

Refined Cauchy/Littlewood Identities and Their Applications to KPZ Models

Wednesday, March 6, 2024 10:20 AM (50 minutes)

The Cauchy identity is a formula about a sum of a product of two Schur functions over partitions and plays an important role in combinatorics, representation theory, and integrable probability. Some generalizations about such as sums of Macdonald polynomials and skew Schur functions are also known.

In this talk, I will report our recent works[1,2] with Matteo Mucciconi (Warwick University) and Tomohiro Sasamoto (Tokyo Institute of Technology) on the identities connecting the sums about the q -Whittaker functions (the case $t = 0$ of the Macdonald polynomial) and the skew Schur functions. They can be considered as refinements of the Cauchy/Littlewood identities. We give a proof of them based on algebraic combinatorics: We introduce a deterministic time evolutions called the skew RSK dynamics and show that one can linearize the dynamics by using some techniques of the affine crystal. The combinatorial objects obtained from the linearized one can be seen as building blocks of sum about the q -Whittaker functions while those from the skew RSK dynamics itself are associated to the sum about the skew Schur functions.

In the language of the integrable probability, the identities can be regarded as relations between two probability measures, the full space/half space q -Whittaker measures and the periodic/free boundary Schur measures. The former measures are related to various KPZ models while the latter ones are typical models of determinantal/Pfaffian and point processes. From these relations we can immediately get the Fredholm determinant/Pfaffian formulas for distribution functions of certain random variables for KPZ models.

[1] T. Imamura, M. Mucciconi, and T. Sasamoto, Forum of Mathematics, Pi 11(e27) 1-101

[2] T. Imamura, M. Mucciconi, and T. Sasamoto, arXiv:2204.08420

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