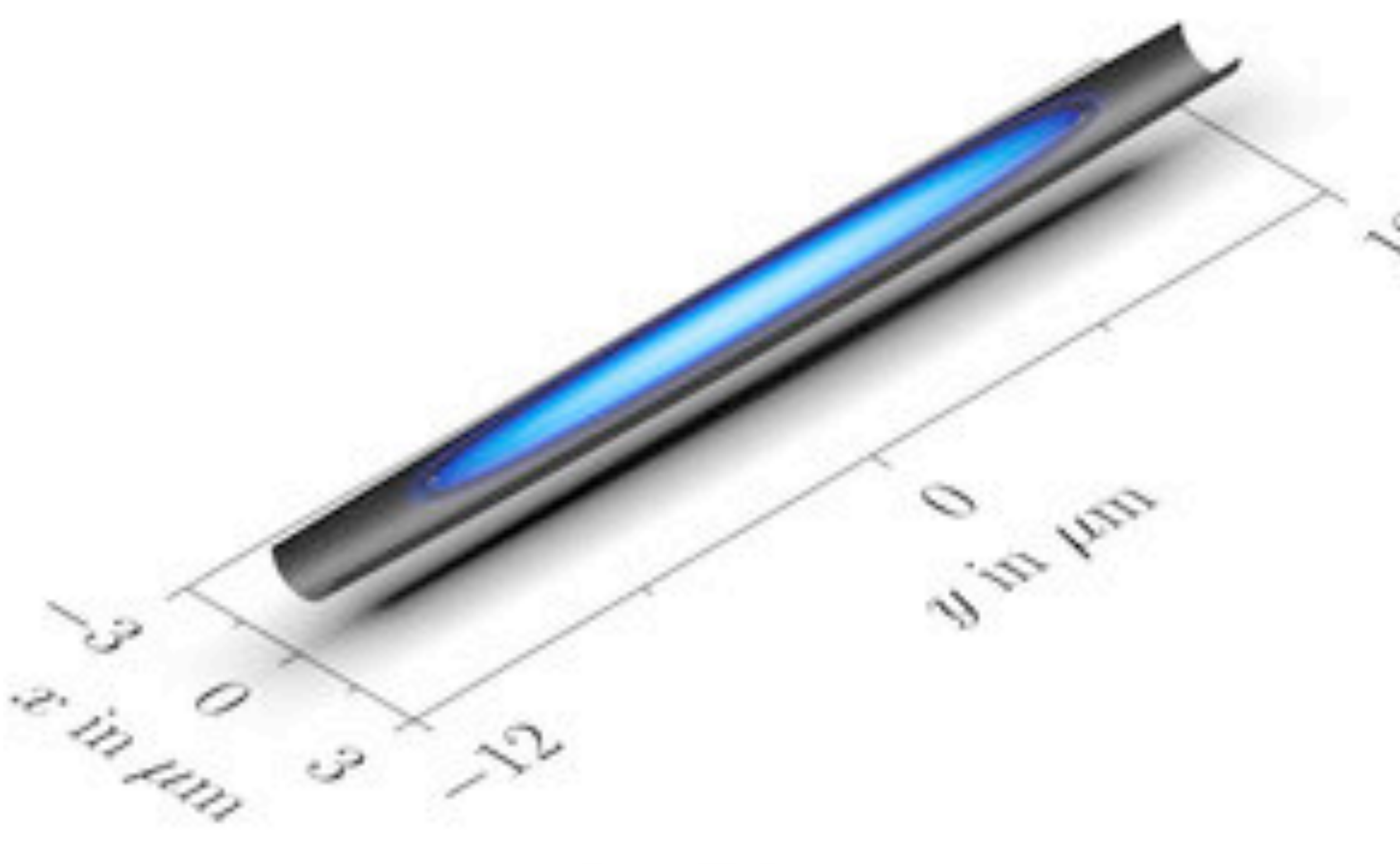
$$J_\varepsilon / c_s = \Pi c_s = -\frac{\mathscr{C}_g}{2} \varepsilon_{\mathscr{C}}(t)$$

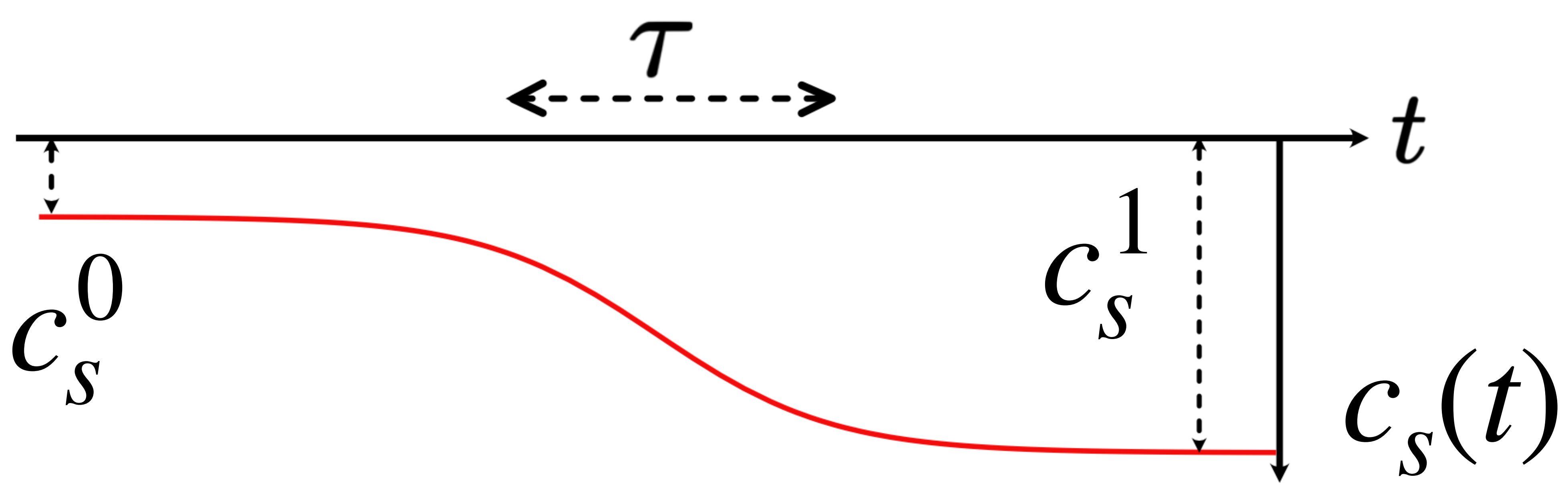
Instantaneous Casimir energy density: $\varepsilon_{\mathscr{C}}(t)$

$$\varepsilon_{\mathscr{C}}(t) = \frac{\pi \hbar c_s(t)}{6L^2}$$

1+1D anomalous Casimir effect in velocity modulated cavities

What if the velocity c_s depends on time?





Curved spacetime description

$$\mathrm{d}s^2 = \frac{c_s(t)}{c_0} c_0^2 \mathrm{d}t^2 - \frac{c_0}{c_s(t)} \mathrm{d}x^2 \qquad \mathscr{R} = -\frac{1}{c_s^2}\frac{\partial_t^2 c_s}{c_s} + 2\left(\frac{\partial_t c_s}{c_s}\right)^2$$

In the presence of gravitational anomalies

$$\varepsilon = \frac{\mathscr{C}_w}{2}\left(-\varepsilon_{\mathscr{C}}(t) + \varepsilon_{\overline{\mathscr{R}}}\right)$$

$$p = \frac{\mathscr{C}_w}{2}\left(-\varepsilon_{\mathscr{C}}(t) + \varepsilon_{\overline{\mathscr{R}}} - 2\varepsilon_{\mathscr{R}}\right)$$

$$J_\varepsilon / c_s = \Pi c_s = \frac{\mathscr{C}_g}{2}\left(\varepsilon_{\mathscr{C}}(t) + \varepsilon_{\overline{\mathscr{R}}} - \varepsilon_{\mathscr{R}}\right)$$

2 new energy scales

$$\varepsilon_{\mathscr{R}} = \frac{\hbar}{48\pi c_s}\left[\frac{\partial_t^2 c_s}{c_s} - 2\left(\frac{\partial_t c_s}{c_s}\right)^2\right] \qquad \varepsilon_{\overline{\mathscr{R}}} = -\frac{\hbar}{96\pi c_s}\left(\frac{\partial_t c_s}{c_s}\right)^2$$

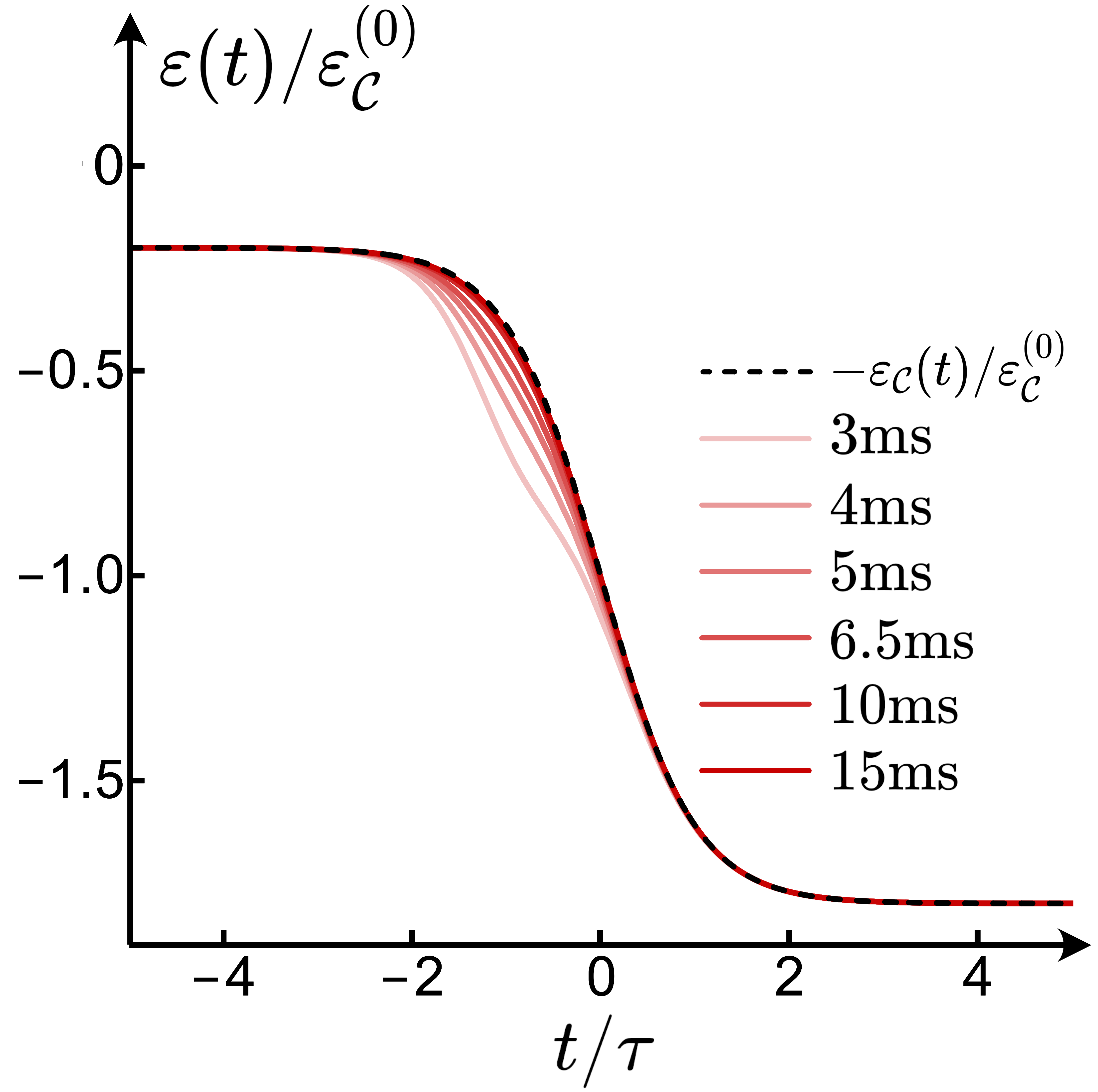
1+1D anomalous Casimir effect in velocity modulated cavities

Consequences on energy:

Experimentally relevant parameters
$$L = 10\mu\text{m}$$
$$c_s^0 = 8.10^{-4}\text{m.s}^{-1}$$
$$c_s^1 = 7.10^{-3}\text{m.s}^{-1}$$

Typical for Bose-Einstein condensates experiments

S.Eckel et. Al. (2021)

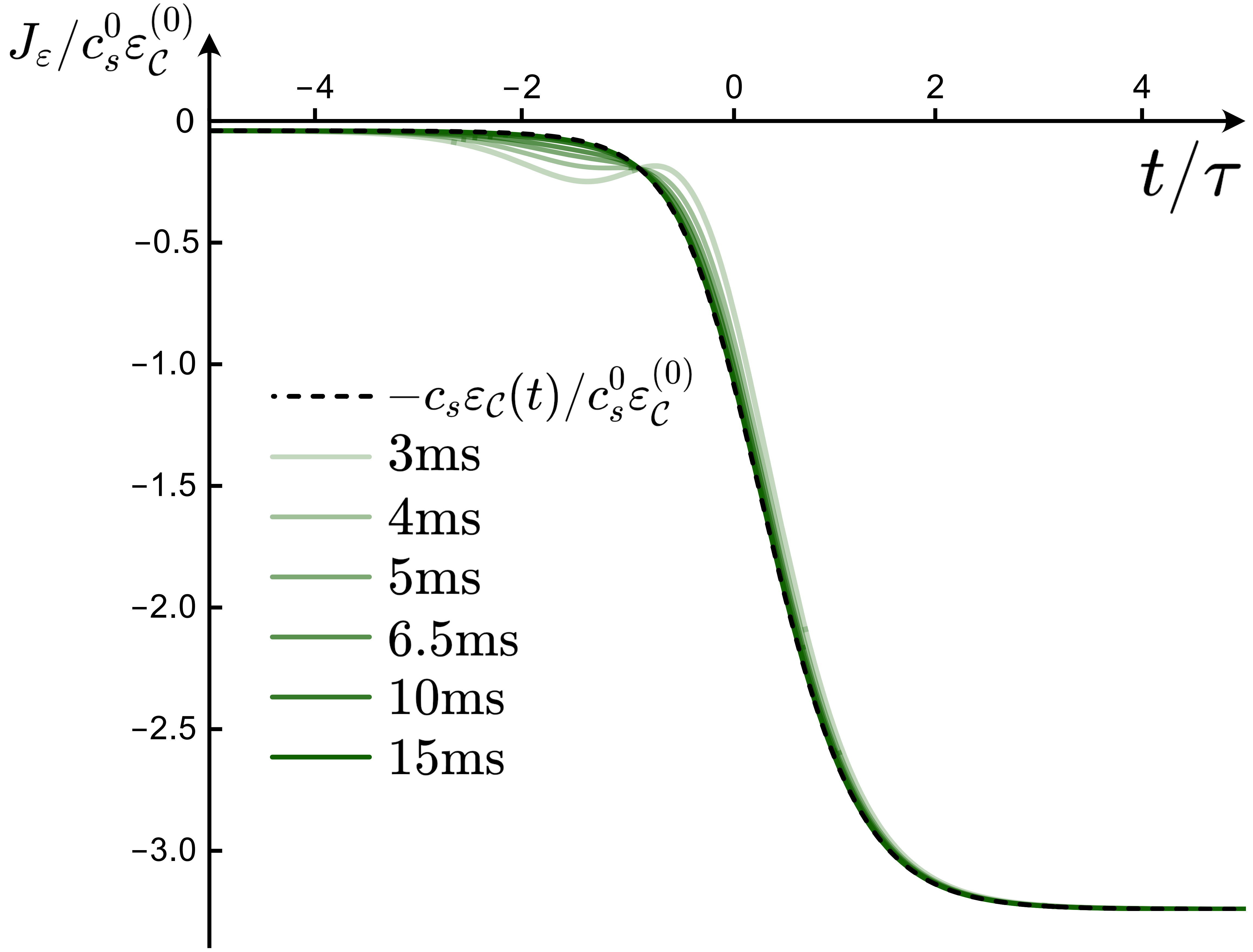


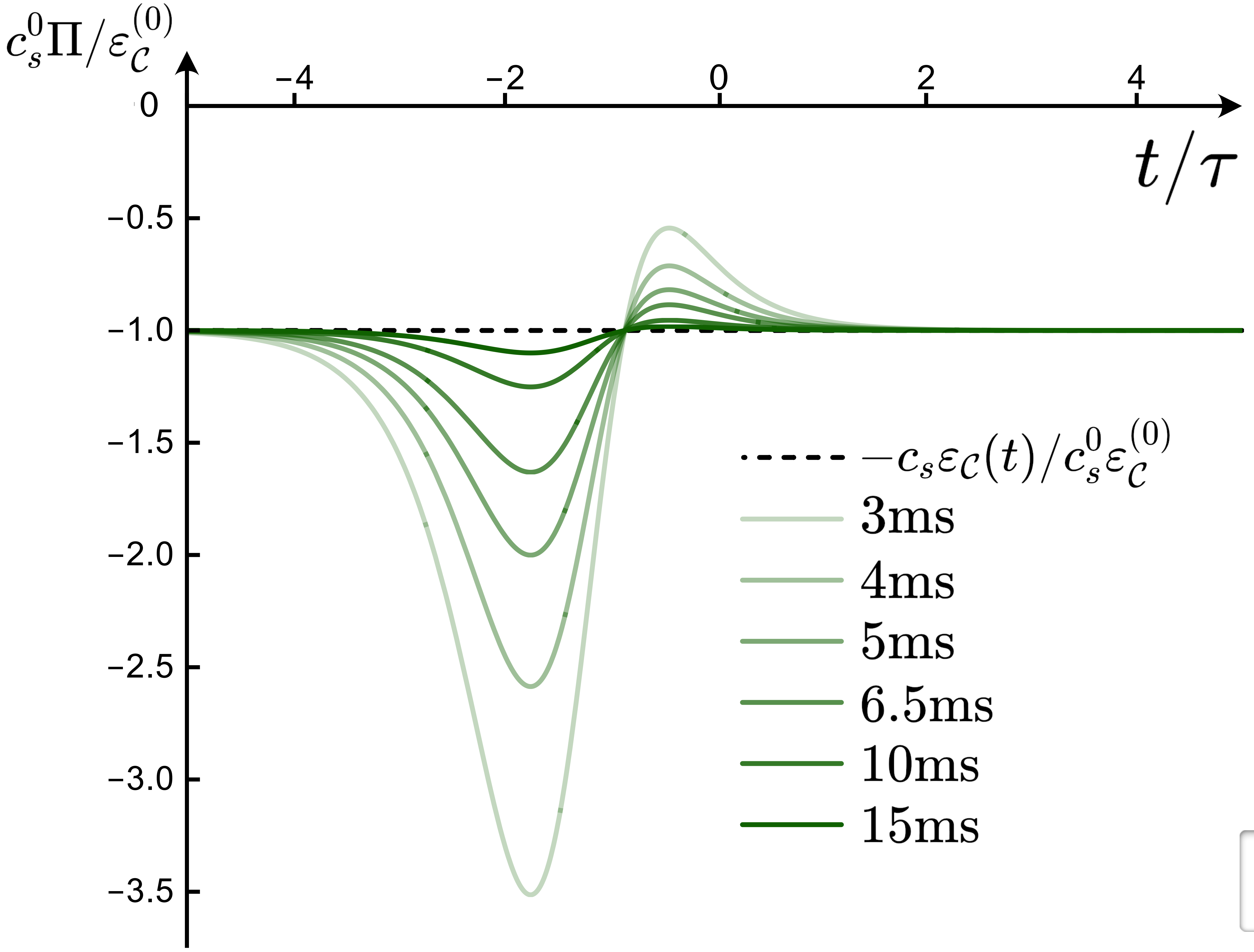
1+1D anomalous Casimir effect in velocity modulated cavities

Consequences on energy currents in chiral systems ($\mathscr{C}_g \neq 0$):

Experimentally relevant parameters $L = 10\mu\text{m}$, $c_s^0 = 8.10^{-4}\text{m.s}^{-1}$, $c_s^1 = 7.10^{-3}\text{m.s}^{-1}$ — Typical for Bose-Einstein condensates experiments

S.Eckel et. Al. (2021)





1+1D anomalous Casimir effect in velocity modulated cavities

Consequences on pressure:

Experimentally relevant parameters $\left.\begin{aligned} L &= 10\mu\text{m} \\ c_s^0 &= 8.10^{-4}\text{m.s}^{-1} \\ c_s^1 &= 7.10^{-3}\text{m.s}^{-1} \end{aligned}\right\}$ Typical for Bose-Einstein condensates experiments

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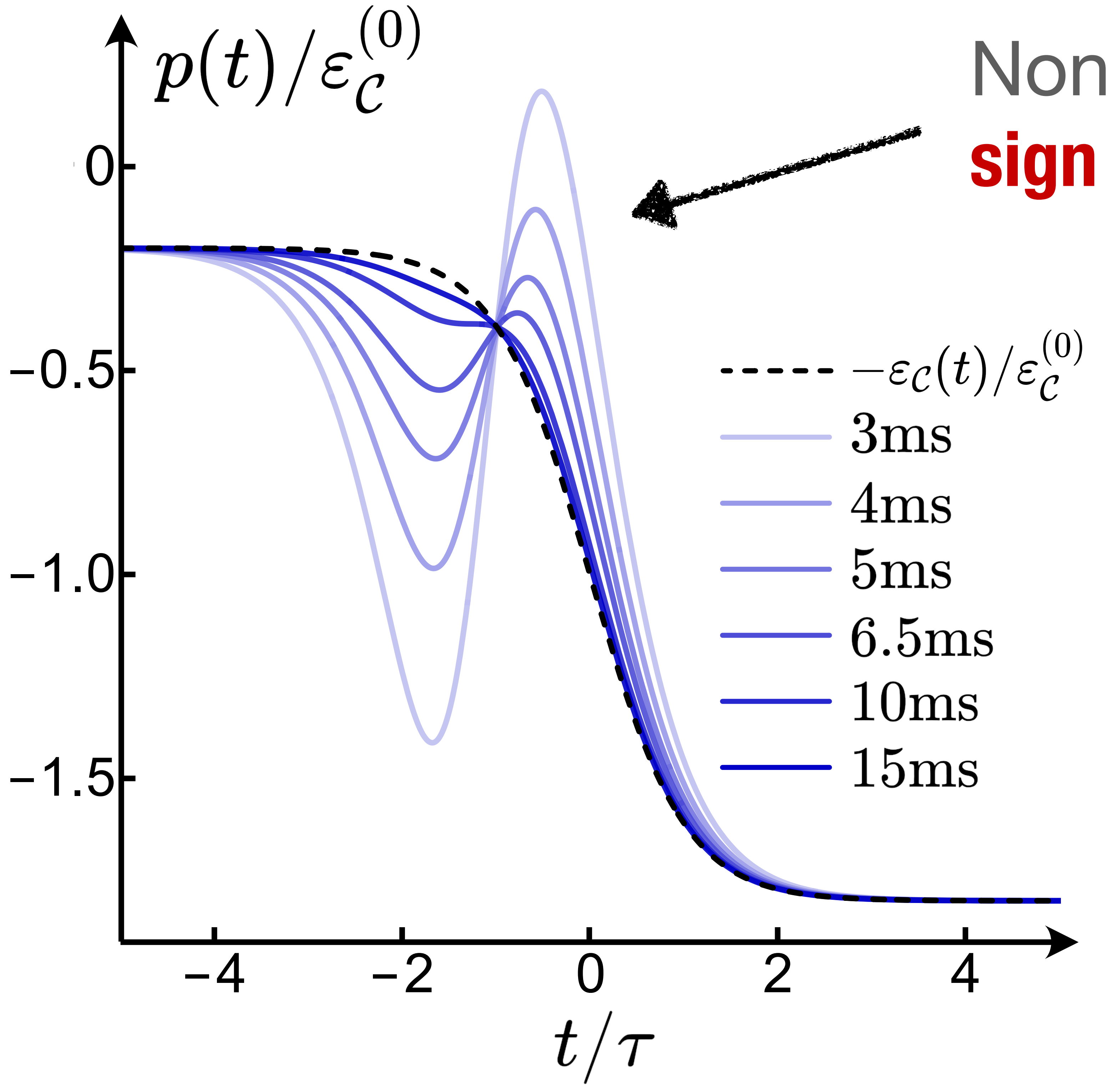

Non monotonous pressure
sign of an instability

1+1D anomalous Casimir effect in velocity modulated cavities

Consequences on pressure:

Experimentally relevant parameters $L = 10\mu\text{m}$, $c_s^0 = 8.10^{-4}\text{m.s}^{-1}$, $c_s^1 = 7.10^{-3}\text{m.s}^{-1}$ — Typical for Bose-Einstein condensates experiments

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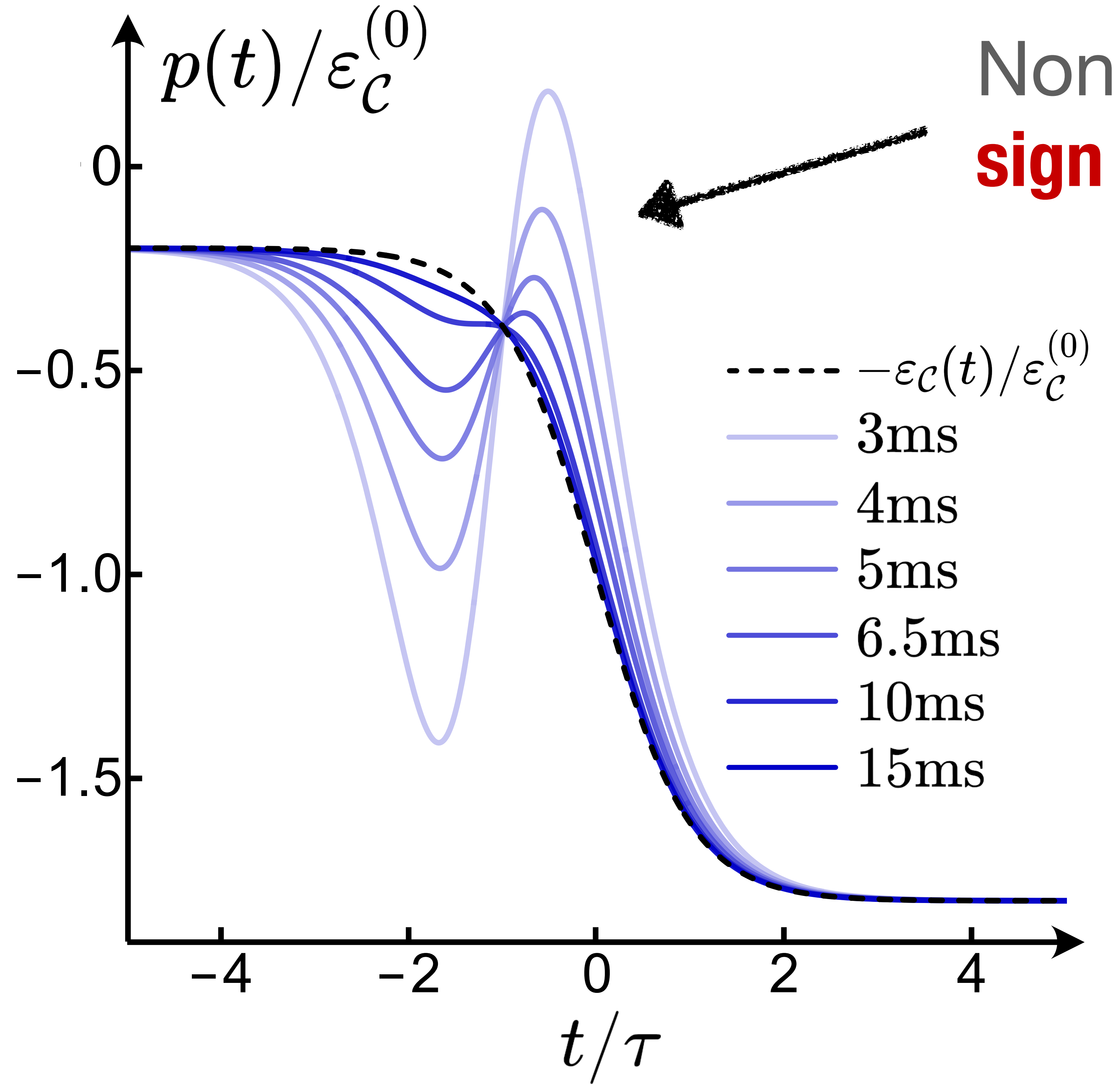
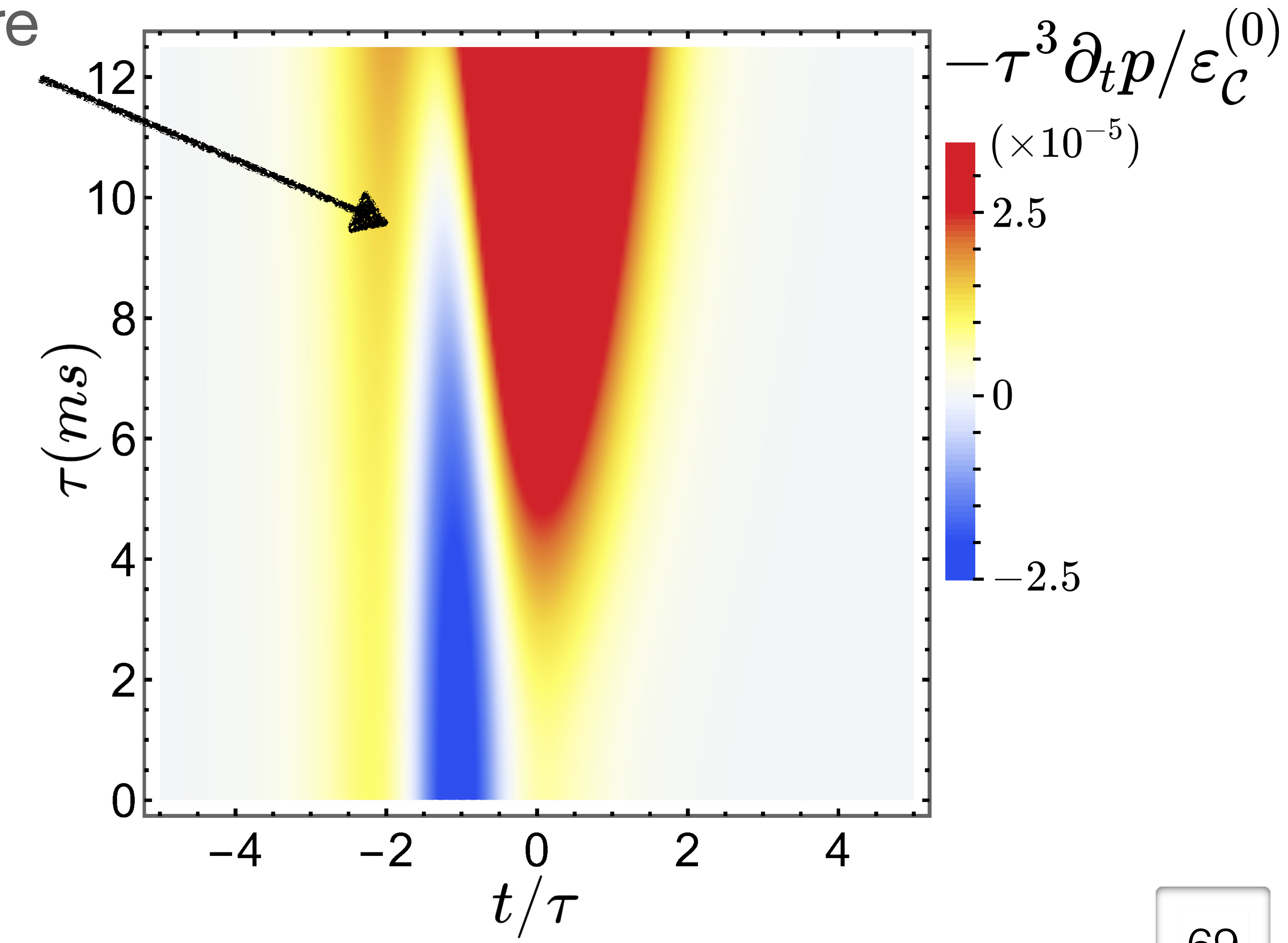


Non monotonous pressure

sign of an instability



1+1D anomalous Casimir effect in velocity modulated cavities

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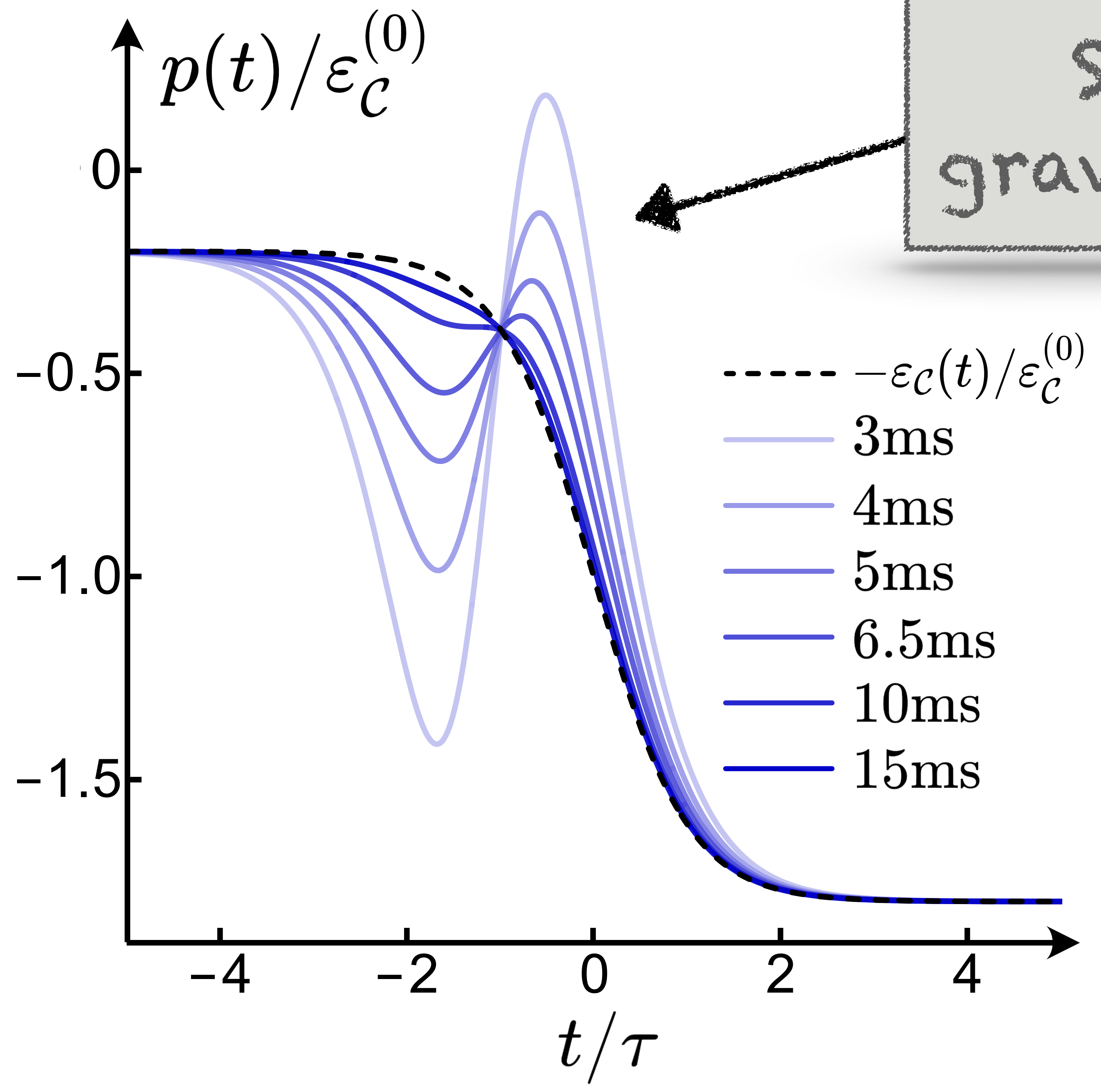
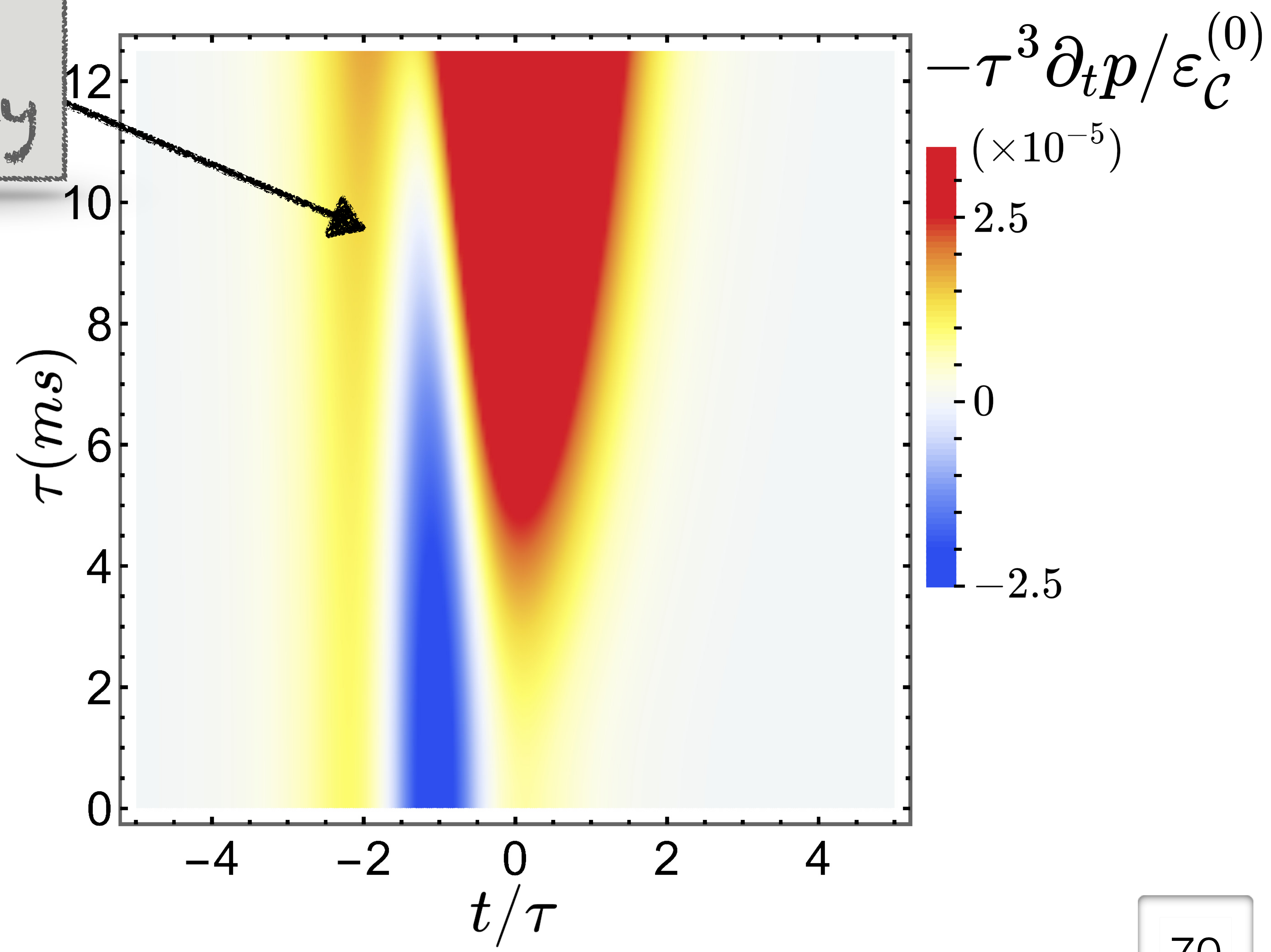
Typical for Bose-Einstein condensates experiments

S.Eckel et. Al. (2021)

Signature of the gravitational anomaly





Conclusion:

- **Curved spacetimes arise naturally in condensed matter**

  In this presentation: Curved spacetimes in classical and quantum fluids

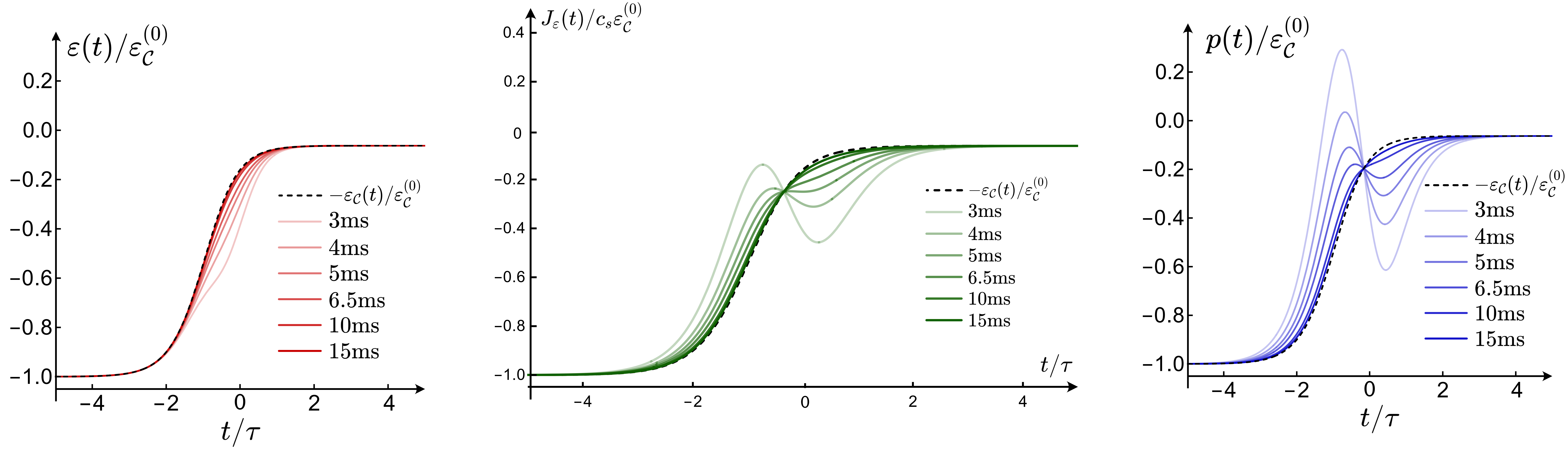
  Curved spacetimes in chiral systems: Quantum Hall edges

- **Conformal anomalies induce sizable corrections to out-of equilibrium thermodynamics**

  In this presentation: Within generalized hydrodynamics

  System driven out-of-equilibium by its geometry

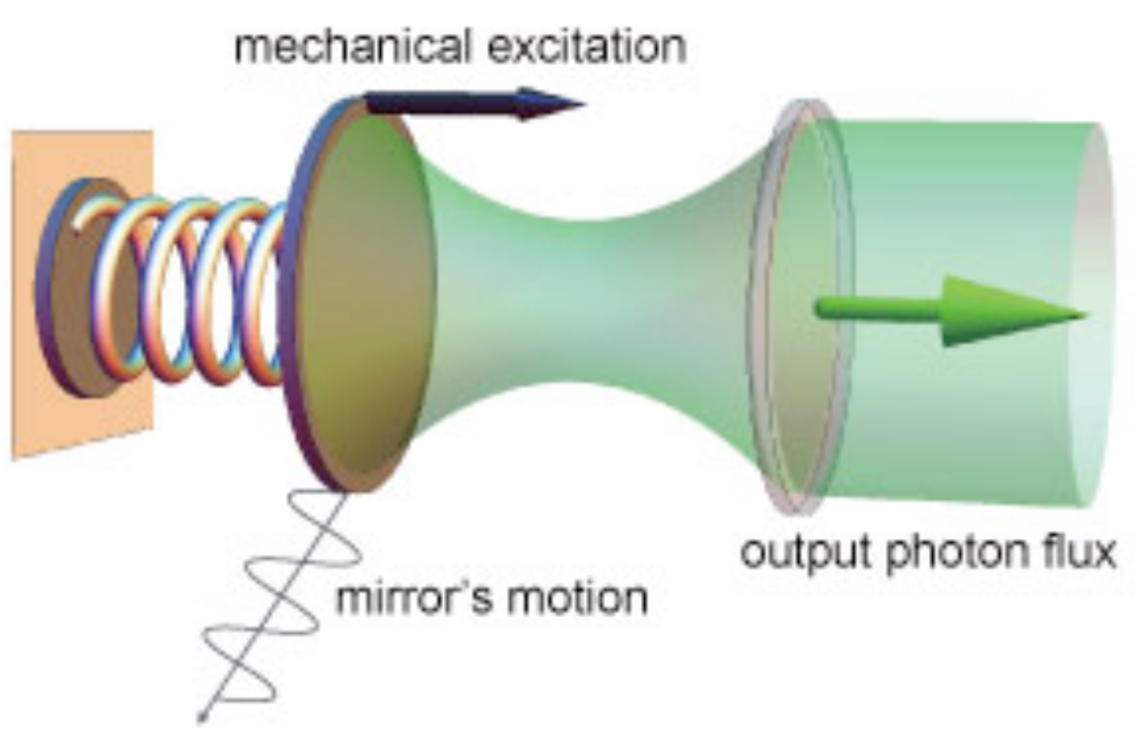
  Thermodynamic instabilities due to anomalies



Conclusion:

- **Comparisons to previous work on dynamical Casimir effects?**

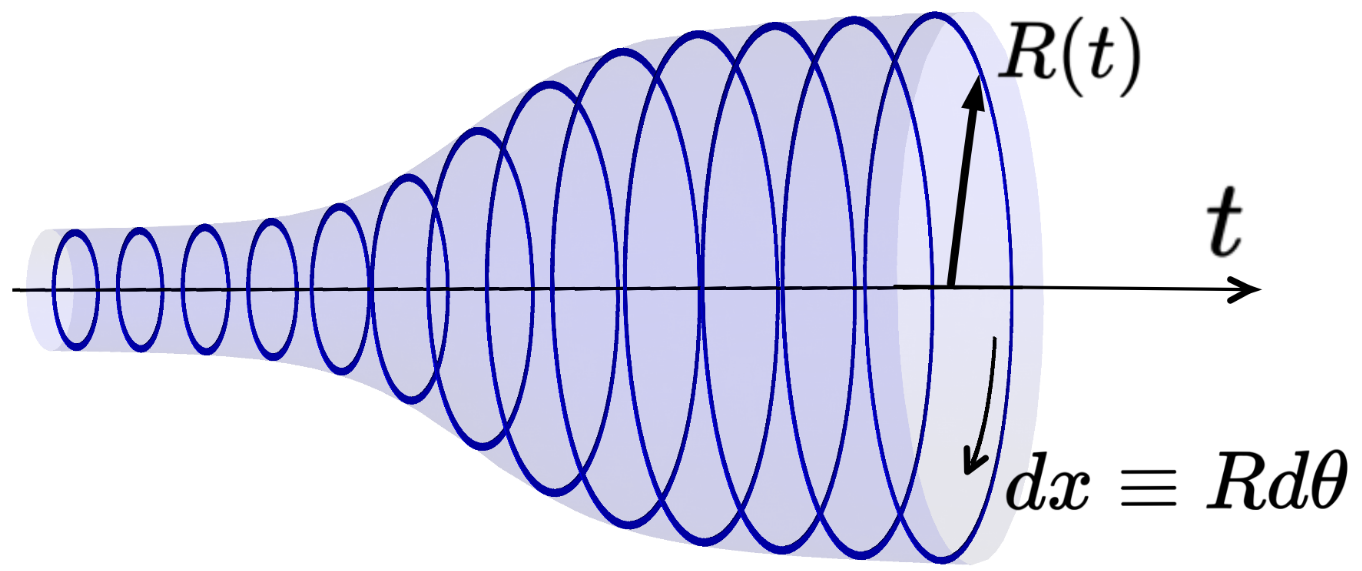
Dynamical Casimir effect



$\mathscr{R} = 0$

Anomalous Dynamical Casimir effect

Spatial curvature varying in time



$\mathscr{R} = -\frac{2}{c_s^2}\frac{\partial_t^2 R}{R}$

C. Fulling and P. Davies on the relationship between dynamical Casimir effect and anomalies

> The relation of that effect, which involves a failure of the usual tracelessness of $\mathscr{T}_{\mu\nu}$ to the present work (dynamical Casimir effect) is unclear
>
> C. Fulling & P. Davies. (1976)