Anomaly Effective Actions for Topological Matter and Axion-Like Interpolating States

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Tours June 5-7 2024

The CFT analysis of chiral interactions indicates that they are mediated by an axion-like excitation, which appears in the conformal limit of the chiral vertex, as a massless exchange.

To illustrate this, we demonstrate how conformal Ward identities (CWIs) in momentum space allow us to reconstruct the entire chiral anomaly interaction in its longitudinal and transverse sectors just by the inclusion of such excitation in the longitudinal sector.

A similar behaviour is uncovered for a Chern-Simons current.

In the presence of fermion mass corrections, the massless exchange transforms into a cut, but the absorption amplitude in the axial-vector channel satisfies mass-independent sum rules related to the anomaly in any chiral interaction.

The detection of an axion-like/quasiparticle in these materials may rely on a combined investigation of these sum rules, along with the measurement of the angle of rotation of the plane of polarization of incident light when subjected to a chiral perturbation.

This phenomenon serves as an analogue of a similar one in ordinary axion physics, in the presence of an axion-like condensate, that we rederive using axion electrodynamics.

Perturbative and Non perturbative analysis in CFT

The Gravitational Chiral Anomaly at Finite Temperature and Density e-Print: 2404.06272 [hep-th]

S. Lionetti, M. Maglio, CC

Axion-like Quasiparticles and Topological States of Matter: Finite Density Corrections of the Chiral Anomaly Vertex e-Print: 2402.03151 [hep-ph]

Creti', S. Lionetti, R. Tommasi, CC to appear in PRD

Parity-violating CFT and the gravitational chiral anomaly S. Lionetti, M. Maglio, CC

Published in: *Phys.Rev.D* 109 (2024) 4, 045004 • e-Print: 2309.05374 [hep-th]

Axion-like Interactions and CFT in Topological Matter, Anomaly Sum Rules and the Faraday Effect Advanced Phys Res, Wiley

M. Creti, S. Lionetti, R. Tommasi, CC

1. Axions in their original formulation

2. Stuckelberg axions (axion-like particles)

3. The nonlocal effective action from the perturbative analysis and conformal symmetry: the centrality of anomaly poles

4. The local effective action and the Kerr Faraday effect for an axion condensate

5. The behaviour of the photon propagator in the presence of an axion condensate

6. The sum rule phenomenon for chiral and conformal anomalies

7. Open issues in condensed matter: can we see an axion-like particle in topological materials ?

1. Axions in their original formulation

Strong CP Problem

The Peccei-Quinn axion is a hypothetical elementary particle postulated to solve the strong CP problem in quantum chromodynamics (QCD). This particle arises from the Peccei-Quinn theory, which introduces a new global symmetry to explain why the CP-violating term in QCD is extremely small.

- Quantum Chromodynamics (QCD) predicts a term that could violate CP symmetry.
- Experimental observations show no such violation, implying the term's coefficient, θ, is very small ($\theta < 10^{-10}$).
- The strong CP problem is the question of why θ is so small.

Peccei-Quinn Theory

global anomalies

- Proposed by Roberto Peccei and Helen Quinn in 1977.
- Introduces a new global $U(1)_{\mathrm{PQ}}$ symmetry.
- This symmetry is spontaneously broken, leading to the emergence of a new pseudo-Nambu-Goldstone boson: the axion.

- Very light and weakly interacting.
- Axion mass and interaction strength are inversely related to the symmetry breaking scale.
- Potential dark matter candidate.
- Axion searches include haloscopes, helioscopes, and light-shining-through-wall experiments.

The PQ axion: where the story commences

Why CP is conserved in Strong Interactions ?

$$L_\theta = \frac{\alpha_s \theta}{8\pi} G^a_{\mu\nu} \tilde{G}^{a\mu\nu} \qquad L_m = \frac{\alpha_s Arg(\det M)}{8\pi} G^a_{\mu\nu} \tilde{G}^{a\mu\nu}$$

$$\bar{\theta} = \theta + Arg(\det M) \Rightarrow L_{\bar{\theta}} = \frac{\alpha_s \bar{\theta}}{8\pi} \vec{G}_{\mu\nu} \cdot \tilde{\vec{G}}^{\mu\nu}$$

$$L = \sum_{n=1}^{N} \bar{\psi}_n \left(i\gamma^\mu D_\mu - m_n \right) \psi_n - \frac{1}{4} G^{a\mu\nu} G^a_{\mu\nu} + \frac{\alpha_s \bar{\theta}}{8\pi} G^a_{\mu\nu} \tilde{G}^{a\mu\nu}$$

We do not observe any CP violating effect in the strong interactions, and this shows that THETA has to be small.

Smallness of the neutron electrical dipole moment shows that THETA has to be zero.

Indeed if we calculate the neutron electric dipole moment we get: $d_n \approx 10^{-16} \bar{\theta}$ e·cm

Given the experimental limit: $d_n < 0.63 \times 10^{-25}$ e·cm

→ $\bar{\theta} \approx 10^{-9}$

PECCEI and QUINN proposed in 1977 an extension of the SM With an anomalous U(1) symmetry U(1)_PQ

<u>This is a symmetry of the theory at the lagrangean level, broken</u>

Spontaneously at a large scale (f_a)

and

Explicitly at the QCD hadron transition

The axion is massless until the QCD transition. Instanton effects are held responsible for the generation of the axion potential

$$\mathcal{L}_{\mathrm{QCD+a}} = -\frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu} + \frac{1}{2}\partial_\mu a \partial^\mu a + \sum_q \bar{q}(i\gamma^\mu\partial_\mu - m_q)q + \frac{g_s^2}{32\pi^2}(\theta + \frac{a}{f_a})G^a_{\mu\nu}\tilde{G}^{a\mu\nu}$$

The axion interacts with photons, electrons, hadrons with a strength ~ f_a^{-1}

Initially f_a was tied to the Electroweak scale and the CP breaking with the Electroweak breaking

The realization that the axion scale can be very big (and its coupling very weak) initiated the 'invisible axion' models

The periodic instanton potential and the axion mass

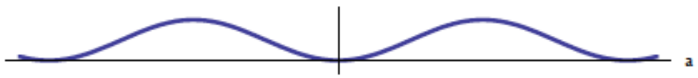
$$m_a(T) = \begin{cases} m_a b(\frac{\Lambda}{T})^4, & T \gtrsim \Lambda, \\ m_a, & T \lesssim \Lambda. \end{cases}$$

$$m_a^2 = \frac{m_\pi^2 f_\pi^2}{f_a^2} \frac{m_u m_d}{(m_u + m_d)^2}$$

b=0.018

Periodic potential and oscillations
Due to “vacuum misalignment”

$$V(\theta) = -C(T)\cos(\theta), \text{ where } \theta \equiv a/F_a.$$



Peccei & Quinn(1977), Weinberg(1978), Wilczek, (1978), J. E. Kim(1979),Shifman, Vainstein & Zakharov, (1980),Dine, Fischler & Srednicki(1981),Zhitnitskii(1980).

In both the DFSZ and KSVZ scenarios a global anomalous $U(1)_{PQ}$ symmetry gets broken at some large scale v_{PQ}, with the generation of a Nambu-Goldstone mode from the CP-odd scalar sector. In the KSVZ case the theory includes a heavy quark Q which acquires a large mass by a Yukawa coupling with the scalar Φ. In this case the Lagrangian of Q takes the form

$$\mathscr{L} = |\partial\Phi|^2 + i\bar{Q}\not{D}\, Q + \lambda\Phi\bar{Q}_L Q_R + h.c. - V(\Phi)$$

scalar Higgs + a heavy quark

with a global $U(1)_{PQ}$ chiral symmetry of the form

$$\begin{aligned}\Phi &\rightarrow e^{i\alpha}\Phi \\ Q &\rightarrow e^{-\frac{i}{2}\alpha\gamma_5}Q\end{aligned}$$

with an $SU(3)_c$ covariant derivative (D) containing the QCD color charge of the heavy fermion Q. The scalar PQ potential can be taken from the usual Mexican-hat form and it is $U(1)_{PQ}$ symmetric. Parameterising the PQ field with respect to its broken vacuum

$$\Phi = \frac{\phi + v_{pq}}{\sqrt{2}} e^{i\frac{a(x)}{v_{PQ}}} + \ldots$$

the Yukawa coupling of the heavy quark Q to the CP-odd phase of Φ, $a(x)$, takes the form

$$e^{i\gamma_5 \frac{a(x)}{2v_{PQ}}} Q_{L/R} \equiv Q'_{L/R}.$$

At this stage one assumes that there is a decoupling of the heavy quark from the low energy spectrum by assuming that v_{PQ} is very large. The standard procedure in order to extract the low energy interaction of the axion field is to first redefine the field Q on order to remove the exponential with the axion in the Yukawa coupling

This amounts to a chiral transformation which leaves the fermionic measure non-invariant

$$\mathrm{D}\bar{Q}\mathrm{D}Q \to e^{i\int d^4x \frac{6a(x)}{32\pi^2 v_{PQ}} G(x)\tilde{G}(x)} \mathrm{D}\bar{Q}\mathrm{D}Q$$

The kinetic term of Q is not invariant under this field redefinition and generates a derivative coupling of $a(x)$ to the axial vector current of Q. For n_f triplets, for instance, the effective action of the axion, up to dimension-5 takes the form

$$\mathscr{L}_{eff} = \frac{1}{2}\partial_\mu a(x)\partial^\mu a(x) + \frac{6n_f}{32\pi^2 v_{PQ}} a(x) G\tilde{G} + \frac{1}{v_{PQ}} \partial_\mu a \bar{Q}\gamma^\mu \gamma_5 Q + \ldots$$

In the case of the DFSZ axion, the solution to the strong CP problem is found by introducing a scalar Φ together with two Higgs doublets H_u and H_d. In this case one writes down a general potential, function of these three fields, which is $SU(2) \times U(1)$ invariant and possesses a global symmetry

$$H_u \to e^{i\alpha X_u} H_u, \qquad H_d \to e^{i\alpha X_d} H_d, \qquad \Phi \to e^{i\alpha X_\Phi} \Phi$$

with $X_u + X_d = -2X_\Phi$. It is given by a combination of terms of the form

$$V = V(|H_u|^2, |H_d|^2, |\Phi|^2, |H_u H_d^\dagger|^2, |H_u \cdot H_d|^2, H_u \cdot H_d, \Phi^2)$$

DFSZ	KSVZ
• $f_a >> f_{ew}$	• $f_a >> f_{ew}$
• Two Higgs fields and	• One Higgs field,
• one scalar field.	• one scalar field
• Fermions: carry PQ charge.	• one exotic quark with PQ charge.

The axion dark matter scenario

Peccei-Quinn symmetry breaks at $T \sim f_a \leq 10^9$GeV

The axions acquire mass at $T \sim \Lambda_{QCD}$

2. Stuckelberg axions (axion-like particles)

Abelian extensions from intersecting branes provide another realization

$$U(N_1) \times U(N_2) \times ... \times U(N_k) = SU(N_1) \times U(1) \times SU(N_2) \times U(1) \times ... \times SU(N_k) \times U(1)$$

(o) **The extra U(1)'s can be organized in terms of a U(1) of hypercharge times some extra U(1)'s which are anomaly free.**

(oo) **Several anomalous U(1)'s**

(ooo) **Just one physical axion**

The physical axion emerges only AFTER electroweak symmetry breaking
Due to the appearance of an "extra potential" in the CP odd sector

The anomalous U(1)'s are in a "broken" form (Stuckelberg symmetry)

$$\mathcal{L}_{St} = \frac{1}{2}(\partial_\mu b - M_1 B_\mu)^2 + \frac{1}{2}(\partial_\mu c - M_2 C_\mu)^2 ,$$

$$\delta_B B_\mu = \partial_\mu \theta_B \qquad \delta b = M_1 \theta_B$$

$$\delta_C C_\mu = \partial_\mu \theta_C \qquad \delta c = M_2 \theta_C ,$$

On the effective theory of low scale orientifold string vacua *Nucl.Phys.B* 746 (2006)

N. IRGES, E. KIRITSIS, CC

Stuckelberg axions and the effective action of anomalous Abelian models.

N. Irges, S. Morelli, CC *JHEP* 07 (2007, 2008) 1,2

a class of models containing a gauge structure of the form

SM x U(1) x U(1) x U(1)

$SU(3) \times SU(2) \times U(1)_Y \times U(1)$.....

from which the hypercharge is assigned in a given string construction, corresponding to a certain class of vacua in string theory (Minimal Low Scale orientifold Models).

Anomalous $U(1)$ extension of the Standard Model

(2024)

see also recent work by

Anastasopoulos, Antoniadis, Benakli, Rondeau

Axion topological field theory of topological superconductors

Xiao-Liang Qi, Edward Witten, and Shou-Cheng Zhang

Phys. Rev. B **87**, 134519 — Published 25 April 2013

$$S_{\text{eff}} = \int d^4x \left[\frac{\theta_L - \theta_R}{64\pi^2} \epsilon^{\mu\nu\sigma\tau} F_{\mu\nu} F_{\sigma\tau} - \frac{1}{4e^2} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2}\rho_L \left(\partial_\mu \theta_L - 2A_\mu\right)^2 + \frac{1}{2}\rho_R \left(\partial_\mu \theta_R - 2A_\mu\right)^2 + J \cos\left(\theta_L - \theta_R\right) \right]. \quad (23)$$

Stuckelberg action for a U(1)

$$\mathcal{L}_A = -\frac{1}{4g^2} F^{\mu\nu} F_{\mu\nu} - \frac{1}{2}(M B_\sigma - \partial_\sigma b)^2$$

There is no potential for b(x). This sets a difference wrt the PQ axion.

Stuckelberg symmetries are “dual” descriptions of “A-F” theories (using a 2-form A coupled to a gauge field B)

$$\mathcal{L} = -\frac{1}{12} H^{\mu\nu\rho} H_{\mu\nu\rho} - \frac{1}{4g^2} F^{\mu\nu} F_{\mu\nu} + \frac{M}{4} \epsilon^{\mu\nu\rho\sigma} A_{\mu\nu} F_{\rho\sigma},$$

$$H_{\mu\nu\rho} = \partial_\mu A_{\nu\rho} + \partial_\rho A_{\mu\nu} + \partial_\nu A_{\rho\mu}, \qquad F_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

$H = dA$ **(constraint imposed by a multiplier, b(x))**

$$\mathcal{L}_0 = -\frac{1}{12} H^{\mu\nu\rho} H_{\mu\nu\rho} - \frac{1}{4g^2} F^{\mu\nu} F_{\mu\nu} - \frac{M}{6} \epsilon^{\mu\nu\rho\sigma} H_{\mu\nu\rho} B_\sigma + \frac{1}{6} b(x) \epsilon^{\mu\nu\rho\sigma} \partial_\mu H_{\nu\rho\sigma}.$$

$$H^{\mu\nu\rho} = -\epsilon^{\mu\nu\rho\sigma} (M B_\sigma - \partial_\sigma b).$$

a simple example

U(1)xU(1) Model

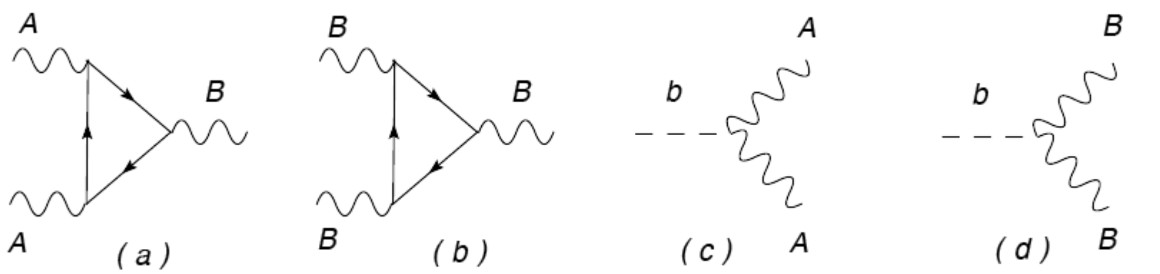
$$\mathcal{L}_0 \;=\; -\frac{1}{4}F_A^2 - \frac{1}{4}F_B^2 + \frac{1}{2}(\partial_\mu b + M_1\, B_\mu)^2 + \overline{\psi} i\gamma^\mu(\partial_\mu + ieA_\mu + ig_B\gamma^5 B_\mu)\psi,$$

$$b \,\rightarrow\, b - M_1\theta_B \qquad \delta B_\mu = \partial_\mu \theta_B$$

$$\mathcal{L}_{gf} = -\xi\left(\partial\cdot B + \frac{M_1}{2\xi}b\right)^2$$

In the $A - B$ model the WZ counterterms are

$$\mathcal{L}_{WZ} = \frac{C_{AA}}{2!M_1} bF_A \wedge F_A + \frac{C_{BB}}{2!M_1} bF_B \wedge F_B,$$



In general, in the presence of an underlying anomalous symmetry, the Stuckelberg action is modified by the presence of Wess-Zumino counterterms for the restoration of gauge invariance.

There are some important differences between the PQ axion and a gauged axion.

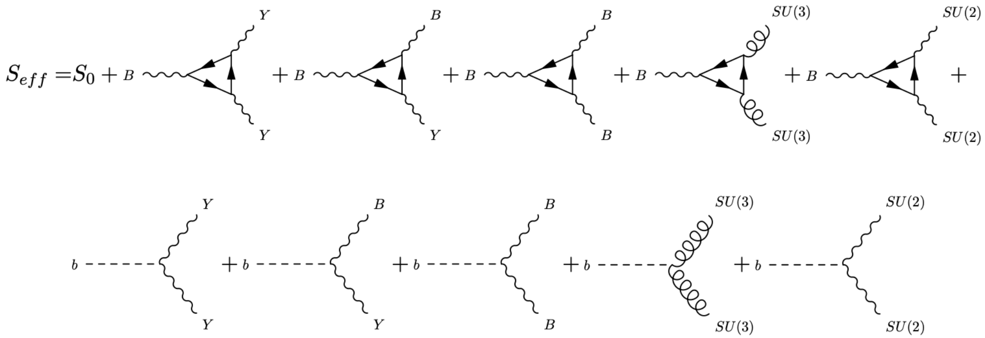
1) In the PQ case the axion is a Nambu-Goldstone mode (of a global symmetry). As such cannot be gauged away. It remains as such until the QCD transition when it acquires its mass by an instanton potential.
2) A gauged axion is not a physical degree of freedom before the electroweak phase transition, when it becomes physical due to a phase-dependent potential

HIGGS-AXION MIXING

(Irges, Kiritsis, CC, 2006)

An extension of the Standard Model
(Irges, Morelli, CC)

f	Q	u_R	d_R	L	e_R
q^B	q^B_Q	$q^B_{u_R}$	$q^B_{d_R}$	q^B_L	$q^B_{e_R}$



f	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	$U(1)_B$
Q	3	2	1/6	q^B_Q
u_R	3	1	2/3	$q^B_Q + q^B_u$
d_R	3	1	$-1/3$	$q^B_Q - q^B_d$
L	1	2	$-1/2$	q^B_L
e_R	1	1	-1	$q^B_L - q^B_d$
H_u	1	2	1/2	q^B_u
H_d	1	2	1/2	q^B_d

$$\begin{pmatrix} G^1_0 \\ G^2_0 \\ \chi \end{pmatrix} = O^\chi \begin{pmatrix} \mathrm{Im} H^0_d \\ \mathrm{Im} H^0_u \\ b \end{pmatrix},$$

$$G^1_0 = \frac{1}{\sqrt{v_u^2 + v_d^2}} (v_d, v_u, 0)$$

$$G^2_0 = \frac{1}{\sqrt{g_B^2 (q_d - q_u)^2 v_d^2 v_u^2 + 2M^2 \left(v_d^2 + v_u^2\right)}} \left(- \frac{g_B (q_d - q_u) v_d v_u^2}{\sqrt{v_u^2 + v_d^2}}, \frac{g_B (q_d - q_u) v_d^2 v_u}{\sqrt{v_d^2 + v_u^2}}, \sqrt{2} M \sqrt{v_u^2 + v_d^2} \right)$$

$$\chi = \frac{1}{\sqrt{g_B^2 (q_d - q_u)^2 v_u^2 v_d^2 + 2M^2 (v_d^2 + v_u^2)}} \left(\sqrt{2} M v_u, -\sqrt{2} M v_d, g_B (q_d - q_u) v_d v_u \right) \tag{14}$$

$$\begin{pmatrix} G_0^1 \\ G_0^2 \\ \chi \end{pmatrix} = O^\chi \begin{pmatrix} \mathrm{Im}H_d^0 \\ \mathrm{Im}H_u^0 \\ b \end{pmatrix},$$

$$O^\chi = \begin{pmatrix} \frac{v_d}{v} & \frac{v_u}{v} & 0 \\ -\frac{g_B(q_d-q_u)v_d v_u^2}{v\sqrt{g_B^2(q_d-q_u)^2 v_d^2 v_u^2+2M^2v^2}} & \frac{g_B(q_d-q_u)v_d^2 v_u}{v\sqrt{g_B^2(q_d-q_u)^2 v_d^2 v_u^2+2M^2v^2}} & \frac{\sqrt{2}Mv}{\sqrt{g_B^2(q_d-q_u)^2 v_d^2 v_u^2+2M^2v^2}} \\ \frac{\sqrt{2}Mv_u}{\sqrt{g_B^2(q_d-q_u)^2 v_u^2 v_d^2+2M^2v^2}} & -\frac{\sqrt{2}Mv_d}{\sqrt{g_B^2(q_d-q_u)^2 v_u^2 v_d^2+2M^2v^2}} & \frac{g_B(q_d-q_u)v_d v_u}{\sqrt{g_B^2(q_d-q_u)^2 v_u^2 v_d^2+2M^2v^2}} \end{pmatrix}$$

$$b = O_{13}^\chi G_0^1 + O_{23}^\chi G_0^2 + O_{33}^\chi \chi,$$

$$\chi = O_{31}^\chi \mathrm{Im}H_d + O_{32}^\chi \mathrm{Im}H_u + O_{33}^\chi b.$$

It can be made ultralight if the SSB scale is the GUT scale

Chi is the Axi-Higgs, obtaining its mass from a SSB and a periodic potential.
It defines te template for an axion-like particle

This allows a fundamental description of axion like particles without any reference to moduli from strings as in Hui, Ostriker, Witten

Ultralight scalars as cosmological dark matter

The description of this phenomenon in the clearest way is in
Cosmological Properties of a Gauged Axion (2010)
Lazarides, Mariano, Guzzi, CC

$$V_{\not P \not Q} = \lambda_0 (H_u^\dagger H_d e^{-i g_B (q_u - q_d) \frac{b}{2M}}) + \lambda_1 (H_u^\dagger H_d e^{-i g_B (q_u - q_d) \frac{b}{2M}})^2 + \lambda_2 (H_u^\dagger H_u)(H_u^\dagger H_d e^{-i g_B (q_u - q_d) \frac{b}{2M}}) + \lambda_3 (H_d^\dagger H_d)(H_u^\dagger H_d e^{-i g_B (q_u - q_d) \frac{b}{2M}}) + \text{h.c.}, \quad (12)$$

PQ symmetric potential

$$V' = 4 v_u v_d \left(\lambda_2 v_d^2 + \lambda_3 v_u^2 + \lambda_0\right) \cos\left(\frac{\chi}{\sigma_\chi}\right) + 2\lambda_1 v_u^2 v_d^2 \cos\left(2\frac{\chi}{\sigma_\chi}\right),$$

non-perturbative periodic potential

$$V' = 4 v_u v_d \left(\lambda_2 v_d^2 + \lambda_3 v_u^2 + \lambda_0\right) \cos\left(\frac{\chi}{\sigma_\chi}\right) + 2\lambda_1 v_u^2 v_d^2 \cos\left(2\frac{\chi}{\sigma_\chi}\right),$$

periodicity in a gauge invariant linear combination of Higgs + Stuckelberg field CP odd phases

$$m_\chi^2 = \frac{2 v_u v_d}{\sigma_\chi^2}\left(\bar{\lambda}_0 v^2 + \lambda_2 v_d^2 + \lambda_3 v_u^2 + 4\lambda_1 v_u v_d\right) \approx \lambda v^2.$$

mass of the axion is controlled by the Higgs vev and the strength of the periodic potential

STUCKELBERG AXIONS

gauge anomalies

- The Stückelberg mechanism involves introducing a pseudoscalar field to cancel gauge anomalies.
- This mechanism is crucial in models with extra $U(1)$ gauge symmetries.
- The pseudoscalar field couples to the gauge fields, ensuring anomaly cancellation.

- Stückelberg axions differ from traditional PQ axions by being linked to gauge symmetries.
- They play a role in anomaly cancellation, similar to the Green-Schwarz mechanism in string theory.
- These models can provide new insights into dark matter and other unresolved issues in particle physics.

- Stückelberg axion models can be embedded into Grand Unified Theories (GUTs).
- An example is the $E_6 \times U(1)_X$ model.
- These models often feature multiple axions with distinct roles and properties.

Similar anomalous gauge structures in condensed matter physics

Axion topological field theory of topological superconductors

Xiao-Liang Qi, Edward Witten, and Shou-Cheng Zhang

Phys. Rev. B **87**, 134519 — Published 25 April 2013

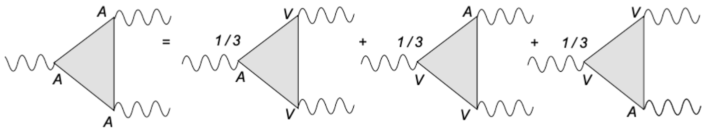
$$S_{\text{eff}} = \int d^4x \left[\frac{\theta_L - \theta_R}{64\pi^2} \epsilon^{\mu\nu\sigma\tau} F_{\mu\nu} F_{\sigma\tau} - \frac{1}{4e^2} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2}\rho_L \left(\partial_\mu \theta_L - 2A_\mu\right)^2 + \frac{1}{2}\rho_R \left(\partial_\mu \theta_R - 2A_\mu\right)^2 + J\cos\left(\theta_L - \theta_R\right) \right]. \tag{23}$$

The Nonlocal action

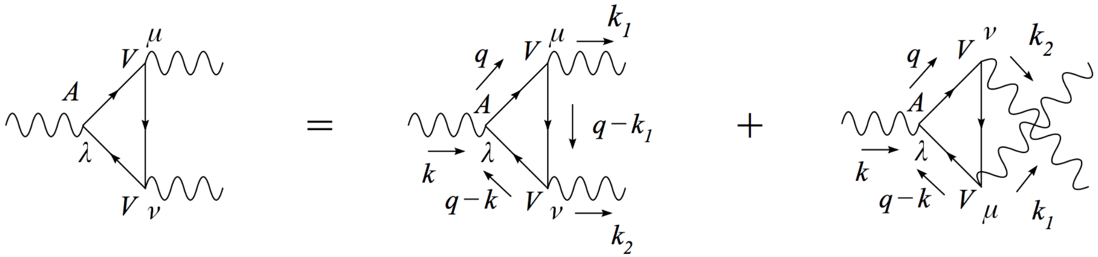
These actions are local, **with a local interaction to the anomaly.**

However, the 1PI manifestation of the anomaly is in the form of **an anomaly pole**. **This is the form of the action that we are going to explore in order to investigate how these axionic degrees of freedom couple to the anomaly**

Signatures of Chiral and Conformal anomalies



AVV diagram



$$\Delta_0^{\lambda\mu\nu} = A_1(k_1,k_2)\varepsilon[k_1,\mu,\nu,\lambda] + A_2(k_1,k_2)\varepsilon[k_2,\mu,\nu,\lambda] + A_3(k_1,k_2)\varepsilon[k_1,k_2,\mu,\lambda]{k_1}^{\nu}$$
$$+ A_4(k_1,k_2)\varepsilon[k_1,k_2,\mu,\lambda]k_2^{\nu} + A_5(k_1,k_2)\varepsilon[k_1,k_2,\nu,\lambda]k_1^{\mu} + A_6(k_1,k_2)\varepsilon[k_1,k_2,\nu,\lambda]k_2^{\mu}.$$

If we change the parameterization of the loop momentum, A1 and A2 change.

$$A_3(k_1,k_2) = -A_6(k_2,k_1) = -16\pi^2 I_{11}(k_1,k_2),$$
$$A_4(k_1,k_2) = -A_5(k_2,k_1) = 16\pi^2\left[I_{20}(k_1,k_2) - I_{10}(k_1,k_2)\right],$$

Some are finite by power counting.
A1 and A2 are not

This is not the only parameterization. A second one is the **longitudinal/transverse (LT) decomposition**

$$W^{\lambda\mu\nu} = \frac{1}{8\pi^2}\left[W^{L\,\lambda\mu\nu} - W^{T\,\lambda\mu\nu}\right],$$

De Rafael et al

developed in the study of g-2 of the muon

$$W^{L\,\lambda\mu\nu} = w_L\, k^\lambda \varepsilon[\mu,\nu,k_1,k_2]$$

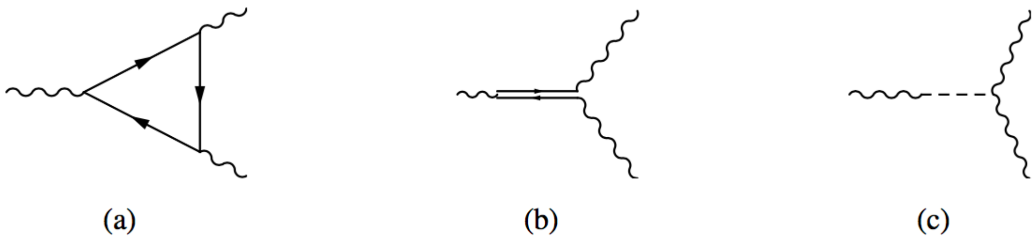
Only the L part contributes to the Ward Identity

$$\begin{aligned} {W^T}_{\lambda\mu\nu}(k_1,k_2) \;&=\; w_T^{(+)}\left(k^2,k_1^2,k_2^2\right)\, t^{(+)}_{\lambda\mu\nu}(k_1,k_2) + w_T^{(-)}\left(k^2,k_1^2,k_2^2\right)\, t^{(-)}_{\lambda\mu\nu}(k_1,k_2) \\ &+ \widetilde{w}_T^{(-)}\left(k^2,k_1^2,k_2^2\right)\, \widetilde{t}^{(-)}_{\lambda\mu\nu}(k_1,k_2), \end{aligned}$$

$$\begin{aligned} t^{(+)}_{\lambda\mu\nu}(k_1,k_2) &= k_{1\nu}\,\varepsilon[\mu,\lambda,k_1,k_2] - k_{2\mu}\,\varepsilon[\nu,\lambda,k_1,k_2] - (k_1\cdot k_2)\,\varepsilon[\mu,\nu,\lambda,(k_1-k_2)] \\ &\quad + \frac{k_1^2+k_2^2-k^2}{k^2}\; k_\lambda\; \varepsilon[\mu,\nu,k_1,k_2]\;, \\ t^{(-)}_{\lambda\mu\nu}(k_1,k_2) &= \left[(k_1-k_2)_\lambda - \frac{k_1^2-k_2^2}{k^2}\; k_\lambda\right]\varepsilon[\mu,\nu,k_1,k_2] \\ \widetilde{t}^{(-)}_{\lambda\mu\nu}(k_1,k_2) &= k_{1\nu}\,\varepsilon[\mu,\lambda,k_1,k_2] + k_{2\mu}\,\varepsilon[\nu,\lambda,k_1,k_2] - (k_1\cdot k_2)\,\varepsilon[\mu,\nu,\lambda,k]. \end{aligned}$$

Tensor structures involved
In the LT parameterization

$$
\begin{aligned}
w_L(s_1, s_2, s) &= -\frac{4i}{s} \\
w_T^{(+)}(s_1, s_2, s) &= i\frac{s}{\sigma} + \frac{i}{2\sigma^2}\Big[(s_{12}+s_2)(3s_1^2 + s_1(6s_{12}+s_2) + 2s_{12}^2)\log\frac{s_1}{s} \\
&+ (s_{12}+s_1)(3s_2^2 + s_2(6s_{12}+s_1) + 2s_{12}^2)\log\frac{s_2}{s} \\
&+ s(2s_{12}(s_1+s_2) + s_1 s_2(s_1+s_2+6s_{12}))\Phi(s_1, s_2)\Big]
\end{aligned}
$$



The triangle diagram in the fermion case (a), the collinear fermion configuration responsible for the anomaly (b) and a diagrammatic representation of the exchange via an intermediate state (dashed line) (c).

The signature of the chiral anomaly is in the the generation of 1 pole in the axial vector channel

$$\mathcal{S}_{eff} = -\frac{e^2}{16\pi^2} \int d^4x \int d^4y \, [\epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}]_x \, \Box^{-1}_{xy} \, [\partial^\lambda B_\lambda]_y,$$

$$\mathcal{S}_{eff}[\eta, \chi; A, B] = \int d^4x \left\{ (\partial^\mu \eta) \, (\partial_\mu \chi) - \chi \, \partial^\mu B_\mu + \frac{e^2}{8\pi^2} \, \eta \, F_{\mu\nu} \tilde{F}^{\mu\nu} \right\}$$

Giannotti-Mottola

$$\Box \, \eta = -\partial^\lambda B_\lambda \,,$$

$$\Box \, \chi = \frac{e^2}{8\pi^2} \, F_{\mu\nu} \tilde{F}^{\mu\nu} =$$

$$S_{\text{eff}} = \int d^4x \left[\frac{\theta_L - \theta_R}{64\pi^2} \epsilon^{\mu\nu\sigma\tau} F_{\mu\nu} F_{\sigma\tau} - \frac{1}{4e^2} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} \rho_L \left(\partial_\mu \theta_L - 2A_\mu \right)^2 + \frac{1}{2} \rho_R \left(\partial_\mu \theta_R - 2A_\mu \right)^2 + J \cos\left(\theta_L - \theta_R \right) \right]. \tag{23}$$

$$\chi = \frac{1}{\sqrt{2}} \bar{\chi} M \qquad \eta = \frac{1}{\sqrt{2}} \frac{\bar{\eta}}{M},$$

$$\mathcal{S}_{eff}[\bar{\eta}, \bar{\chi}; A, B] = \int d^4x \, \{ (\partial^\mu \bar{\eta}) \, (\partial_\mu \bar{\chi}) + M \partial^\mu \bar{\chi} \, B_\mu + \alpha \frac{\bar{\eta}}{M} \, F_{\mu\nu} \tilde{F}^{\mu\nu} \}$$

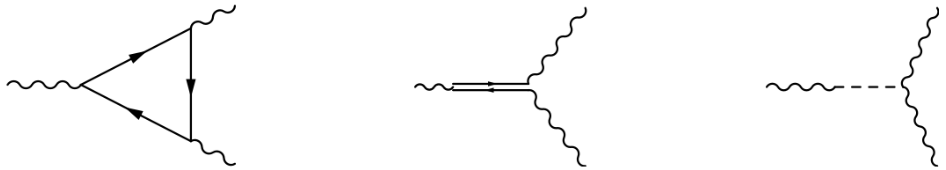
Creti, Lionetti Tommasi, CC

$$\bar{\eta} = a(x) - b(x) \qquad \bar{\chi} = b(x) + a(x)$$

$$\mathcal{S}_{eff}[a, b; A, B] = \int d^4x \left(\frac{1}{2} (\partial_\mu a - \bar{M} B_\mu)^2 - \frac{1}{2} (\partial_\mu b - \bar{M} B_\mu)^2 + \frac{a - b}{\bar{M}} F \tilde{F} \right)$$

Parity-Odd 3-Point Functions from CFT in Momentum Space and the Chiral Anomaly

S. Lionetti, M. Maglio, C.C.


(a) (b) (c)

Bzowski McFadden Skenderis for parity even, extended to parity odd cases + anomalies

$$\langle J^{\mu_1}(p_1)J^{\mu_2}(p_2)J_5^{\mu_3}(p_3)\rangle = \frac{1}{8\pi^2}\left(W_L^{\mu_1\mu_2\mu_3} - W_T^{\mu_1\mu_2\mu_3}\right)$$

$$\nabla_\mu \langle J^\mu\rangle = 0, \qquad \nabla_\mu \langle J_5^\mu\rangle = a\,\varepsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}$$

$$p_{i\mu_i}\,\langle J^{\mu_1}(p_1)J^{\mu_2}(p_2)J_5^{\mu_3}(p_3)\rangle = 0, \qquad i=1,2$$

$$p_{3\mu_3}\,\langle J^{\mu_1}(p_1)J^{\mu_2}(p_2)J_5^{\mu_3}(p_3)\rangle = -8\,a\,i\,\varepsilon^{p_1p_2\mu_1\mu_2}$$

Longitudinal/Transverse decomposition

$$J^\mu(p) = j^\mu(p) + j^\mu_{loc}(p),$$
$$j^\mu = \pi^\mu_\alpha(p)\,J^\alpha(p), \qquad \pi^\mu_\alpha(p) \equiv \delta^\mu_\alpha - \frac{p_\alpha\,p^\mu}{p^2},$$
$$j^\mu_{loc}(p) = \frac{p^\mu}{p^2}\,p\cdot J(p)$$

$$J_5^\mu(p) = j_5^\mu(p) + j^\mu_{5loc}(p),$$
$$j_5^\mu = \pi^\mu_\alpha(p)\,J_5^\alpha(p),$$
$$j^\mu_{5loc}(p) = \frac{p^\mu}{p^2}\,p\cdot J_5(p)$$

$$\langle J^{\mu_1}(p_1)J^{\mu_2}(p_2)J_5^{\mu_3}(p_3)\rangle = \langle j^{\mu_1}(p_1)j^{\mu_2}(p_2)j_5^{\mu_3}(p_3)\rangle + \langle J^{\mu_1}(p_1)J^{\mu_2}(p_2)\, j^{\mu_3}_{5\ \text{loc}}\,(p_3)\rangle$$

reconstruction

$$\langle J^{\mu_1}(p_1)J^{\mu_2}(p_2)\, j^{\mu_3}_{5\ \text{loc}}\,(p_3)\rangle = \frac{p_3^{\mu_3}}{p_3^2}\,p_{3\,\alpha_3}\,\langle J^{\mu_1}(p_1)J^{\mu_2}(p_2)J_5^{\alpha_3}(p_3)\rangle = -\frac{8\,a\,i}{p_3^2}\varepsilon^{p_1p_2\mu_1\mu_2}\,p_3^{\mu_3}$$

solve the constraint
with a pole

transverse sector parameterised by form factors

$$\begin{aligned}\langle j^{\mu_1}(p_1)j^{\mu_2}(p_2)j_5^{\mu_3}(p_3)\rangle = \pi^{\mu_1}_{\alpha_1}(p_1)\pi^{\mu_2}_{\alpha_2}(p_2)\pi^{\mu_3}_{\alpha_3}(p_3)\Big[&A_1(p_1,p_2,p_3)\,\varepsilon^{p_1p_2\alpha_1\alpha_2}p_1^{\alpha_3} + A_2(p_1,p_2,p_3)\,\varepsilon^{p_1p_2\alpha_1\alpha_3}p_3^{\alpha_2}\\ &+ A_3(p_1,p_2,p_3)\,\varepsilon^{p_1p_2\alpha_2\alpha_3}p_2^{\alpha_1} + A_4(p_1,p_2,p_3)\,\varepsilon^{p_1\alpha_1\alpha_2\alpha_3} + A_5(p_1,p_2,p_3)\,\varepsilon^{p_2\alpha_1\alpha_2\alpha_3}\Big]\end{aligned}$$

$$\left(\sum_{i=1}^{3}\Delta_i - 2d - \sum_{i=1}^{2}p_i^{\mu}\frac{\partial}{\partial p_i^{\mu}}\right)\langle J^{\mu_1}(p_1)J^{\mu_2}(p_2)J_5^{\mu_3}(p_3)\rangle = 0.$$

impose CWIs

$$\begin{aligned}0 = \sum_{j=1}^{2}&\left[-2\frac{\partial}{\partial p_{j\kappa}} - 2p_j^{\alpha}\frac{\partial^2}{\partial p_j^{\alpha}\,\partial p_{j\kappa}} + p_j^{\kappa}\frac{\partial^2}{\partial p_j^{\alpha}\,\partial p_{j\alpha}}\right]\langle J^{\mu_1}(p_1)J^{\mu_2}(p_2)J_5^{\mu_3}(p_3)\rangle\\ &+2\left(\delta^{\mu_1\kappa}\frac{\partial}{\partial p_1^{\alpha_1}} - \delta^{\kappa}_{\alpha_1}\frac{\partial}{\partial p_{1\mu_1}}\right)\langle J^{\alpha_1}(p_1)J^{\mu_2}(p_2)J_5^{\mu_3}(p_3)\rangle\\ &+2\left(\delta^{\mu_2\kappa}\frac{\partial}{\partial p_2^{\alpha_2}} - \delta^{\kappa}_{\alpha_2}\frac{\partial}{\partial p_{2\mu_2}}\right)\langle J^{\mu_1}(p_1)J^{\alpha_2}(p_2)J_5^{\mu_3}(p_3)\rangle \equiv \mathcal{K}^{\kappa}\,\langle J^{\mu_1}(p_1)J^{\mu_2}(p_2)J_5^{\mu_3}(p_3)\rangle\,.\end{aligned}$$

$$
\begin{aligned}
&\pi^{\lambda_1}_{\mu_1}(p_1)\pi^{\lambda_2}_{\mu_2}(p_2)\pi^{\lambda_3}_{\mu_3}(p_3)\left(\mathcal{K}^\kappa \left\langle j^{\mu_1}(p_1) j^{\mu_2}(p_2) j_5^{\mu_3}(p_3)\right\rangle\right) = \\
&= \pi^{\lambda_1}_{\mu_1}(p_1)\pi^{\lambda_2}_{\mu_2}(p_2)\pi^{\lambda_3}_{\mu_3}(p_3)\bigg[p_1^\kappa\left(C_{11}\, \varepsilon^{\mu_1\mu_2\mu_3 p_1} + C_{12}\, \varepsilon^{\mu_1\mu_2\mu_3 p_2} + C_{13}\, \varepsilon^{\mu_1\mu_2 p_1 p_2} p_1^{\mu_3}\right) \\
&\quad + p_2^\kappa\left(C_{21}\, \varepsilon^{\mu_1\mu_2\mu_3 p_1} + C_{22}\, \varepsilon^{\mu_1\mu_2\mu_3 p_2} + C_{23}\, \varepsilon^{\mu_1\mu_2 p_1 p_2} p_1^{\mu_3}\right) + C_{31}\varepsilon^{\kappa\mu_1\mu_2\mu_3} + C_{32}\varepsilon^{\kappa\mu_1\mu_2 p_1} p_1^{\mu_3} \\
&\qquad + C_{33}\varepsilon^{\kappa\mu_1\mu_2 p_2} p_1^{\mu_3} + C_{34}\varepsilon^{\kappa\mu_1 p_1 p_2}\delta^{\mu_2\mu_3} + C_{35}\varepsilon^{\kappa\mu_2 p_1 p_2}\delta^{\mu_1\mu_3} + C_{36}\varepsilon^{\kappa\mu_3 p_1 p_2}\delta^{\mu_1\mu_2}\bigg],
\end{aligned}
$$

project the CWIs

$$
\begin{aligned}
&K_{31}\, A_1 = 0, && K_{32}\, A_1 = 0, \\
&K_{31}\, A_2 = 0, && K_{32}\, A_2 = \left(\frac{4}{p_1^2} - \frac{2}{p_1}\frac{\partial}{\partial p_1}\right) A_2(p_1 \leftrightarrow p_2) + 2A_1, \\
&K_{31}\, A_2(p_1 \leftrightarrow p_2) = \left(\frac{4}{p_2^2} - \frac{2}{p_2}\frac{\partial}{\partial p_2}\right) A_2 - 2A_1, && K_{32}\, A_2(p_1 \leftrightarrow p_2) = 0,
\end{aligned}
$$

derive diff. eqs.

$$
K_i = \frac{\partial^2}{\partial p_i^2} + \frac{(d+1-2\Delta_i)}{p_i}\frac{\partial}{\partial p_i}, \qquad K_{ij} = K_i - K_j.
$$

$$
I_{\alpha\{\beta_1\beta_2\beta_3\}}\left(p_1, p_2, p_3\right) = \int dx x^\alpha \prod_{j=1}^{3} p_j^{\beta_j} K_{\beta_j}\left(p_j x\right)
$$

solutions expressed in terms of 3K integrals

$$K_\nu(x) = \frac{\pi}{2}\frac{I_{-\nu}(x) - I_\nu(x)}{\sin(\nu\pi)}, \qquad \nu \notin \mathbb{Z} \qquad\qquad I_\nu(x) = \left(\frac{x}{2}\right)^\nu \sum_{k=0}^{\infty} \frac{1}{\Gamma(k+1)\Gamma(\nu+1+k)}\left(\frac{x}{2}\right)^{2k}$$

$$J_{N\{k_j\}} = I_{\frac{d}{2}-1+N\{\Delta_j - \frac{d}{2}+k_j\}}$$

$$A_1 = \alpha_1\, J_{3\{0,0,0\}},$$
$$A_2 = \alpha_2\, J_{1\{0,0,0\}} + \alpha_3\, J_{2\{0,1,0\}}.$$

form factors

$$\langle j^{\mu_1}(p_1) j^{\mu_2}(p_2) j_5^{\mu_3}(p_3)\rangle = \pi^{\mu_1}_{\alpha_1}(p_1)\pi^{\mu_2}_{\alpha_2}(p_2)\pi^{\mu_3}_{\alpha_3}(p_3)\Big[8ia\Big(-2\,J_{1\{0,0,0\}} + J_{2\{0,1,0\}}\Big)\varepsilon^{p_1\alpha_1\alpha_2\alpha_3} - 8ia\Big(-2\,J_{1\{0,0,0\}} + J_{2\{1,0,0\}}\Big)\varepsilon^{p_2\alpha_1\alpha_2\alpha_3}\Big]$$

$$\begin{aligned} A_2^{(P)} =& \frac{-e^3\, p_2^2}{2\pi^2\lambda^2}\Bigg\{ -\lambda\,(p_1^2 - p_2^2 + p_3^2) + p_1^2\Big[\left(p_1^2 - p_2^2\right)^2 + 4p_3^2 p_2^2 - p_3^4\Big]\log\left(\frac{p_1^2}{p_2^2}\right) \\ &+ p_3^2\Big[p_1^4 - 4p_1^2p_2^2 - \left(p_2^2 - p_3^2\right)^2\Big]\log\left(\frac{p_2^2}{p_3^2}\right) + 4p_1^2p_3^2\Big(p_1^2 - p_3^2\Big)\log\left(\frac{p_1^2}{p_3^2}\right) \\ &- 2p_1^2p_3^2\Big[p_1^2\left(p_2^2 - 2p_3^2\right) + p_1^4 + p_2^2p_3^2 - 2p_2^4 + p_3^4\Big]C_0\left(p_1^2, p_2^2, p_3^2\right)\Bigg\}. \end{aligned}$$

example

They exactly reproduce the perturbative result.
The result is proportional to the residue at the pole in the longitudinal sector that solves the longitudinal anomalous WI

The correlator is compleely fixed by CWIs + the residue at a single pole

The analysis can be repeated with few additional technical steps for the AAA correlator

$$\left\langle J_{5\,\text{loc}}^{\mu_1}(p_1)\, J_5^{\mu_2}(p_2)\, j_5^{\mu_3}(p_3)\right\rangle = -\frac{8\,a' i}{p_1^2}\varepsilon^{p_1 p_2 \mu_2 \mu_3} p_1^{\mu_1}$$

$$\left\langle J_5^{\mu_1}(p_1)\, J_{5\,\text{loc}}^{\mu_2}(p_2)\, j_5^{\mu_3}(p_3)\right\rangle = \frac{8\,a' i}{p_2^2}\varepsilon^{p_1 p_2 \mu_1 \mu_3} p_2^{\mu_2}$$

$$\left\langle J_5^{\mu_1}(p_1)\, J_5^{\mu_2}(p_2)\, j_{5\,\text{loc}}^{\mu_3}(p_3)\right\rangle = -\frac{8\,a' i}{p_3^2}\varepsilon^{p_1 p_2 \mu_1 \mu_2} p_3^{\mu_3}.$$

scale WI

$$\sum_{i=1}^{3} p_i \frac{\partial \tilde{A}}{\partial p_i} - (\Delta_3 - 5)\,\tilde{A} = 0$$

$$\sum_{i=1}^{3} p_i \frac{\partial A}{\partial p_i} - (\Delta_3 - 3)\, A = 0.$$

$$\langle j_5^{\mu_1}(p_1)\, j_5^{\mu_2}(p_2)\, j_5^{\mu_3}(p_3)\rangle = \pi_{\alpha_1}^{\mu_1}(p_1)\,\pi_{\alpha_2}^{\mu_2}(p_2)\,\pi_{\alpha_3}^{\mu_3}(p_3)\Big[\tilde{A}(p_1,p_2,p_3)\varepsilon^{p_1 p_2 \alpha_1 \alpha_2} p_1^{\alpha_3} + A(p_1,p_2,p_3)\varepsilon^{p_1 \alpha_1 \alpha_2 \alpha_3} - A(p_2,p_1,p_3)\varepsilon^{p_2 \alpha_1 \alpha_2 \alpha_3}\Big].$$

Parity-Violating CFT and the Gravitational Chiral Anomaly

S. Lionetti, M. Maglio, CC

$$J^{\lambda}_{5f} = \bar{\psi}\gamma_5\gamma^{\lambda}\psi$$

$$\nabla_\mu \langle J^\mu_5 \rangle = a_1\, \varepsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} + a_2\, \varepsilon^{\mu\nu\rho\sigma} R^{\alpha\beta}{}_{\mu\nu} R_{\alpha\beta\rho\sigma},$$

$$J^{\lambda}_{CS} = \epsilon^{\lambda\mu\nu\rho} V_\mu \partial_\nu V_\rho,$$

- **Diffeomorphism invariance**

We start from diffeomorphism invariance.
Under a diffeomorphism the fields transform with a Lie derivative

$$\delta g_{\mu\nu} = \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu$$
$$\delta A_\mu = \xi^\nu \nabla_\nu A_\mu + \nabla_\mu \xi^\nu A_\nu.$$

$$\nabla^\mu \langle T_{\mu\nu} \rangle - F_{A\nu\mu} \langle J^\mu_5 \rangle + A_\nu \nabla_\mu \langle J^\mu_5 \rangle = 0.$$

$$0 = p_{i\mu_i} \left\langle T^{\mu_1\nu_1}(p_1)\, T^{\mu_2\nu_2}(p_2)\, J^{\mu_3}_5(p_3) \right\rangle,$$

- **Gauge invariance** $\qquad \delta g_{\mu\nu} = 0, \qquad \delta A_\mu = \partial_\mu \alpha.$

$$\nabla_\alpha \langle J^\alpha_5 \rangle = a_1\, \varepsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} + a_2\, \varepsilon^{\mu\nu\rho\sigma} R_{\alpha\beta\mu\nu} R^{\alpha\beta}{}_{\rho\sigma},$$

$$p_{3\mu_3} \langle T^{\mu_1\nu_1}(p_1) T^{\mu_2\nu_2}(p_2) J^{\mu_3}_5(p_3) \rangle = 4\, i\, a_2\, (p_1 \cdot p_2) \left\{ \left[\varepsilon^{\nu_1\nu_2 p_1 p_2} \left(g^{\mu_1\mu_2} - \frac{p_1^{\mu_2} p_2^{\mu_1}}{p_1 \cdot p_2} \right) + (\mu_1 \leftrightarrow \nu_1) \right] + (\mu_2 \leftrightarrow \nu_2) \right\}.$$

- **Weyl invariance**

$$\delta g_{\mu\nu} = 2 g_{\mu\nu}\sigma,$$
$$\delta A_\mu = 0.$$

$$\left(\sum_{i=1}^{3}\Delta_i - 2d - \sum_{i=1}^{2} p_i^\mu \frac{\partial}{\partial p_i^\mu}\right)\langle T^{\mu_1\nu_1}(p_1) T^{\mu_2\nu_2}(p_2) J_5^{\mu_3}(p_3)\rangle = 0.$$

CWIs

$$\begin{aligned}
0 =& \mathcal{K}^\kappa \langle T^{\mu_1\nu_1}(p_1)\, T^{\mu_2\nu_2}(p_2)\, J_5^{\mu_3}(p_3)\rangle \\
=& \sum_{j=1}^{2}\left(2(\Delta_j - d)\frac{\partial}{\partial p_{j\kappa}} - 2 p_j^\alpha \frac{\partial}{\partial p_j^\alpha}\frac{\partial}{\partial p_{j\kappa}} + (p_j)^\kappa \frac{\partial}{\partial p_j^\alpha}\frac{\partial}{\partial p_{j\alpha}}\right)\langle T^{\mu_1\nu_1}(p_1)\, T^{\mu_2\nu_2}(p_2)\, J_5^{\mu_3}(p_3)\rangle \\
&+ 4\left(\delta^{\kappa(\mu_1}\frac{\partial}{\partial p_1^{\alpha_1}} - \delta^\kappa_{\alpha_1}\delta^{(\mu_1}_\lambda \frac{\partial}{\partial p_{1\lambda}}\right)\left\langle T^{\nu_1)\alpha_1}(p_1)\, T^{\mu_2\nu_2}(p_2)\, J_5^{\mu_3}(p_3)\right\rangle \\
&+ 4\left(\delta^{\kappa(\mu_2}\frac{\partial}{\partial p_2^{\alpha_2}} - \delta^\kappa_{\alpha_2}\delta^{(\mu_2}_\lambda \frac{\partial}{\partial p_{2\lambda}}\right)\left\langle T^{\nu_2)\alpha_2}(p_2)\, T^{\mu_1\nu_1}(p_1)\, J_5^{\mu_3}(p_3)\right\rangle .
\end{aligned}$$

$$T^{\mu_i\nu_i}(p_i) = t^{\mu_i\nu_i}(p_i) + t^{\mu_i\nu_i}_{loc}(p_i),$$
$$J_5^{\mu_i}(p_i) = j_5^{\mu_i}(p_i) + j^{\mu_i}_{5\,loc}(p_i),$$

L/T decomposition

$$t^{\mu_i\nu_i}(p_i) = \Pi^{\mu_i\nu_i}_{\alpha_i\beta_i}(p_i)\, T^{\alpha_i\beta_i}(p_i), \qquad t^{\mu_i\nu_i}_{loc}(p_i) = \Sigma^{\mu_i\nu_i}_{\alpha_i\beta_i}(p)\, T^{\alpha_i\beta_i}(p_i),$$

$$j_5^{\mu_i}(p_i) = \pi^{\mu_i}_{\alpha_i}(p_i)\, J_5^{\alpha_i}(p_i), \qquad j^{\mu_i}_{5\,loc}(p_i) = \frac{p_i^{\mu_i}\, p_{i\,\alpha_i}}{p_i^2}\, J_5^{\alpha_i}(p_i),$$

$$\langle T^{\mu_1\nu_1}T^{\mu_2\nu_2}J_5^{\mu_3}\rangle = \langle t^{\mu_1\nu_1}t^{\mu_2\nu_2}j_5^{\mu_3}\rangle + \langle T^{\mu_1\nu_1}T^{\mu_2\nu_2}j_{5\,loc}^{\mu_3}\rangle = \langle t^{\mu_1\nu_1}t^{\mu_2\nu_2}j_5^{\mu_3}\rangle + \langle t^{\mu_1\nu_1}t^{\mu_2\nu_2}j_{5\,loc}^{\mu_3}\rangle\,.$$

$$\langle t^{\mu_1\nu_1}t^{\mu_2\nu_2}j_{5\,loc}^{\mu_3}\rangle = 4ia_2\frac{p_3^{\mu_3}}{p_3^2}\,(p_1\cdot p_2)\left\{\left[\varepsilon^{\nu_1\nu_2p_1p_2}\left(g^{\mu_1\mu_2}-\frac{p_1^{\mu_2}p_2^{\mu_1}}{p_1\cdot p_2}\right)+(\mu_1\leftrightarrow\nu_1)\right]+(\mu_2\leftrightarrow\nu_2)\right\}. \quad (61$$

$$\begin{aligned}\langle t^{\mu_1\nu_1}(p_1)t^{\mu_2\nu_2}(p_2)\,j_5^{\mu_3}(p_3)\rangle = \Pi^{\mu_1\nu_1}_{\alpha_1\beta_1}(p_1)\,\Pi^{\mu_2\nu_2}_{\alpha_2\beta_2}(p_2)\,\pi^{\mu_3}_{\alpha_3}(p_3)\Big[\\
A_1\varepsilon^{p_1\alpha_1\alpha_2\alpha_3}p_2^{\beta_1}p_3^{\beta_2} - A_1(p_1\leftrightarrow p_2)\varepsilon^{p_2\alpha_1\alpha_2\alpha_3}p_2^{\beta_1}p_3^{\beta_2}\\
+A_2\varepsilon^{p_1\alpha_1\alpha_2\alpha_3}\delta^{\beta_1\beta_2} - A_2(p_1\leftrightarrow p_2)\varepsilon^{p_2\alpha_1\alpha_2\alpha_3}\delta^{\beta_1\beta_2}\\
+A_3\varepsilon^{p_1p_2\alpha_1\alpha_2}p_2^{\beta_1}p_3^{\beta_2}p_1^{\alpha_3} + A_4\varepsilon^{p_1p_2\alpha_1\alpha_2}\delta^{\beta_1\beta_2}p_1^{\alpha_3}\\
+A_5\varepsilon^{p_1p_2\alpha_1\alpha_3}p_2^{\beta_1}p_3^{\alpha_2}p_3^{\beta_2} + A_5(p_1\leftrightarrow p_2)\varepsilon^{p_1p_2\alpha_2\alpha_3}p_3^{\beta_2}p_2^{\alpha_1}p_2^{\beta_1}\\
+A_6\varepsilon^{p_1p_2\alpha_1\alpha_3}p_3^{\alpha_2}\delta^{\beta_1\beta_2} + A_6(p_1\leftrightarrow p_2)\varepsilon^{p_1p_2\alpha_2\alpha_3}p_2^{\alpha_1}\delta^{\beta_1\beta_2}\\
+A_7\varepsilon^{p_1p_2\alpha_1\alpha_2}p_2^{\beta_1}\delta^{\beta_2\alpha_3} - A_7(p_1\leftrightarrow p_2)\varepsilon^{p_1p_2\alpha_1\alpha_2}p_3^{\beta_2}\delta^{\beta_1\alpha_3}\Big]\end{aligned}$$

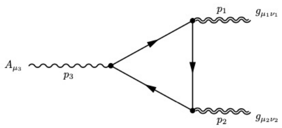
the analysis is rather involved

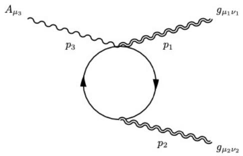
$$A_1 = -4\,i\,a_2\,p_2^2\,I_{5\{2,1,1\}}$$

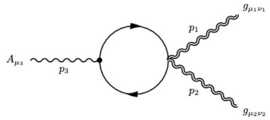
$$A_2 = -8\,i\,a_2\,p_2^2\left(p_3^2\,I_{4\{2,1,0\}}-1\right)$$

$$A_3 = 0$$

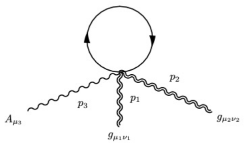
$$A_4 = 0.$$







in agreement with the perturbative realization



$$S_0 = \int d^d x\, e \left[\frac{i}{2} \bar{\psi} e^{\mu}_{a} \gamma^{a} \left(\partial_\mu \psi \right) - \frac{i}{2} \left(\partial_\mu \bar{\psi} \right) e^{\mu}_{a} \gamma^{a} \psi - g A_\mu \bar{\psi} e^{\mu}_{a} \gamma^{a} \gamma_5 \psi + \frac{i}{4} \omega_{\mu a b} e^{\mu}_{c} \bar{\psi} \gamma^{abc} \psi \right]$$

Therefore: CWIs + an anomaly pole allow to reconstruct the entire CP-odd anomalous correlators for chiral and gravitationla anomalies

PARITY ODD TRACE ANOMALIES

$$g_{\mu\nu}\langle T^{\mu\nu}\rangle = b_1\, E_4 + b_2\, C^{\mu\nu\rho\sigma}C_{\mu\nu\rho\sigma} + b_3\, \nabla^2 R + b_4\, F^{\mu\nu}F_{\mu\nu},$$

$$C^{\mu\nu\rho\sigma}C_{\mu\nu\rho\sigma} = R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma} - 2R^{\mu\nu}R_{\mu\nu} + \frac{1}{3}R^2,$$

$$E_4 \equiv E = R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma} - 4R^{\mu\nu}R_{\mu\nu} + R^2.$$

$$\mathcal{A} = b_1\, E_4 + b_2\, C^{\mu\nu\rho\sigma}C_{\mu\nu\rho\sigma} + b_3\, \nabla^2 R + b_4\, F^{\mu\nu}F_{\mu\nu} + f_1\, \varepsilon^{\mu\nu\rho\sigma} R_{\alpha\beta\mu\nu}R^{\alpha\beta}{}_{\rho\sigma} + f_2\, \varepsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma},$$

in perturbation theory, if nonvanishing, would break CPT
with significant implications for physics BSM

Is there a parity-odd trace anomaly?

perturbative analysis differ in the result

NO:
Delle Rose, Armillis, CC 2011
The Trace Anomaly and the Gravitational Coupling of an Anomalous U(1)

F. Bastianelli and Coll.
S. Franchino-Vinas and Coll. 2202.10813 [hep-th]
Trace anomaly of Weyl fermions in the Breitenlohner--Maison scheme for $\gamma * \gamma *$
R. Zwicky and Coll. 2024

YES: L. Bonora et al

CFT in momentum space: if we assume an anomaly then **yes**, there is one surviving sector in the corelator: the anomaly pole

CFT correlators and CP-violating trace anomalies

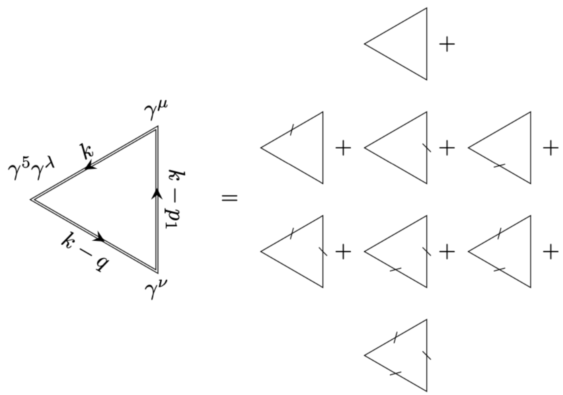
S. Lionetti, M. Maglio CC e-Print: 2307.03038 [hep-th]

DENSITY AND THERMAL EFFECTS

Axion-like Quasiparticles and Topological States of Matter: Finite Density Corrections of the Chiral Anomaly Vertex

PRD, 2024

M. CRETI' S. LIONETTI,R. TOMMASI



complete general classification of the vertex AVV at finite T and chemical potential

$$S_F(k,\beta,\mu) \equiv (\not{k}+m)G_F(k,\beta,\mu) =$$
$$(\not{k}+m)\left\{\frac{1}{k^2-m^2}+2\pi i\delta\left(k^2-m^2\right)\left[\frac{\theta\left(k_0\right)}{e^{\beta(E-\mu)}+1}+\frac{\theta\left(-k_0\right)}{e^{\beta(E+\mu)}+}\right.\right.,$$

$$\begin{aligned}
\Gamma^{\lambda\mu\nu}(p_1,p_2,\eta) = {} & \chi_1^{\lambda\mu\nu}(p_1,p_2,\eta)\,\bar{B}_1(p_1,p_2,\eta)+\chi_1^{\lambda\mu\nu}(p_2,p_1,\eta)\,\bar{B}_1(p_2,p_1,\eta)\\
&+\chi_2^{\lambda\mu\nu}(p_1,p_2,\eta)\,\bar{B}_2(p_1,p_2,\eta)+\chi_2^{\lambda\mu\nu}(p_2,p_1,\eta)\,\bar{B}_2(p_2,p_1,\eta)\\
&+\chi_3^{\lambda\mu\nu}(p_1,p_2,\eta)\,\bar{B}_3(p_1,p_2,\eta)+\chi_3^{\lambda\mu\nu}(p_2,p_1,\eta)\,\bar{B}_3(p_2,p_1,\eta)\\
&+\chi_4^{\lambda\mu\nu}(p_1,p_2,\eta)\,\bar{B}_4(p_1,p_2,\eta)+\chi_4^{\lambda\mu\nu}(p_2,p_1,\eta)\,\bar{B}_4(p_2,p_1,\eta)\\
&+\chi_5^{\lambda\mu\nu}(p_1,p_2,\eta)\,\bar{B}_5(p_1,p_2,\eta)+\chi_5^{\lambda\mu\nu}(p_2,p_1,\eta)\,\bar{B}_5(p_2,p_1,\eta)\\
&+\chi_6^{\lambda\mu\nu}(p_1,p_2,\eta)\,\bar{B}_6(p_1,p_2,\eta)+\chi_6^{\lambda\mu\nu}(p_2,p_1,\eta)\,\bar{B}_6(p_2,p_1,\eta)\\
&+\chi_7^{\lambda\mu\nu}(p_1,p_2,\eta)\,\bar{B}_7(p_1,p_2,\eta)+\chi_7^{\lambda\mu\nu}(p_2,p_1,\eta)\,\bar{B}_7(p_2,p_1,\eta)\\
&+\chi_8^{\lambda\mu\nu}(p_1,p_2,\eta)\,\bar{B}_8(p_1,p_2,\eta)+\chi_8^{\lambda\mu\nu}(p_2,p_1,\eta)\,\bar{B}_8(p_2,p_1,\eta)\\
&+\chi_9^{\lambda\mu\nu}(p_1,p_2,\eta)\,\bar{B}_9(p_1,p_2,\eta)+\chi_{10}^{\lambda\mu\nu}(p_1,p_2,\eta)\,\bar{B}_{10}(p_1,p_2,\eta),
\end{aligned}$$

$$i\gamma^\mu\partial_\mu\psi - m\psi - \mu\gamma^0\psi = 0,$$

the pole is untouched

$$\begin{aligned}
iS(x'-x) &= \\
\sum_{\lambda,k}\Big[\theta(t'-t)\Big([1-f^+(E)]\psi^{(+)}_{\lambda\kappa}(\mathbf{x}',t')\overline{\psi}^{(+)}_{\lambda k}(\mathbf{x},t) &+ [1-f^+(-E)]\psi^{(-)}_{\lambda k}(\mathbf{x}',t')\overline{\psi}^{(-)}_{\lambda k}(\mathbf{x},t)\Big)\\
-\theta(t-t')\Big(f^+(-E)\psi^{(-)}_{\lambda k}(\mathbf{x}',t')\overline{\psi}^{(-)}_{\lambda k}(\mathbf{x},t) &+ f^+(E)\psi^{(+)}_{\lambda k}(\mathbf{x}',t')\overline{\psi}^{(+)}_{\lambda k}(\mathbf{x},t)\Big)\Big].
\end{aligned}$$

implications for the chiral magnetic effect

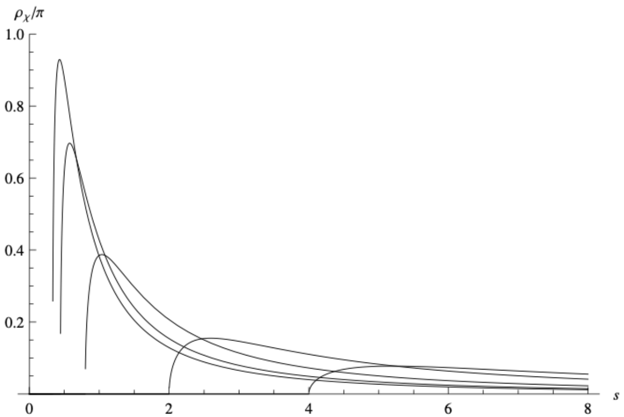
The Dispersive sum rule and the spectral density flow

$$\frac{1}{\pi}\int_0^\infty \rho(s,m^2)ds = f, \qquad \Gamma^{\mu\alpha\beta}(p,q) = i\frac{g^2}{12\pi^2}\,\phi_1(k^2,m^2)\,\frac{k^\mu}{k^2}\varepsilon[p,q,\alpha,\beta]\,,$$

$$\chi(k^2,m^2) \equiv \Phi_1(k^2,m^2)/k^2. \qquad \Phi_1(k^2,m^2) = -1 - 2\,m^2\,\mathcal{C}_0(k^2,m^2)\,,$$

$$\text{Disc}\left(\frac{\mathcal{C}_0(k^2,m^2)}{k^2}\right) = -2i\frac{\pi}{(k^2)^2}\log\frac{1+\sqrt{\tau(k^2,m^2)}}{1-\sqrt{\tau(k^2,m^2)}}\theta(k^2-4m^2) + i\frac{\pi}{m^2}\delta(k^2).$$

$$\lim_{m\to 0}\rho_\chi(s,m^2) = \lim_{m\to 0}\frac{2\pi m^2}{s^2}\log\left(\frac{1+\sqrt{\tau(s,m^2)}}{1-\sqrt{\tau(s,m^2)}}\right)\theta(s-4m^2) = \pi\delta(s)$$



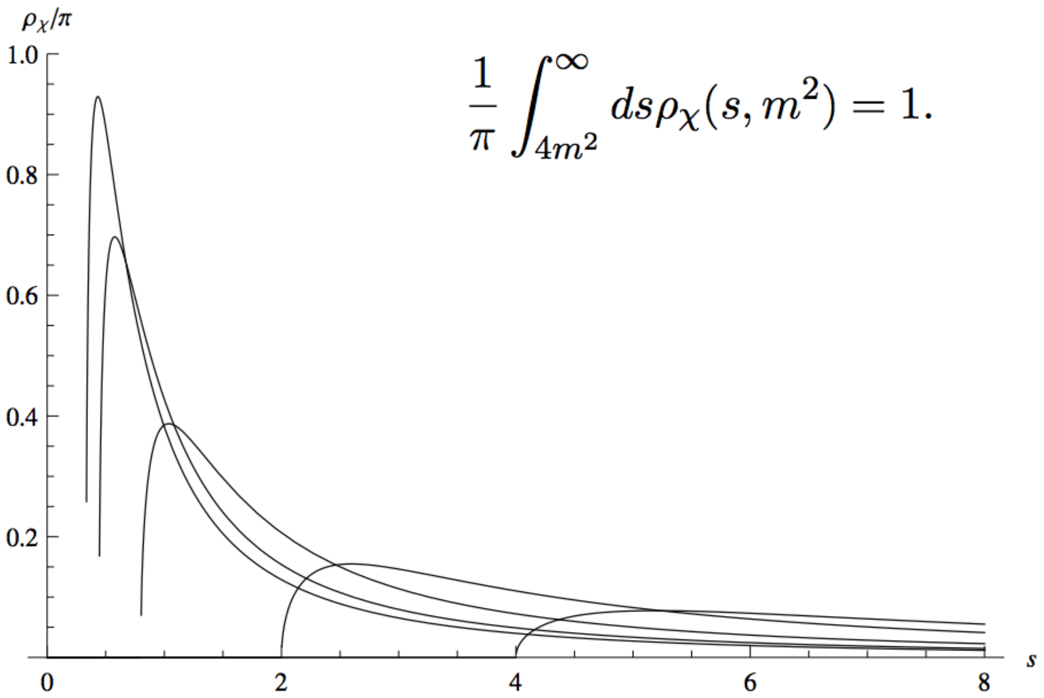
$$\langle 0|J_f^\mu|\gamma\gamma\rangle = f_1(q^2)\frac{q^\mu}{q^2}F_{\kappa\lambda}\tilde{F}^{\kappa\lambda}(q)$$
$$\langle 0|J_f^\mu|gg\rangle = f_2(q^2)\frac{q^\mu}{q^2}R_{\kappa\lambda\rho\sigma}\tilde{R}^{\kappa\lambda\rho\sigma}(q)$$
$$\langle 0|J_{CS}^\mu|gg\rangle = f_3(q^2)\frac{q^\mu}{q^2}R_{\kappa\lambda\rho\sigma}\tilde{R}^{\kappa\lambda\rho\sigma}(q),$$

SIMILAR SUM RULES
in other form factors

$$\begin{aligned}
\int_{4m^2}^\infty ds\,\Delta_{AVV}(s,m) &= 2\,d_{AVV}\\
\int_{4m^2}^\infty ds\,\Delta_{TTJ_f}(s,m) &= \frac{2}{3}\,d_{TTJ_f}\\
\int_{4m^2}^\infty ds\,\Delta_{TTJ_{CS}}(s,m) &= \frac{14}{45}\,d_{TTJ_{CS}}
\end{aligned}$$

$$\begin{aligned}
\Delta_{AVV}(q^2,m) \equiv \mathrm{Im} f_1(q^2) &= \frac{d_{AVV}}{q^2}(1-v^2)\log\frac{1+v}{1-v}\\
\Delta_{TTJ_f}(q^2,m) \equiv \mathrm{Im} f_2(q^2) &= \frac{d_{TTJ_f}}{q^2}(1-v^2)^2\log\frac{1+v}{1-v}\\
\Delta_{TTJ_{CS}}(q^2,m) \equiv \mathrm{Im} f_3(q^2) &= \frac{d_{TTJ_{CS}}}{q^2}v^2(1-v^2)^2\log\frac{1+v}{1-v},
\end{aligned}$$

with $v = \sqrt{1-4m^2/q^2}$ and $d_{AVV} = -1/2\,\alpha_{em}$, $d_{TTJ_f} = 1/(192\pi)$ and $d_{TTJ_{CS}} = 1/(96\pi)$



$$\frac{1}{\pi}\int_{4m^2}^{\infty} ds\,\rho_\chi(s,m^2) = 1.$$

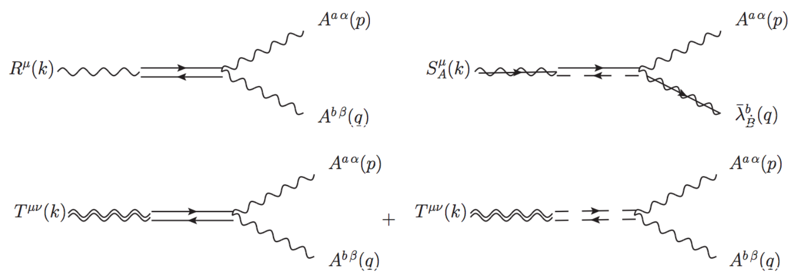
EXACT SUM RULE

The form factor that
carries the chiral
and conformal anomaly
away from the critical point
shows a branch cut.

The spectral dentity exhibits a pole
As m→0

$$\lim_{m\to 0}\rho_\chi(s,m^2) = \lim_{m\to 0}\frac{2\pi m^2}{s^2}\log\left(\frac{1+\sqrt{\tau(s,m^2)}}{1-\sqrt{\tau(s,m^2)}}\right)\theta(s-4m^2) = \pi\delta(s)$$

Delle Rose, CC



THESE ANALYSIS
ARE PURELY
PERTURBATIVE.

Four-point functions of gravitons and conserved currents of CFT in momentum space: testing the nonlocal action with the TTJJ

Claudio Corianò (INFN, Lecce and Salento U.), Matteo Maria Maglio (Heidelberg U.), Riccardo Tommasi (INFN, Lecce and Salento U.) (Dec 24, 2022)

Published in: *Eur.Phys.J.C* 83 (2023) 5, 427 • e-Print: 2212.12779 [hep-th]

A discussion of the relation beween the prediction for the effective action as derived from CWIs and those form the variational solution. This solution does not reproduce the correct hierarchies that are consistently derived from flat space. Technical details are discussed in this paper.

Conclusions

The breaking of conformal symmetry is associated to the the propagation of massless effective states in the effective action.

For chiral anomalies, the interactions can be reconstructed by a combination of the Anomaly pole + CWIs. We have shown it in the case of the AVV, for the J5TT (work in preparation)

For parity breaking trace/conformal anomalies, we have also shown that
the reconstruction can also be based entirely on the selection of an anomaly pole to solve the CWIs.

We have used the TTJJ correlator to show that the anomaly induced actions either in the Riegert form or in the Fradkin-Vilkovisky form miss crucial Weyl invariant terms in order to be consistent with the CWIs and identified such terms

Applications

Condensed Matter theory: application of this class of nonlocal actions in the context of topological Materials (via Luttinger formula)