Conformal symmetry and fundamental fields: Black holes and the Big Bang

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P. Beltrán-Palau, A. del Río and J. N.-S., PRD 2023
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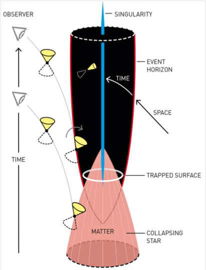
Motivation and plan of the talk

- General relativity predicts two different types of spacetime singularities: inside black holes and at the beginning of time
- The Event Horizon protects the outside world from black hole singularities [Penrose's cosmic censorship hypothesis (1969)]



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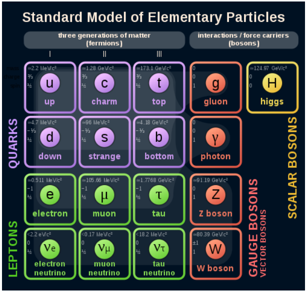
Penrose's Weyl curvature hypothesis (1979) protects the smoothness of the big bang singularity, which is assumed to be conformally regular

- These issues have been traditionally dissociated in discussions of the specific and detailed structure of the Standard Model of particle physics



- Matter: spin-$1/2$ fields $N_{1/2} = 15$ Weyl fermions (for each generation);
- Interactions: spin-1 fields $N_1 = 8 + 3 + 1 = 12$ for the gauge bosons;
- Gauge group: $SU(3) \times SU(2) \times U(1)$. Each generation is free of gauge anomalies
- 3 generations
- Higgs: spin-0 field $N_0 = 4$ scalar fields (complex Higgs doublet)

- In this talk, we will try to combine concepts related to GR (black holes event horizons, Weyl curvature hypothesis) with the properties of the field content of the SM

- The link between both areas is local conformal symmetry

$$\boxed{g_{\mu\nu}(x) \to \Omega^2(x) g_{\mu\nu}(x)}$$

$$\psi(x) \to \Omega^{-3/2}(x)\psi(x) \ , \qquad A_\mu(x) \to A_\mu(x) \quad \cdots$$

- In a first approximation, ignoring interactions and masses, **the Standard Model is a conformally invariant theory**

Semiclassical Schwarzschild geometry and conformal fields

- Let us first recall the well-known description of the Schwarzschild black hole

$$ds^2 = -(1-2M/r)dt^2 + (1-2M/r)^{-1}dr^2 + r^2 d\Omega$$

- It is the (static) spherical vacuum solution of Einstein's equations

$$G_{\mu\nu} = 8\pi T_{\mu\nu} = 0$$

- Things get much more complicated when a quantum field is added to the problem
- The vacuum expectation values $\langle T_{\mu\nu} \rangle$ of a conformal field in the static (Boulware) vacuum state behave [Christensen-Fulling 1977, Candelas 1980]

$$\langle B|T^{\mu}_{\ \nu}|B\rangle \sim_{r\to 2M} -\frac{(2N_1 + \frac{7}{2}N_{1/2} + N_0)}{30\ 2^{12}\pi^2 M^4} \frac{\hbar}{(1-2M/r)^2} \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 \\ 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & \frac{1}{3} \end{pmatrix}$$

$$\langle B|T^{\mu}_{\ \nu}|B\rangle \sim_{r\to\infty} \mathcal{O}(r^{-5})$$

- We observe that the static state leads to a divergence of the stress tensor at the event horizon

$$\langle T^{\mu}_{\ \nu}\rangle \sim_{r\to 2M} -\frac{\left(2N_1+\frac{7}{2}N_{1/2}+N_0\right)}{30\ 2^{12}\pi^2 M^4}\frac{\hbar}{(1-2M/r)^2}\begin{pmatrix}-1 & 0 & 0 & 0\\ 0 & \frac{1}{3} & 0 & 0\\ 0 & 0 & \frac{1}{3} & 0\\ 0 & 0 & 0 & \frac{1}{3}\end{pmatrix}$$

- This is the main argument often used to exclude the horizon as a physical portion of the static (Boulware) state

- However, the above claim is circular in the sense that it assumes the existence of a horizon that persists in the quantum corrected, backreacted geometry

- Therefore, in order to make progress, we must study the backreaction problem

- At a primary level, we can assume the semiclassical Einstein equations

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi G\langle T_{\mu\nu}\rangle$$

- Solving these equations is a daunting task because we do not have sufficient analytical control over the general vev [Renormalization in curved space is very complex]

$$\langle T_{\mu\nu}(g_{\rho\sigma})\rangle$$

[Candelas-Howard; Brown-Ottewill; Anderson-Hiscock-Samuel; Levi-Ori; Taylor-Breen-Ottewill, ...]

- To make progress, we will assume that the classical stress-energy tensor of the assumed fundamental fields is traceless as a consequence of conformal invariance
- However, the trace of $\langle T_{\mu\nu}\rangle$ is not zero [Capper-Duff 74, Deser-Duff-Isham 76]

$$\langle T^{\mu}_{\mu}\rangle = \hbar(c\, C^2 - a\, E + d\, \Box R)\ ,$$

where C^2 is the square of the Weyl curvature, and E is the Euler density

- The problem is still very difficult and it requires some approximations...

S-wave approximation for the quantized field

- Approximation: impose spherical symmetry for the quantized field φ

$$ds^2 = g_{ab}dx^a dx^b + e^{-2\phi}d\Omega^2$$

$$\varphi = \varphi(x^a)$$

$$T_{ab} = \frac{T^{(2)}_{ab}}{4\pi r^2} = \frac{1}{4\pi r^2}(\nabla_a\varphi\nabla_b\varphi - \frac{1}{2}g_{ab}(\nabla\varphi)^2)$$

[very popular in the '90: Callan-Giddings-Harvey-Strominger] referred as 2d dilaton gravity

- The approximated back-reacted geometry can be obtained by solving the semiclassical Einstein equations sourced by the effective 2d stress-energy tensor

$$\langle T^{(2)}_{ab}\rangle$$

- The most relevant part of $\langle T^{(2)}_{ab}\rangle$ is determined by the corresponding trace anomaly [C is the central charge]

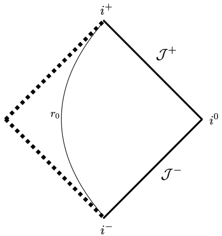
$$\langle T^{a(2)}_{a}\rangle = \frac{C\hbar}{24\pi}R^{(2)} ,$$

S-wave approximation

- The trace anomaly is sufficient to reconstruct $\langle T_{ab}\rangle$ with the boundary condition [static (Boulware) vacuum]

$$\langle T^a_b\rangle \sim_{r\to\infty} 0$$

- We have all the ingredients to obtain the backreacted Schwarzschild geometry in the s-wave approximation
- The quantum corrected geometry has the properties of a non-symmetric wormhole [Fabbri-Farese-N.-S.-Olmo-Sanchis, hep-th/0512167; PRD'06]



Improved approximations

- This picture is consistent with results obtained via the effective action [Berthiere, Sakar, Solodukhin PLB'18], order-reduced semiclassical gravity [Arrechea-Barcelo-Carballo-Rubio-Garay, PRD'23]
- It is interesting to see the approximated form of the metric around the classical horizon $r = 2M$. The perturbative solution in powers of $\hbar$ gives: (For simplicity we take $N_0 = 1, N_{1/2} = 0, N_1 = 0$). [Beltrán-Palau-Del Río-N.-S. PRD'23].

$$ds^2 = -[1 - \frac{2M}{r} - \frac{\hbar r}{13440\pi M^2(r-2M)} + \cdots]dt^2 + [1 - \frac{2M}{r} - \frac{\hbar r}{4480\pi M^2(r-2M)} + \cdots]^{-1}dr^2 + r^2 d\Omega^2 \, .$$

- The classical Schwarzschild coordinate singularity at $r = 2M$ get shifted to the value

$$r_0 = 2M + \mathcal{O}(l_P)$$

$$g_{rr}^{-1}(r_0) = 0 \qquad g_{tt}(r_0) \neq 0$$

- The static quantum-corrected spacetime is horizonless and does not define a black hole geometry. An asymptotically flat branch connects the throat $r = r_0$, and a curvature singularity develops beyond it. [Cosmic censorship is broken]

The weak point in the argument

- We have assumed a time-independent background.
- In a physical gravitational collapse, there is no way to avoid the time dependence and thus the creation of particles.
- At late times, and for a fixed Schwarzschild background, the stress-energy tensor does not vanish at infinity [Unruh's vacuum state]

$$\langle U \,|T^{\mu}_{\nu}|\, U\rangle \sim_{r\to\infty} \frac{L}{4\pi r^2}\begin{pmatrix} -1 & -1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \neq 0$$

  $L =$ Hawking's Luminosity of the black hole

- This implies regularity at the classical event horizon H^+

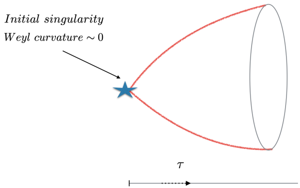
$$\langle U \,|T^{\mu}_{\nu}|\, U\rangle \sim_{r\to 2M} \; regular$$

  Quantum backreaction is then expected to maintain the trapped regions

- We will come back to this issue later. Let us jump to the Big Bang

Conformal symmetry and the Big Bang singularity

- The smoothness of the Big Bang singularity can be protected by imposing the Weyl curvature hypothesis (Penrose 1979, P. Tod 2003)



- The Big Bang singularity is assumed to be purely conformal: it can be reabsorbed by a conformal transformation in the metric tensor

$$g_{\mu\nu}(x) \to \Omega^2(x) g_{\mu\nu}(x) \ ,$$

 and invisible to conformally invariant matter

- Penrose, 2011: *There can, however, be an issue with regard to what is referred to as a conformal anomaly, according to which a symmetry of the classical fields (here the strict conformal invariance) may not hold exactly true in the quantum context.*

- While the metric is conformally regular, the rescaled $\langle T_{\mu\nu} \rangle$

$$\langle T_{\mu\nu} \rangle \to \Omega^{-2} \langle T_{\mu\nu} \rangle + \mathcal{O}(\hbar)$$

 is not longer regular at the Big Bang due to conformal anomalies

- This is somewhat in tension with Penrose's Weyl curvature hypothesis.
- The easiest way to resolve this tension is to require exact conformal symmetry

$$\boxed{\sum_{\text{fundamental fields}} \langle T_\mu^\mu \rangle = 0}$$

- Furthermore [R.P.A.C. Newman 1993]: a perfect fluid spacetime, with $\langle T_\mu^\mu \rangle = 0$ which evolves from a spacelike conformal singularity, having a vanishing Weyl tensor

$$C_{\mu\nu\rho\sigma} = 0 \ ,$$

is necessarily FLRW near the singularity

$$ds^2 \sim_{\tau \to 0} \tau^2 (-d\tau^2 + h_{ij} dx^i dx^j)$$

Trace anomaly in four dimensions

- The contribution of the known fields of the Standard Model to $\langle T^\mu_\mu \rangle$ (ignoring masses and interactions) is given by

$$\langle T^\mu_\mu \rangle = \hbar (c\, C^2 - a\, E)$$

with

$$a = \frac{1}{360(4\pi)^2}[N_0 + \frac{11}{2} N_{1/2} + 62 N_1] > 0\,, \quad c = \frac{1}{120(4\pi)^2}[N_0 + 3N_{1/2} + 12N_1] > 0\,,$$

- $N_1 = 12$ (electroweak bosons and gluons)
- $N_{1/2} = 3 \times 15$ (three generations of left-handed and right-handed leptons and quarks)
- $N_0 = 4$ (real components of the Higgs doubled)
- As free fields in curved spacetime, their contribution to the conformal anomaly is always additive and cannot be forced to cancel

  [We have ignored contributions of the form $\Box R$ as they are intrinsically ambiguous and can be shifted by local counterterms]

Trace anomaly in four dimensions

- However, in sharp contrast to two-dimensional conformal invariance, it is now possible to introduce a new field with negative contribution to c and a while preserving unitarity
- The simplest way is provided by the so-called "dimensionless scalar field" ξ obeying a 4th order field equation [Bogolubov et al. textbook, 1987]

$$\Box^2 \xi = 0$$

- It is the simplest gauge invariant theory of spin 0:

$$\xi(x) \to \xi(x) + \alpha(x) \ , \qquad \Box \alpha = 0$$

- An important property of this theory is that its physical content consists of a single quantum state: the vacuum [Bogolubov et al. 87]
- Two-point function: scale invariant (like a conventional scalar field in 2d)

$$\langle \xi(x)\xi(y) \rangle = -(4\pi)^{-1} \log |\kappa^2 (x-y)^2| \ ,$$

Trace anomaly in four dimensions

- In curved spacetime, it can be uniquely extended to a conformally invariant theory

$$S = -\frac{1}{2}\int d^4x\sqrt{-g}\,\xi\triangle_4\,\xi\ ,$$

where $\triangle_4$ is the unique conformally-invariant fourth order operator

$$\triangle_4 = \Box^2 + 2R^{\mu\nu}\nabla_\mu\nabla_\nu - \frac{2}{3}R\Box + \frac{1}{3}(\nabla^\mu R)\nabla_\mu\ .$$

- A nice feature of this field is that it contributes negatively to the conformal anomaly

$$a = -\frac{28}{360(4\pi)^2}, \qquad c = -\frac{8}{120(4\pi)^2}\ .$$

[Gusynin'89]

- It opens the door for canceling the entire conformal anomaly !!!

Cancelling the conformal anomaly

- Conformal anomaly cancelled for

  Boyle-Turok, 2021; Miller-Volovik-Zubkov 2022

$$N_{1/2} = 4N_1 \ , \qquad N^{\xi} = 3N_1 \ , \qquad N_0 = 0$$

- The Standard Model (including right-handed neutrinos, 3 generations, excluding the Higgs)$+36$ dimension-zero scalars

$$N_1 = 12 \ , \qquad N_{1/2} = 3 \times 16 \ , \qquad N_0 = 0$$

$N_1 \equiv W^{\pm}, Z, \gamma, \ gluons$

$N_{1/2} \equiv e_R, e_L, \nu_L, \nu_R, u^a_L, u^a_R, d^a_L, d^a_R \times 3 \ generations$

$N_0 \equiv Higgs \sim composite field$

$N^{\xi} \equiv$ 36 Dimension-zero scalars (to fix the vacuum; no new particles)

Physical implication 1: gravitational particle creation and dark matter

- So far, we have been neglecting the masses and the interactions of the basic constituents

- According to the seesaw mechanism [Minkowski 77], the mass matrix of the neutrinos, after symmetry breaking, is given by ($m_D \ll M_R$)

$$\mathcal{M} = \begin{pmatrix} 0 & m_D \\ m_D & M_R \end{pmatrix}$$

- The dominant contribution is the Majorana mass M_R, which is assumed to be several orders of magnitude beyond the electroweak scale
- The heavier ν_R is the natural candidate for dark matter (decoupled from all of the other particles in the SM).

Physical implication 1: gravitational particle creation and dark matter

- It can only be produced gravitationally, via the mixing-frequency mechanism. The predicted number density is [Beltran-Palau, Nadal-Gisbert, N.-S., Pla, 2023]

$$n_{k,h} = \frac{1}{2} - \frac{e^{-\pi\kappa}\sinh(2\pi\kappa)\sqrt{\kappa}}{4\pi}\left(e^{-2i\Theta_k}e^{i\frac{\pi}{4}}\Gamma(i\kappa)\Gamma(\tfrac{1}{2}-i\kappa) + e^{2i\Theta_k}e^{-i\frac{\pi}{4}}\Gamma(-i\kappa)\Gamma(\tfrac{1}{2}+i\kappa)\right)$$

$$\langle n(t)\rangle = \alpha(\Theta_k)\left(\frac{M_{\nu_R}}{t}\right)^{3/2} ,$$

- Low energy vacuum at the Big Bang $\Theta_k = 0$. To match the observed dark matter density [Beltran-Palau, Nadal-Gisbert, N.-S. and Pla, 2023]

$$M_{\nu_R} = 3 \times 10^8 GeV$$

- Low energy vacuum at late times ($\equiv$ vacuum proposed by Boyle-Finn-Turok (2018))

$$M_{\nu_R} = 5 \times 10^8 GeV$$

Physical implications 2: black holes physics

- The specialness of the 36 dimensionaless scalar fields ξ: no particles in Minkowski space

$$\langle 0_\xi | T^\mu_\nu | 0_\xi \rangle_{Minkowski} = 0$$

- For asymptotically flat spacetime backgrounds (far from a classical black hole)

$$\langle 0_\xi | T^\mu_\nu | 0_\xi \rangle \sim_{r \to \infty} 0$$

- Vacuum "choice" for $|0_\xi\rangle \sim$ Boulware vacuum state
- Vacuum choice for Standard Model fields $|0_{SM}\rangle \sim$ Unruh state

  This is somewhat analogous to the "hybrid quantum states" of 2d dilaton gravity, in the language of Pataux-Sarkar-Solodukhin, PRD'23, PRL'23 [Pataux's talk]

Summary

- Penrose's Weyl curvature hypothesis (WCH) protects the smoothness of the big bang singularity: it is conformally regular
- It requires exact conformal symmetry (anomaly cancellation)
- Anomaly cancelation has major implications for constraining the set of fundamental fields
- Implications 1: A natural candidate for dark matter ν_R, produced by gravitational particle creation
- Implications 2: A new scenario for quantum black holes [Work in progress]

THANKS !!!