Cutoff for the Transience Time of the SSEP with Traps and the FEP

Based on j.w. with B. Massoulié (Ceremade)

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March 6, 2024

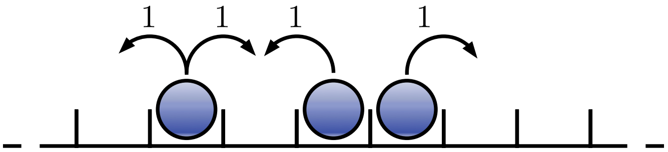
INTRO: SSEP ON THE RING

Consider the discrete ring $\mathbb{T}_K = \{1, \dots, K\}$

- Each of its sites $k \in \mathbb{T}_K$ is either

$$\begin{cases} \text{Occupied by a particle} & \Longrightarrow \quad \sigma_k = 1 \\ \text{Empty} & \Longrightarrow \quad \sigma_k = 0 \end{cases}$$

- Configurations $\sigma \in \{0, 1\}^K$.
- Particles jump at rate 1 to any *empty* neighboring sites (exclusion rule)



I - Transience cutoff for the SSEP with Traps

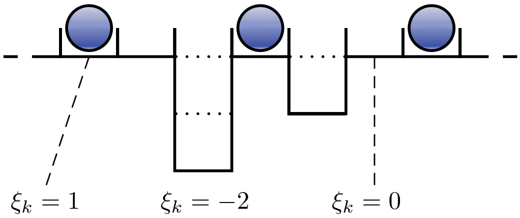
SSEP WITH TRAPS ON THE RING: CONFIGURATION

Consider the discrete ring $\mathbb{T}_K = \{1, \dots, K\}$

- ▷ Each of its sites $k \in \mathbb{T}_K$ is either

$$\begin{cases} \text{Occupied by a particle} & \Longrightarrow \quad \xi_k = 1 \\ \text{Empty} & \Longrightarrow \quad \xi_k = 0 \\ \text{A trap of depth } |a|, \text{ for } a < 0 & \Longrightarrow \quad \xi_x = a. \end{cases}$$

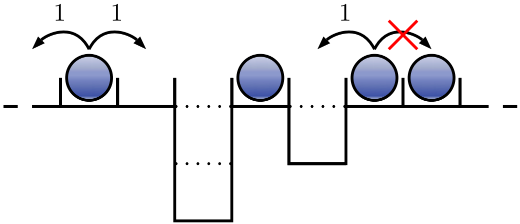
- ▷ Configurations $\xi = (\xi_k)_{k \in \mathbb{T}_K} \in \{1, 0, -1, -2, \dots\}^K$.



SSEP WITH TRAPS ON THE RING: DYNAMICS

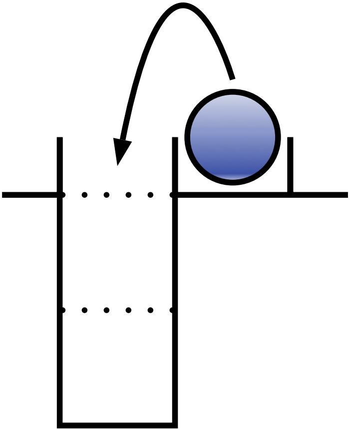
Particles and empty sites behave exactly as in the (nearest-neighbor) SSEP, meaning that particles

- ▷ jump at rate 1 to (nearest-neighbor) empty sites
- ▷ cannot jump to occupied sites (exclusion rule)
- ▷ jump at rate 1 to (nearest-neighbor) traps



SSEP WITH TRAPS ON THE RING: DYNAMICS

▷ When a particle jumps to a trap, it is destroyed, and the trap's depth is reduced by 1



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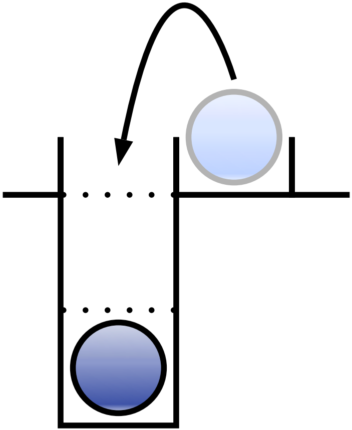

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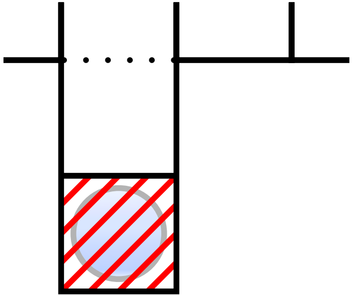

SSEP with traps on the ring: dynamics

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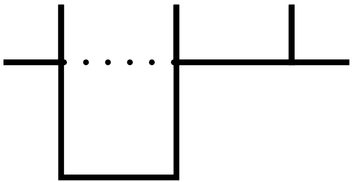

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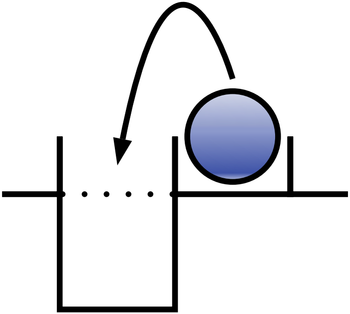
▷ If the trap was of depth one, it simply becomes an "normal" empty site, and the particle is destroyed.



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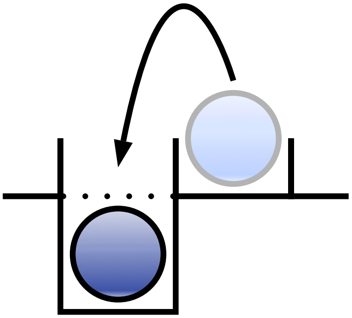

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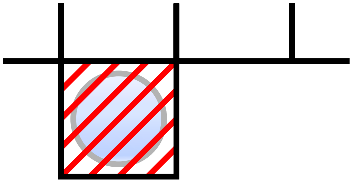

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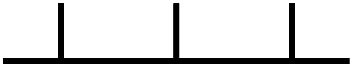

SSEP WITH TRAPS ON THE RING: DYNAMICS

The generator for the SWT is given by

$$\mathcal{L}_K f(\xi) = \sum_{k \in \mathbb{T}_K} \sum_{z=\pm 1} \mathbf{1}_{\{\xi_k = 1, \xi_{k+z} \leq 0\}} \{f(\xi^{k,k+z}) - f(\xi)\},$$

where

$$\begin{cases} \xi_k^{k,k+z} = \xi_k - 1 \\ \xi_{k+z}^{k,k+z} = \xi_{k+z} + 1 \end{cases}$$

defines the configuration where the particle at site k has jumped to $k+z$.

▷ Dynamics very close to a water+ice phase separation model by Funaki [AIHP 91], who studied its *hydrodynamic limit* (Stefan problem)

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Some basic properties of the SWT

- The depth of any trap can only decrease, as well as the total number of particles in the system.
- The SWT dynamics is *attractive* : one can couple the evolution of two SWT $\xi(\cdot)$, $\tilde{\xi}(\cdot)$ starting from $\xi \leq \tilde{\xi}$, in such a way that at any time $t > 0$, we still have
$$\xi(t) \leq \tilde{\xi}(t)$$
- When the *last particle gets trapped*, the SWT becomes frozen.
- When the *last trap gets filled*, the SWT becomes a standard SSEP.
- Until one of those two things occur, the system is in a transient state. We denote by
$$\mathcal{T}_K = \Big\{\xi,\ \exists k, k' \in \mathbb{T}_K \mid \xi_k = 1, \xi_{k'} < 0\Big\}$$
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Estimation of the transience time I

Question: How long does the SWT stay transient in the worst case scenario (worst initial configuration) ?

Given a time $t > 0$, $\varepsilon \in [0,1]$, define the *maximal transience probability* at time t, and the associated *ε-transience time*

$$p_K(t) = \sup_{\xi} \mathbb{P}_{\xi}(\xi(t) \in \mathcal{T}_K),$$

$$\theta_K(\varepsilon) = p_K^{-1}(\varepsilon) := \inf\left\{t \geq 0 : \ p_K(t) \leq \varepsilon\right\}$$

Theorem (E', Massoulié 24+)

The transience probability vanishes uniformly over large times of order $K^2 \log K$, meaning that

$$\lim_{t\to\infty} \sup_{K \geq 0} \ p_K(tK^2 \log K) = 0. \qquad (Easy)$$

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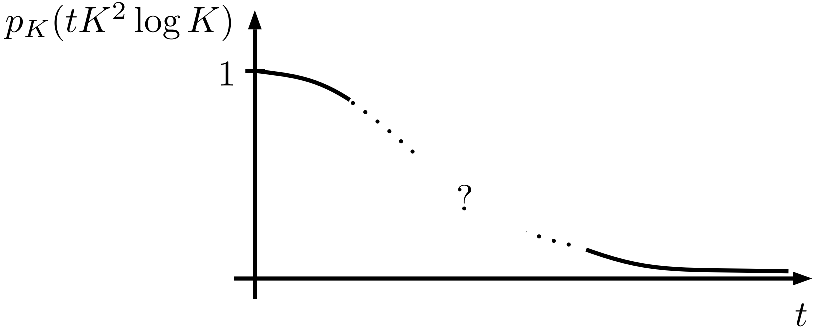
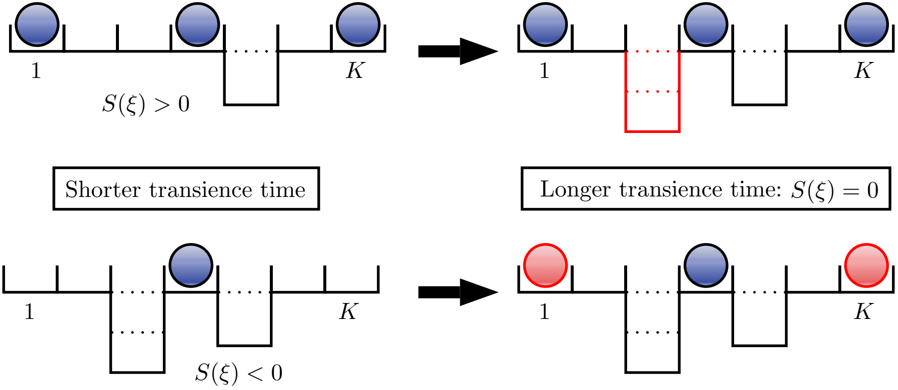
First Theorem



Attractiveness and critical transience

Define $S(\xi) = \sum_{k\in\mathbb{T}_K} \xi_k$ the number of excess particles, $S(\xi) = 0$ for critical conf., becomes identically 0 at the end of the transience time

▷ Attractiveness $\Longrightarrow$ worst transience probability for a critical configuration



HEURISTICS: FULL EXPLORATION TIME FOR K PARTLCLES

Assume for illustration that the particles move independantly. Each particle fully explores $\mathbb{T}_K$ before time t w.p.

$$1 - e^{-ct/K^2} \leq q_K(t) \leq 1 - e^{-Ct/K^2}.$$

▷ Still transient at t+indep particles $\Longrightarrow$ Upper bound

$$p_K(t) \leq 1 - (1 - e^{-ct/K^2})^K \simeq K e^{ct/K^2}.$$

▷ For the lower bound, it is enough to consider the configuration with K particles at the origin, and a trap of depth K on the other side:

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Theorem (E', Massoulié 24+)

The transience time exhibits cutoff, meaning that it goes sharply from 1 *to* 0 *at time*

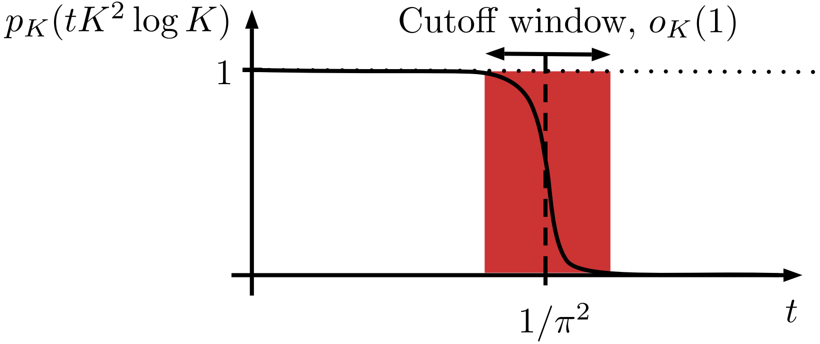
$$t_K^\star := \frac{K^2}{\pi^2} \log K,$$

i.e.

$$\theta_K(\varepsilon) = t_K^\star + \mathcal{O}_\varepsilon(K^2 \log \log K). \qquad (Hard)$$

Estimation of the transience time II

Second Theorem



Consequence on mixing time (supercritical case)

For $S(\xi) = s$, the stationary state for the SWT is the uniform state $\pi_{K,s}$ over

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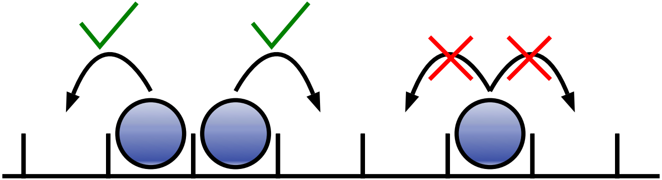
II - Transience cutoff for the Facilitated Exclusion Process

The facilitated exclusion process (FEP)

Exclusion process on $\mathbb{T}_N$, site x is either empty ($\eta_x = 0$) or occupied ($\eta_x = 1$)

▷ jump at rate 1 to (nearest-neighbor) empty sites *IF* the other neighbor is occupied (the particle is *active*) $\longmapsto$ isolated particles cannot jump



▷ If all particles are isolated: absorbing –frozen– state ($\in \mathcal{F}_N$)

▷ Empty sites isolated: ergodic state ($\in \mathcal{E}_N$), –two empty sites cannot *become* neighbors–

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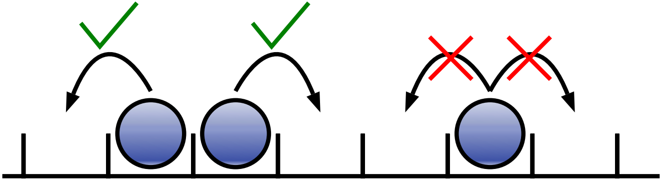

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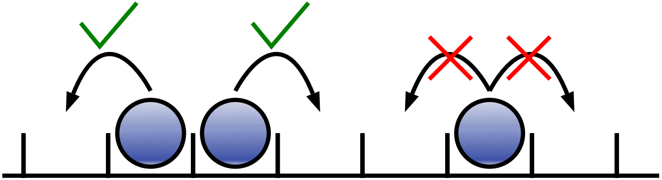

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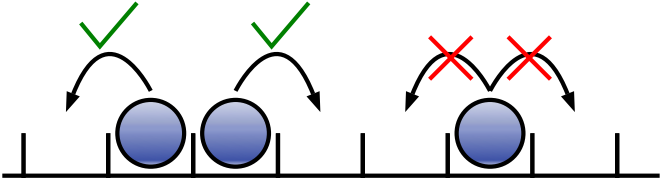

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- ▷ Until frozen or ergodic: transient state ($\in \mathcal{T}_N$), with both *active particles* and pairs of *neighboring empty sites*.

Transience cutoff for the FEP

Question: How long does the SWT stay transient in the worst case scenario (worst initial configuration) ?

$$p_N(t) = \sup_\eta \mathbb{P}_\eta(\eta(t) \in \mathcal{T}_N),$$

$$\theta_N(\varepsilon) = p_N^{-1}(\varepsilon) := \inf\left\{t \geq 0, \mathbb{P}_\eta(\eta(t) \in \mathcal{T}_N) \leq \varepsilon\right\}$$

Theorem (E', Massoulié 24+)

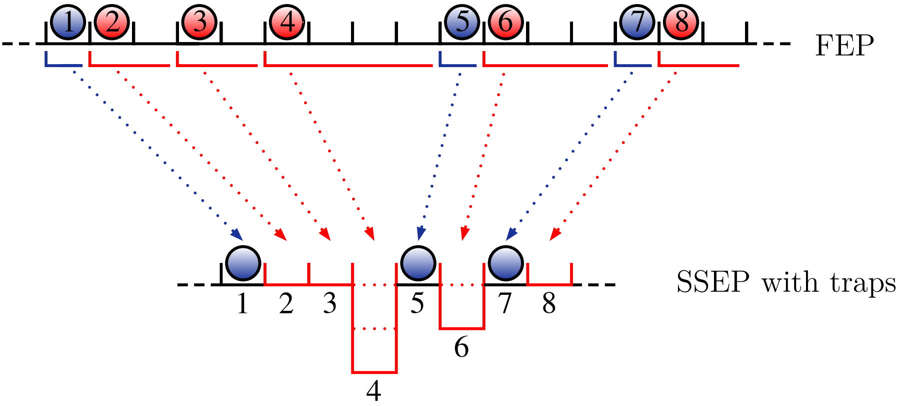
The FEP's transience time exhibits cutoff, meaning that it goes sharply from 1 *to* 0 *at time*

$$t^\star_{N/2} := \frac{N^2}{4\pi^2}\log N,$$

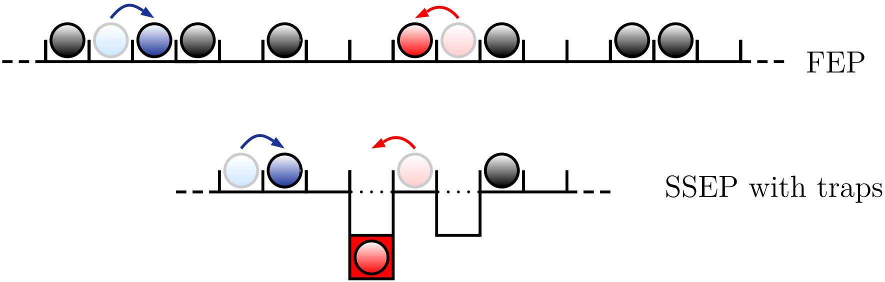
i.e.

$$\theta_N(\varepsilon) = t^\star_{N/2} + \mathcal{O}_\varepsilon(N^2 \log\log N).$$

Configuration mapping SWT ↔ FEP



Dynamical mapping SWT $\leftrightarrow$ FEP



$\mapsto$ For a critical ($K = N/2$) FEP configuration $\eta^\star$, the transience time $\theta_{\eta^\star}$ is that of the mapped critical SWT $\xi^\star$:

$$\sup_{\eta^\star \text{critical}} \theta_{\eta^\star} = \sup_{\xi^\star \text{critical}} \theta_{\xi^\star} \simeq t^\star_{N/2}$$

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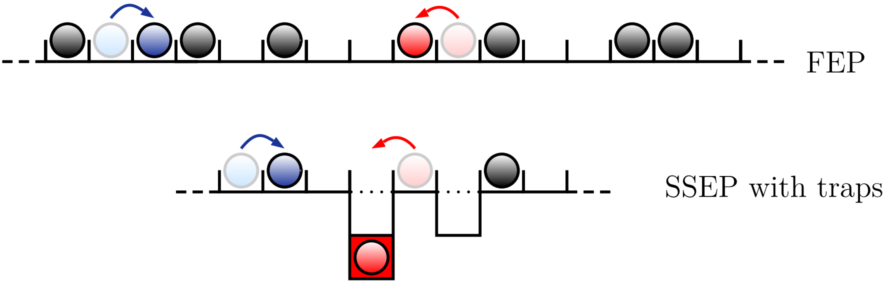

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Criticality bounds: freezing time

▷ For η with $K \leq N/2$ particles, $\xi := \xi_\eta$ the mapped SWT

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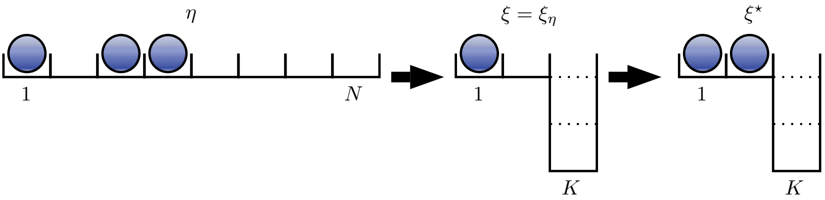
for $\xi^\star \geq \xi$ a critical SWT.



$\mapsto$ Subcritical configurations freeze “faster” than critical ones.

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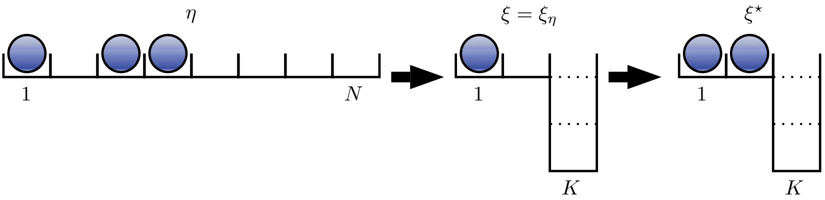

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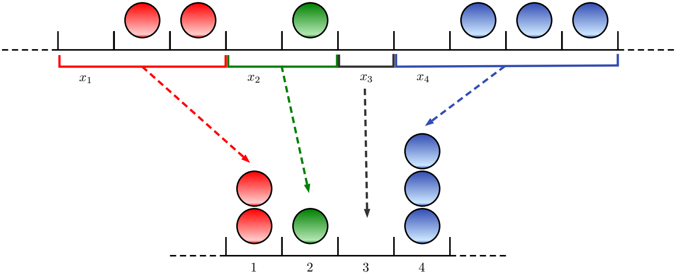
Criticality bounds: "ergodic" time

- For $K \geq N/2$, last inequality no longer true. We use another mapping, with the zero-range process ω on $\mathbb{T}_{N-K}$.



This rate 1 zero-range process is also constrained and attractive. A particle can jump away from a site *if it is not alone* on the site.

$$\theta_\eta = \theta_\omega \leq \theta_{\omega^\star} = \theta_{\xi^\star} \simeq t^\star_{N-K} \leq t^\star_{N/2}$$

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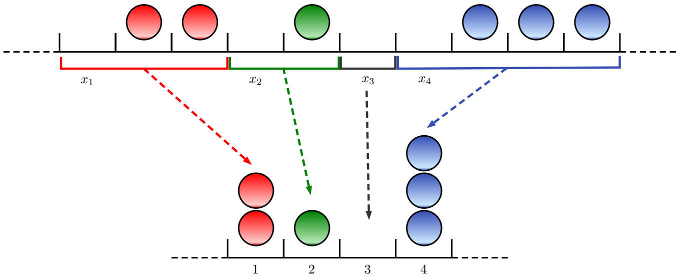

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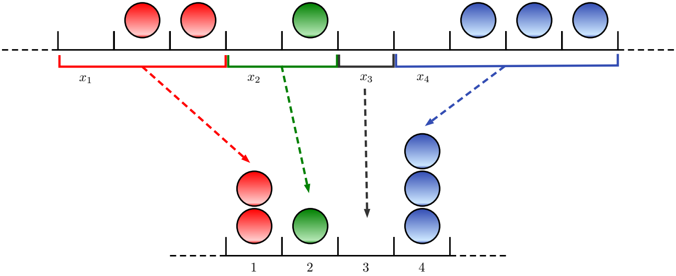

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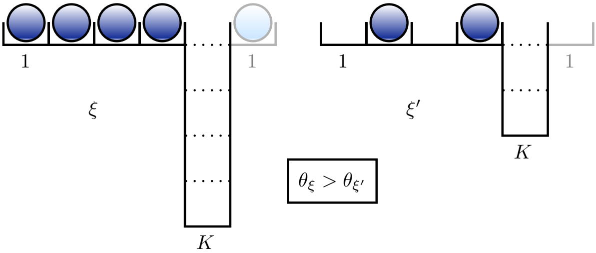
FEP mixing time cutoff (Ongoing work)

- Transience time is identical (trajectory per trajectory) between SWT and FEP
- Not the case a priori for mixing time
- The SWT configuration can be in its stationary state (SSEP), but the mapped FEP is not
- Need to understand the joint distribution of a tagged particle/current at the origin AND the configuration (tricky)

III -Main ideas of the proof

Transience cutoff for the SWT: unique trap

We take a critical configuration ξ with a single trap, we can assume all other sites are occupied



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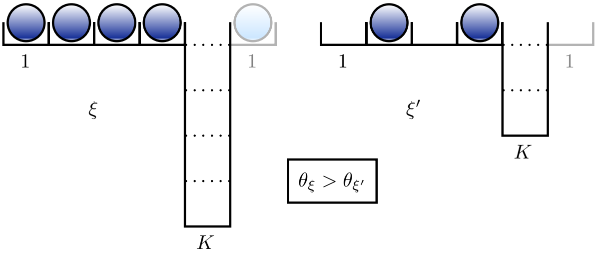

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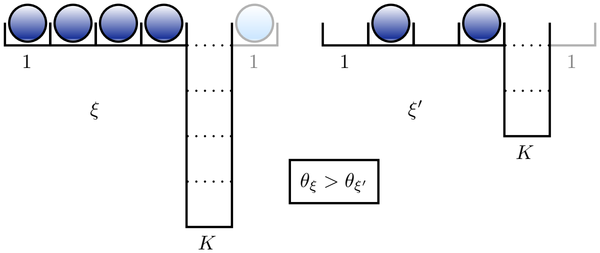

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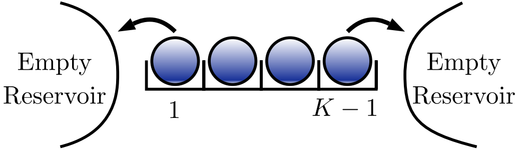

Transience cutoff for the SWT: general case

General case : all about understanding where a remaining trap is to break periodicity.

1) Break $\mathbb{T}_K$ into $Q := \log K$ pieces $A_1, \dots A_Q$
2) Then,

$$\xi(t) \text{ is transient } \Rightarrow \exists i,\ A_i \text{ still contains a trap at time } t.$$

We are left estimating

$$p_K(t) \leq Q \sup_{\xi} \mathbb{P}_\xi\Big(\xi(t) \text{ has a trap in } A_1\Big).$$

3) If there is still a trap in A in $\xi(t)$, no particle can have fully crossed it in either direction. We couple the SWT with SSEP with empty reservoirs on a larger bulk

Take-home message and ongoing projects

- SWT interesting on its own, easily generalized
- Huge improvement (sharp estimate) on the previous transience bounds (product distribution, based on zero-range mapping)
- Cutoff in a new setting than mixing time
- Mixing time for the FEP ? tricky
- Worst critical SWT configuration ? Conjecture : single trap
- Transience and mixing time for boundary-driven FEP ?

Thanks for your attention !

(And check out Brune's poster ;-))