# Defects and scaling in disordered Tomonaga-Luttinger liquids

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University of Bonn

Galeski et al. Phys. Rev. Lett. 128, 237201

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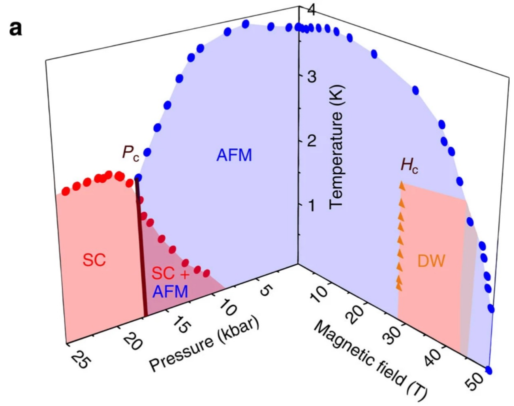
Interactions

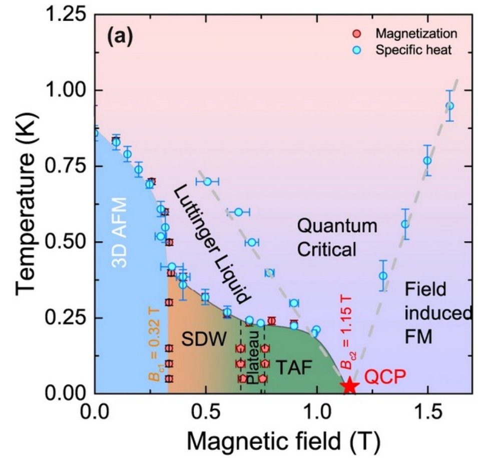
Competing orders

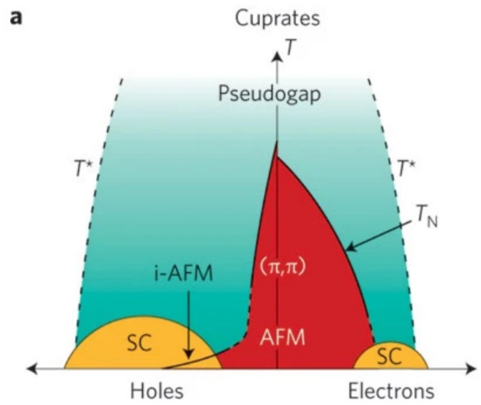
Quantum critical points

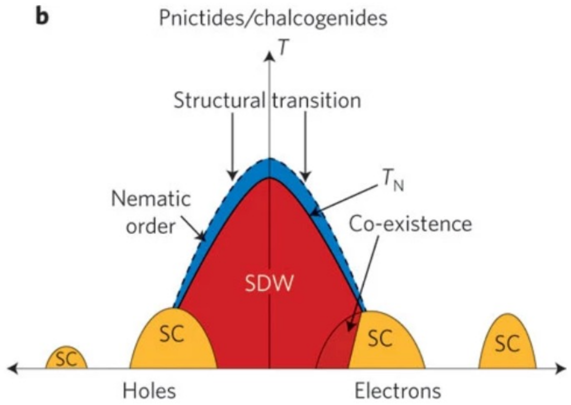
Heavy Fermions




Quantum Magnets



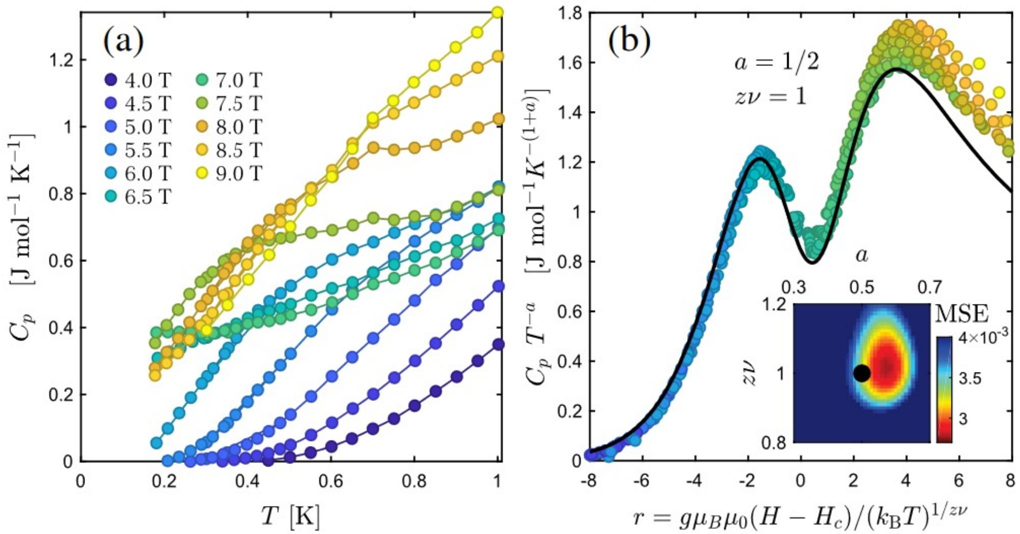

*Moll et al. Nat Commun* **6**, 6663 (2015).

Basov, D., Chubukov, A. *Nature Phys* **7**, 272–276 (2011).

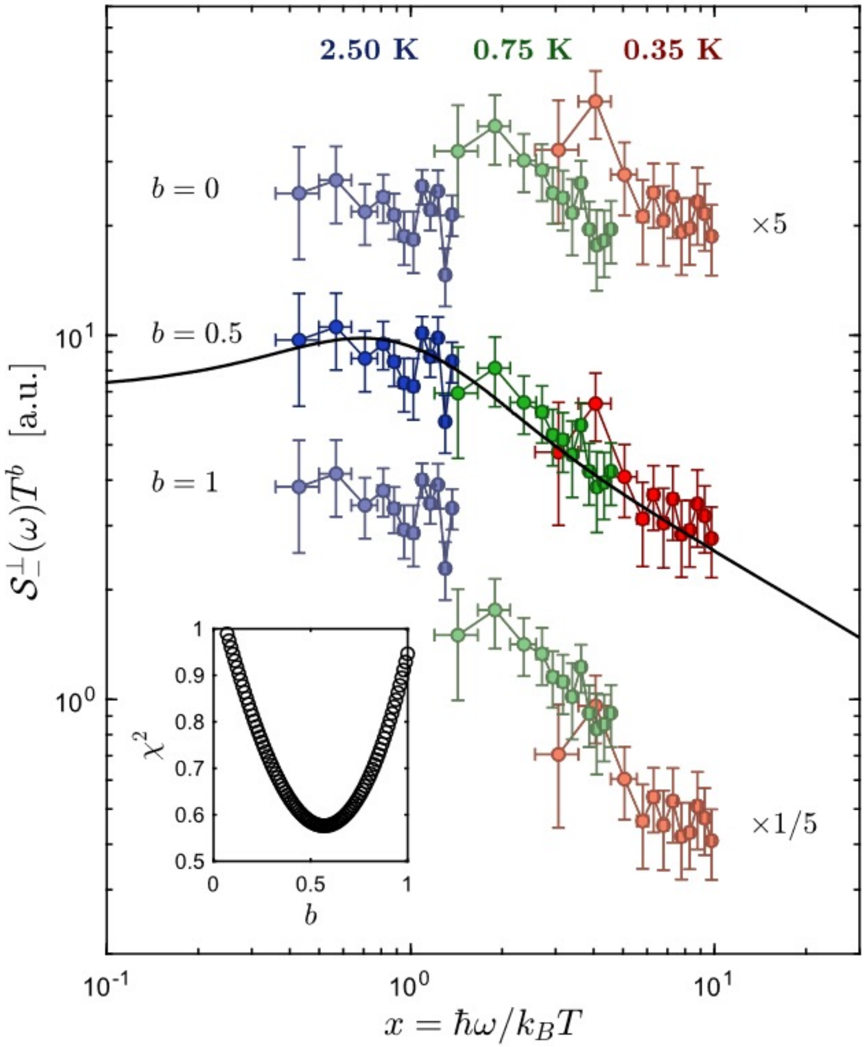
*Kuchler et al. Rev. Sci. Instrum.* 94, 045108 (2023)

- Temperature – only energy scale
- All observables become: Universal functions of E/kT

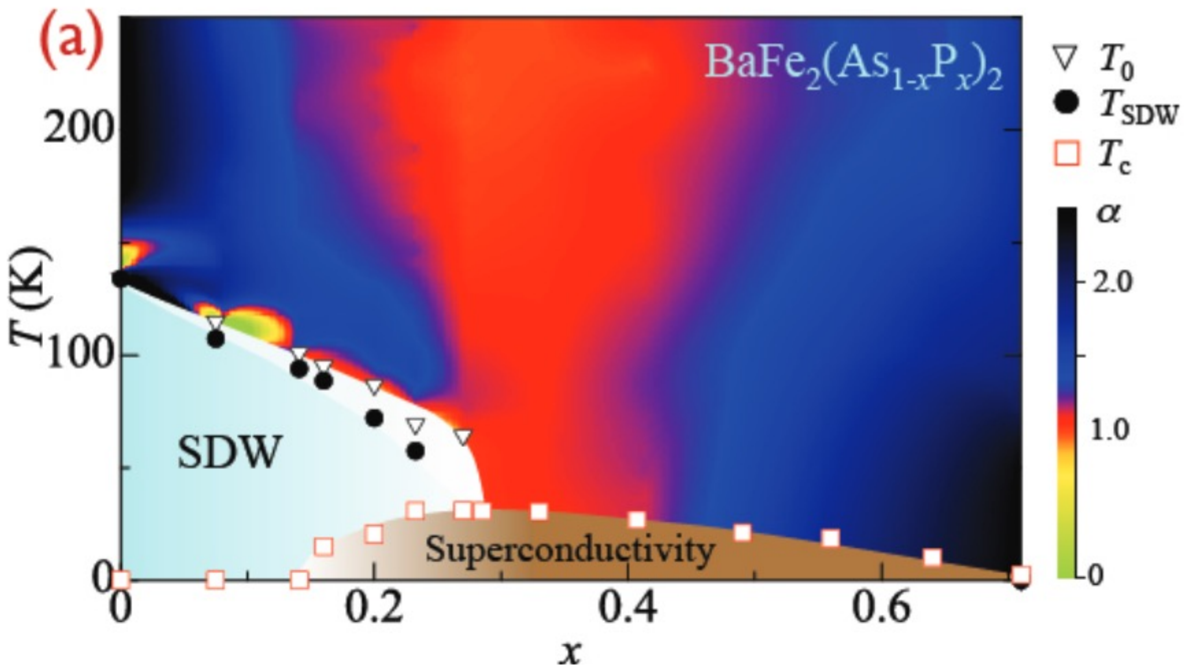
$$\chi(q_{0,}\omega) = T^{\alpha} F(\omega/_{kT})$$




Blosser et al. Phys. Rev. Lett. 121, 247201



- QCP – often require doping
- Criticality emerges from a disordered state!
- We need to understand disorder **and** strong correlations
- Hopelessly difficult…

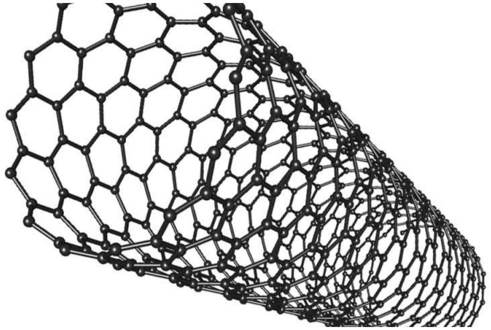


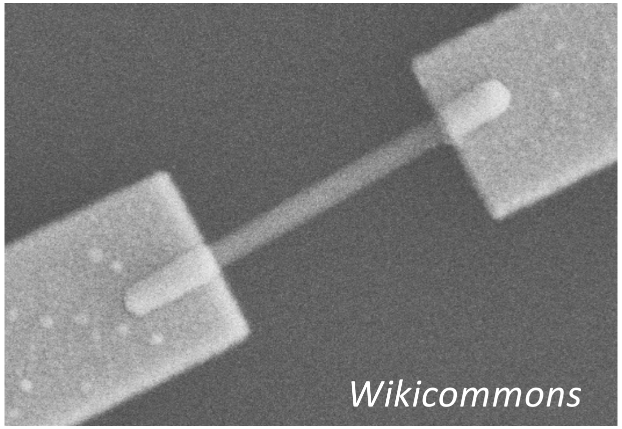

S. Kasahara et al., Phys. Rev. B 81, 184519 (2010)

# Tomonaga Luttinger Liquid:

- 1D analogue of Fermi liquid
  - Strongly correlated system
  - Linear low energy spectrum
  - Power law correlations
- Typically, itinerant electrons:
  - QH Edge, CNT, Nanowires etc

$$H = \frac{v}{2} \int dx \left[ \frac{1}{K} (\partial_x \phi)^2 + K (\partial_x \tilde{\phi})^2 \right],$$





## Table of content:

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- **Introduction – done!**
- Tomonaga-Luttinger liquid in spin chains
- Spin ladders:
  - Impurities at B=0
  - Disorder in the Tomonaga-Luttinger phase
- Conclusions and outlook

# Tomonaga Luttinger Liquid: Spin chains!

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XXY Spin chain:

$$H = J_{xy} \sum \{S_n^x S_{n+1}^x + S_n^y S_{n+1}^y\} + J_{xy} \sum S_n^z S_{n+1}^z - h \sum S_n^z$$

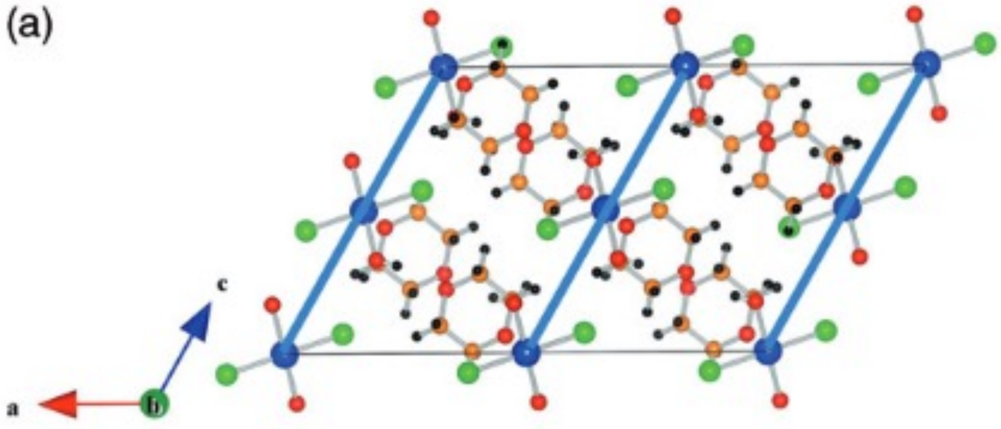
Jordan-Wigner transform:

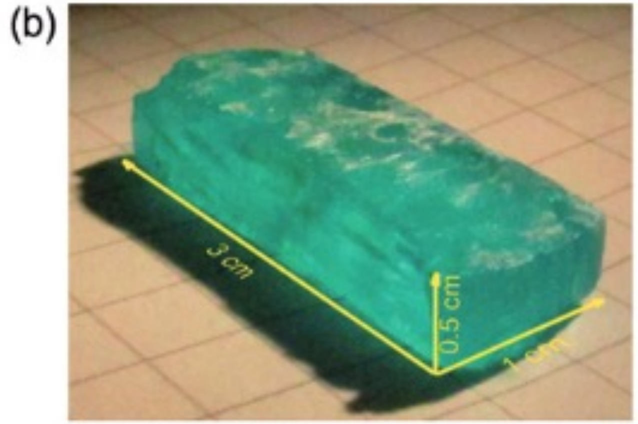
$$H = \sum \frac{J_{xy}}{2} (c_n^+ c_{n+1} + c_{n+1}^+ c_n) + J_z c_n^+ c_n \, c_{n+1}^+ c_{n+1} - c_n^+ c_n \, (h + J_z)$$

Bosonization: Luttinger Liquid

$$H = \frac{v}{2} \int dx \left[ \frac{1}{K} (\partial_x \phi)^2 + K (\partial_x \tilde{\phi})^2 \right],$$

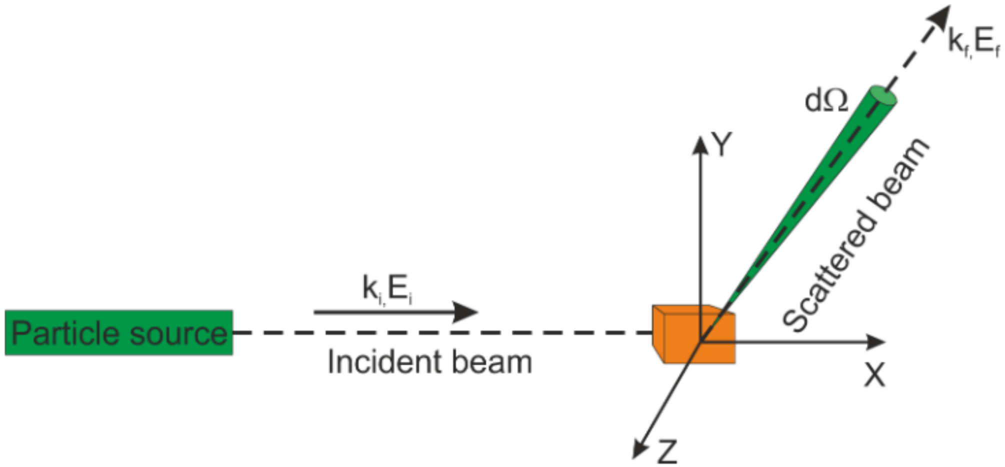
**Example:** copper pyrazine dinitrate (CuPzN)




*Yankova et al. Philosophical Magazine*, *92*(19–21), 2629–2647

# Machinery of neutron scattering:




Scattering cross-section:

$$\frac{d^2\sigma}{d\Omega dE_f} = \frac{k_f}{k_i}(r_0\gamma)^2 |F(\boldsymbol{Q})|^2 \exp\left(-2W(\boldsymbol{Q})\right) \sum_{\alpha\beta}\left(\delta_{\alpha\beta} - \frac{Q_\alpha Q_\beta}{Q^2}\right) S^{\alpha\beta}(\boldsymbol{Q},\omega)$$

Dynamic structure factor:

$$\mathcal{S}(q_\parallel,\omega) \propto T^{1/2K-2}\mathrm{Im}\left\{\left[1-\exp\left(-\frac{\hbar\omega}{k_B T}\right)\right]^{-1} \times \Phi\left(\frac{\hbar\omega}{k_B T},\frac{u(q_\parallel-\pi)}{k_B T}\right)\right\},$$

$$\Phi(x,y) = \frac{\Gamma\left(\frac{1}{8K} - i\frac{x-y}{4\pi}\right)}{\Gamma\left(1-\frac{1}{8K} - i\frac{x-y}{4\pi}\right)} \frac{\Gamma\left(\frac{1}{8K} - i\frac{x+y}{4\pi}\right)}{\Gamma\left(1-\frac{1}{8K} - i\frac{x+y}{4\pi}\right)}$$

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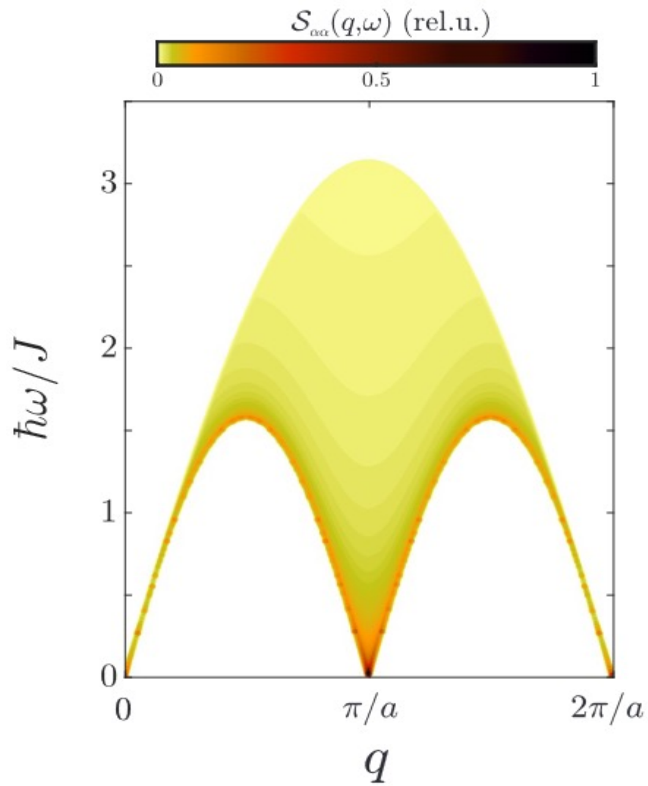
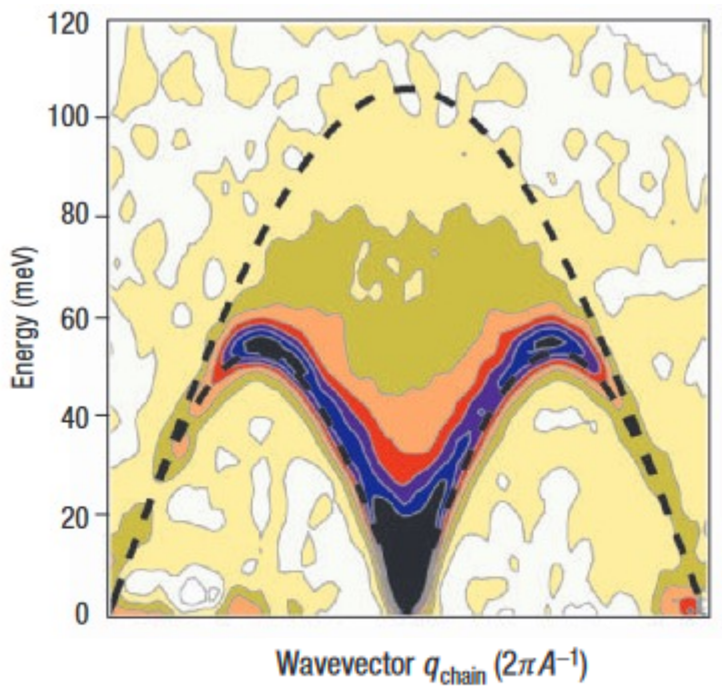
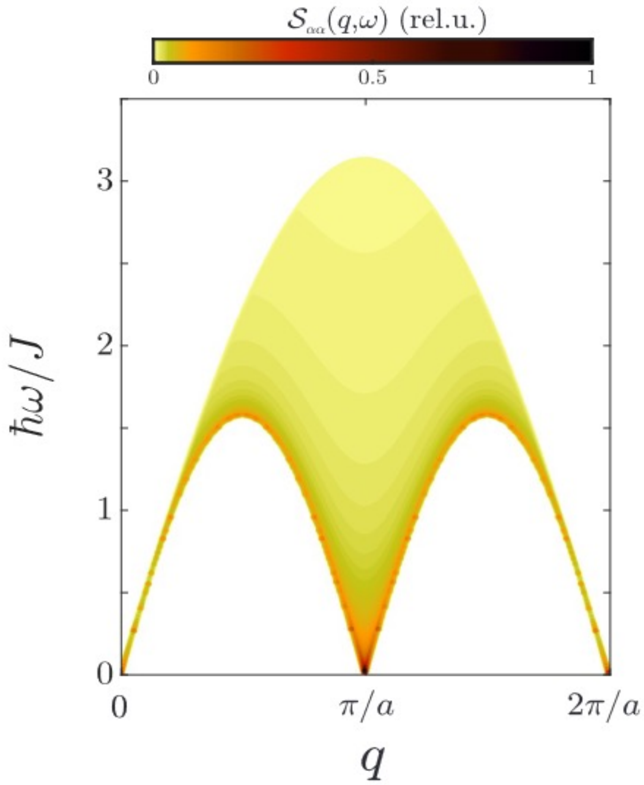
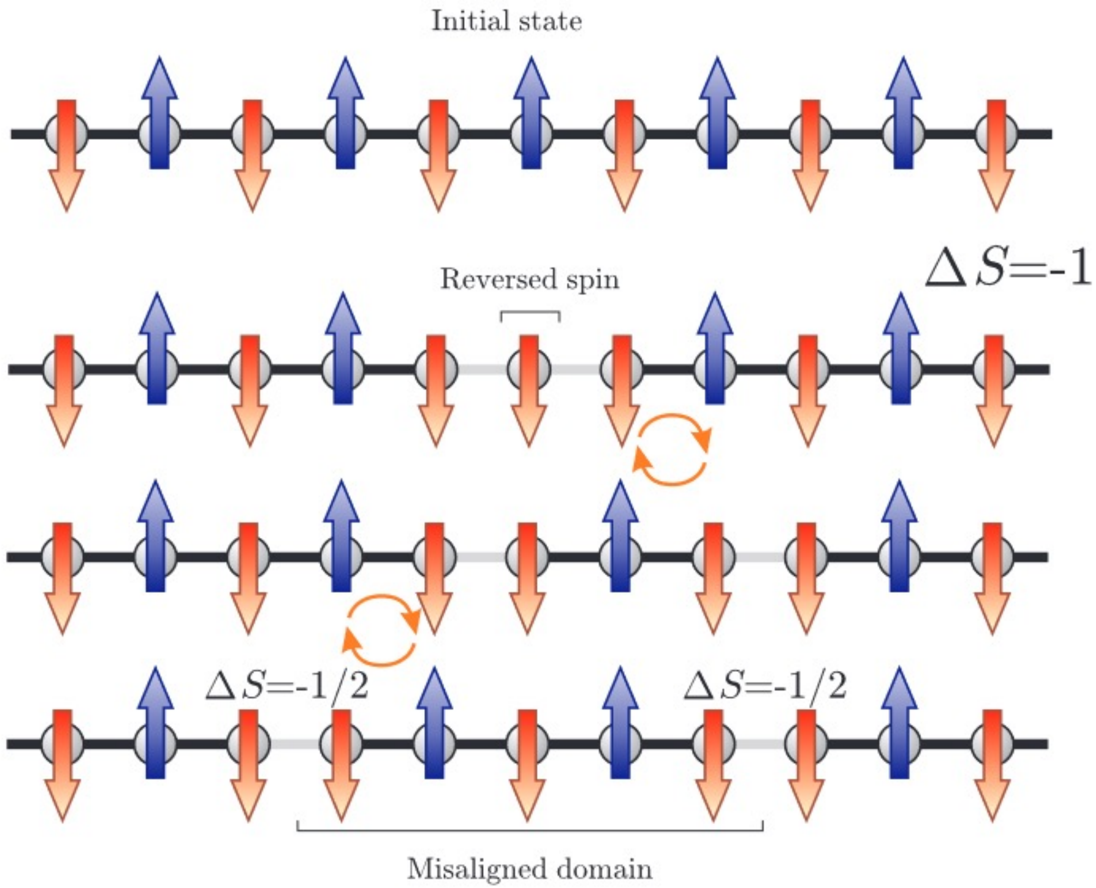



*Lake et al. Nature Mater* **4**, 329–334 (2005)

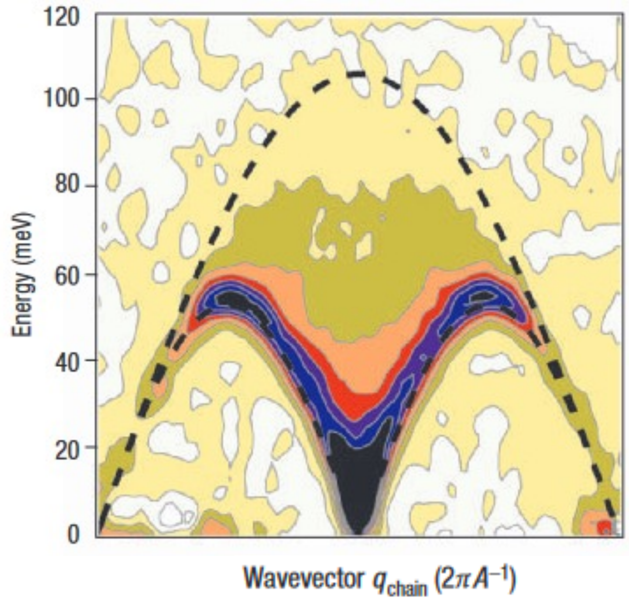
# Does it work?

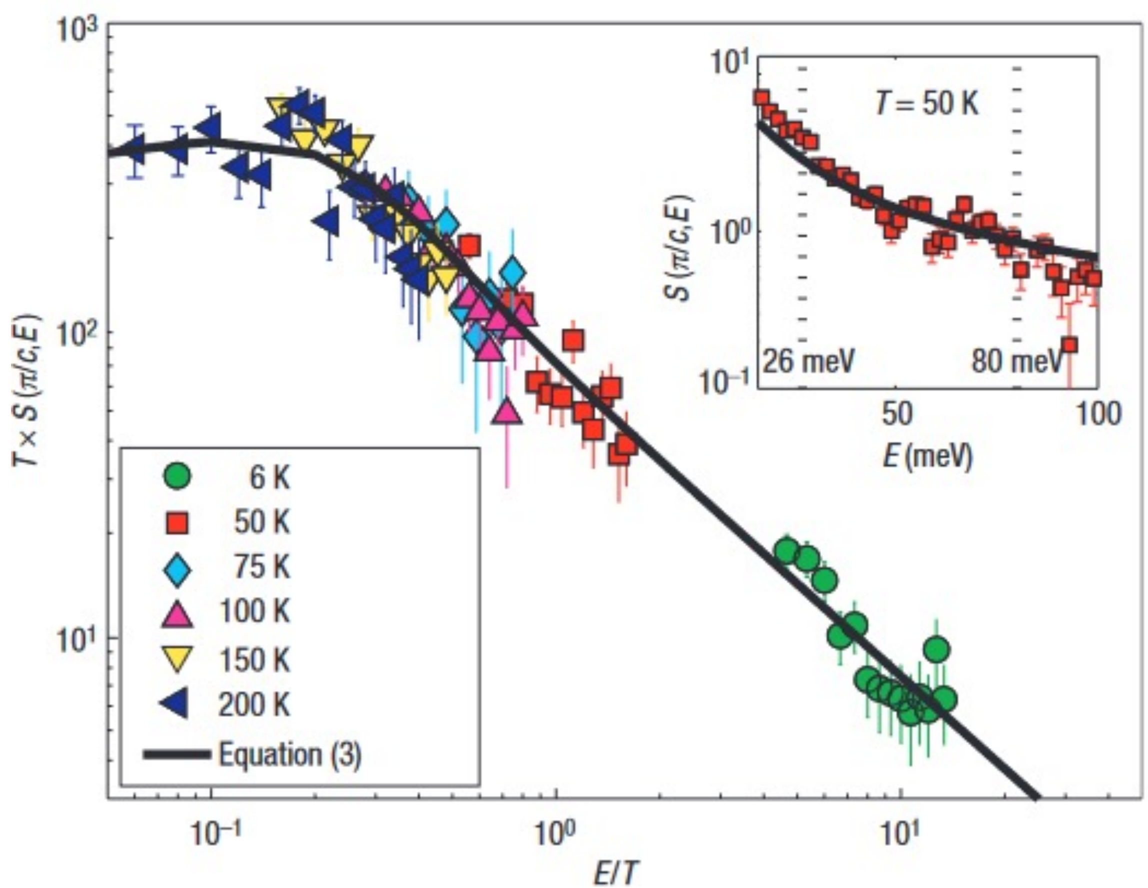



Povarov, Quantum Magnetism, ETH Unpublished
*Lake et al. Nature Mater* **4**, 329–334 (2005)

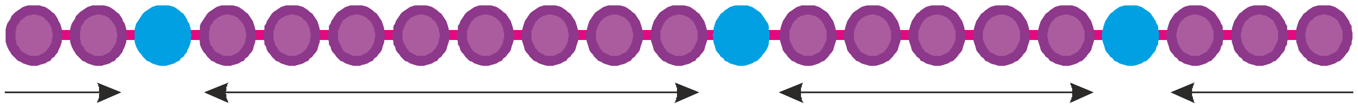
$$\chi(q_0,\omega) = T^{\alpha}\mathcal{F}(\frac{\omega}{T})$$



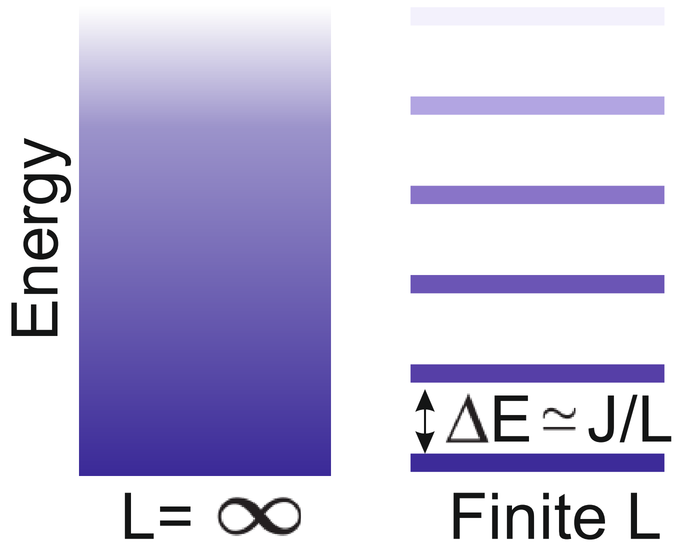




*Lake et al. Nature Mater* **4**, 329–334 (2005)

# Disorder in spin chains:

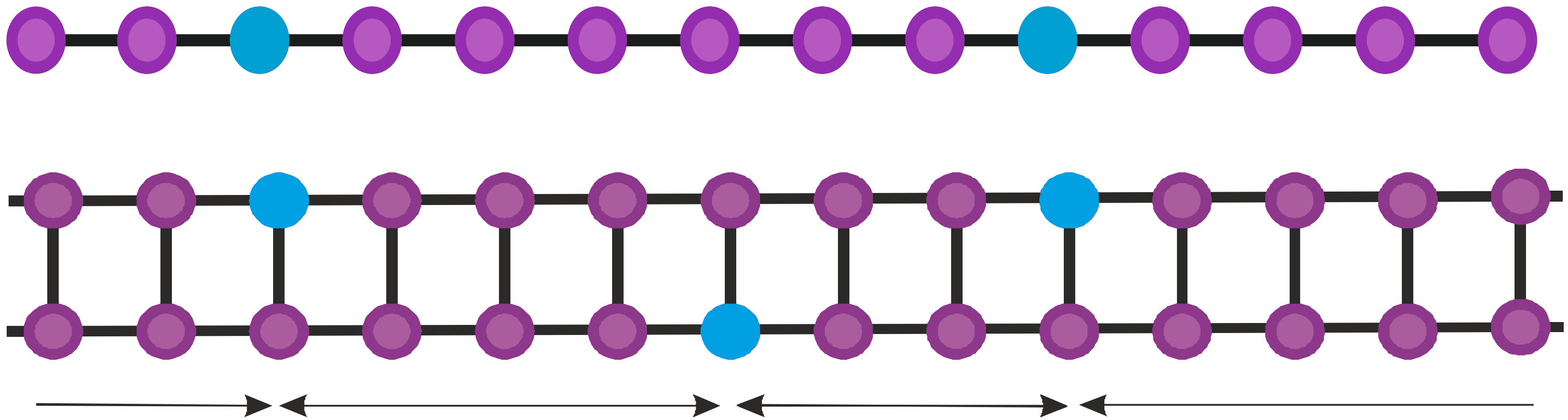


- Impurities cut the chain
- New length scale:
  - Discrete energy levels
  - Spinon confiement
- But only repulsive interaction
- Bohrdt et al.:



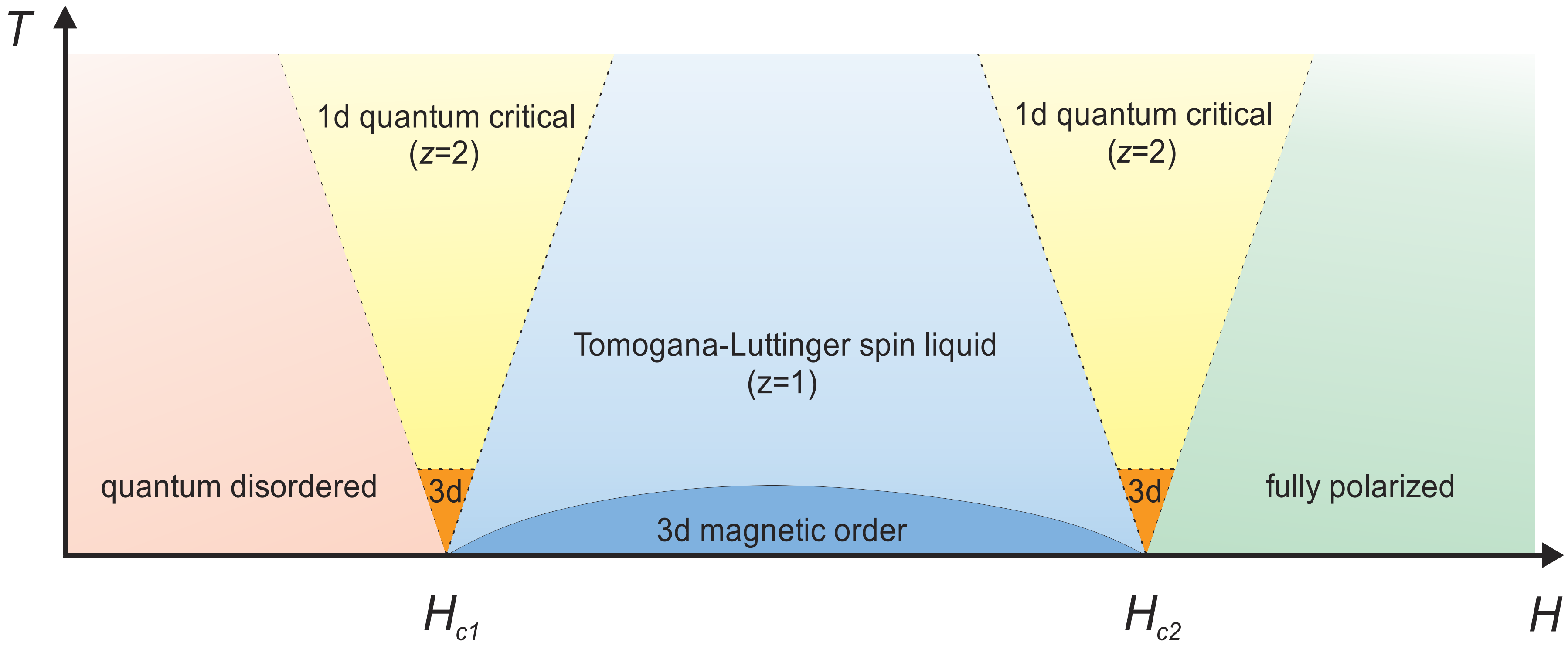

$$\chi_{st}^{\pm} = \left(\frac{T}{v}\right)^{\frac{1}{2K}-1} F_K\left(\frac{T}{v}\right) \Rightarrow \chi_{st}^{\pm} = \left(\frac{T}{v}\right)^{\frac{1}{2K}-1} F'_K\left(\frac{LT}{v}\right)$$

Ladders have a much less restrictive geometry: defects do not break continuity!

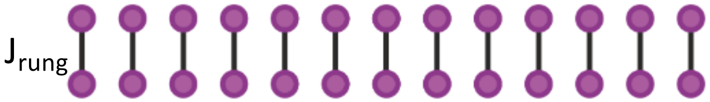


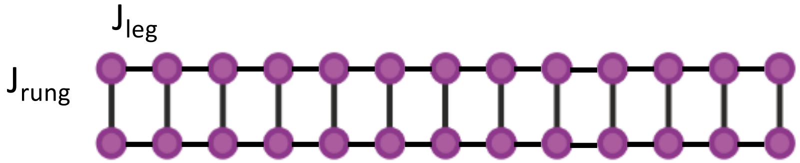
$$\hat{\mathcal{H}} = \sum_n J_{\|}(\hat{\mathbf{S}}_{n,1}\hat{\mathbf{S}}_{n+1,1} + \hat{\mathbf{S}}_{n,2}\hat{\mathbf{S}}_{n+1,2}) + J_{\perp}\hat{\mathbf{S}}_{n,1}\hat{\mathbf{S}}_{n,2}$$

# Phase diagram of the spin ladder

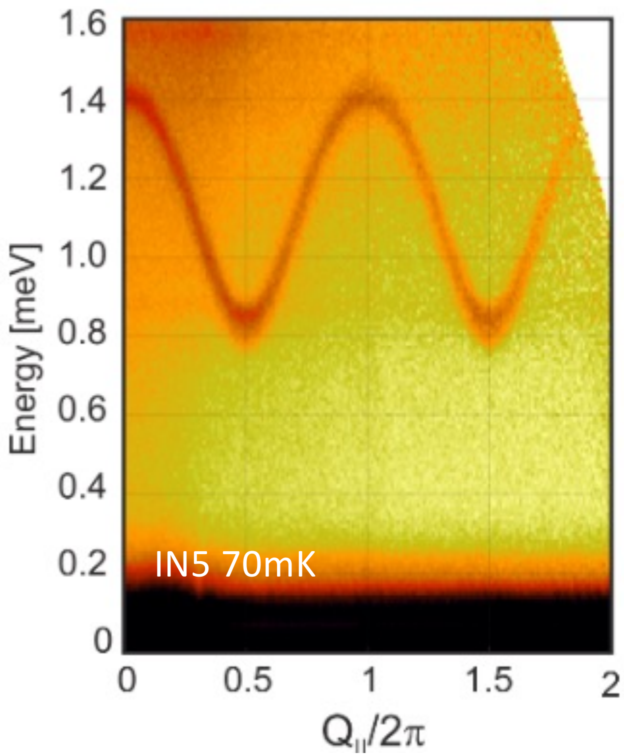


# Phase diagram of the spin ladder: gapped phase





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# Phase diagram of the spin ladder: Luttinger liquid

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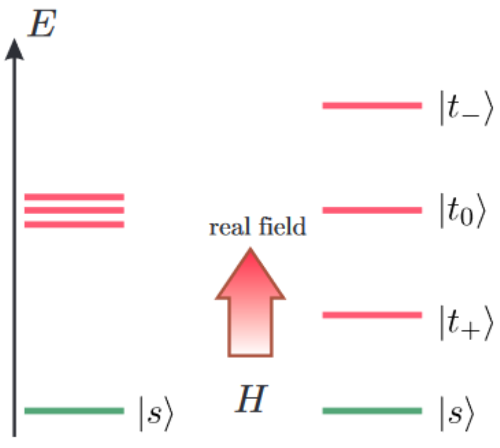


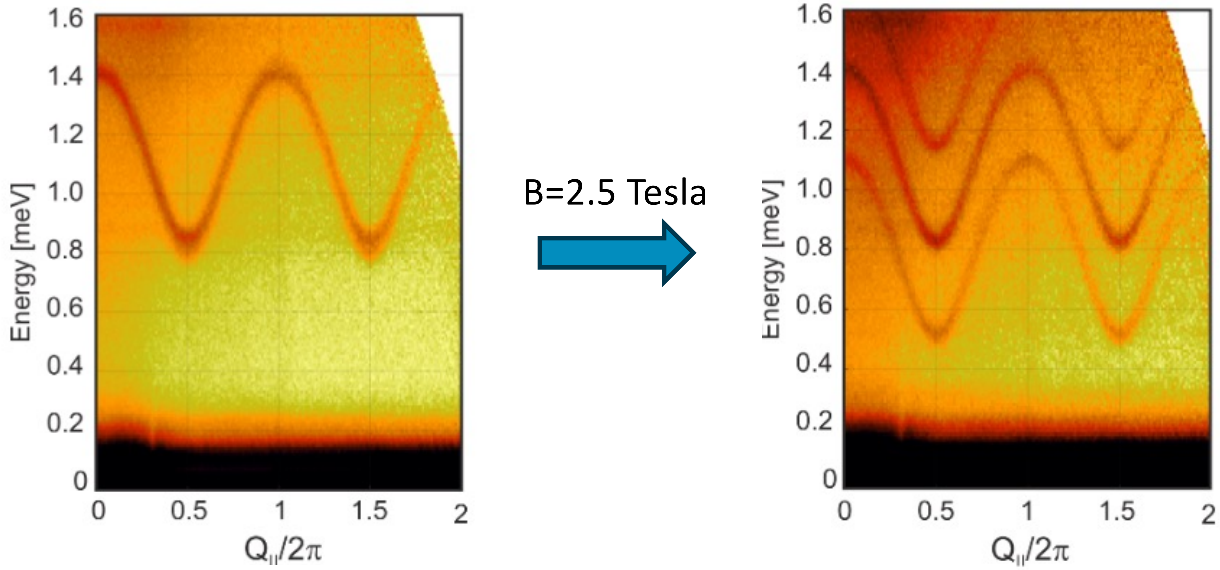


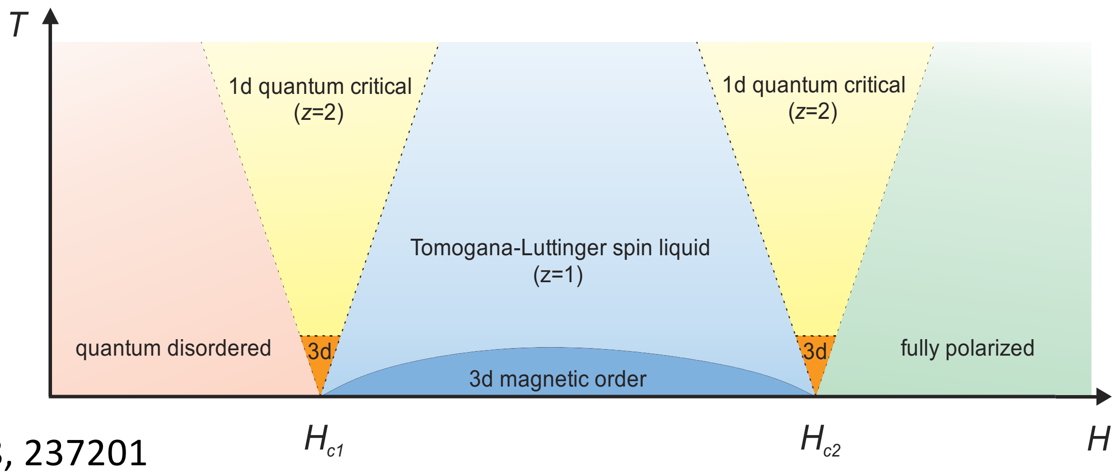


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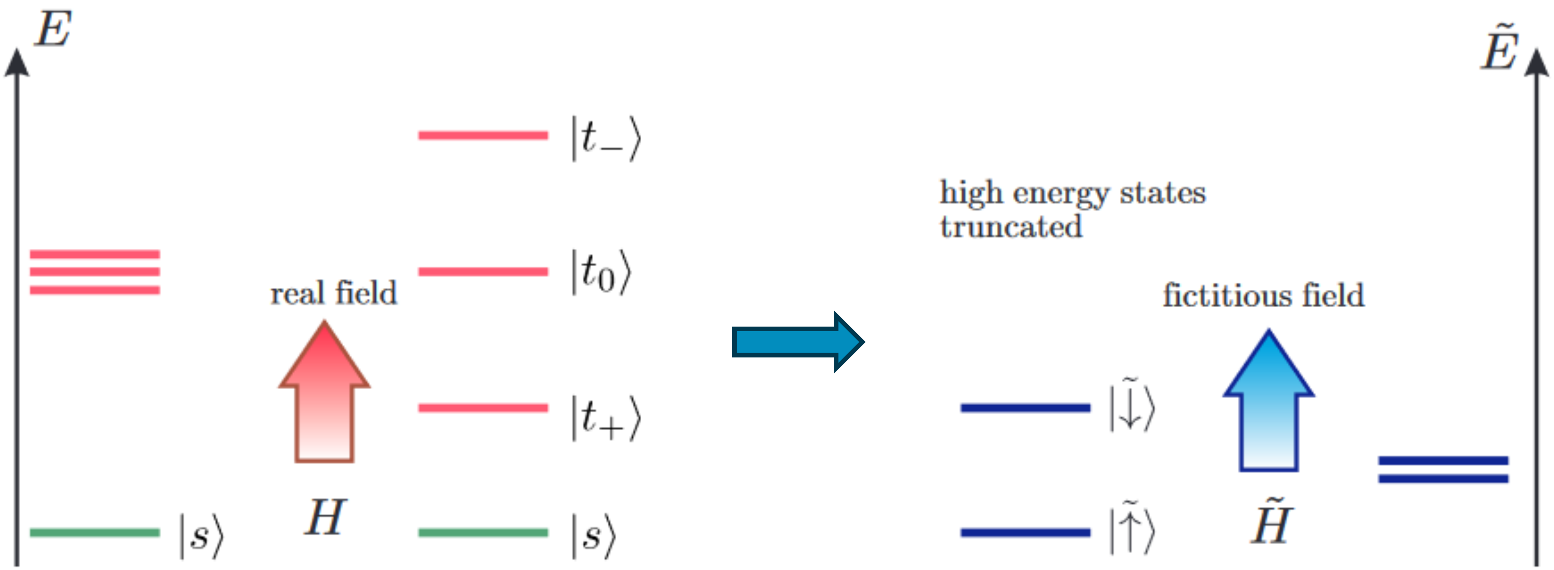
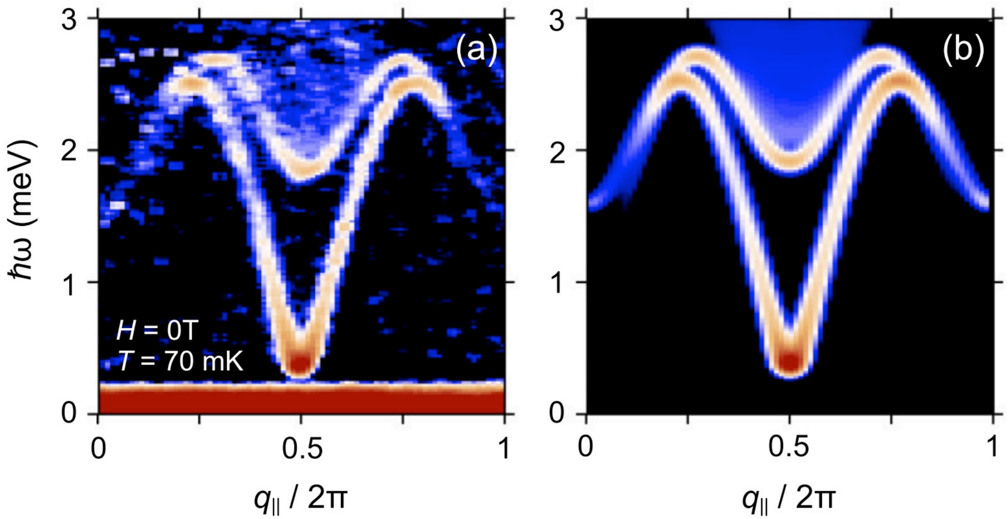


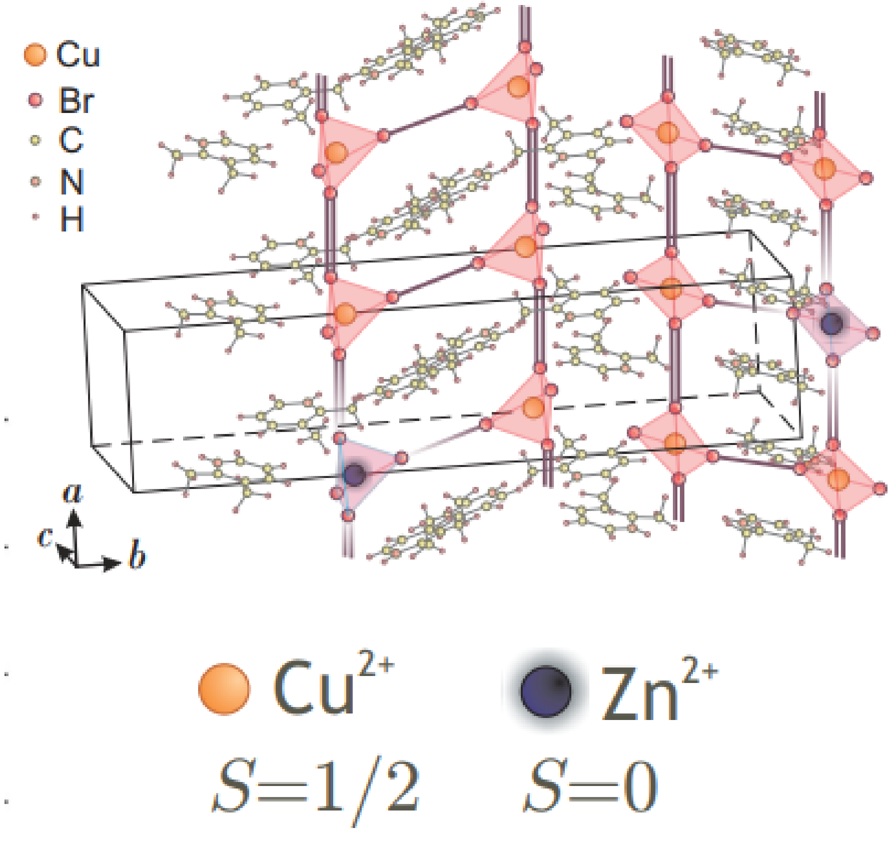

$$\hat{\mathcal{H}} = \sum_n J_{\parallel}(\hat{\mathbf{S}}_{n,1}\hat{\mathbf{S}}_{n+1,1} + \hat{\mathbf{S}}_{n,2}\hat{\mathbf{S}}_{n+1,2}) + J_{\perp}\hat{\mathbf{S}}_{n,1}\hat{\mathbf{S}}_{n,2}$$

$$\hat{\tilde{\mathcal{H}}} = \sum_n J_{\parallel}(\hat{\tilde{S}}^x_n\hat{\tilde{S}}^x_{n+1} + \hat{\tilde{S}}^y_n\hat{\tilde{S}}^y_{n+1} + \frac{1}{2}\hat{\tilde{S}}^z_n\hat{\tilde{S}}^z_{n+1}) - g\mu_B\tilde{H}\hat{\tilde{S}}^z_n, \text{ with } \tilde{H} = H - \frac{J_{\perp} + \frac{1}{2}J_{\parallel}}{g\mu_B}.$$

**New materials:**

- DIMPY:
  - $J_{leg}$=16.7K
  - $J_{rung}$=9.5K
  - ξ≈6.2
- BPCB:
  - $J_{leg}$=3.6K
  - $J_{rung}$=12.96K
  - ξ≈1

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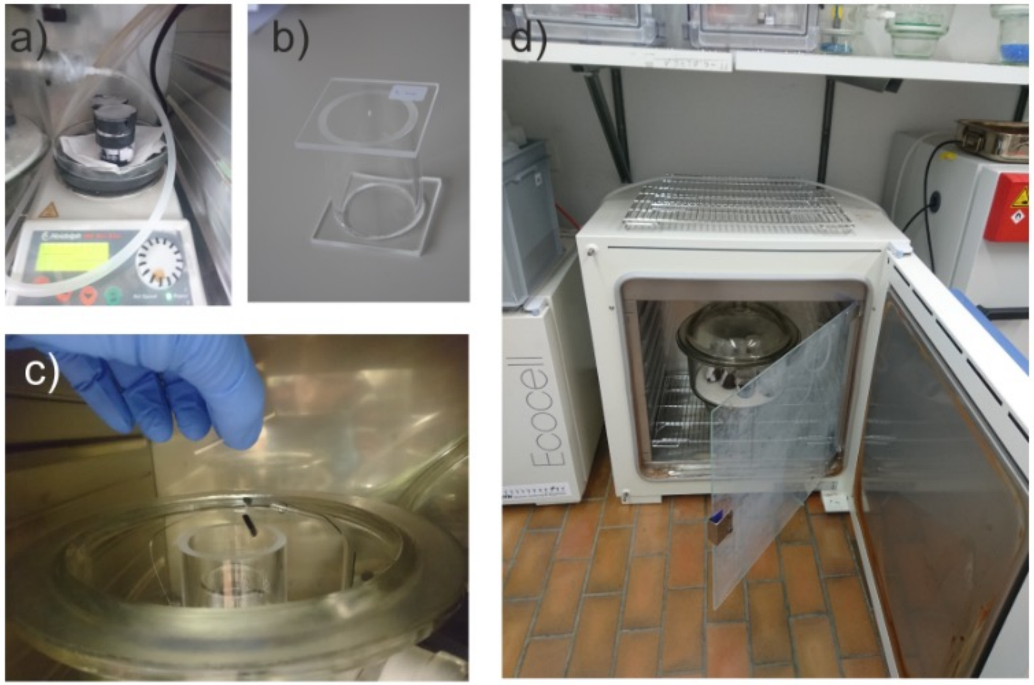
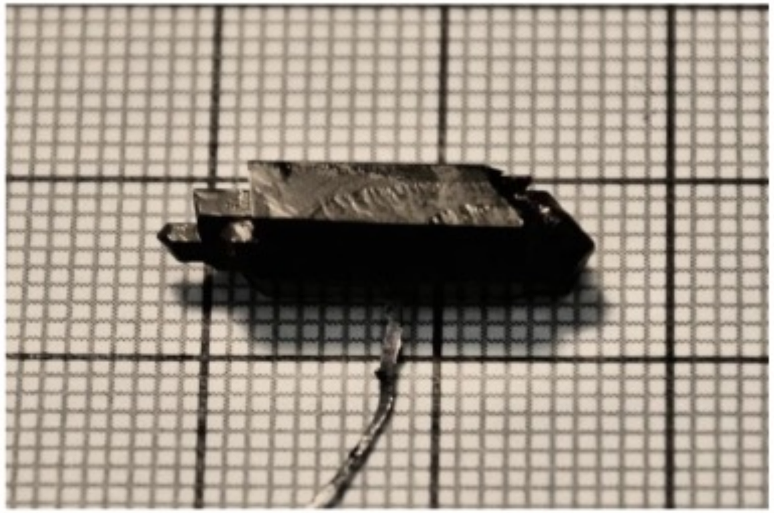
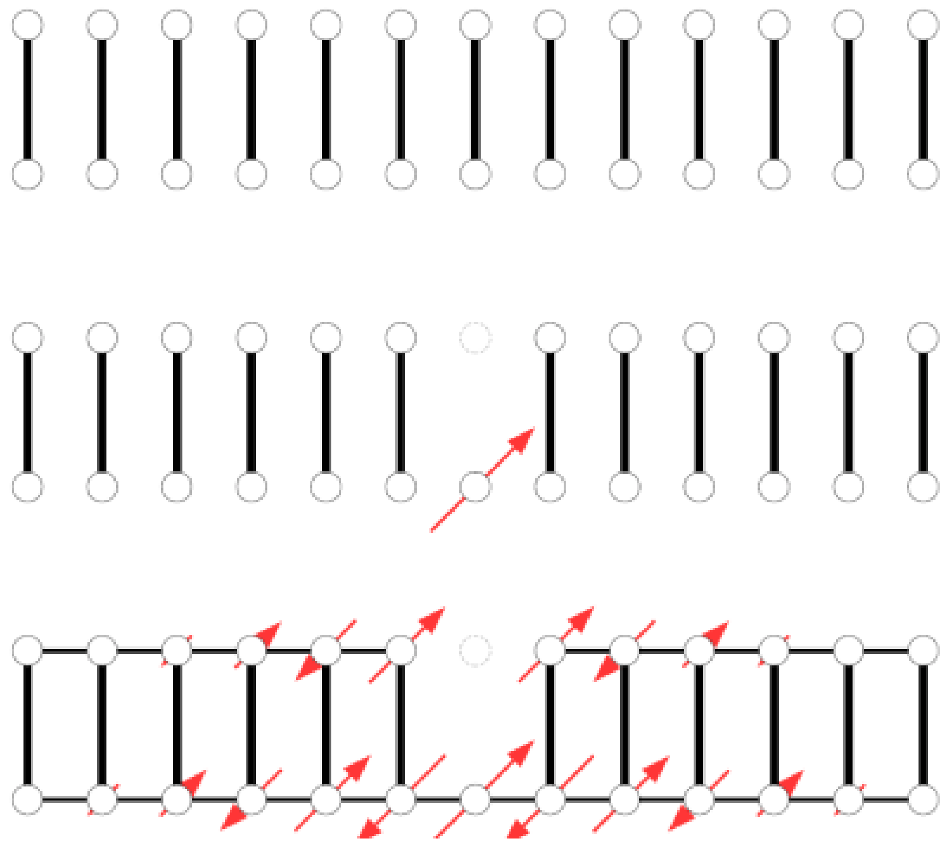
# Spin ladders: material growth



# Depleted Spin Ladders: Spin islands

- Single rung:
- Singlet → Free spin
- In the ladder:
  - Emergent extended object
  - Correlation length ->Size
- Strong rung: small objects
- Strong leg: extended objects



# Non-interactingspin islands: BPCB

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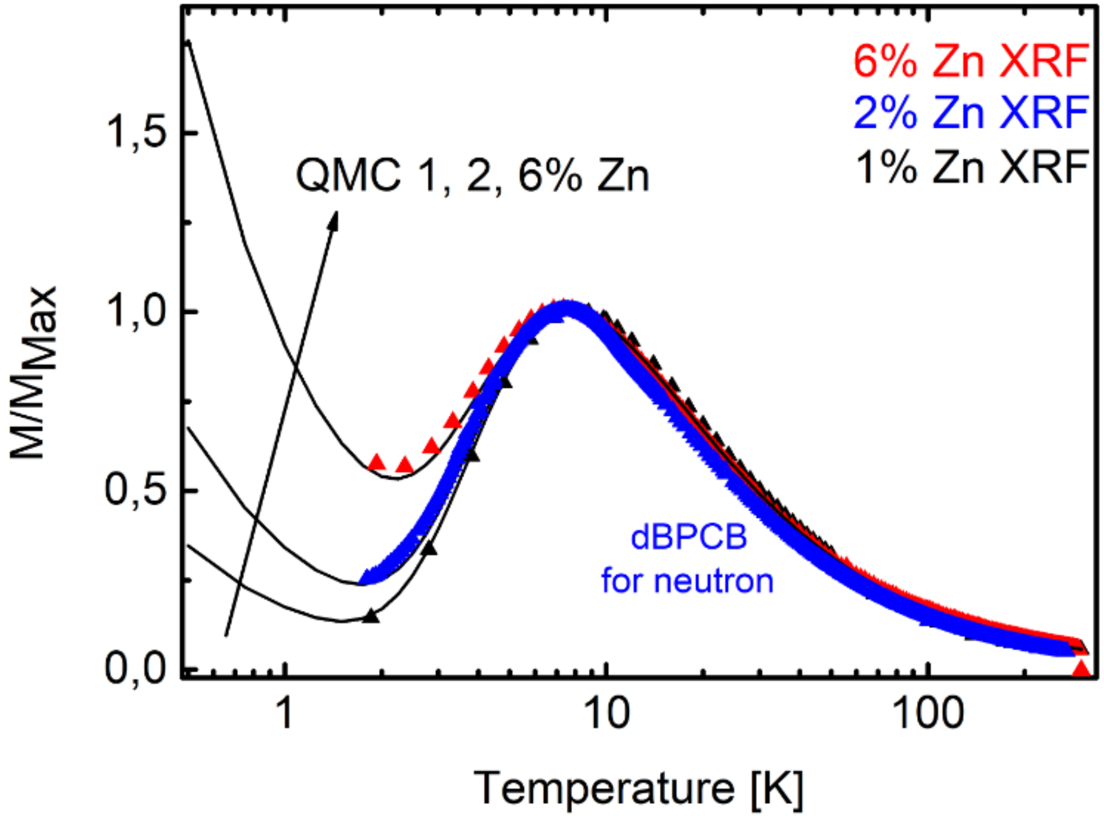
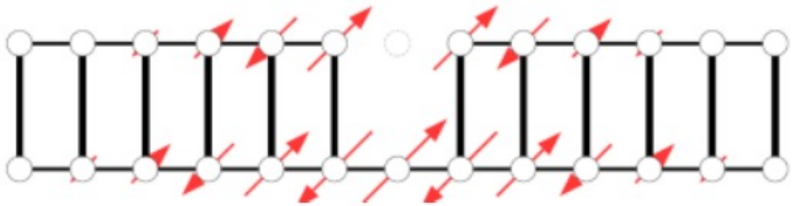


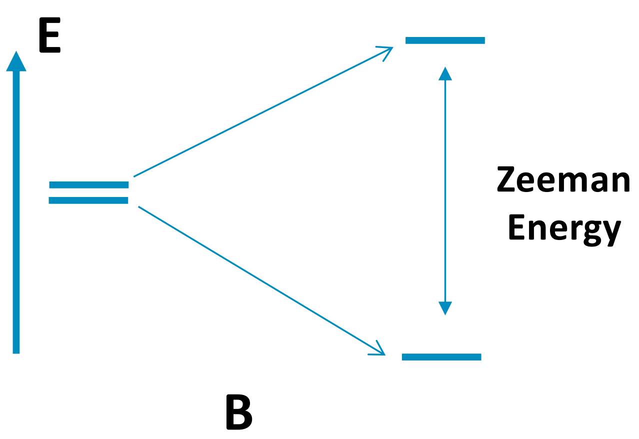

ALPS

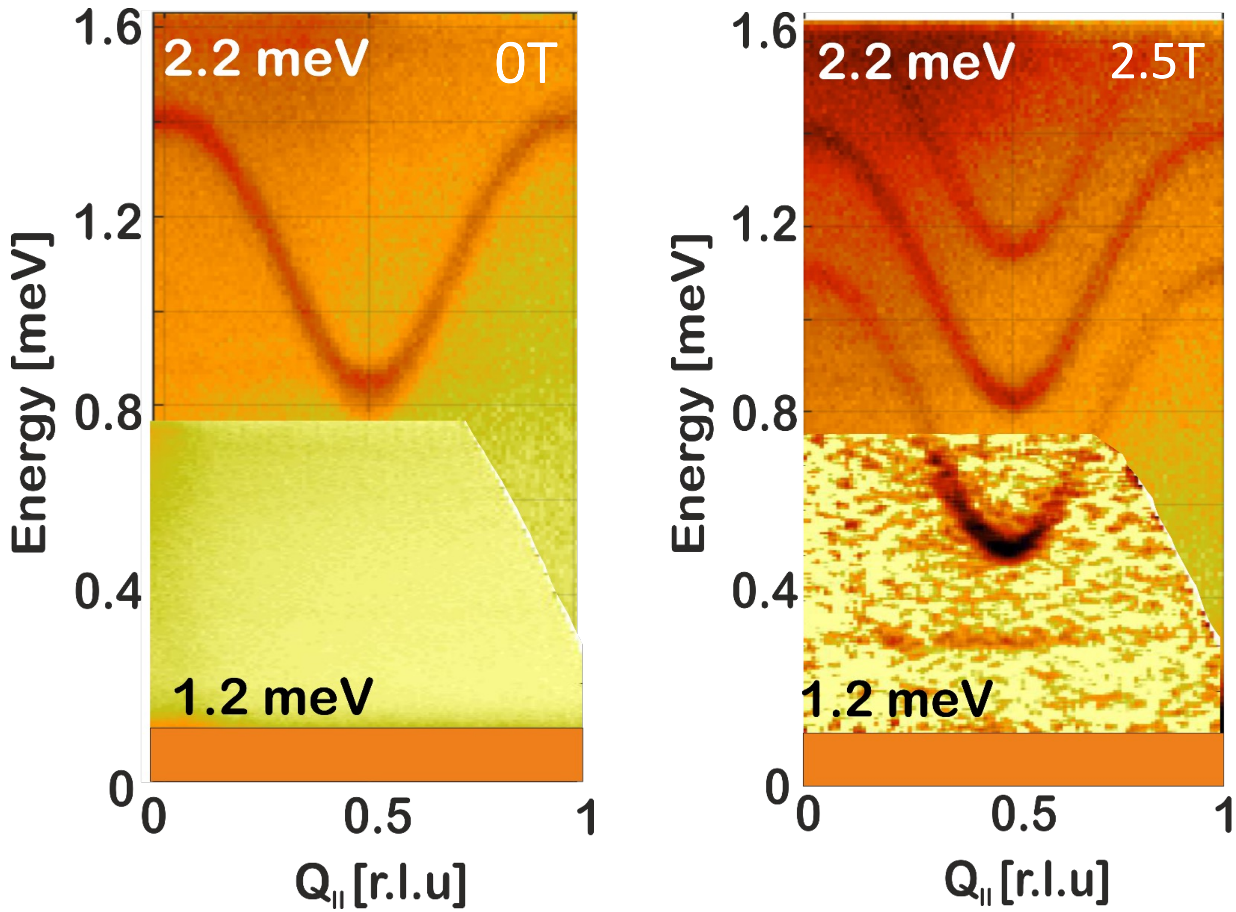
Applications and Libraries for Physics Simulations

# Non-interacting spin islands: BPCB



Spin in field:

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# Interacting spin islands: DIMPY

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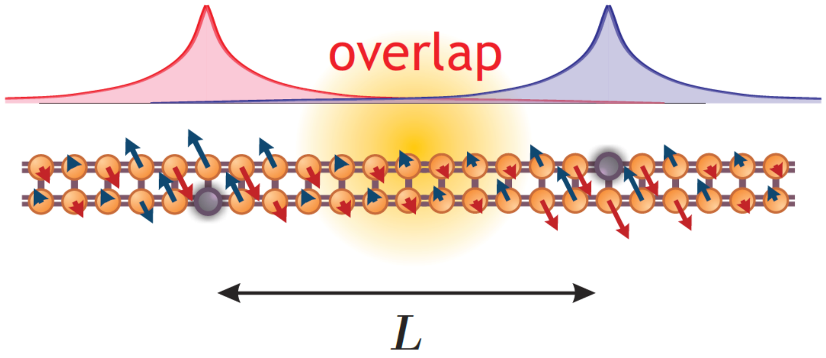
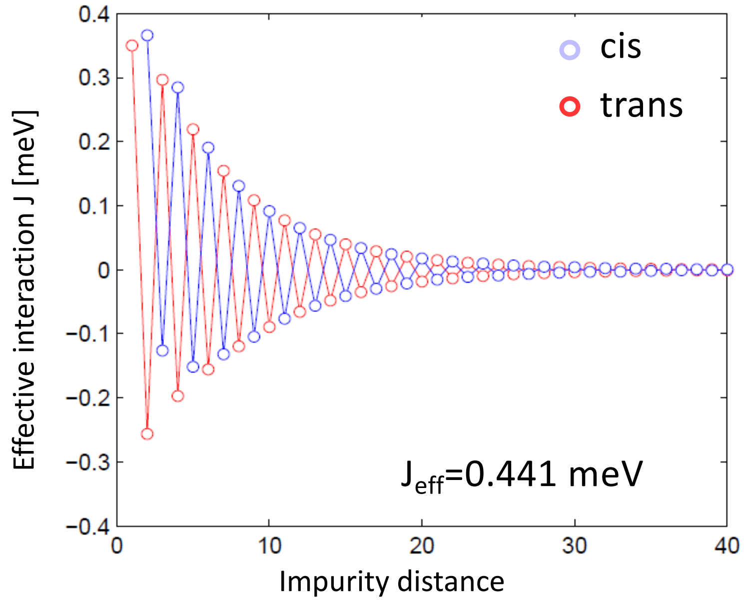
- **Strong leg ladder**
  - DIMPY: $\xi \approx 6 < L$
  - New many body strongly interacting system
  - Estimating $J_{eff}$:

$$J_{\mathrm{eff}}(L)=J_0(-1)^L e^{-L/\xi}$$

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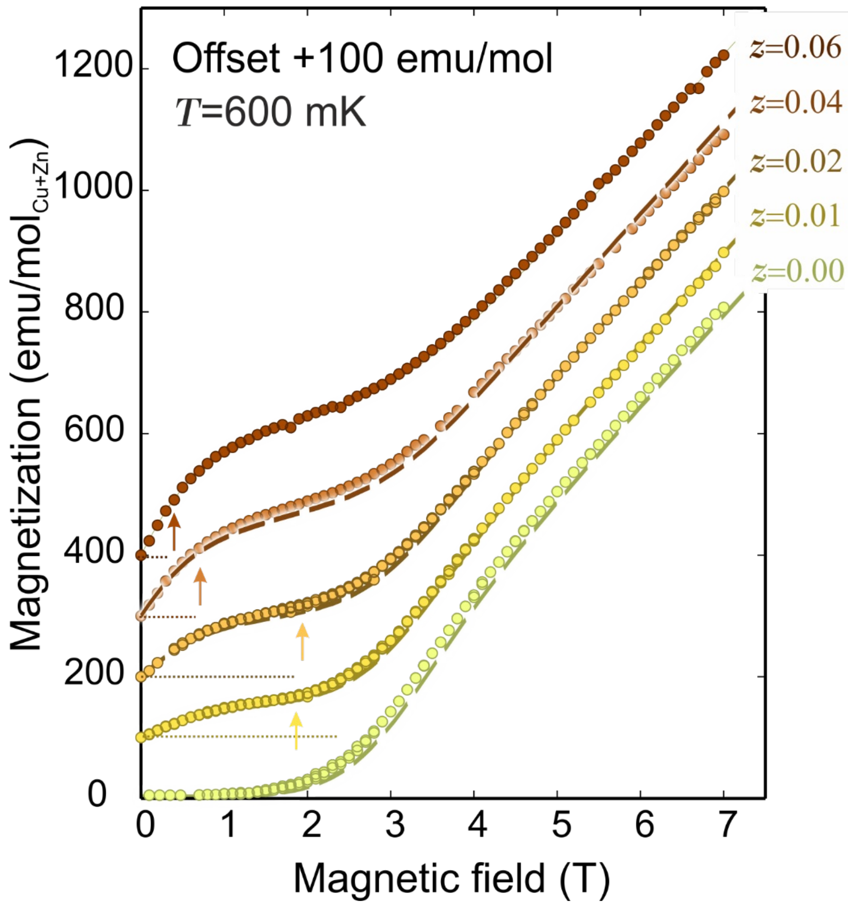
# DIMPY – Magnetic properties



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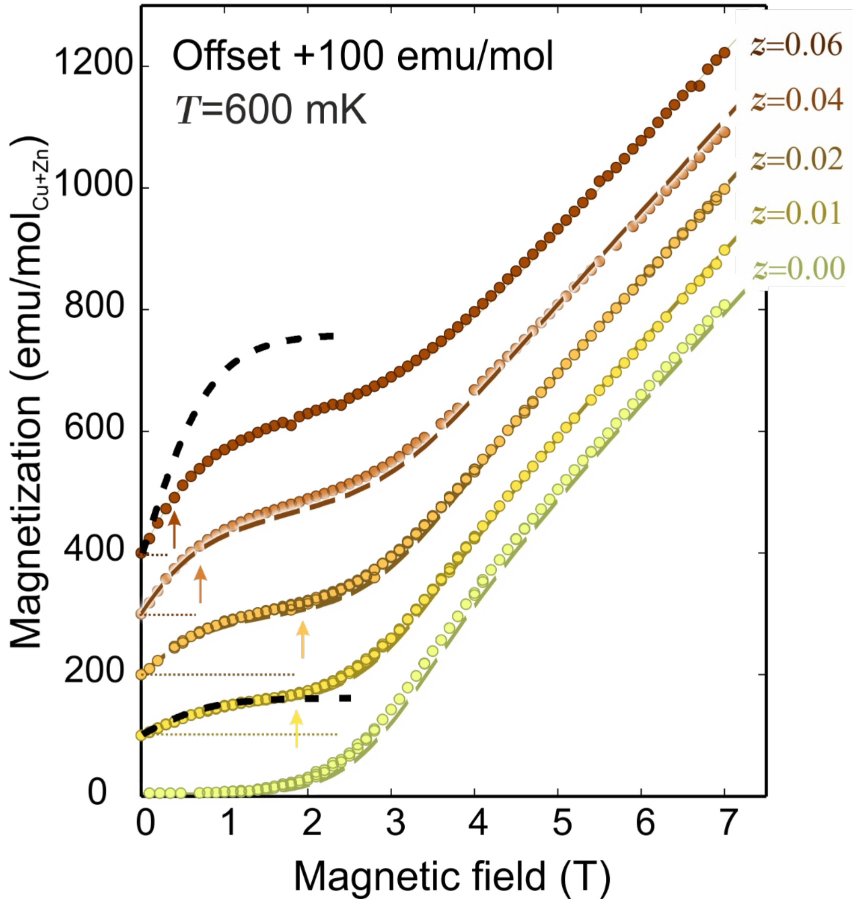
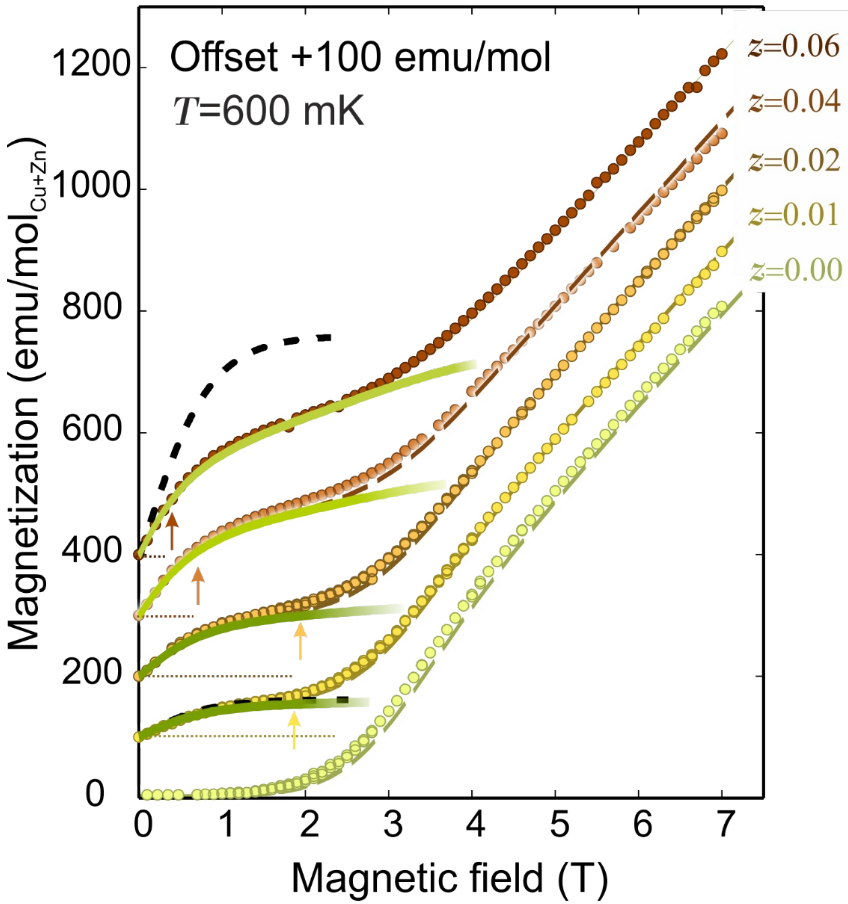
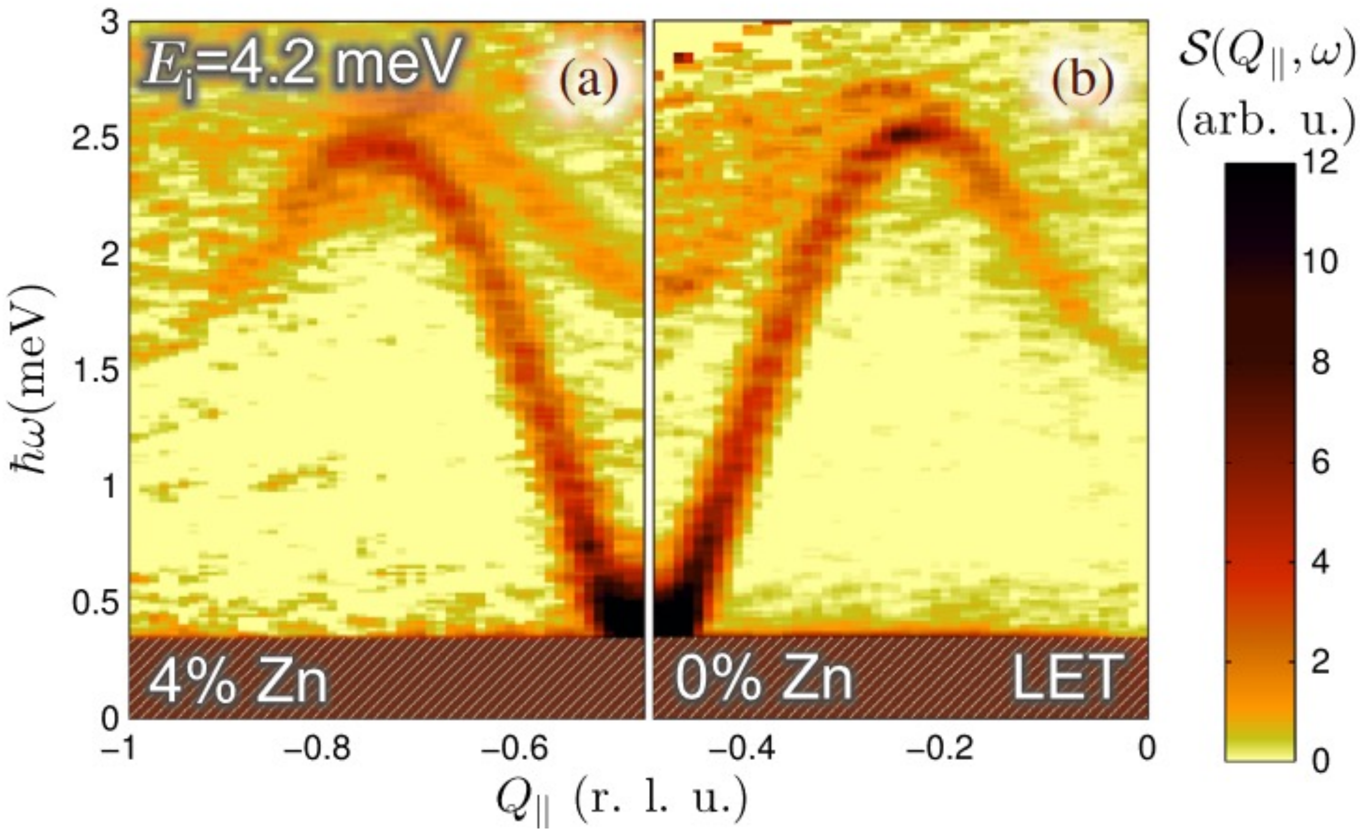




Offset +100 emu/mol
T=600 mK
z=0.06
z=0.04
z=0.02
z=0.01
z=0.00
Magnetization (emu/mol$_{Cu+Zn}$)
Magnetic field (T)
0
200
400
600
800
1000
1200
0 1 2 3 4 5 6 7

# DIMPY – Dynamics

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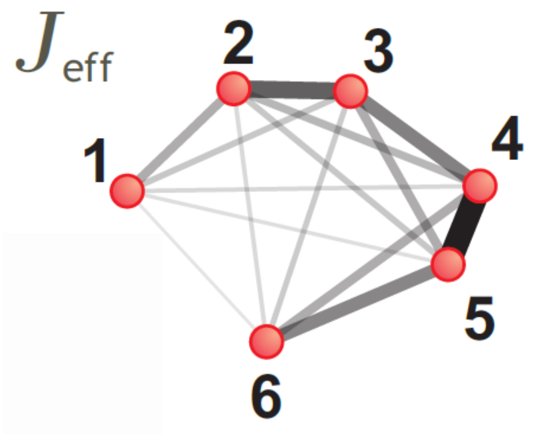
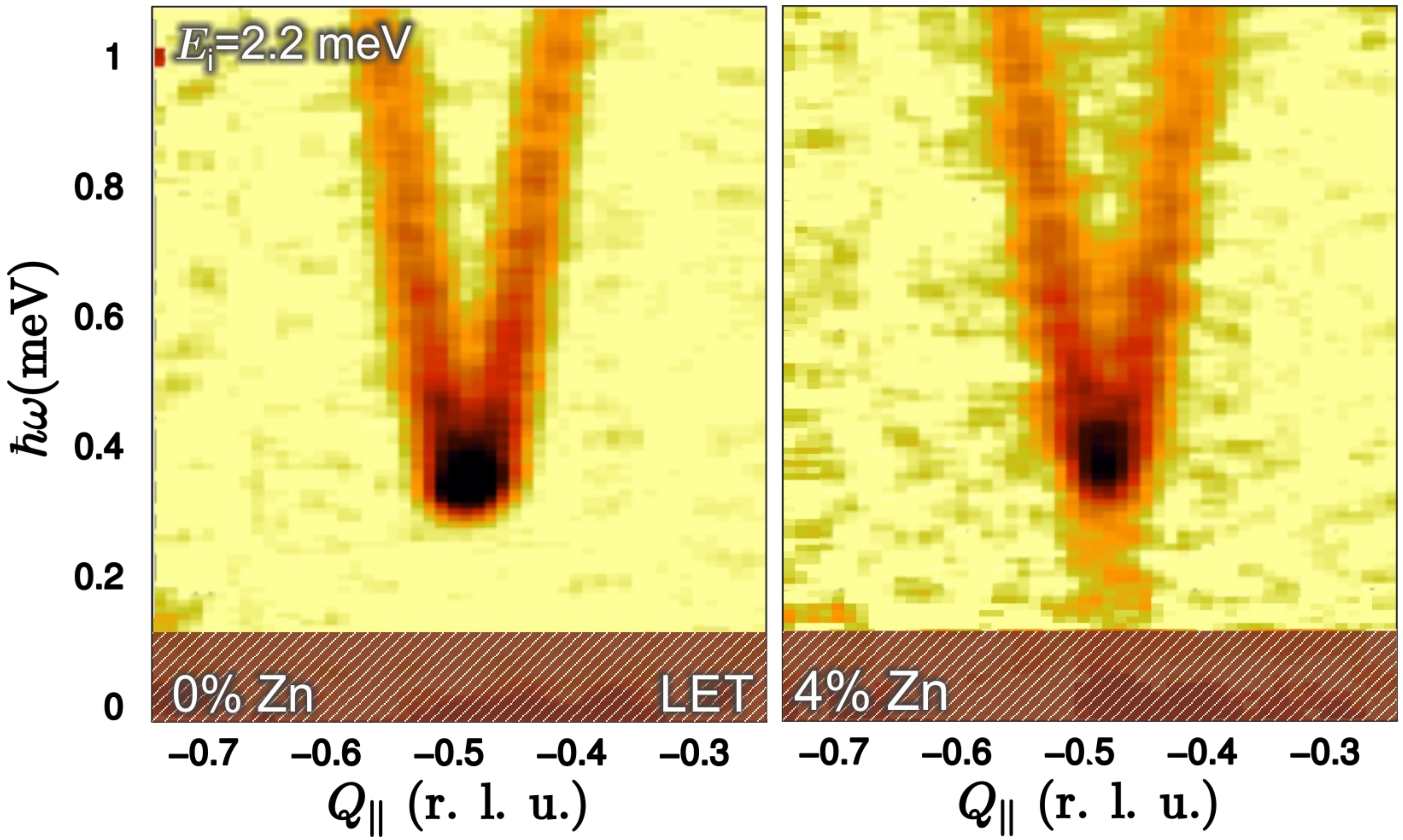
# DIMPY – Dynamics




$$J_{eff}(L) = J_0(-1)^L e^{-L/\xi}$$

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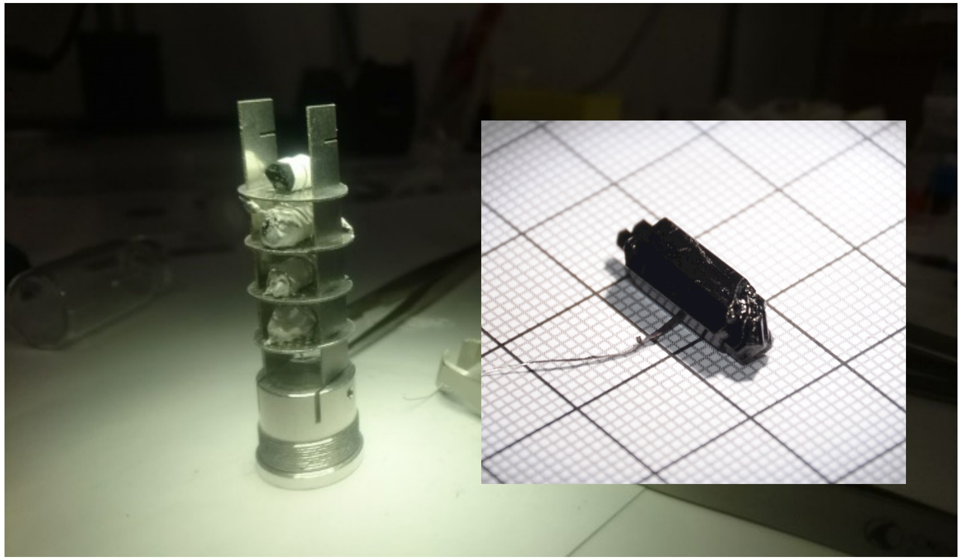

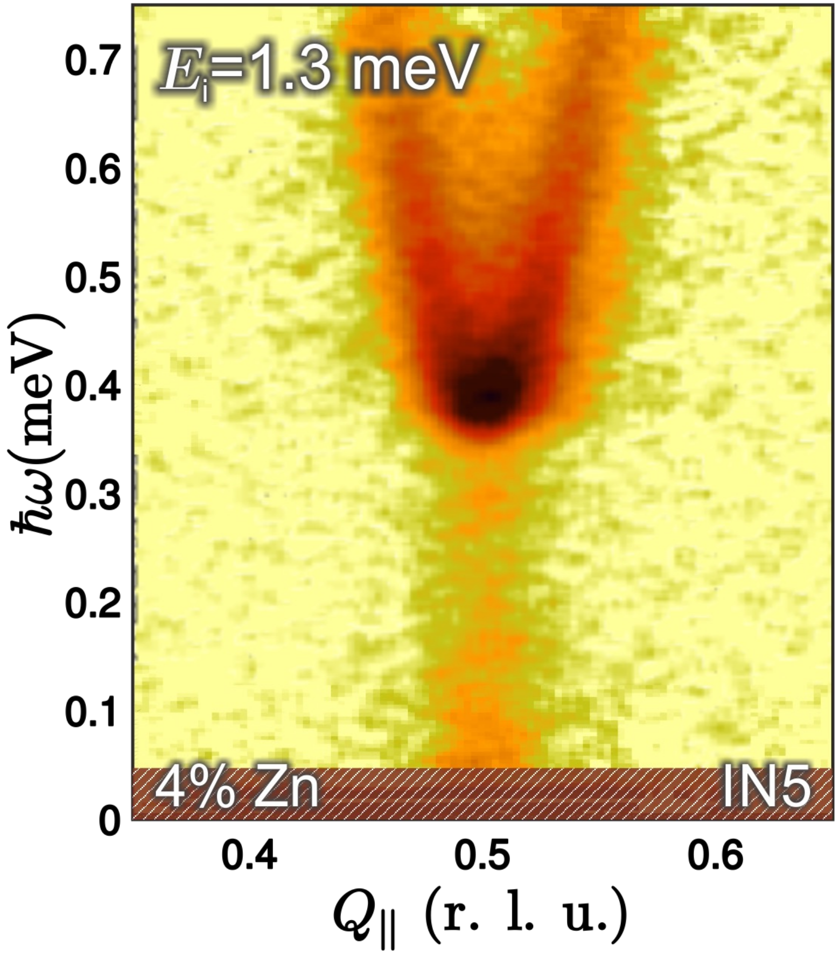


- Deuterated chemicals
- 2g crystal assembly
- 4% Zn substitution
- ξ≈6, L ≈ 12

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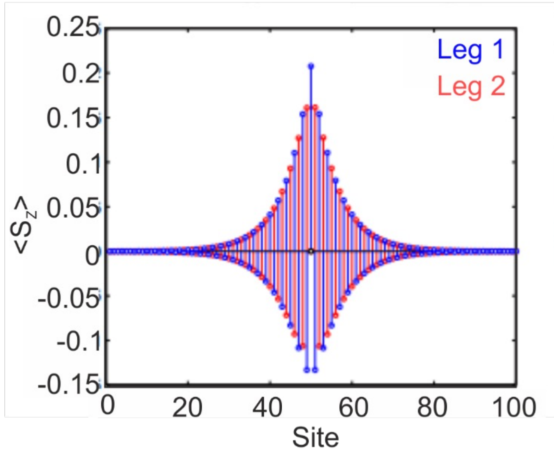


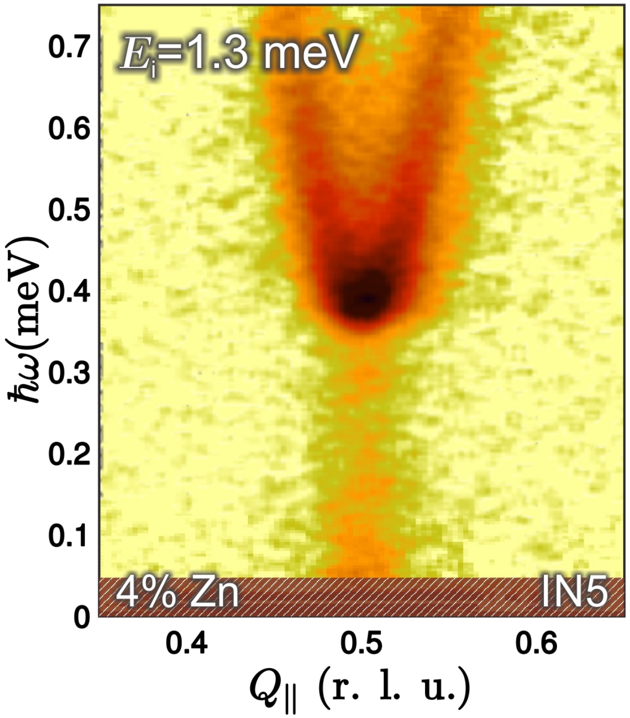


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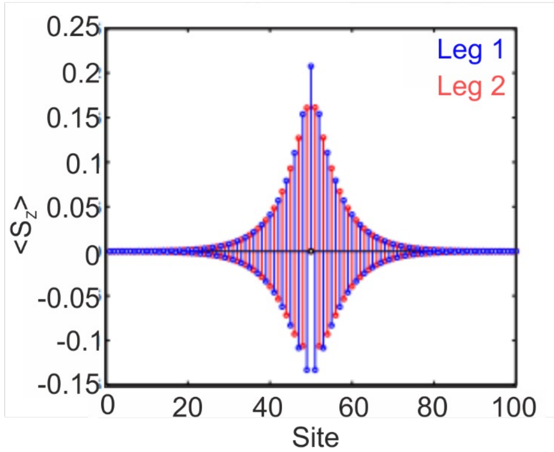


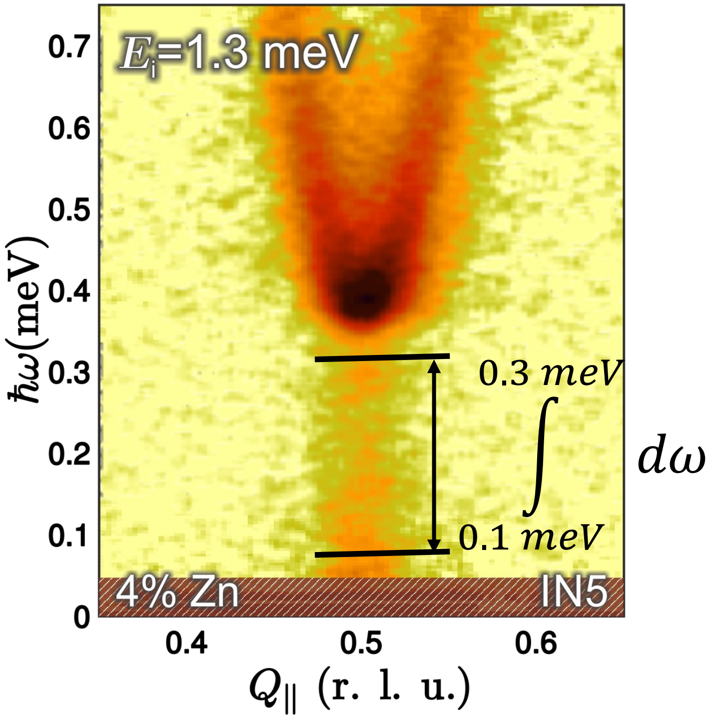


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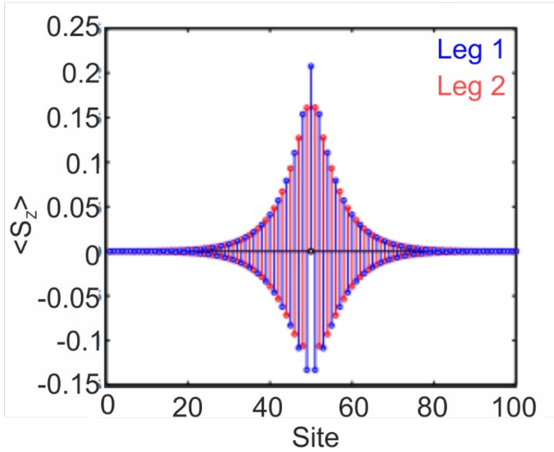
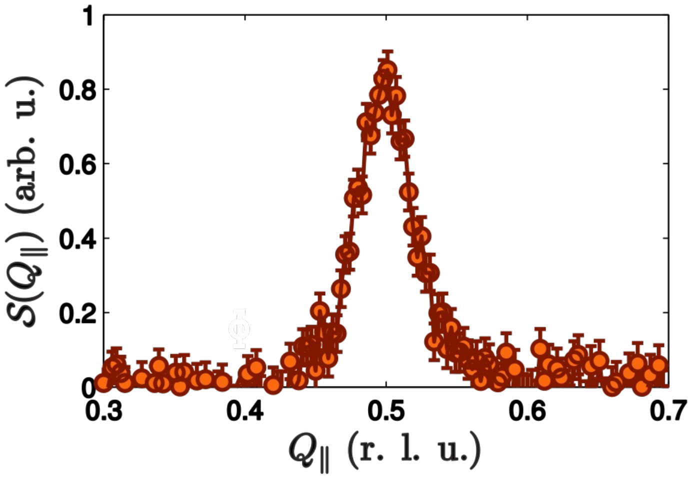
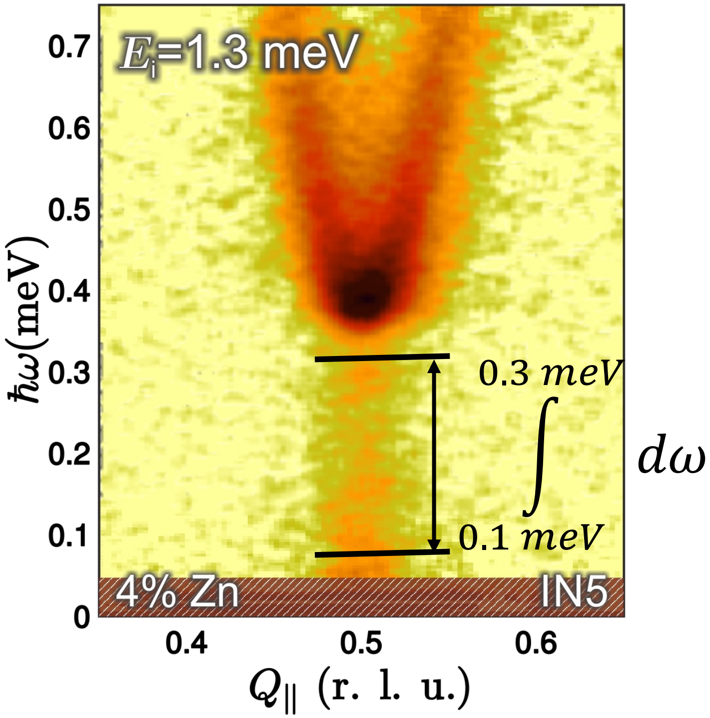
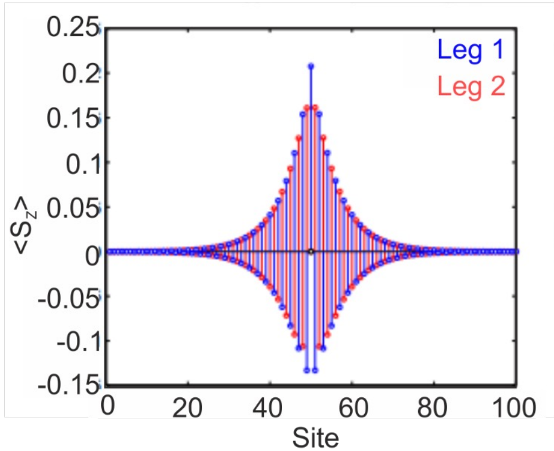
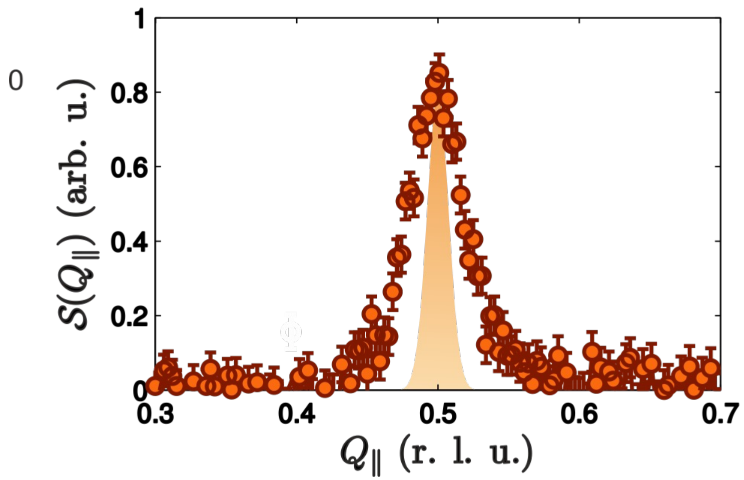


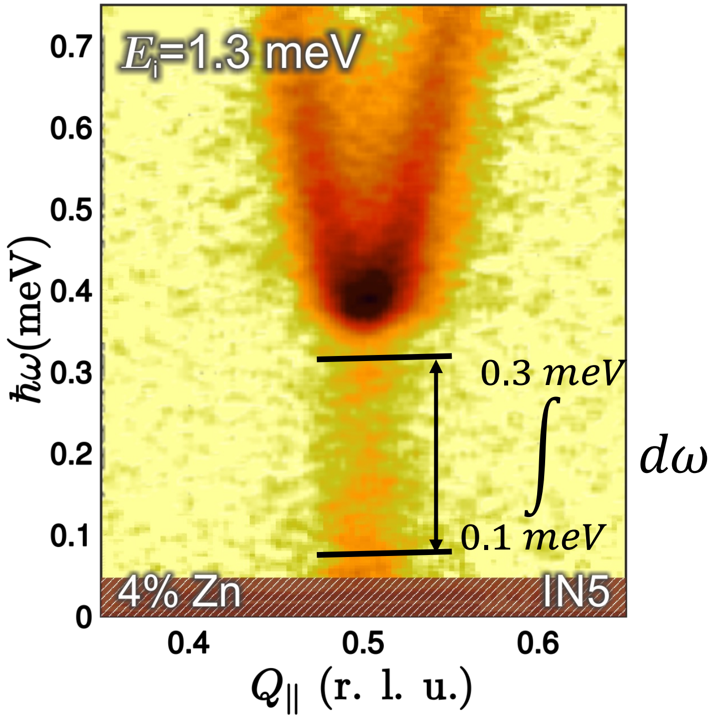




Schmidiger, Povarov, Galeski et al Phys. Rev. Lett. 116, 257203




$$\left|F(Q_{\parallel}, Q_{\perp})\right|^2 = \left|\frac{(1 - e^{-i2\pi Q_{\perp}})\sinh(\xi^{-1})}{\cos(2\pi Q_{\parallel}) + \cosh(\xi^{-1})} - 1\right|^2$$

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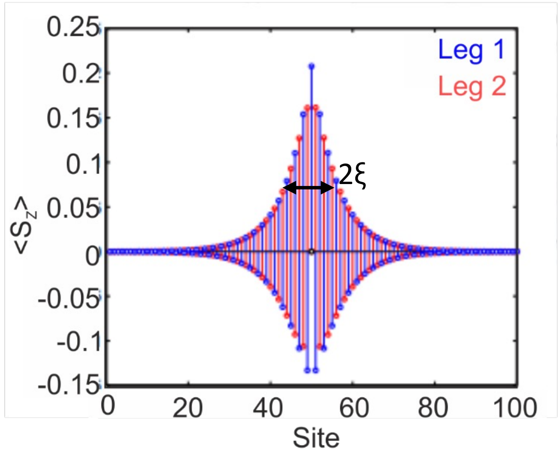


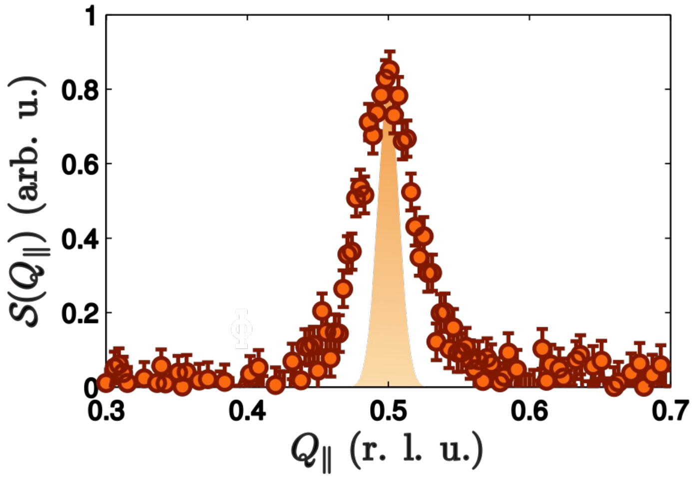


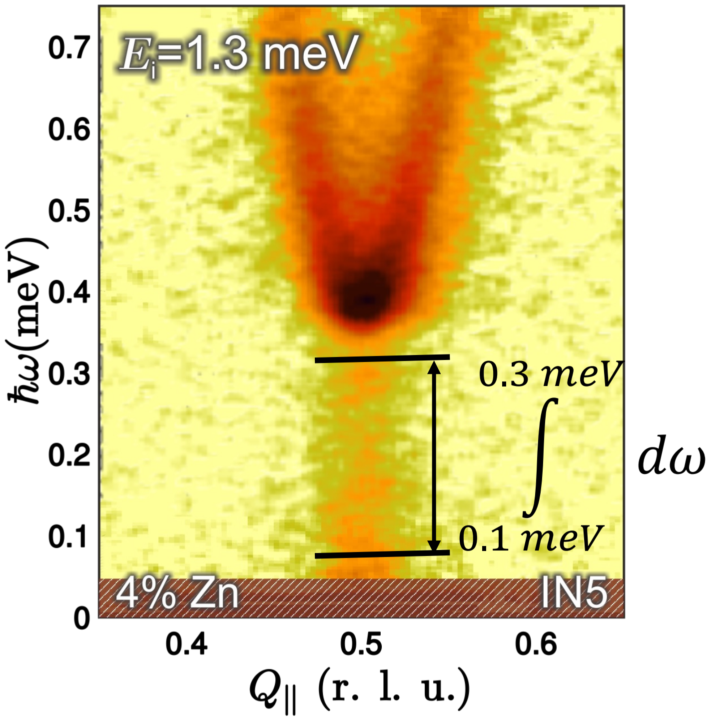


$$\left|F(Q_\parallel, Q_\perp)\right|^2 = \left|\frac{(1-\mathrm{e}^{-\mathrm{i}2\pi Q_\perp})\sinh(\xi^{-1})}{\cos(2\pi Q_\parallel)+\cosh(\xi^{-1})} - 1\right|^2$$

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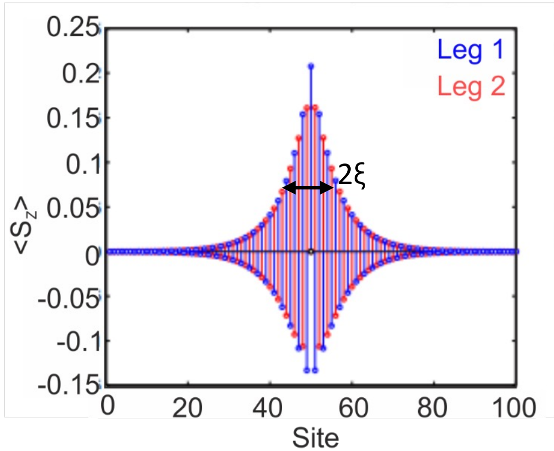
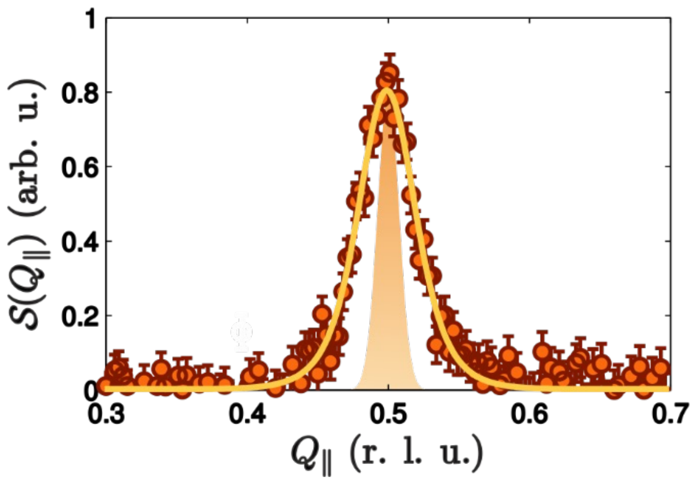
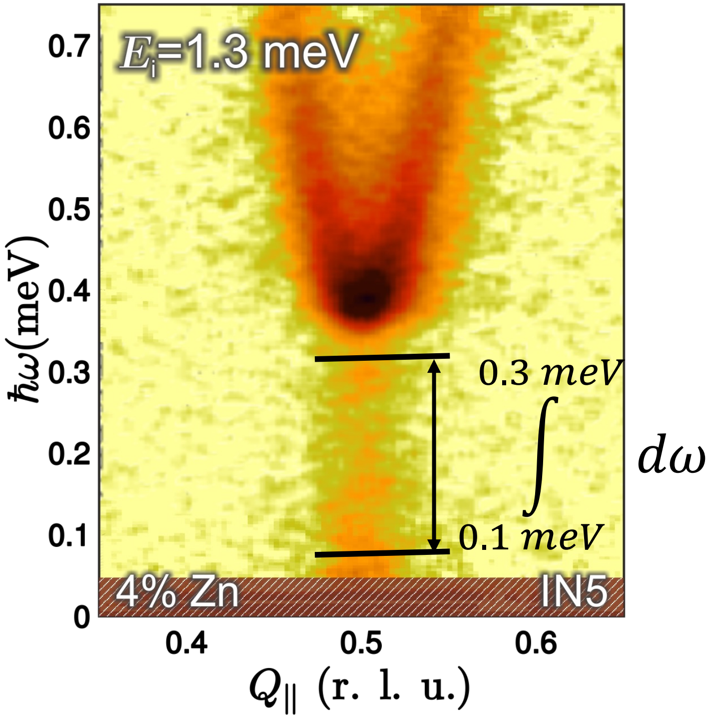
# DIMPY – Dynamics






$$|F(Q_{\parallel}, Q_{\perp})|^2 = \left| \frac{(1 - \mathrm{e}^{-\mathrm{i}2\pi Q_{\perp}}) \sinh(\xi^{-1})}{\cos(2\pi Q_{\parallel}) + \cosh(\xi^{-1})} - 1 \right|^2$$

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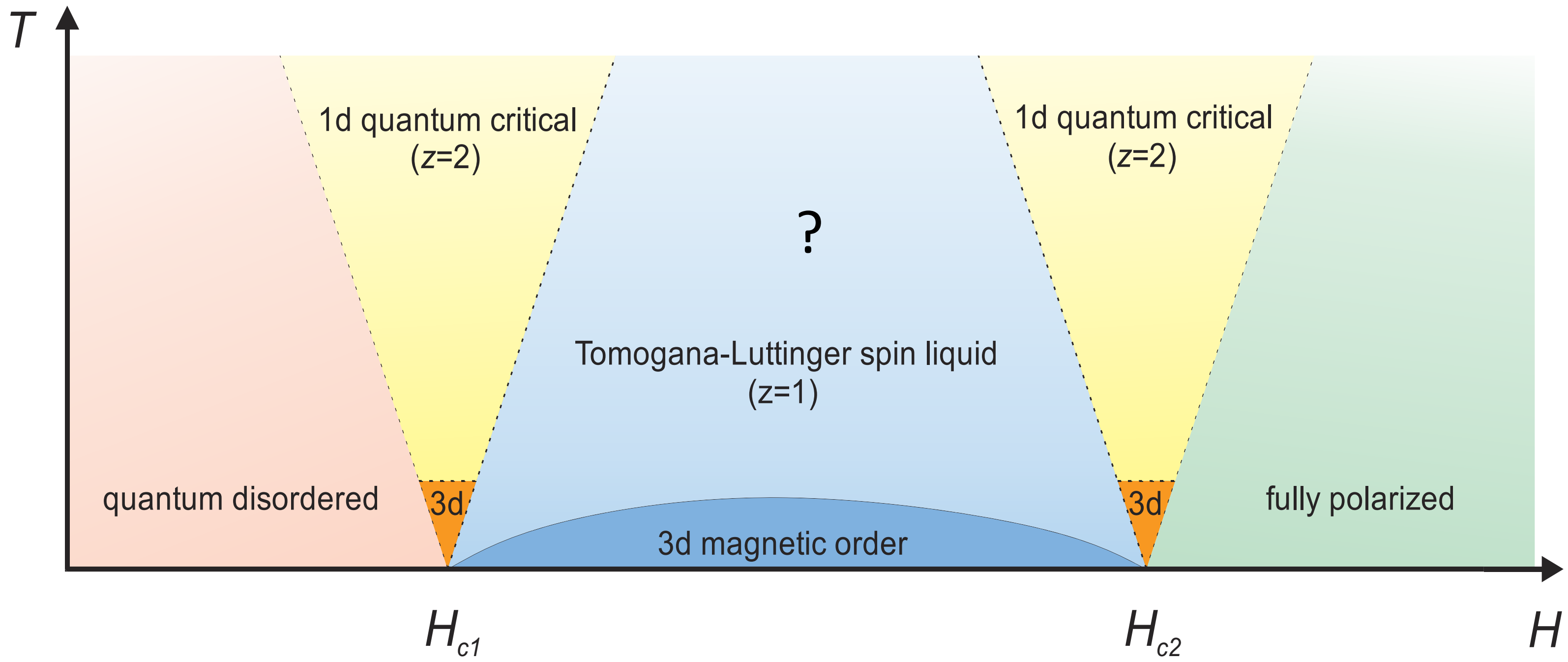
# Disordered Luttinger liquid

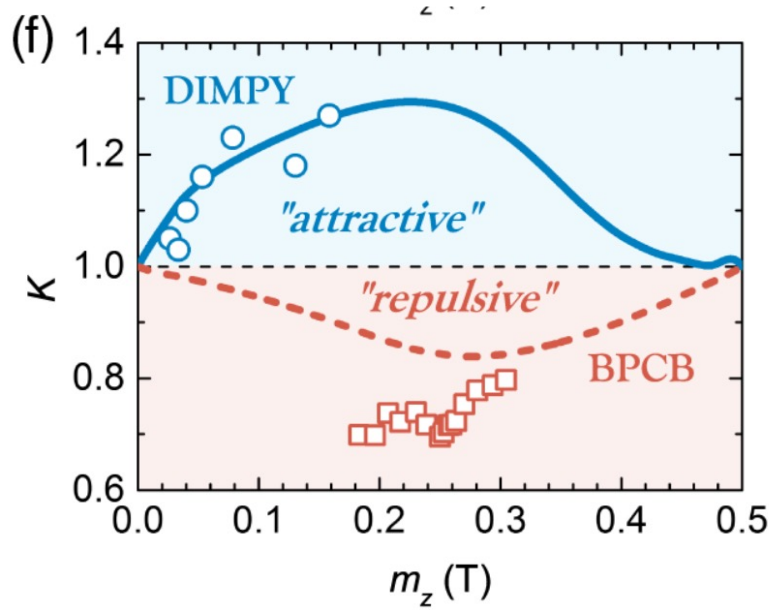
- Spin island picture collapses:
  - Exponential -> algebraic correlations



- Transverse staggered suscetibility
  - Scaling for finite segments:
  - Using results for G+-(LT) of Bohrdt et al.

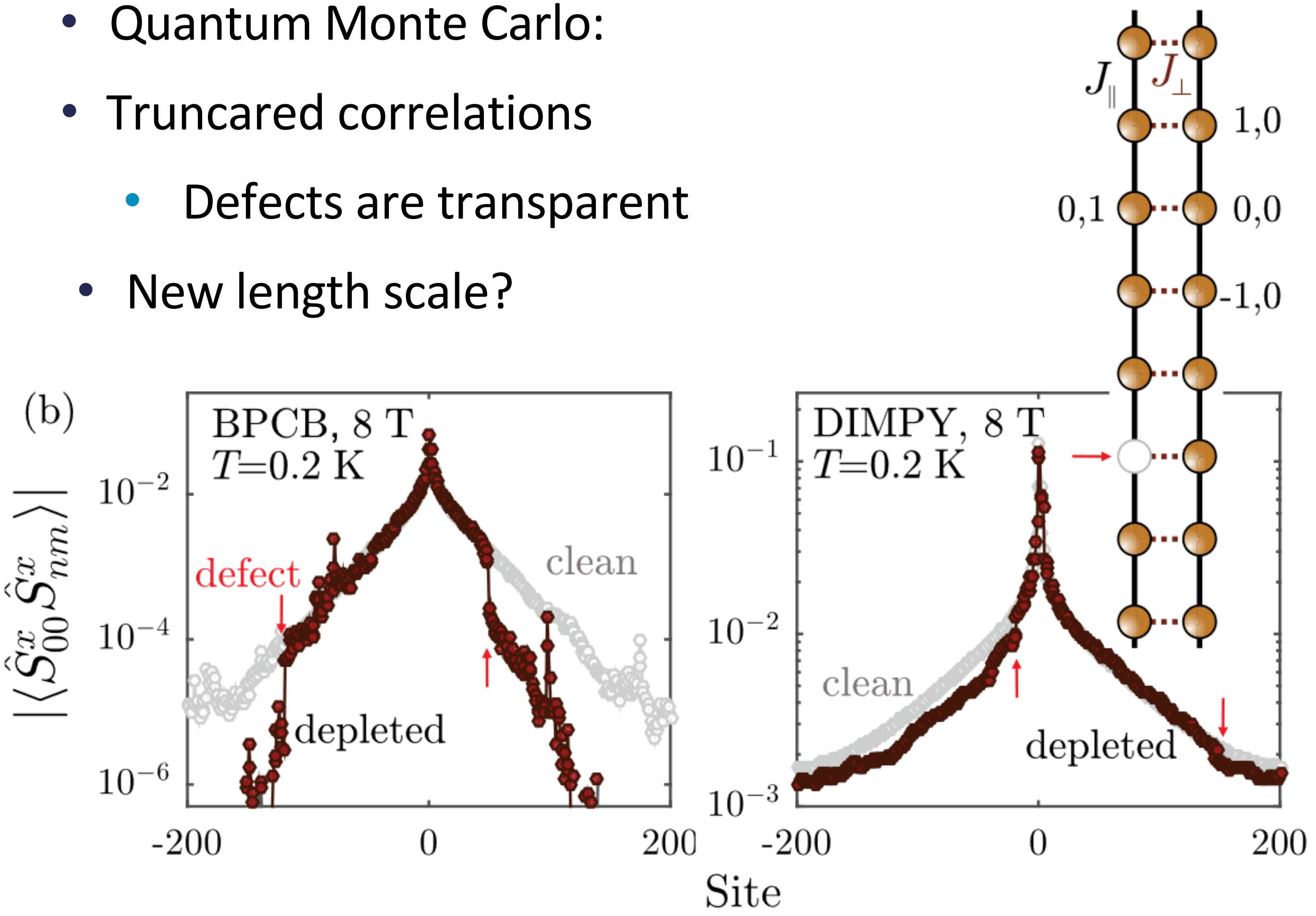
$$\chi_{st}^{\pm} = \left(\frac{T}{v}\right)^{\frac{1}{2K}-1} F_K\left(\frac{T}{v}\right) \Rightarrow \chi_{st}^{\pm} = \left(\frac{T}{v}\right)^{\frac{1}{2K}-1} F'_K\left(\frac{LT}{v}\right)$$

- Chains are always repulsive
- Interactions?
  - BPCB - repulsive
  - DIMPY - attractive




Jeong et al. Phys. Rev. Lett. 117, 106402

- Quantum Monte Carlo:
- Truncared correlations
    - Defects are transparent
- New length scale?

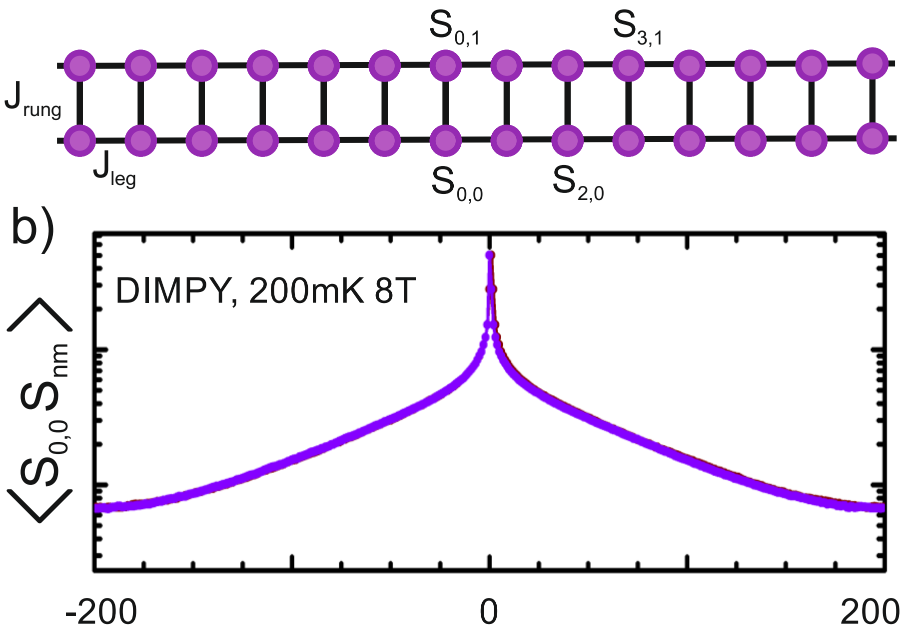


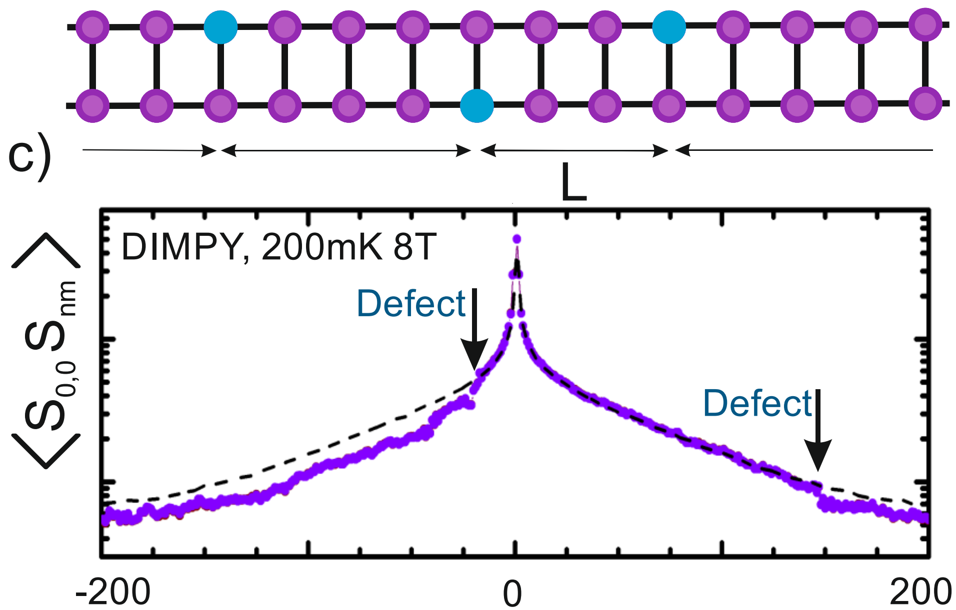

Galeski et al. Phys. Rev. Lett. 128, 237201

# Luttinger liquid: Spin Ladders

UNIVERSITÄT BONN

- Quantum Monte Carlo:
- Truncared correlations
  - Defects are transparent
- New length scale?

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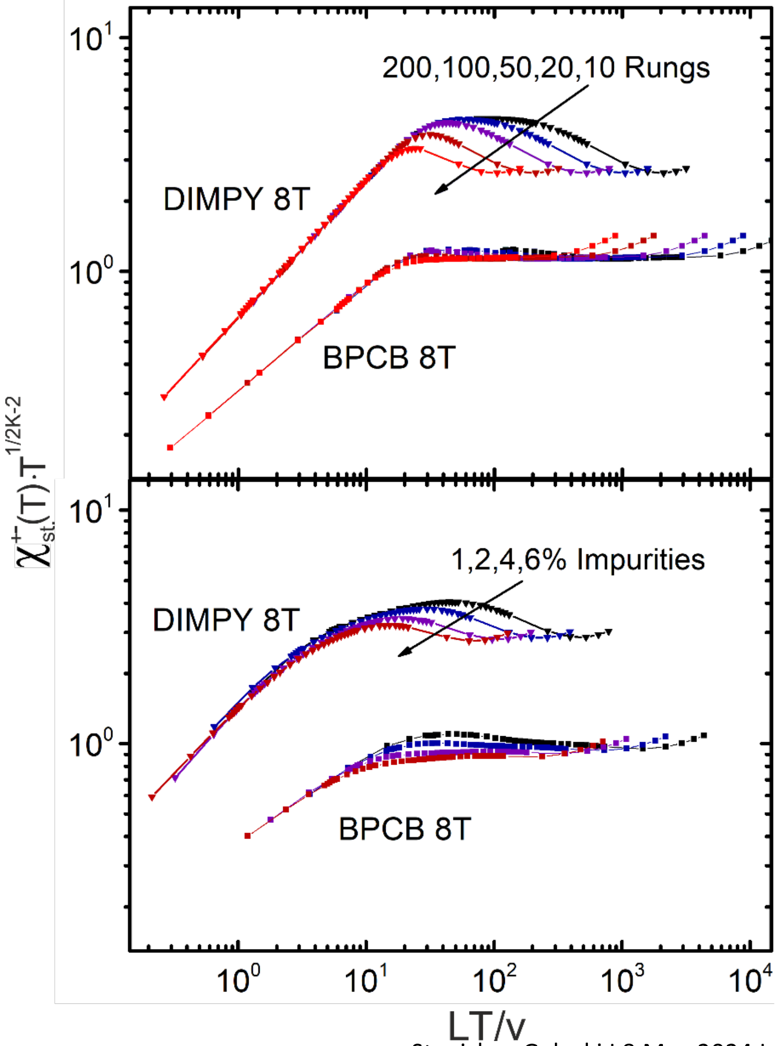
# Luttinger liquid: Spin Ladders – LT Scaling

UNIVERSITÄT BONN

- Transverse staggered suscetibility
  - Scaling for finite segments:
  - Using results for $G^{+-}(LT)$ of Bohrdt et al.

$$\chi_{st}^{\pm} = \left(\frac{T}{v}\right)^{\frac{1}{2K}-1} F_K\left(\frac{LT}{v}\right)$$

- Interactions?
  - BPCB - repulsive
  - DIMPY - attractive

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# Experimental test?

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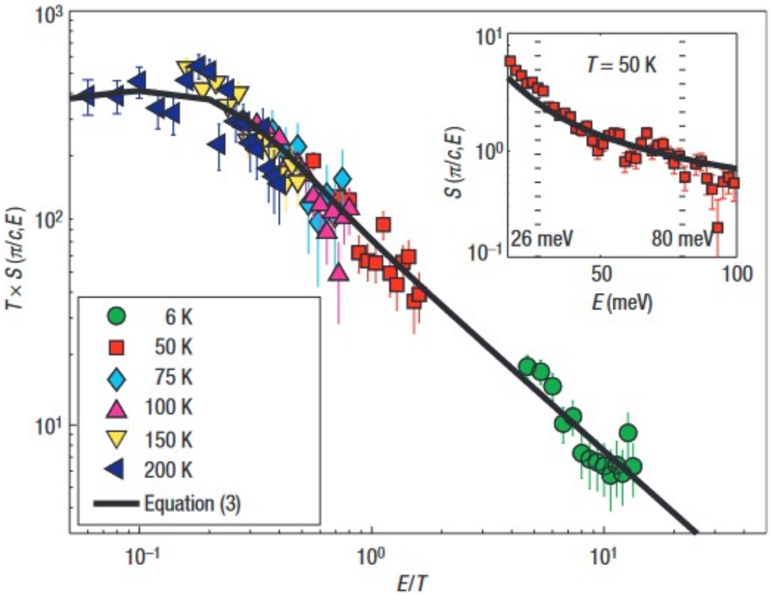
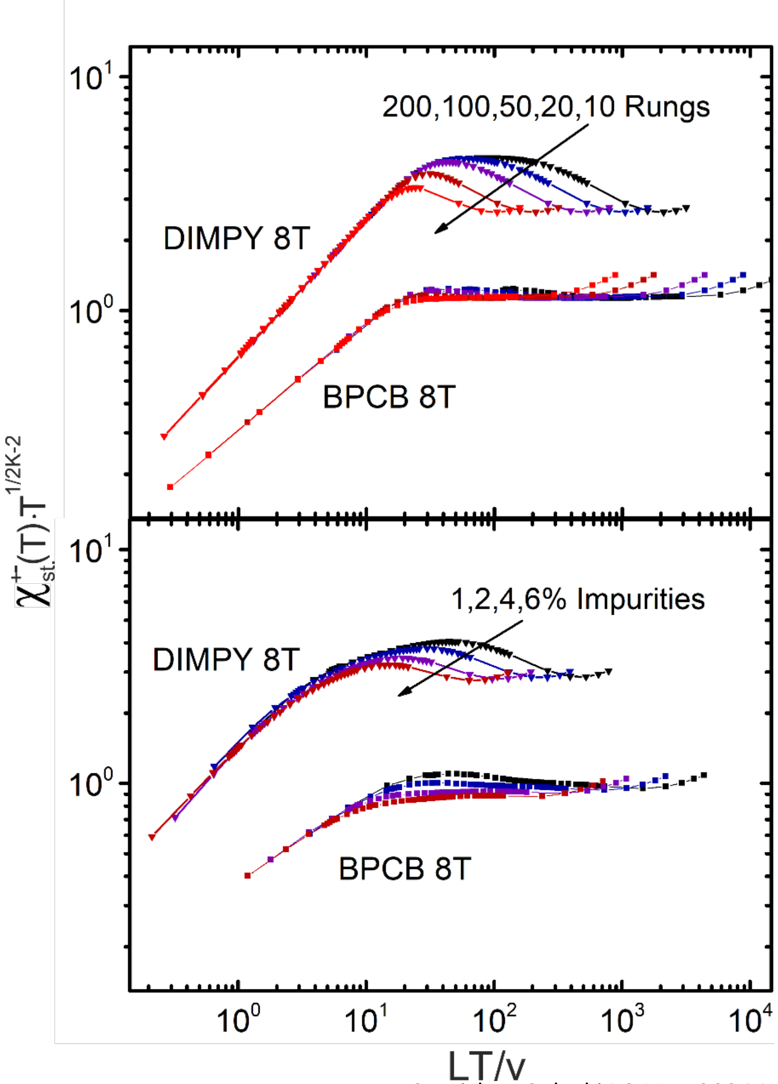
- Neutron scattering?
  - Many different deuterated crystals
  - Months of measurement time
  - No chance....

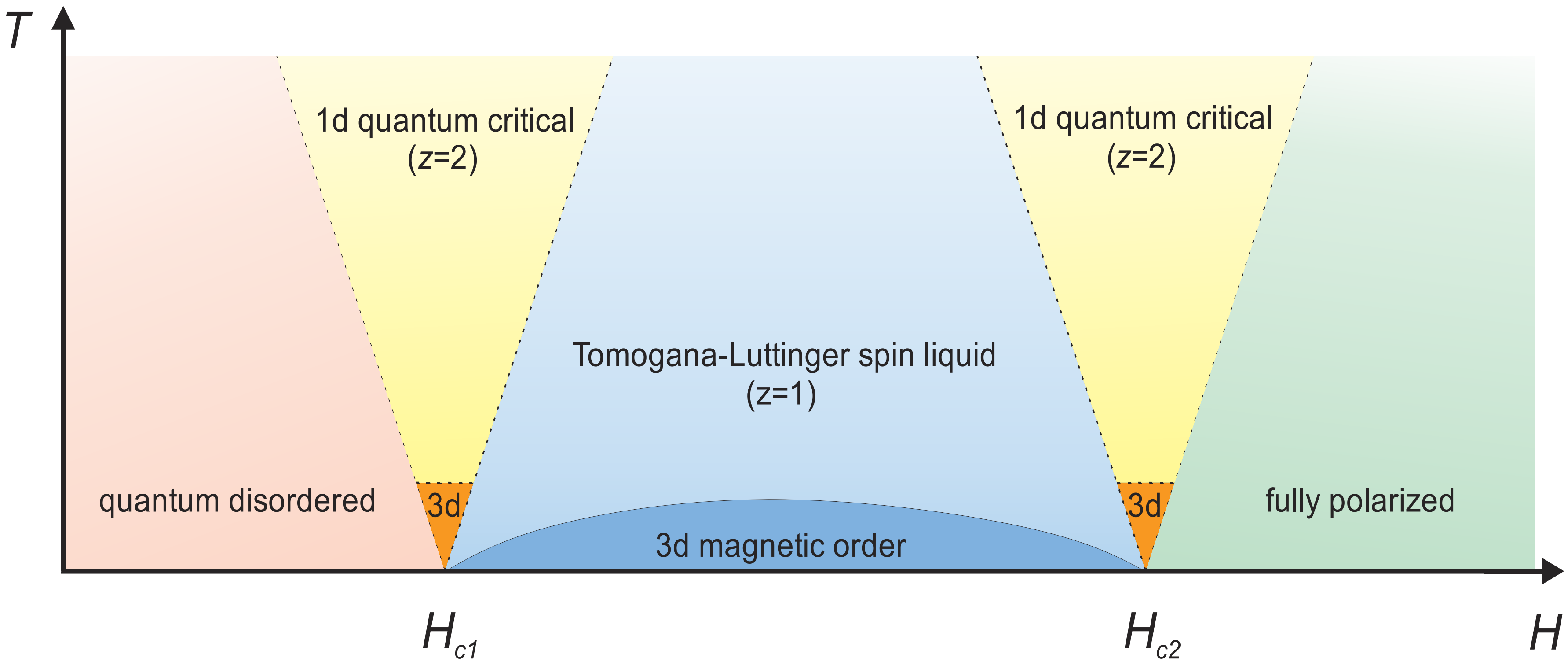



*Lake et al. Nature Mater* **4**, 329–334 (2005)
Galeski et al. Phys. Rev. Lett. 128, 237201

# Experimental test: 3D Long Range Order

UNIVERSITÄT BONN



# Experimental test: 3D Long Range Order

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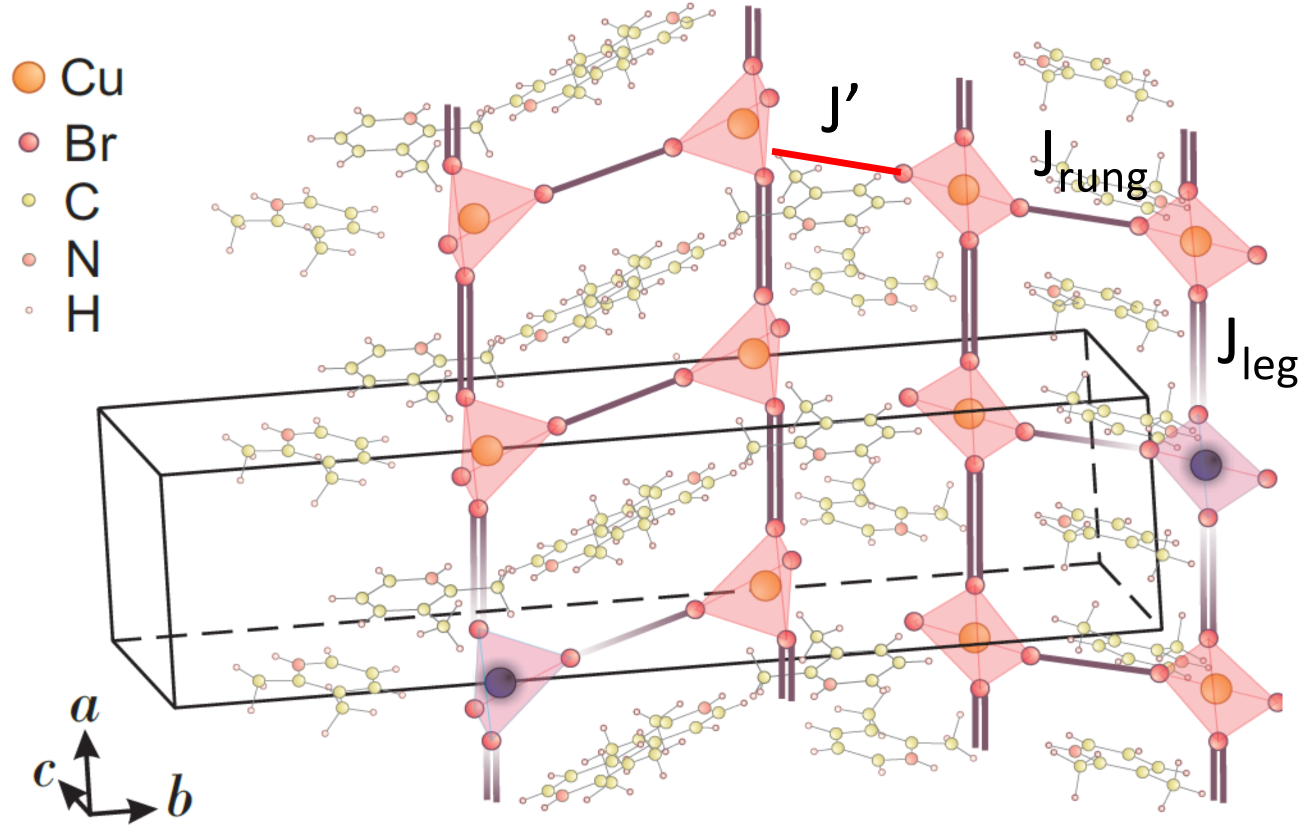
$J_{leg}$~$J_{rung}$~10K
J'~0.1K

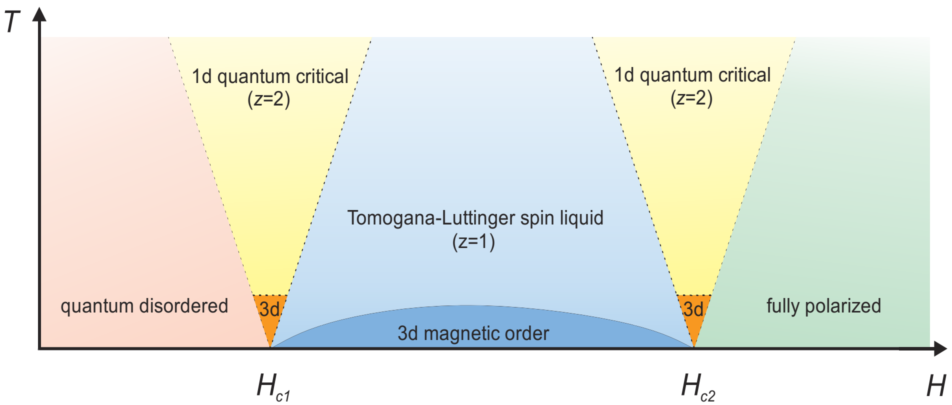


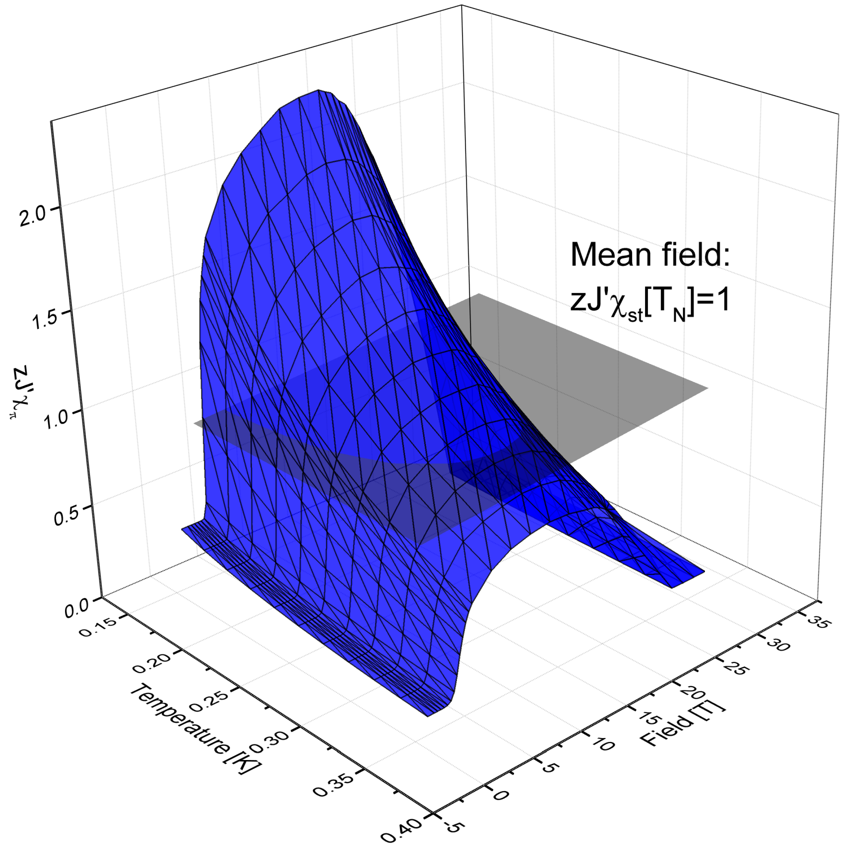
Mean field theory for

coupled 1D objects

$$\langle J_{3D}\rangle \chi^{\pm}_{st}(T_N) = 1$$

Indirect way of measuring $\chi^{\pm}_{st.}$(T)

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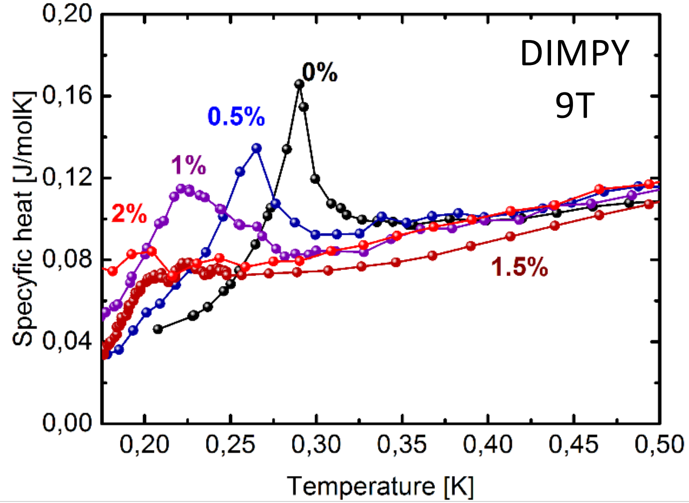
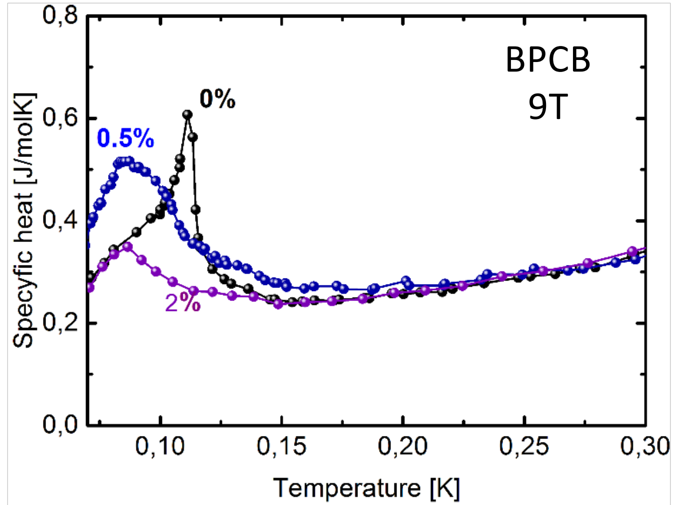
# Experimental test: 3D Long Range Order

UNIVERSITÄT BONN

- Substracted nuclear contribution
  - Depletion reduces $T_N$
  - Regardless of interaction parameter K
- Transition broadens
  - For 2% order fully supressed
  - Possible break down of MFT

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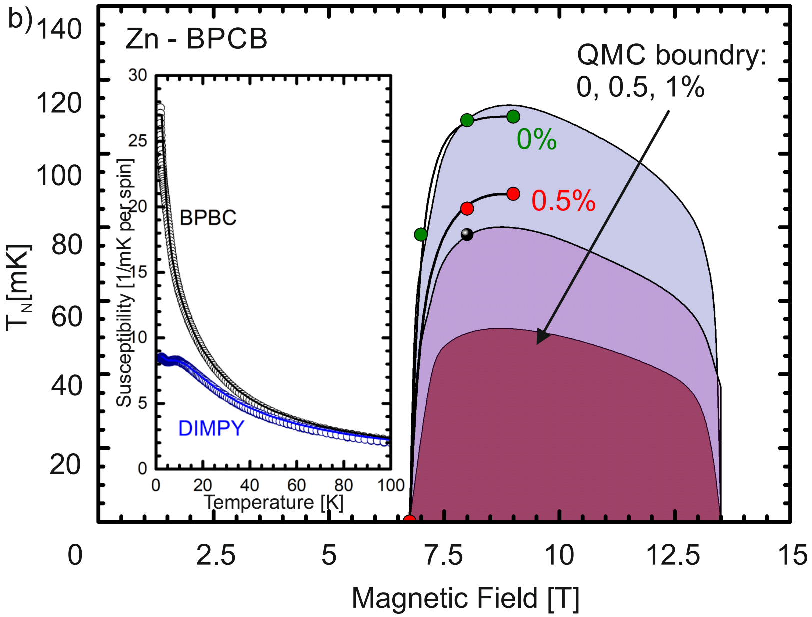
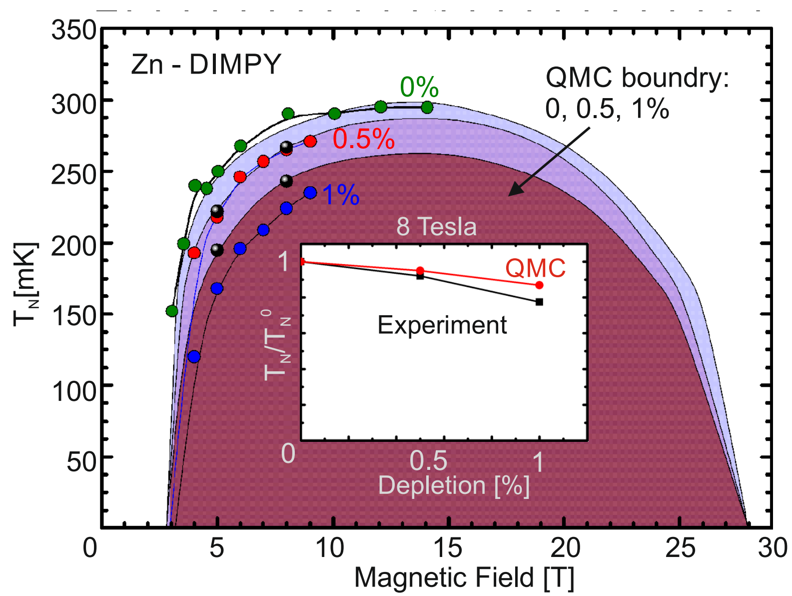
# Experimental test: 3D Long Range Order

UNIVERSITÄT BONN

- Experiment: Maximum of specyfic heat anomally
- QMC: months of simulations
- Agreements within 10%! → LT scaling confirmed!

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# Summary:

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J. Ollivier

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FÜR CHEMISCHE PHYSIK FESTER STOFFE

A. Zheludev

D. Blosser

D. Schmidiger

ETH

NEUTRONS FOR SOCIETY

J. Gooth

S .Gvasaliya

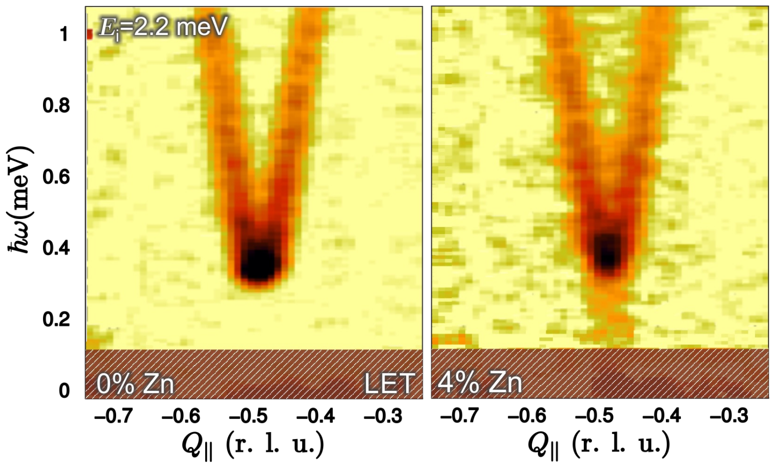
Laboratory for Solid State Physics
Neutron Scattering & Magnetism

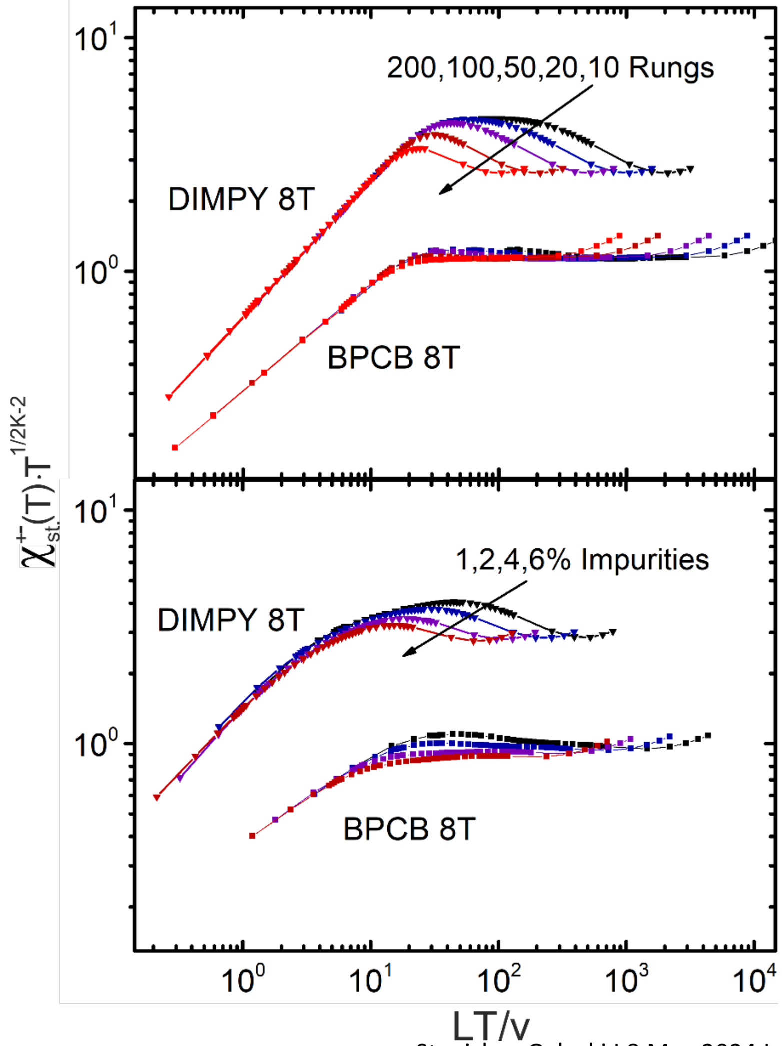
## Disorder in the Haldane gapped phase:

- New emergent degrees of freedom
- New strongly disordered magnetic sub-system

## Disorder in the TLL phase:

- New lengths cale
- Regardless of interaction sign
- Even if lattice continuity is preserved

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