# Nonlinear Quantum Graphs

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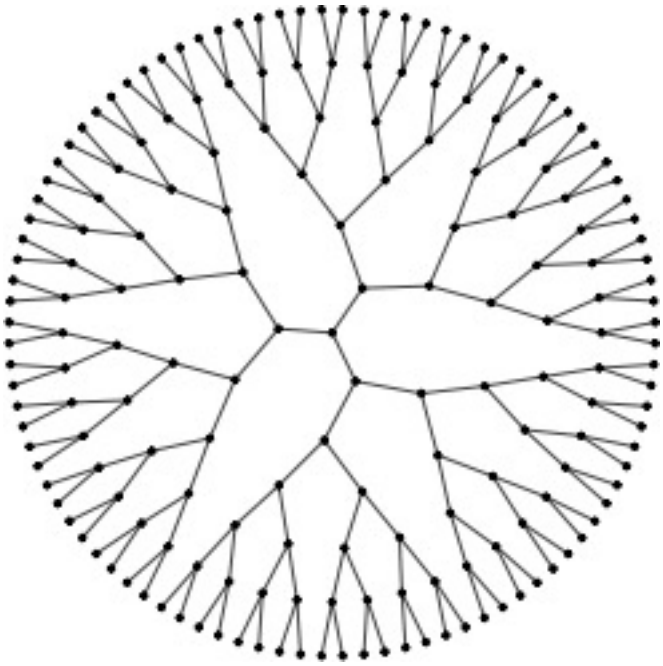
Institut de Mathématiques de Toulouse



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## On the existence of prescribed L^2 norm solutions for nonlinear Schrödinger equations on metric graphs: the mass supercritical case

**Auteur:** louis Jeanjean[1]

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In this talk we discuss the existence of prescribed L^2 norm solutions to nonlinear Schrödinger equations set on metric graphs. A common strategy employed to find such a solution is to search for a constrained critical point of the associated energy functional. Some geometric properties of the functional vary depending on the exponent in the nonlinear term of the equation. In the so-called mass subcritical case, the functional is bounded from below and coercive on the constraint, so one may search for a critical point as a global minimum. As such, in the last years, this case has been extensively studied.
However, in the complementary case, known as the mass supercritical case, the energy functional is no longer bounded from below on the constraint and presents a lack of a priori bounds on the possible critical points. As a result, very little is yet known about this case. Through the presentation of some of the few existing results, we shall discuss the main obstacles that need to be overcome to treat this case under general assumptions. We will also present some of the tools that have already been developed for this purpose.

This talk is based on some joint works with J. Borthwick (Besancon), X. Chang (Changchun) and N. Soave (Torino).

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## Existence of infinitely many normalized solutions for mass-supercritical nonlinear Schrödinger equations on noncompact metric graph.

**Auteur:** Pablo Carrillo[1]

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This talk builds on the one of L. Jeanjean. In the mass-supercritical case we consider nonlinear Schrödinger equations on a noncompact metric graph with a localized nonlinearity. We show, for any prescribed L^2 norm, the existence of infinitely many solutions having this norm.
The usual procedure to obtain one solution is to prove that the associated energy functional possesses, on the constraint, what is known as a mountain pass geometry. We present a generalisation of this approach which permits to obtain infinitely many distinct solutions. Among the new challenges, we find the difficulties of having to handle functions which now may be sign-changing and making sure that the solutions obtained are indeed distinct.

This talk is based on a joint work with D. Galant (Mons et Valenciennes), L. Jeanjean (Besançon) et C. Troestler (Mons).

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## Propriété spectrale du Laplacien sur des graphes

**Auteur:** Serge Nicaise[1]

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La connaissance de certaines propriétés spectrales de l'opérateur de Laplace sur des graphes a de nombreuses conséquences, en particulier dans l'étude de la stabilité asymptotique des petites ondes stationnaires pour NLS sur les graphes. L'objectif principal de mon exposé sera de rappeler quelques anciens résultats sur des graphes finis et de présenter quelques nouveaux résultats pour des graphes infinis.

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## Constant sign and sign changing NLS ground states on noncompact metric graphs

**Auteur:** Colette De Coster[None]

**Co-auteurs:** Christophe Troestler ; Damien Galant ; Enrico Serra ; Simone Dovetta

We investigate existence and nonexistence of positive and nodal action ground states for the nonlinear Schrödinger equation on noncompact metric graphs with rather general boundary conditions. For noncompact graphs with finitely many edges, we detect purely topological sharp conditions preventing the existence of ground states or of nodal ground states. We also investigate analogous conditions of metrical nature. The negative results are complemented by several sufficient conditions to ensure existence, either of topological or metrical nature, or a combination of the two. For graphs with infinitely many edges, all bounded, we focus on periodic graphs and infinite trees. In these cases, our results completely describe the phenomenology.

Furthermore, we study nodal domains and nodal sets of nodal ground states and we show that the situation on graphs can be totally different from that on domains of IR^N.

This is a joint work with Simone Dovetta (Politecnico di Torino (Italie)), Damien Galant (UPHF et UMons (Belgique)), Enrico Serra (Politecnico di Torino (Italie)), Christophe Troestler (UMons (Belgique)).

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## Qualitative properties of solutions of NLS on metric graphs

**Auteur:** Damien Galant[1]

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In this talk, we will study qualitative properties of solutions to the Nonlinear Schrödinger Equation -u" + au = $|u|^{p-2}u$ on compact metric graphs.
Among the questions of interest are:

- if the graph possesses a symmetry, what about its positive solutions? What about action ground states? Can one identify and describe symmetry-breaking situations depending on parameters (lengths of edges, exponent p,...)?
- what can we say about nodal sets and nodal zones of sign-changing solutions? Can nontrivial solutions vanish identically on some edges? What about nodal ground states?
- when the parameter p gets close to 2, one expects the nonlinear problem to get close to a linear problem, leading to the spectral study of the graph. How to make this heuristic precise?

We will discuss those issues by presenting both general results and example studies.
In particular, we will provide a framework to rigorously study the behavior of the problem in the regime "p close to 2", which will be a powerful tool to understand qualitative properties of solutions.

The talk will end by presenting several open questions and research perspectives.

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## Some aspects of the multiscale finite element method

**Auteur:** Alexei LOZINSKI[1]

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We shall talk about computational methods for multiscale partial differential equations, and in particular the multiscale finite element method (MsFEM). This is a finite element type method that performs a Galerkin approximation of the PDE on a problem-dependent basis. We shall discuss two aspects of these methods based on our recent work with Rutger Biezemans, Claude Le Bris, and Frédéric Legoll (ENPC & INRIA).

First, the intrusiveness of the MsFEM is considered. Since the MsFEM uses a problem-dependent basis, it cannot easily be implemented in generic industrial codes and this hinders its adoption beyond academic environments. A generic methodology is proposed to translate the MsFEM into an effective problem that can be solved by generic codes.

Second, a new convergence analysis for the MsFEM is presented that establishes convergence under minimal regularity hypotheses. This bridges an important gap between the theoretical understanding of the method and its field of application, where the usual regularity hypotheses are rarely satisfied.

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## GRAFIDI: une librairie Python pour faire des simulations sur les graphes quantiques

**Auteur:** Romain Duboscq[1]

**Co-auteurs:** Christophe Besse [2]; Stefan Le Coz

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Dans cet exposé, je présenterai mes travaux en collaboration avec Christophe BESSE et Stefan LE COZ autour de la simulation numérique pour les graphes quantiques (nonlinéaires). En particulier, je parlerai de la librairie Python GRAFIDI qui permet de résoudre numériquement des problèmes tels que le calcul d'états stationnaires ou bien encore l'évolution d'une solution de l'équation de Schrödinger sur un graphe à l'aide d'une discrétisation par différences finies. Certaines études numériques que nous avons menées seront abordées et motiveront des questions plus théoriques.

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## Blow-up on a star-graph

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We construct a finite time blow-up solution to the mass-critical focusing nonlinear Schrödinger equation on a metric star graph.