Hybrid Quantum State in 2d Dilaton Gravity

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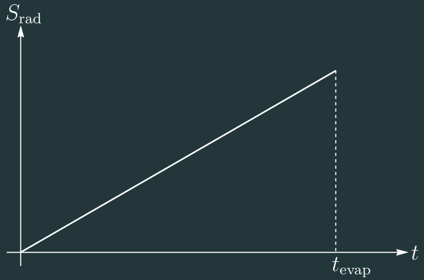
Black hole thermodynamics: a brief recap

- Bekenstein ('72-73): black holes have an entropy $\propto$ surface area of horizon.
- Hawking ('74-75): black holes emit thermal radiation and eventually evaporate.

$\hookrightarrow$ Bekenstein-Hawking entropy: $S_{BH} = \frac{A}{4}$.

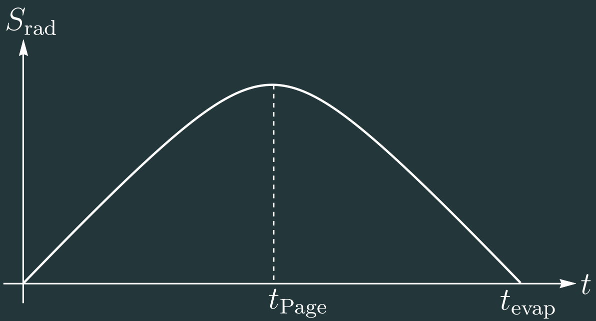
- "Information paradox": a black hole formed from a pure state ($S_{\mathrm{initial}} = 0$) will lead to a steady increase in entropy until its full evaporation ($S_{\mathrm{final}} > 0$).



$\hookrightarrow$ Non-unitary evolution, in disagreement with quantum mechanics.

The Page curve

- Page ('93): if evolution is unitary the entanglement entropy of the radiation will start decreasing around $t_{\mathrm{Page}} \simeq \frac{1}{2} t_{\mathrm{evap}}$.



$\hookrightarrow$ Entanglement between outside radiation and the inside of the black hole: $S_{\mathrm{outside}} = S_{inside}$ so when the black hole shrinks $S \to 0$.

$\hookrightarrow$ Which curve is the correct one ?

A possible way towards the resolution

- ’t Hooft, Susskind (’93): holographic principle, information contained in a volume of space is encoded on its boundary.

$\hookrightarrow$ Realized by AdS/CFT correspondence (Maldacena, 1997).

$\hookrightarrow$ In this context, a CFT on the boundary will evolve unitarily, thus an interior black hole will do too.

$\hookrightarrow$ Provides an argument in favor of unitary evolution, however it does not explain the exact process.

- A possible explanation is the ”island” rule for black hole entropy.

$\hookrightarrow$ Penington ’19, Almheiri et al. ’19.

$\hookrightarrow$ For a nice introduction and review see ”The entropy of Hawking radiation”, Almheiri et al., ’20.

A semi-classical approach

- Hawking's calculation was semi-classical: quantized fields propagate on a classical background spacetime.
- Berthiere, Sarkar, Solodukhin ('17): consider conformally coupled fields propagating on a $4\mathrm{d}$ static spherically symmetric metric.

$\hookrightarrow$ Quantum effective action determined by the conformal anomaly.

$\hookrightarrow$ Two important result from this analysis:

i) There is no static solution with an horizon of finite temperature.

ii) The classical horizon is replaced by the throat of a Damour-Solodukhin wormhole (black hole mimicker).

$\hookrightarrow$ This suggests that the back-reaction of the fields can modify largely the geometry near the horizon.

$\hookrightarrow$ Complicated to lead a full analysis in $4\mathrm{d}$.

Two-dimensional semi-classical gravity

- Working in 2d will give us more analytical control: curvature has only 1 independent component and every metric is conformally flat:

$$\mathrm{d}s^2 = -e^{2\rho}\mathrm{d}x^+\mathrm{d}x^- \,. \tag{1}$$

- If we consider conformally coupled fields, which contribute via the conformal anomaly to the energy-momentum tensor, the effective action is known (Polyakov, '81)
- However in 2d the Einstein-Hilbert action is topological (Gauss-Bonnet):

$$S_{EH} = \frac{1}{16\pi G}\int_M \mathrm{d}^2x\,\sqrt{-g}\,R = \frac{1}{8G}\chi(M) \tag{2}$$

$\hookrightarrow$ we need another action.

Alternative actions

- We can either take a function of the curvature:

$$S = \int \mathrm{d}^2 x \sqrt{-g}\, f(R)\,, \tag{3}$$

or introduce a scalar field ϕ (dilaton) and take an action $S[g_{\mu\nu}, \phi]$.

- Jackiw-Teitelboim gravity:

$$S_{JT} = \int \mathrm{d}^2 x \sqrt{-g}\, \phi(R - \Lambda)\,, \tag{4}$$

$\hookrightarrow$ studied in the context of the information paradox (Almheiri et al. '19, Pedraza et al. '19)

- CGHS model (Callan, Giddings, Harvey, Strominger, 1991):

$$S_{CGHS} = \frac{1}{2\pi} \int \mathrm{d}^2 x \sqrt{-g} \left\{ e^{-2\phi} \left[R + 4(\nabla\phi)^2 + 4\lambda^2 \right] - \frac{1}{2} \sum_{i=1}^{N} (\nabla f_i)^2 \right\}, \tag{5}$$

$\hookrightarrow$ we will focus on this "string inspired" integrable model.

Interpretation of the dilaton ϕ as a radius

- Consider a $4d$ spherically symmetric metric

$$\tilde{g} = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu + r^2(x^0, x^1)\mathrm{d}\Omega^2\,, \quad \mu,\nu = 0,1\,. \tag{6}$$

Plugging this into the Einstein-Hilbert action and setting $r \equiv \frac{1}{\lambda}e^{-\phi}$ gives

$$S_{EH}[\tilde{g}] = \frac{1}{2\pi}\int \mathrm{d}^2x\,\sqrt{-g}\,e^{-2\phi}\left[R + 2(\nabla\phi)^2 + 2\lambda^2 e^{2\phi}\right], \tag{7}$$

which is similar to the CGHS action.

$\hookrightarrow$ One can think of $e^{-\phi}$ as the radius of a transverse sphere.

- Trapped point: the area of the transverse sphere decreases in both null directions:

$$\partial_\pm e^{-\phi} < 0 \Leftrightarrow \partial_\pm\phi > 0\,. \tag{8}$$

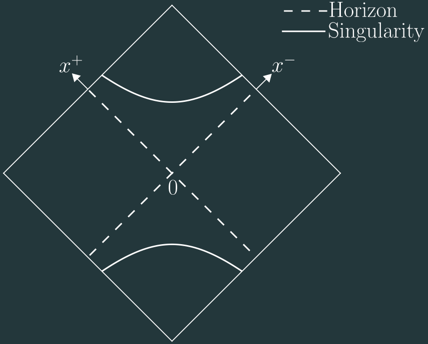
$\hookrightarrow$ Apparent horizon $=$ boundary of a region of trapped points.

Classical eternal black hole

- General static solution of CGHS without matter:

$$ds^2 = -e^{2\phi}dx^+dx^- = \frac{dx^+dx^-}{\frac{M}{\lambda} - \lambda^2 x^+ x^-}\,, \tag{9}$$

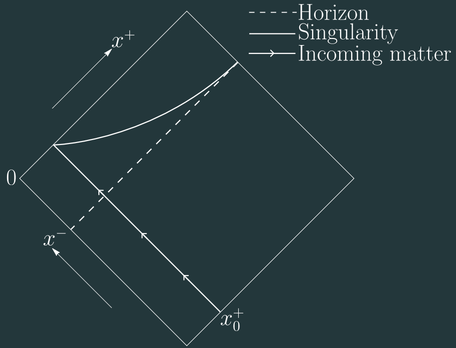
with M the ADM mass.



$\hookrightarrow$ Schwarzschild-like geometry.

Classical black hole formation

- Start from Minkowski and send in a shock wave of mass m along a null line $x^+ = x_0^+$:



$\hookrightarrow$ From the conformal anomaly and energy conservation one can compute Hawking radiation at $\mathcal{I}^+$, radiation is thermal but we have neglected the back-reaction.

$\hookrightarrow$ We need to include quantum corrections directly into the model.

The semi-classical RST model

- Russo, Susskind, Thorlacius (1992): extend the CGHS model by adding the effective Polyakov action:

$$\mathcal{S}_{RST} = \mathcal{S}_{CGHS} - \frac{\kappa}{2\pi} \int \mathrm{d}^2x \sqrt{-g} \left\{ \frac{1}{2}(\nabla\psi)^2 + (\psi + \phi)R \right\}. \tag{10}$$

$\kappa \sim$ number of fields

$\psi =$ auxiliary scalar field satisfying $\Box\psi = R$

$\hookrightarrow$ different solutions corresponding to different boundary conditions,

$\hookrightarrow$ we interpret these as different quantum states for the fields,

$\hookrightarrow$ we define them by fixing the energy density T of the fields at flat infinity,

$$T^{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta S_{\mathrm{matter}}}{\delta g_{\mu\nu}}. \tag{11}$$

Quantum states

- **Hartle-Hawking** state: thermal energy density at infinity, energy-momentum tensor regular at the classical horizon.

$\hookrightarrow$ Well suited to describe Hawking radiation.

- **Boulware** state: energy density vanishes at infinity, however, when considered on the classical black hole metric, the energy-momentum diverges at the horizon.

$\hookrightarrow$ May describe non-physical particles that should not be detected at infinity (*e.g.* ghosts).

- **Unruh** state: vanishing energy density at past infinity and thermal at future infinity.

$\hookrightarrow$ Most realistic state to describe a black hole formation and evaporation.

A general solution

- For an initially static spacetime perturbed by a shock wave at $x^+ = x_0^+$ the solution is given by

$$\mathrm{d}s^2 = -e^{2\phi}\mathrm{d}x^+\mathrm{d}x^- \,, \tag{12}$$

where $\phi(x^+, x^-)$ is determined by the master equation

$$\Omega(\phi) \equiv e^{-2\phi} + \kappa\phi = -\lambda^2 x^+ x^- + 2\kappa P \ln(-\lambda^2 x^+ x^-) + \frac{M}{\lambda} - \frac{m}{\lambda x_0^+}(x^+ - x_0^+)\theta(x^+ - x_0^+) \,, \tag{13}$$

$P =$ constant linked to asymptotic energy density,

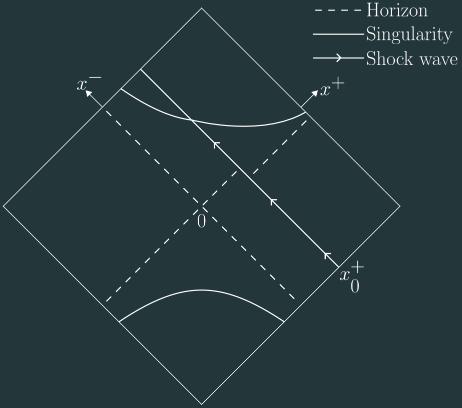
$M =$ mass initially present,

$m =$ mass of incoming shock wave (static solution if $m = 0$)

$\hookrightarrow$ in the limit $\kappa \to 0$ we recover the classical solution.

Hartle-Hawking solution ($P = 0$)

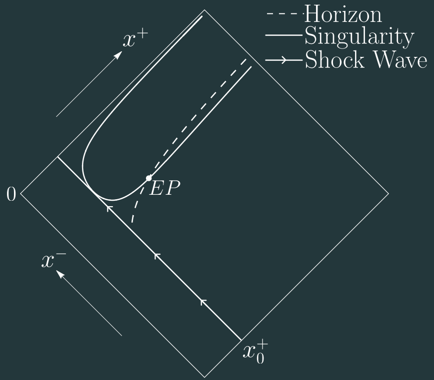
- There is always a singularity, even for $M = 0$, which is spacelike and hidden behind the horizon for large enough κ or M. The shock wave simply displaces it as well as the horizon.



$\hookrightarrow$ Cannot give Minkowski since it does not contain radiation.

Boulware solution ($P = -\frac{1}{4}$)

- $M = 0$ gives Minkowski, a shock wave creates an horizon and a singularity.

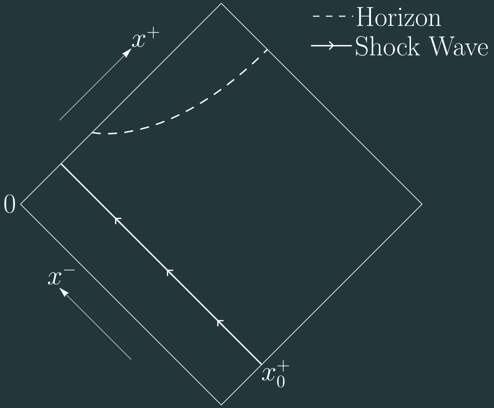


$\hookrightarrow$ A naked singularity appears at a finite point, can be interpreted as the evaporation point, where the model breaks down.

$\hookrightarrow$ Using the island rule one can derive a Page curve for this solution (Hartman et al. '20).

Boulware with negative κ

- A field in the Boulware state does not radiate at $\infty \to$ well suited for non-physical particles (*e.g.* ghosts) that contribute negatively to the central charge $\kappa \to$ consider the Boulware solution with $\kappa < 0$.



$\hookrightarrow$ no singularity, geodesically complete, but no radiation.

Hybrid quantum state

- We assume that 2 types of fields are present, physical and non-physical, and that they are in 2 different quantum states, Hartle-Hawking and Boulware respectively.

$\hookrightarrow$ 2 auxiliary fields ψ_1 and ψ_2 with suitable boundary conditions and associated to 2 central charges $\kappa_1 > 0$ and $\kappa_2 < 0$.

$\hookrightarrow$ this yields

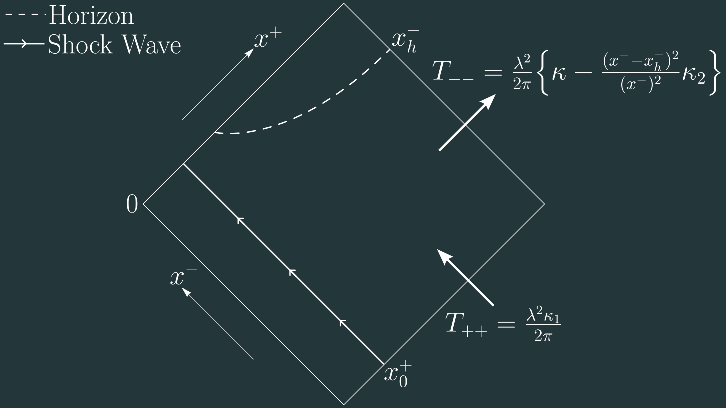
$$e^{-2\phi} + \kappa\phi = -\lambda^2 x^+ x^- - \frac{\kappa_2}{2}\ln(-\lambda^2 x^+ x^-) + \frac{M}{\lambda} - \frac{m}{\lambda x_0^+}(x^+ - x_0^+)\theta(x^+ - x_0^+)\,, \quad (14)$$

where $\kappa = \kappa_1 + \kappa_2$.

- $\kappa < 0$: spacetime is geodesically complete and fully regular, with an horizon and radiation.

Hybrid Hartle-Hawking/Boulware solution

- Representing the energy fluxes at ∞ we get:



$\hookrightarrow$ Deviation from thermality at $\mathcal{I}^+$.

$\hookrightarrow$ We can get rid of radiation at $\mathcal{I}^-$ by using the Unruh state.

Hybrid Unruh/Boulware state

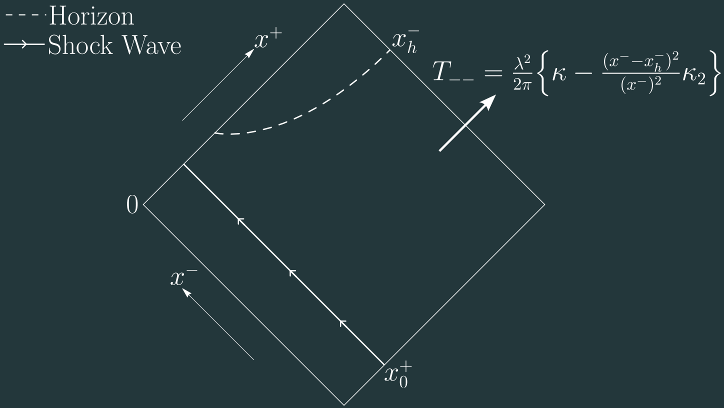
- We can modify the boundary conditions of field ψ_1 to only get radiation at $\mathcal{I}^+$. This allows us to define the Unruh state for physical particles.
- We can then consider the hybrid state where physical particles are in the Unruh state and non-physical ones in the Boulware state:

$$e^{-2\phi}+\kappa\phi = -\lambda^2 x^+ x^- - \frac{\kappa_2}{2}\ln(-\lambda^2 x^+ x^-) - \frac{\kappa_1}{2}\ln(\lambda x^+) + \frac{M}{\lambda} - \frac{m}{\lambda x_0^+}(x^+ - x_0^+)\theta(x^+ - x_0^+)\,, \tag{15}$$

$\hookrightarrow$ ψ_1 satisfies Boulware-type conditions at $\mathcal{I}^-$ and Hartle-Hawking-type at $\mathcal{I}^+$.

$\hookrightarrow$ Same spacetime structure but without radiation at $\mathcal{I}^-$.

Hybrid Unruh/Boulware solution



$\hookrightarrow$ Formation of an apparent horizon, but note that it does not capture the evaporation process as the horizon is eternal.

$\hookrightarrow$ Radiation is not thermal at $\mathcal{I}^+$, let us study the evolution of its entropy.

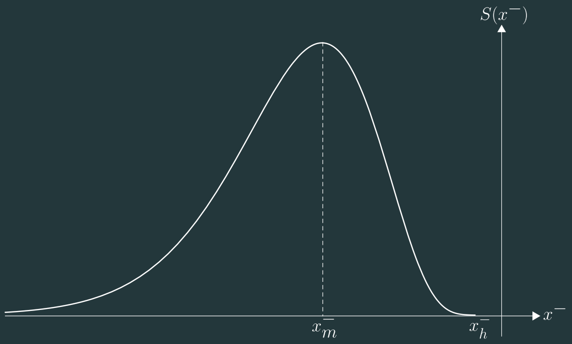
Entropy of radiation

- Define the radiation entropy at $\mathcal{I}^+$ by

$$\partial_- S = \frac{2\pi}{\lambda} T_{--} \tag{16}$$

($\mathrm{d}S = T^{-1}\mathrm{d}E$ in asymptotically flat coordinates).

$\hookrightarrow$ This entropy has a maximum at $x^- = x_m^- \equiv \left(1 - \sqrt{\frac{\kappa}{\kappa_2}}\right)^{-1} x_h^-$ and can be shown to follow a Page curve behavior:



Conclusion and possible continuations

- The back-reaction of particles on the spacetime indeed modifies the geometry.
- The hybrid state solution where non-physical particles dominate ($\kappa < 0$) turns out to be the most interesting one: the singularity disappears and we are left with a geodesically complete causal diamond, and we can derive a Page curve for the radiation entropy.
- However the picture does not seem complete since the horizon does not disappear.
- A possible continuation would be to make the connection with the island procedure by applying it to our hybrid solution.
- Many open questions remain, such as considering massive and/or interacting fields, can this be reproduced in higher dimensions, etc.

Thank you!