Particle Production in Curved Spacetimes

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in collaboration with W.D. van Suijlekom and H. Falcke.

M.F. Wondrak, W.D. van Suijlekom, and H. Falcke, Phys. Rev. Lett. 130 (2023) 221502.

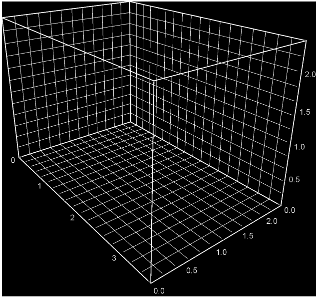
Outline

- Particle production from vacuum fluctuations
- Formalism from effective actions
- Application to electric and gravitational fields
- Discussion & conclusions

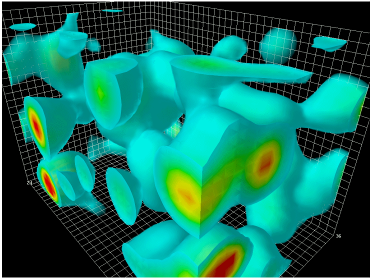
How Do Particles Arise From Vacuum Fluctuations?

Vacuum Fluctuations

Classical vacuum



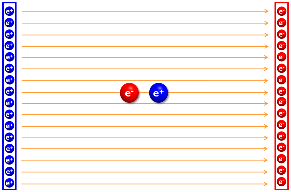
Quantum vacuum



- The quantum vacuum state is populated by short-lived excitations, staying undetectable according to the Heisenberg uncertainty relation, $\Delta E\,\Delta t \geq \hbar/2$.
- The simulation on the right shows gluons in quantum chromodynamics (QCD) arising, interacting, and annihilating.
  (lattice QCD simulation, box size: $2.4 \times 2.4 \times 3.6\ \text{fm}^3$).

http://www.physics.adelaide.edu.au/theory/staff/leinweber/VisualQCD/ImprovedOperators/index.html [last access: 10 February 2024].

Electric Background Field



Particle production in a homogeneous electric field $\vec{E}$ (Schwinger effect).
Here: Free complex scalar field of charge q and mass m.

Does a vacuum state evolve into a non-vacuum state?

$$\langle 0_{\text{out}}|0_{\text{in}}\rangle = \int \mathcal{D}\phi \, \exp\left(\frac{\mathrm{i}}{\hbar} S[\phi, A]\right) =: \exp\left(\frac{\mathrm{i}}{\hbar} W[A]\right).$$

$W = \int \mathrm{d}^4x \sqrt{-g}\, \mathcal{L}_{\text{eff}}$: effective action; A: e.m. background field

The probability for particle production reads

$$1 - |\langle 0_{\text{out}}|0_{\text{in}}\rangle|^2 = 1 - \exp(-2/\hbar\, \Im W[A]).$$

Effective Lagrangian for a constant homogeneous electric background field:

$$\Im(\mathcal{L}_{\text{eff}}) = \frac{(q\vec{E})^2}{16\pi^3\hbar} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} \exp\left(-\frac{n\pi m^2}{\hbar\sqrt{q^2\vec{E}^2}}\right).$$

H.H. Euler, W. Heisenberg, Zeitschr. Physik 98 (1936) 714; J.S. Schwinger, Phys. Rev. 82 (1951) 664.

Electric Background Field

- Diagrammatic language for a vacuum fluctuation: closed loop.
- Obstructions to annihilation:
    - Coulomb force in an electric field (Schwinger effect).
- Reduced probability for annihilation. Both partners turn into real particles.

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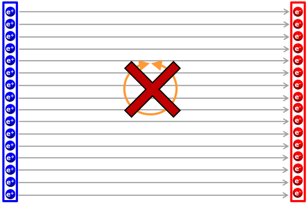

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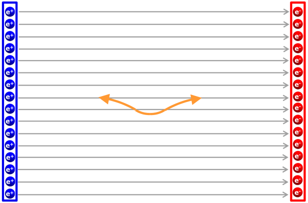

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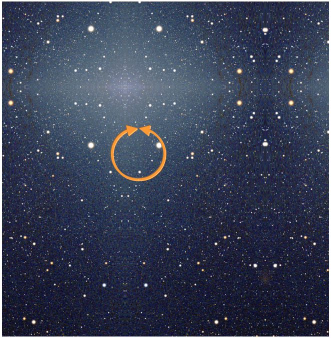

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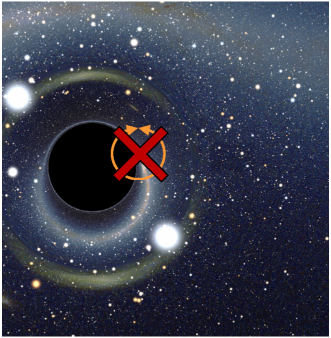

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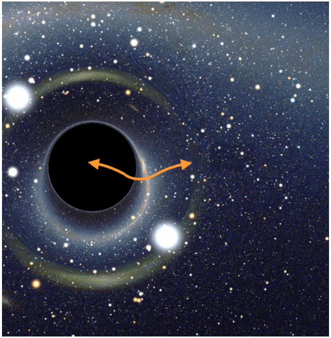

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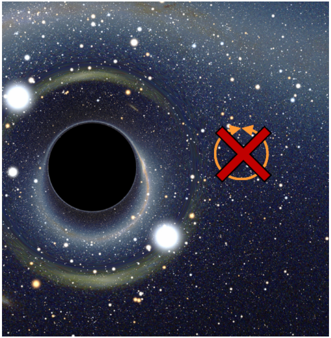

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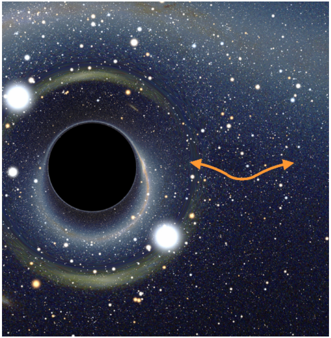

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Effective Action

Does a vacuum state evolve into a non-vacuum state?

Special case of the generating functional

$$\langle 0_{\mathrm{out}} | 0_{\mathrm{in}} \rangle = \int \mathcal{D}\phi \, \exp\left(\frac{\mathrm{i}}{\hbar} S[\phi, g] \right) =: \exp\left(\frac{\mathrm{i}}{\hbar} W[g] \right)$$

introducing the effective action W. The probability for particle production reads

$$1 - |\langle 0_{\mathrm{out}} | 0_{\mathrm{in}} \rangle|^2 = 1 - \exp(-2/\hbar \, \Im W[g]) \, .$$

The action for a free neutral scalar field in Euclidean spacetime (Wick-rotated) is given by

$$S_{\mathrm{E}}[\phi, g_{\mathrm{E}}] = \frac{1}{2} \int \mathrm{d}^D x \, \sqrt{g_{\mathrm{E}}} \, \left(\hbar^2 \, \partial_\mu \phi \, \partial^\mu \phi + m^2 \phi^2 + \hbar^2 \, \xi R \, \phi^2 \right) =: \frac{1}{2} \int \mathrm{d}^D x \, \sqrt{g_{\mathrm{E}}} \, \phi H \phi.$$

ξ: gravitational coupling parameter.
$\xi = 0$: minimal coupling, i.e. the only coupling term between matter and curvature is the metric determinant.
$\xi = 1/6$: conformal coupling, i.e. classical field equations invariant under conformal transformations.

Effective Action

Schwinger representation of the effective action:

The Gaussian form of the path integral shows that the effective action is 1-loop exact, and has the formal solution

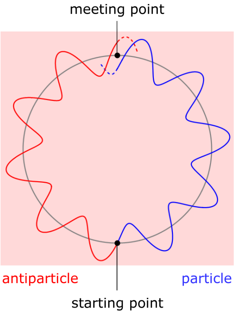
$$\begin{aligned} W_{\mathrm{E}}[g_{\mathrm{E}}] &= -\hbar \ln \int \mathcal{D}\phi \, \exp\left(-\frac{1}{\hbar} S_{\mathrm{E}}[\phi, g_{\mathrm{E}}]\right) \\ &= \frac{\hbar}{2} \operatorname{Tr} \ln H/\tilde{\mu}^2 \\ &= -\frac{\hbar}{2} \left(\frac{\tilde{\mu}}{\hbar}\right)^{2z} \int_0^\infty \frac{\mathrm{d}s}{s^{1-z}} \underbrace{\operatorname{Tr}\left(e^{-s(\Delta+\xi R)}\right)}_{\text{heat trace}} e^{-s(m^2 - \mathrm{i}\epsilon)/\hbar^2} . \end{aligned}$$

ζ-function regularization: arbitrary mass scale $\tilde{\mu}$, parameter $z \in \mathbb{C}$.

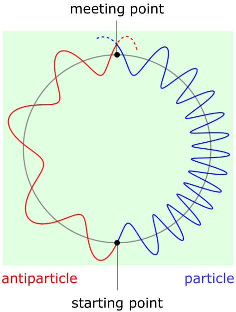
Interpretation via Interference

When re-writing the heat trace in the Schwinger representation (first quantization), the effective action in $D = 4$ dimensions becomes

$$W[g] = -\frac{\hbar}{2}\left(\frac{\tilde{\mu}}{\hbar}\right)^{2z}\int \mathrm{d}^4x\,\sqrt{-g}\int_0^\infty \frac{\mathrm{d}s}{s^{1-z}}\int_{x(0)=x(s)}\mathcal{D}x(\tau)\,\exp\left(\frac{\mathrm{i}}{\hbar}\,S_\mathrm{e}[x(\tau)]\right).$$



Destructive interference.



Constructive interference.

Covariant Perturbation Theory

Expand the heat trace for the scalar field in curvatures (covariant perturbation theory)

$$\mathrm{Tr}\, e^{-s(\Delta+\xi R)} \simeq \frac{1}{(4\pi s)^2} \int \mathrm{d}^4x \sqrt{g_\mathrm{E}} \left[1 - s\left(\xi - \frac{1}{6}\right) R \right.$$
$$+ s^2 \Big(R_{\mu\nu\rho\sigma} \tilde{f}_1(s\Delta) R^{\mu\nu\rho\sigma} - R_{\mu\nu} \tilde{f}_1(s\Delta) R^{\mu\nu} + R f_R(s\Delta) R$$
$$\left. + \Omega_{\mu\nu} f_5(s\Delta) \Omega^{\mu\nu} \Big) \right] + \dots$$

including the case that the particle was also electrically charged.

f: form factors;

$\Omega_{\mu\nu}$: curvature of e.m. gauge connection, in flat spacetime: $\Omega_{\mu\nu} = [\nabla_\mu, \nabla_\nu] = \mathrm{i}qF_{\mu\nu}/\hbar$

The effective action becomes

$$W_\mathrm{E} = -\frac{\hbar}{32\pi^2} \left(\frac{\tilde{\mu}}{\hbar}\right)^{2z} \int \mathrm{d}^4x \sqrt{g_\mathrm{E}} \left[\Gamma(-2+z) \left(\frac{m^2}{\hbar^2} - \mathrm{i}\epsilon\right)^{2-z} - \Gamma(-1+z) \left(\frac{m^2}{\hbar^2} - \mathrm{i}\epsilon\right)^{1-z} \left(\xi - \frac{1}{6}\right) R \right.$$
$$\left. + \Gamma(z) \left(\frac{m^2}{\hbar^2} - \mathrm{i}\epsilon\right)^{-z} \left(\frac{1}{180} \left(R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - R_{\mu\nu}R^{\mu\nu}\right) + \frac{1}{2}\left(\xi - \frac{1}{6}\right)^2 R^2 + \frac{1}{12}\Omega_{\mu\nu}\Omega^{\mu\nu} \right) \right] + \dots$$

A.O. Barvinsky, G.A. Vilkovisky, Nucl. Phys. B333 (1990) 471; A. Codello, O. Zanusso, J. Math. Phys. 54 (2013) 013513.

Covariant Perturbation Theory

When sending the regularization parameter $z \to 0$, an imaginary contribution arises:

$$\left(\frac{\tilde{\mu}}{\hbar}\right)^{2z} \left(\frac{m^2}{\hbar^2} - \mathrm{i}\epsilon\right)^{-z} = 1 - z \ln\left(\frac{m^2}{\tilde{\mu}^2} - \mathrm{i}\epsilon\right) + \mathcal{O}(z^2).$$

The effective action presents an imaginary part in the massless case!

The particle production probability reads:

$$\mathcal{P} = \frac{1}{32\pi} \int \mathrm{d}^4x \sqrt{-g} \left[\frac{1}{2}\left(\xi - \frac{1}{6}\right)^2 R^2 + \frac{1}{180}\left(R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - R_{\mu\nu}R^{\mu\nu}\right) + \frac{1}{12}\,\Omega_{\mu\nu}\Omega^{\mu\nu}\right].$$

Particle Production in Purely Electric Fields

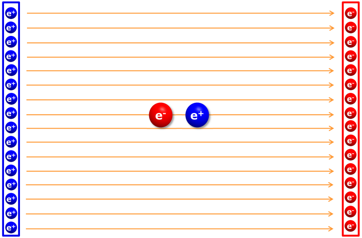
Massless free complex scalar field in a homogeneous electric field $\vec{E}$ (Schwinger effect).

From the treatment above, the probability for particle production events for a massless complex scalar field with charge q reads:

$$\begin{aligned}\mathcal{P} &= \frac{2}{32\pi} \int \mathrm{d}^4x \, \sqrt{-g} \, \frac{1}{12} \, \Omega_{\mu\nu}\Omega^{\mu\nu} \\ &= \frac{2}{32\pi} \int \mathrm{d}^4x \, \sqrt{-g} \, \frac{(q\vec{E})^2}{6\hbar^2}\end{aligned}$$

Consistent with the massless limit of Schwinger's closed-form expression:

$$\mathcal{P} = \frac{2}{32\pi} \int \mathrm{d}^4x \, \sqrt{-g} \, \frac{(q\vec{E})^2}{6\hbar^2} \, \frac{12}{\pi^2} \times \underbrace{\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} \exp\left(-\frac{n\pi m^2}{\hbar\sqrt{q^2\vec{E}^2}}\right)}_{\to \pi^2/12}$$

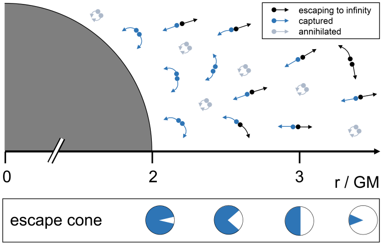


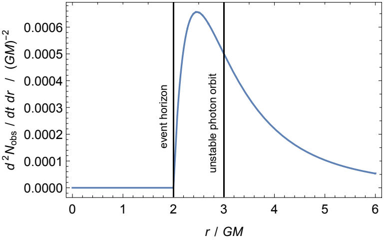
Particle Production in the Schwarzschild Spacetime

Massless scalar field in a Schwarzschild spacetime:

Ricci-flat ($R_{\mu\nu} = 0$). Kretschmann scalar: $R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} = C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} = \frac{48(GM)^2}{r^6}$.

Particle production events for a massless real scalar field: $\mathcal{P} = \frac{1}{32\pi} \int \mathrm{d}^4 x \sqrt{-g}\, \frac{4\,(GM)^2}{15 r^6}$.





- Between horizon and photon orbit:
  At least one particle has to end in the event horzion (see Hawking).
- Emission of entangled pairs outside the photon orbit.
  $\rightarrow$ Hint towards non-thermal emission.

Adjustment Factors

From analogy with the Schwinger effect:

- In a particle production event, an arbitrary number of pairs can be created. In the electric case, $12/\pi^2 \approx 1.22$ pairs are produced per event on average.
- Requiring that the electric and the gravitational field strength lead to an equal particle production event rate density, one finds a formal replacement rule $\Omega_{\mu\nu}\Omega^{\mu\nu} \to 2/30 \times R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$.

$$E_{\text{curv}} = \hbar\,(R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}/30)^{1/4} = \underbrace{6.11 \times 10^{-30}\,\text{J}}_{38\,\text{peV}} \left(\frac{M}{M_\odot}\right)^{1/2} \left(\frac{r}{2.5\,GM}\right)^{-3/2}$$

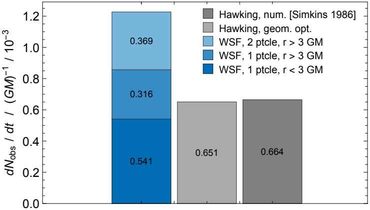
A more intuitive scale is given by the effective temperature for blackbody radiation from matching $\langle E \rangle = E_{\text{curv}}$:

$$T_{\text{eff}}(r) = \frac{30\zeta(3)}{\pi^4}\,\frac{E_{\text{curv}}(r)}{k_B} \approx 164\,\text{nK} \left(\frac{M}{M_\odot}\right)^{-1} \left(\frac{r}{2.5\,GM}\right)^{-3/2}$$

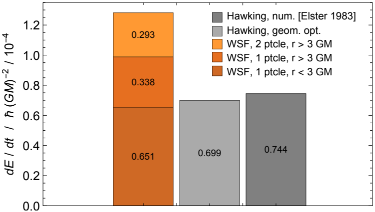
Comparison

	WSF	Hawking effect
Particle Flux	$12.3 \times 10^{-4}\,(GM)^{-1}$ $250\,\mathrm{s}^{-1}\,(M_\odot/M)$	$6.51 \times 10^{-4}\,(GM)^{-1}$ $132\,\mathrm{s}^{-1}\,(M_\odot/M)$
Energy Flux	$12.8 \times 10^{-5}\,\hbar\,(GM)^{-2}$ $5.57 \times 10^{-28}\,\mathrm{J\,s}^{-1}\,(M_\odot/M)^2$	$6.99 \times 10^{-5}\,\hbar\,(GM)^{-2}$ $3.04 \times 10^{-28}\,\mathrm{J\,s}^{-1}\,(M_\odot/M)^2$
Local temperature ($r = 2.5\,GM,\ M = M_\odot$)	164 nK	138 nK

$\Rightarrow$ The predictions correspond to each other up to a relative factor ~ 1.9.



Particle flux.



Energy flux.

Discussion

- A neutral massless scalar field is a model for 1 degree of freedom, photons and gravitons have two polarizations. Photons might not be created, but gravitons might (consider relation to anomalies).
- For mathematical consistency, we worked in Euclidean signature. A Wick rotation is naturally assumed to lead to a stationary Lorentzian spacetime.
- The curvature expansion of the effective action extends to 2^{nd} order. Consideration of the impact of a magnetic field on the Schwinger effect (from the parity-odd term $\Omega_{\mu\nu}\tilde{\Omega}^{\mu\nu}$) requires higher orders.
- The relation of the energy of produced pairs to curvature (Kretschmann scalar $R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$) has not been derived stringently, but from analogy to Schwinger. It might miss terms for more complex spacetimes.
- The local emission is assumed isotropic for a static observer, while tidal forces are anisotropic.

M.N. Chernodub, arXiv:2306.03892 [hep-th]; A. Ferreiro, J. Navarro-Salas, S. Pla, arXiv:2306.07628 [gr-qc]; M.P. Hertzberg, A. Loeb, arXiv:2307.05243 [gr-qc]; M.F. Wondrak, W.D. van Suijlekom, H. Falcke, 2308.12326 [gr-qc].

Conclusions

- Gravitational fields generate particle–anti-particle pairs. A virtual particle pair does not recombine because of tidal forces.
- The predicted particle and energy fluxes are compatible with the exact expression for the Schwinger effect and with the Hawking emission within a factor ~ 2.
- The presence of a black hole event horizon does not enter the calculations.

$\Rightarrow$ This indicates that normal matter (e.g. neutron stars) would decay, not only black holes.

- Resummation of higher-order curvature terms.
- Interpretation of negative values of $\Im W$.
- Back reaction on spacetime.

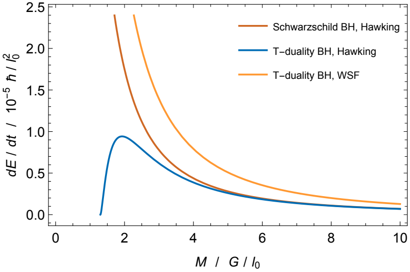
Back Up

Particle Production in the Regular Bardeen Spacetime

Massless scalar field in the Bardeen spacetime:

Static, spherically symmetric, asymptotically flat singularity-free spacetime.
Regulator ℓ_0: minimal length inherited from string theory ("T-duality black hole").

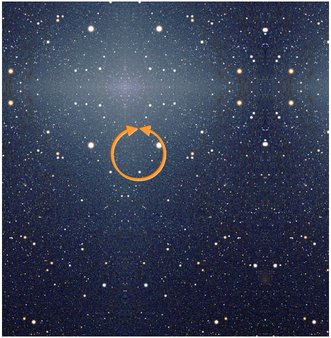
Line element: $ds^2 = -\left(1 - \frac{2G\,Mr^2}{(r^2+\ell_0{}^2)^{3/2}}\right) \mathrm{d}t^2 + \left(1 - \frac{2G\,Mr^2}{(r^2+\ell_0{}^2)^{3/2}}\right)^{-1} \mathrm{d}r^2 + r^2\,\mathrm{d}\Omega^2.$



Similar to the Hawking evaporation process for the singular Schwarzschild black hole, the evaporation process continues for a regular black hole leading to a final explosion.

P. Nicolini, E. Spallucci, M.F. Wondrak, Phys. Lett. B 797 (2019) 134888.

Particle Production



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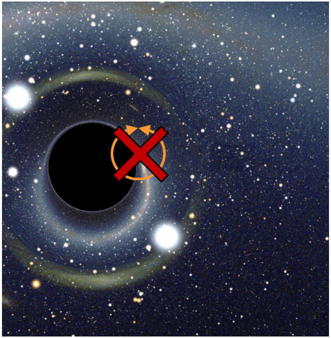

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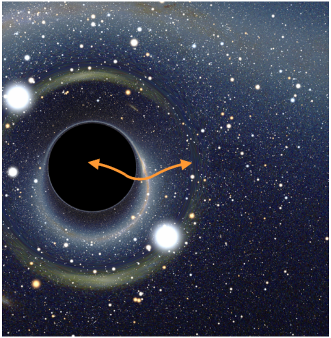

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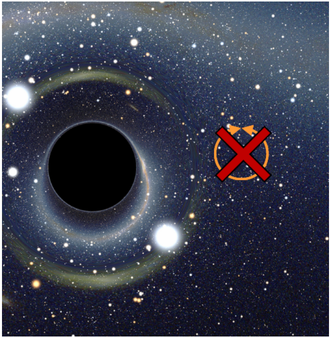

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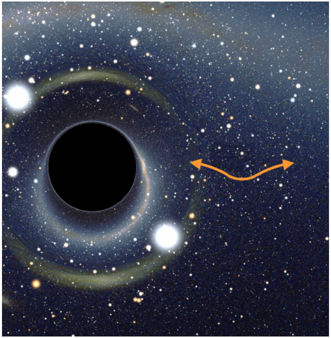

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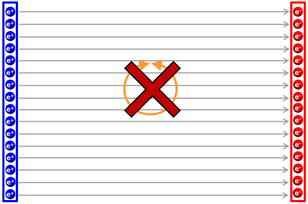

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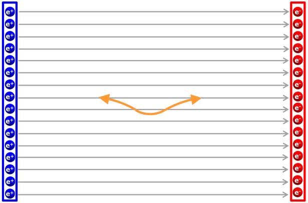

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    - Tidal forces in a gravitational field.
- Reduced probability for annihilation. Both partners turn into real particles.
- A distant observer perceives a flux of particles.

Particle Production in the Schwarzschild Spacetime

Particle production in a Schwarzschild spacetime:

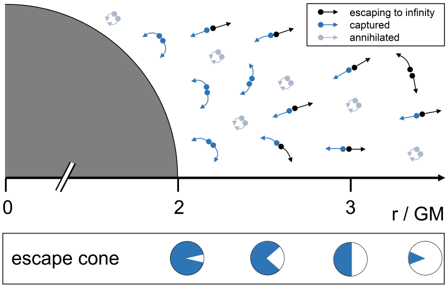
$$ds^2 = -\left(1 - \frac{2GM}{r}\right) dt^2 + \left(1 - \frac{2GM}{r}\right)^{-1} dr^2 + r^2 \left(d\theta^2 + \sin^2\theta \, d\phi^2\right)$$

Ricci-flat solution ($R_{\mu\nu} = 0$).
Kretschmann scalar:

$$R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} = C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} = 48(GM)^2/r^6$$

Particle production events for a massless real scalar field:

$$\mathcal{P} = \frac{1}{32\pi} \int d^4x \sqrt{-g} \, \frac{4\,(GM)^2}{15 r^6}$$



Escape cone similar to graybody factor.
Event horizon at $r = 2GM$: escape probability $= 0\%$,
photon orbit at $r = 3GM$: escape probability $= 50\%$.

Outside the photon orbit, even two (entangled) particles may escape to infinity.
→ Hint towards non-thermal emission.

Radial Distribution

Increasing pair production rate towards $r \to 0$,
increasing escape probability towards $r \to \infty$.

The highest production rate of escaping particles per spherical shell occurs at $\sim 2.5\,GM$.