From Signaling to Interviews in Random Matching Market

Sophie Yu

The Wharton school of the University of Pennsylvania

Joint work: Maxwell Allman, Itai Ashlagi (Stanford), Amin Saberi (Stanford)

Motivations on interviews and signals

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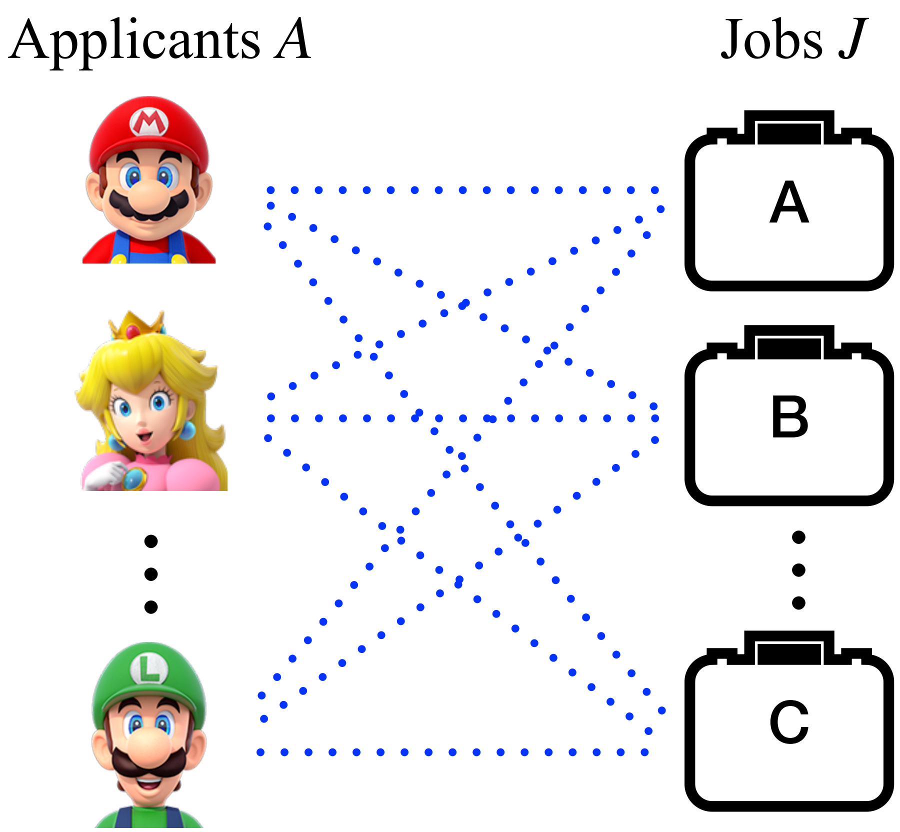
- **Key Insight**:
  - Interviews and signals help participants **narrow down their choices** in complex matching markets, improving the market efficiency.

Model setup



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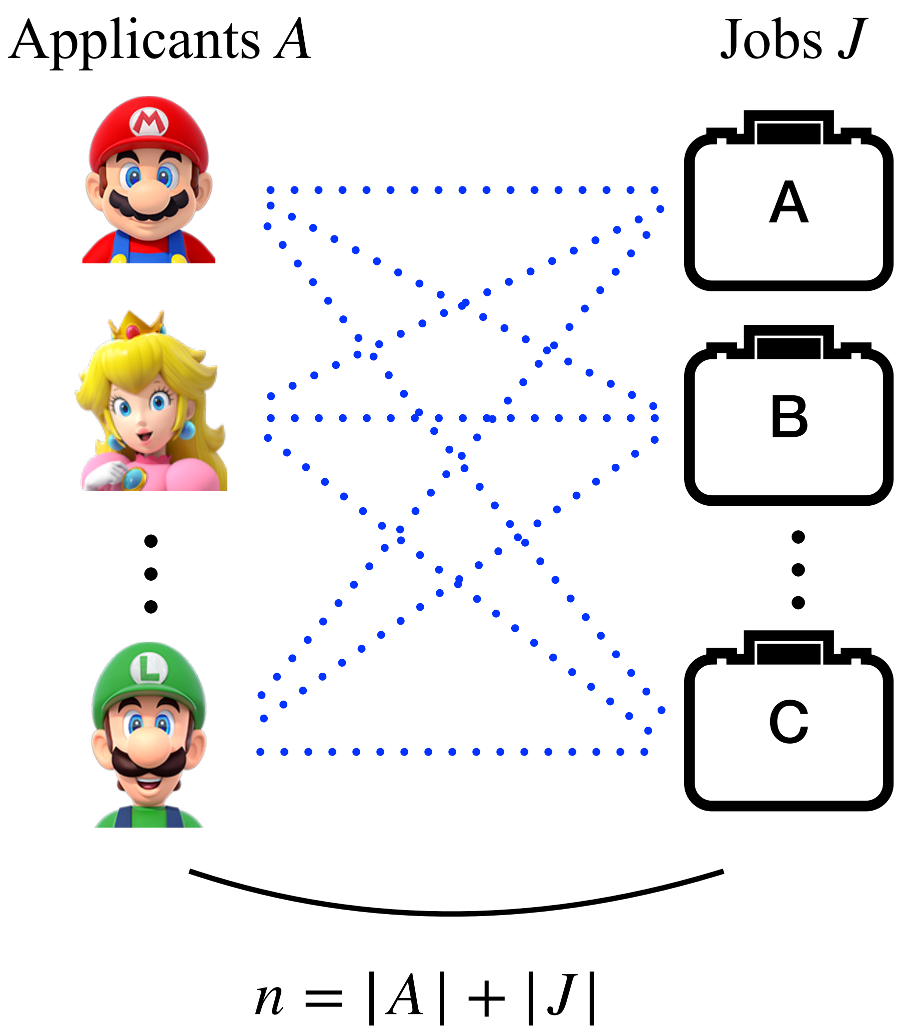

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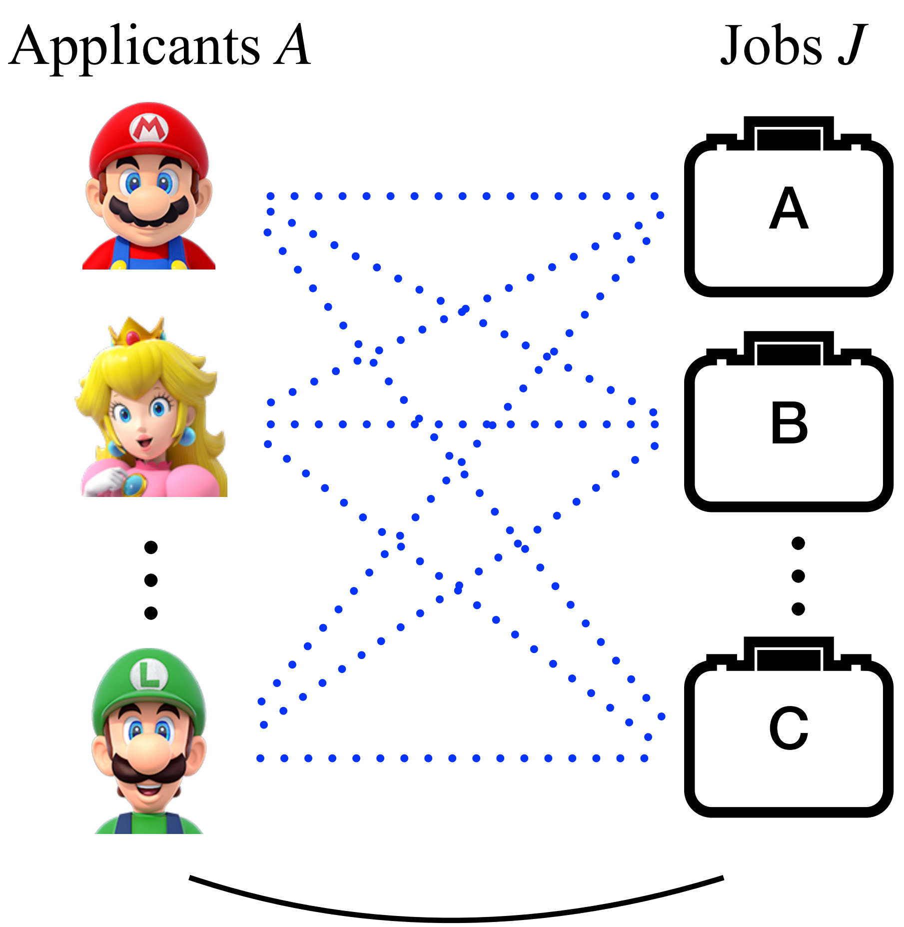

$n = |A| + |J|$

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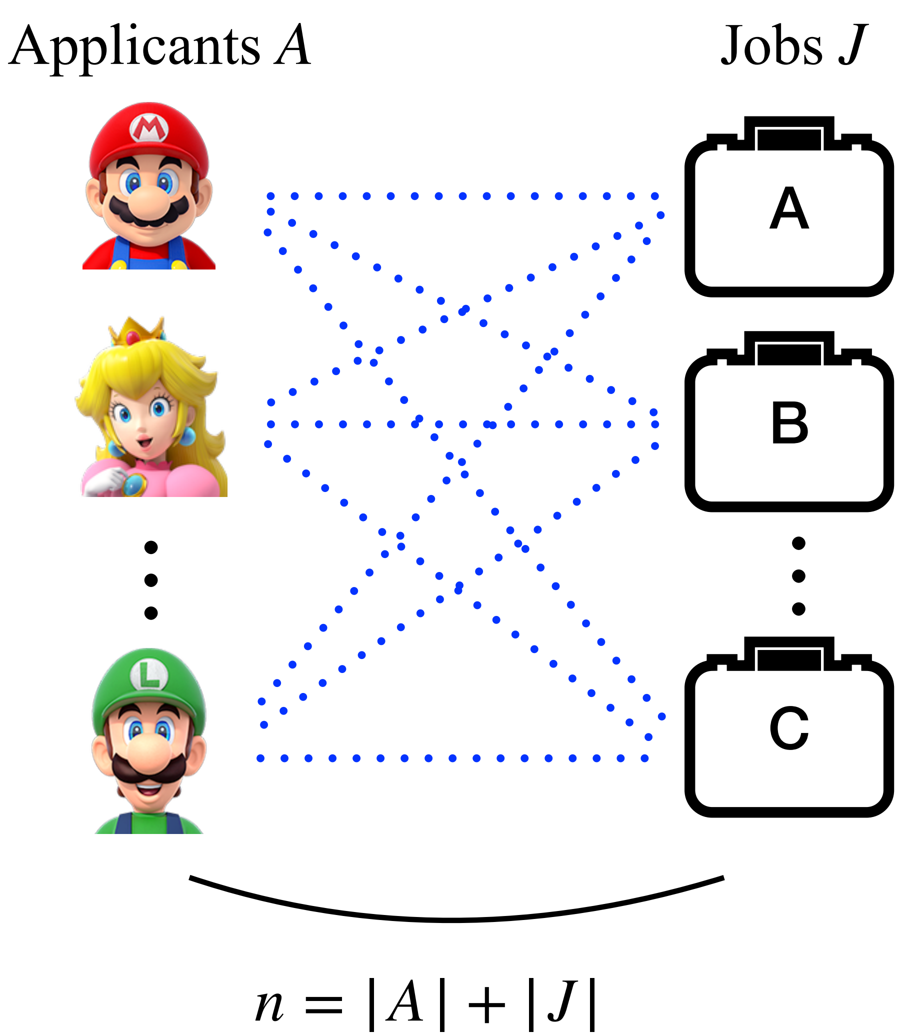

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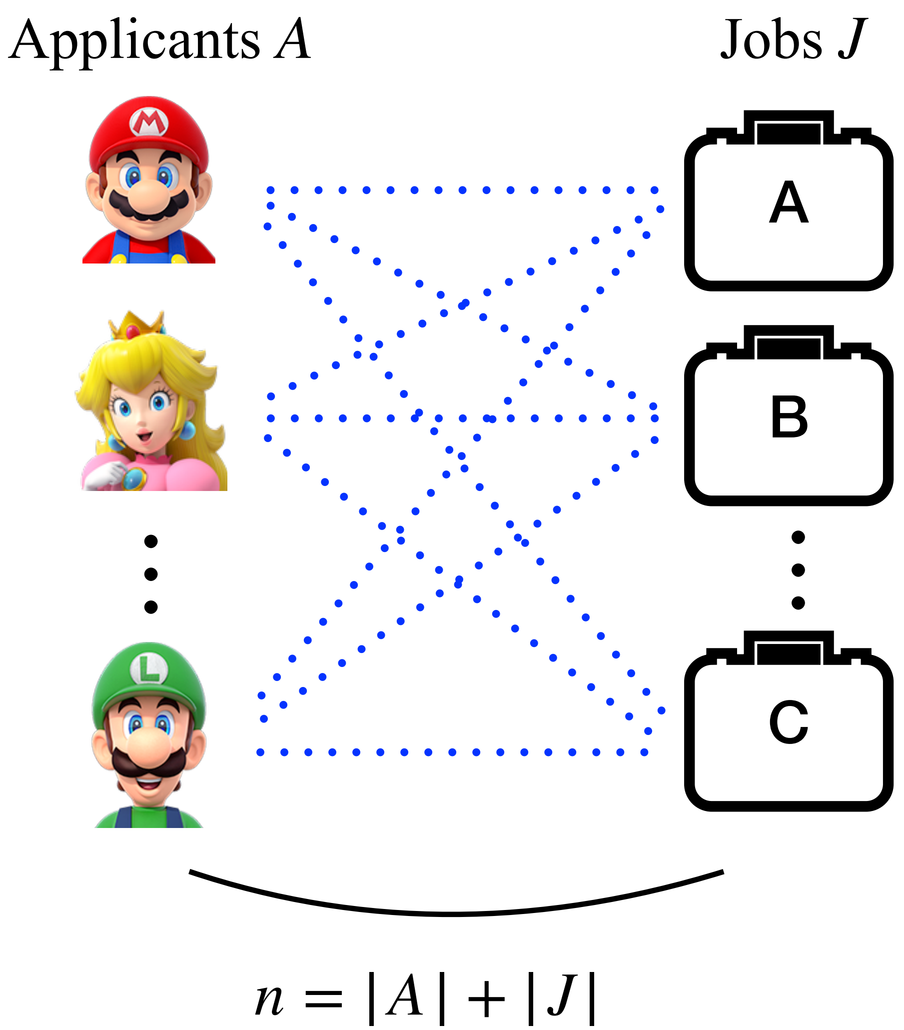

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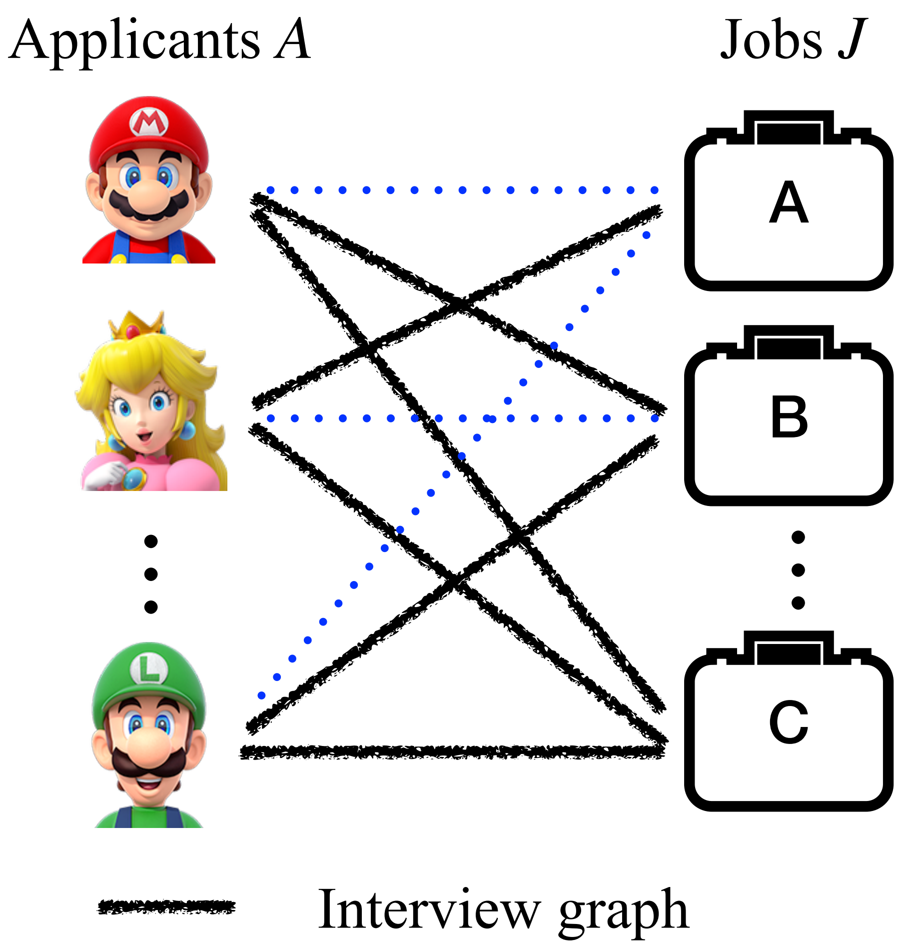

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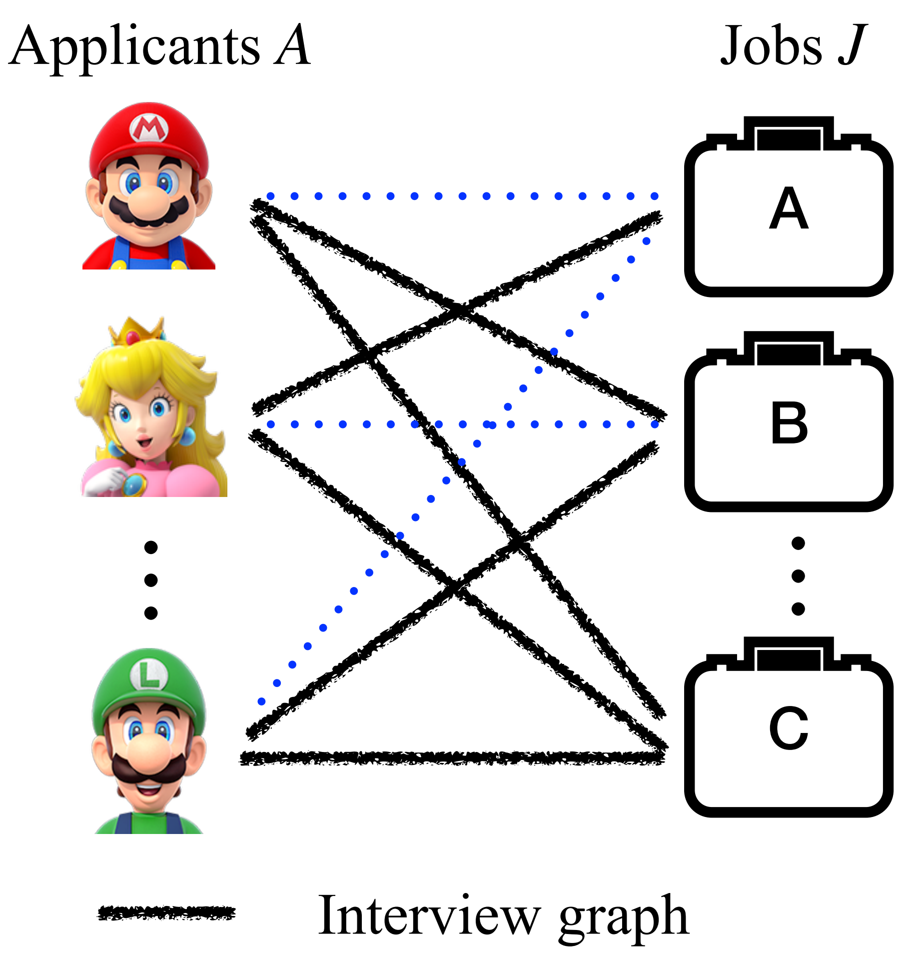

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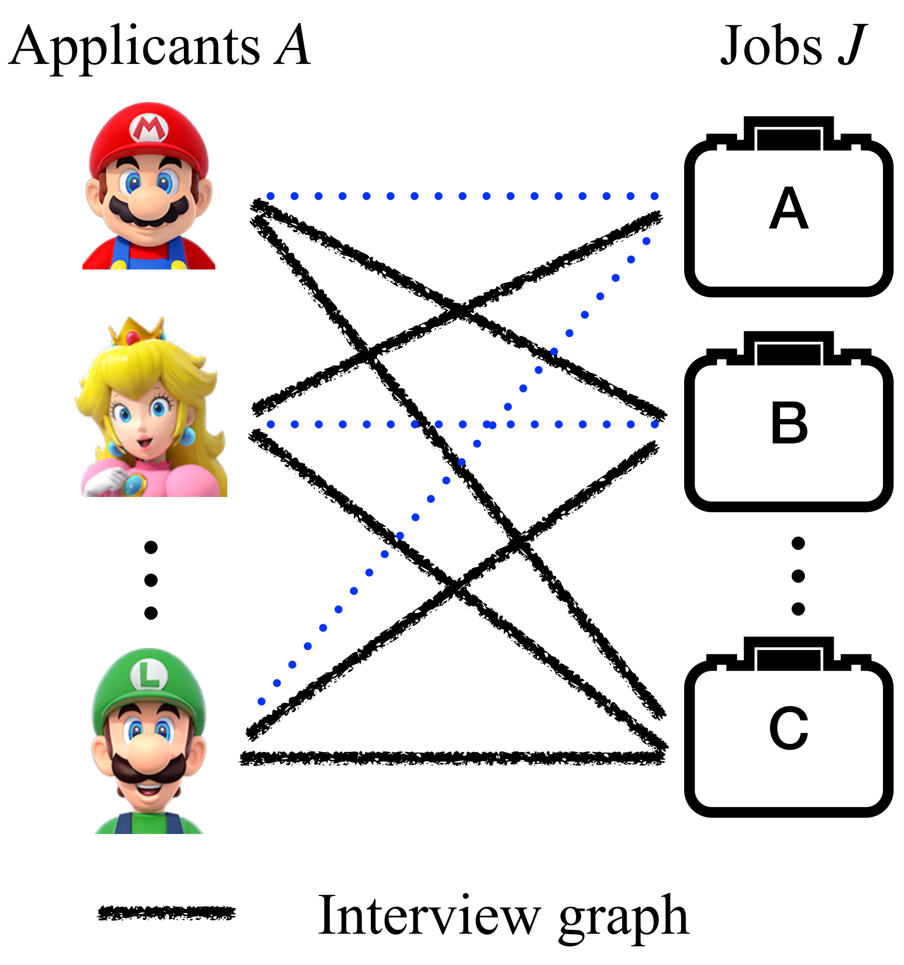

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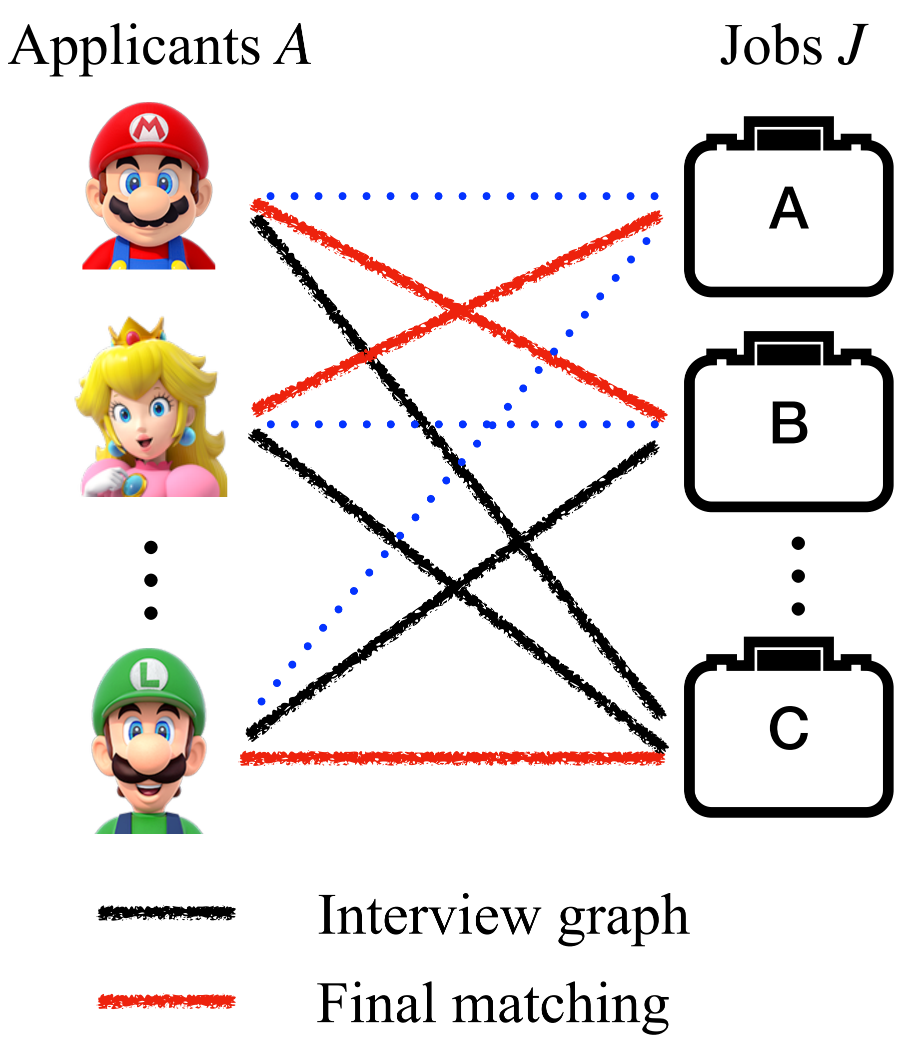

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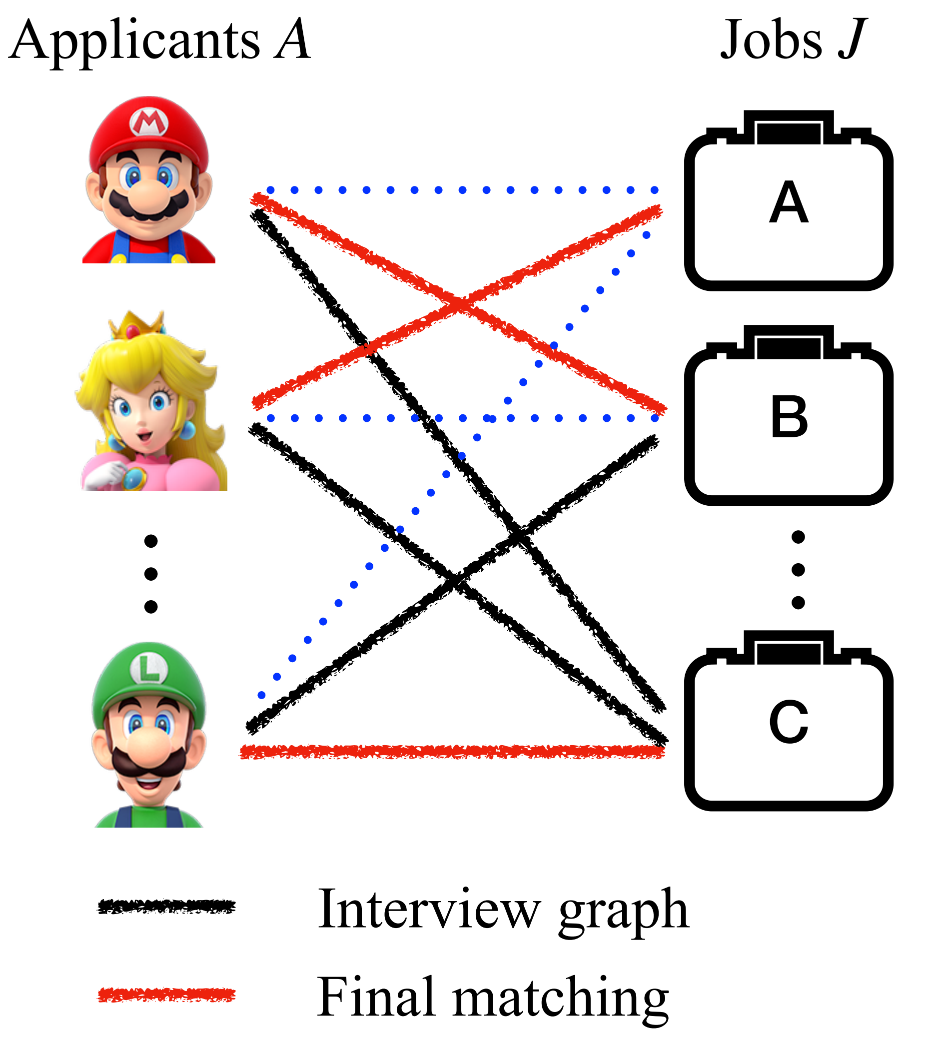

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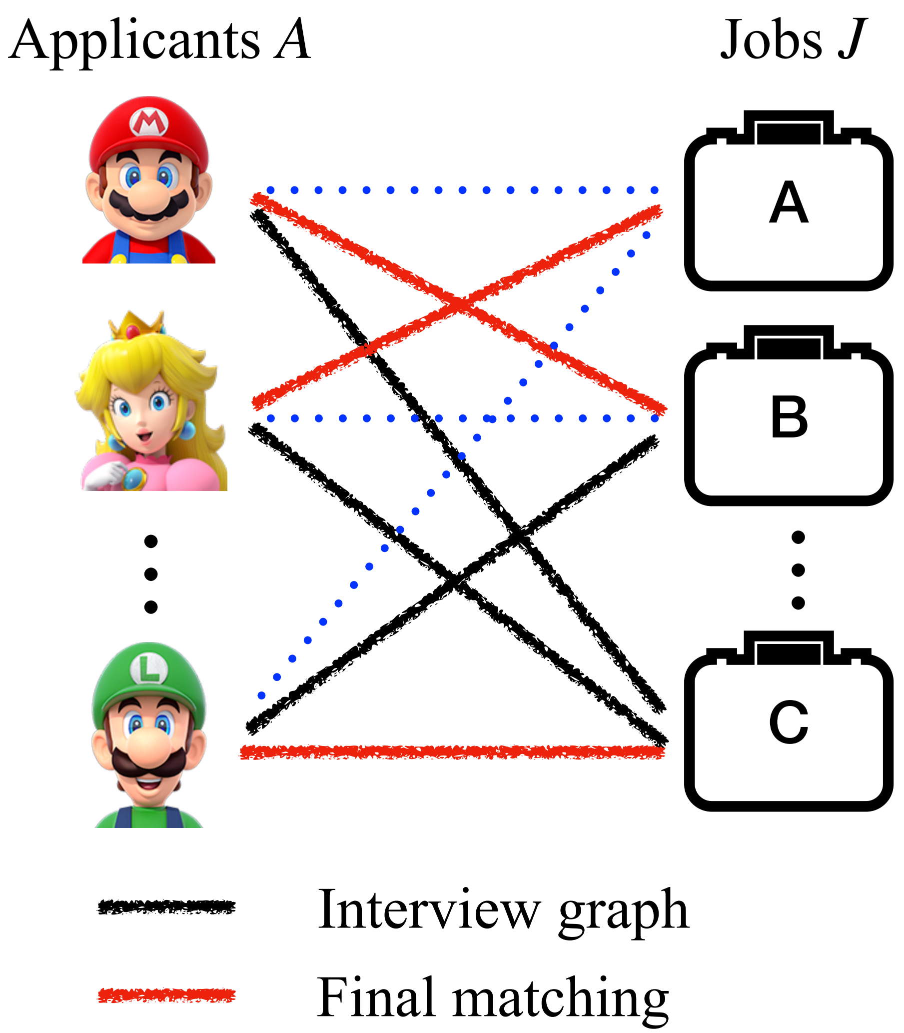

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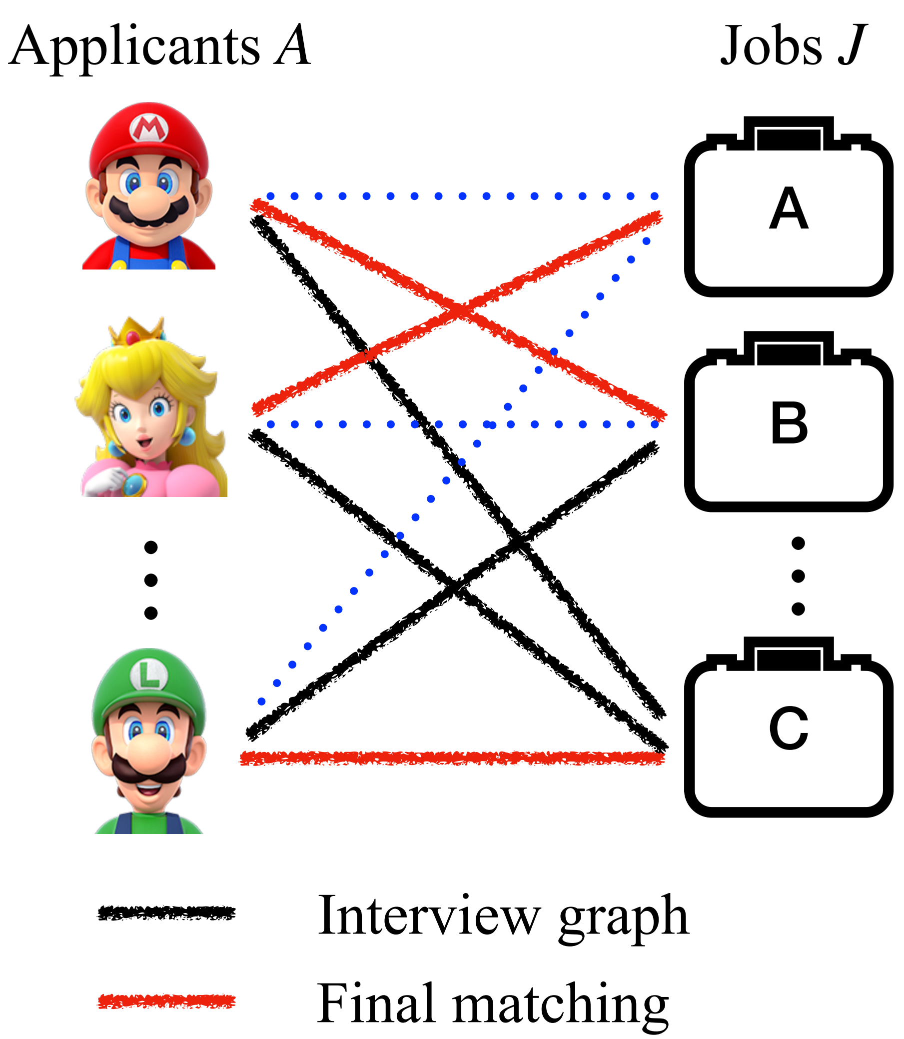

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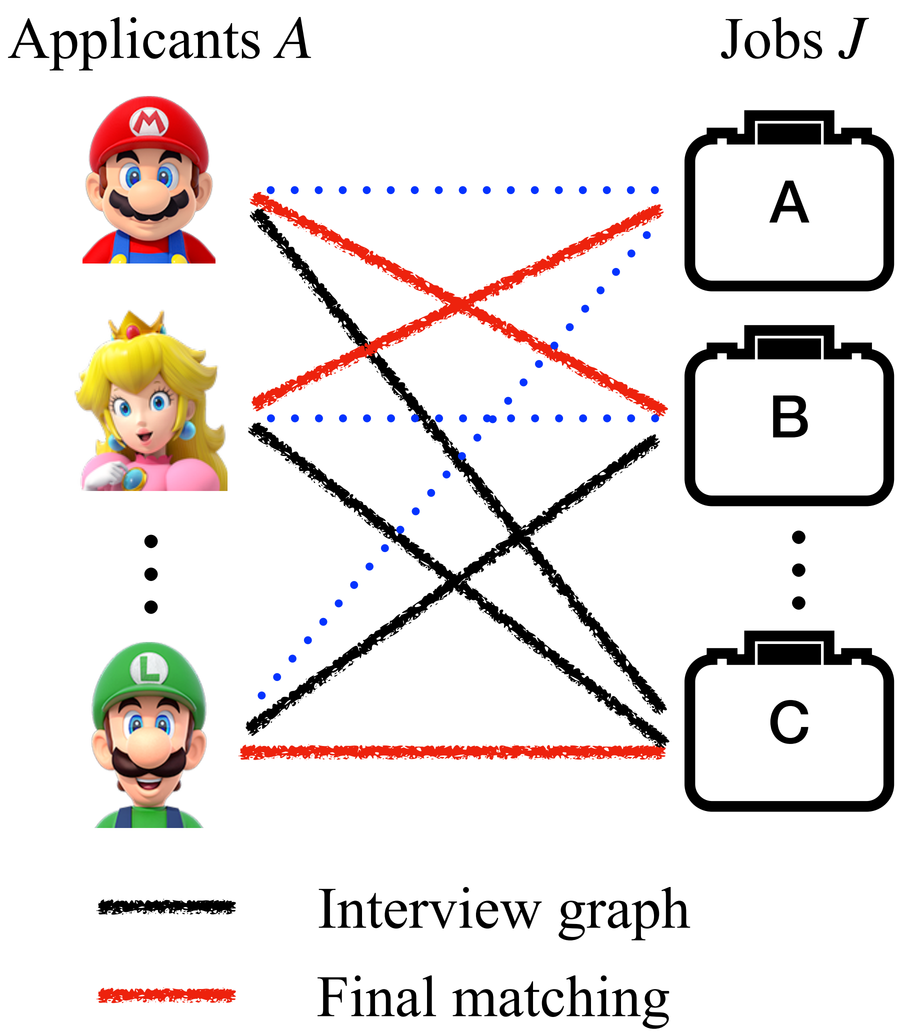

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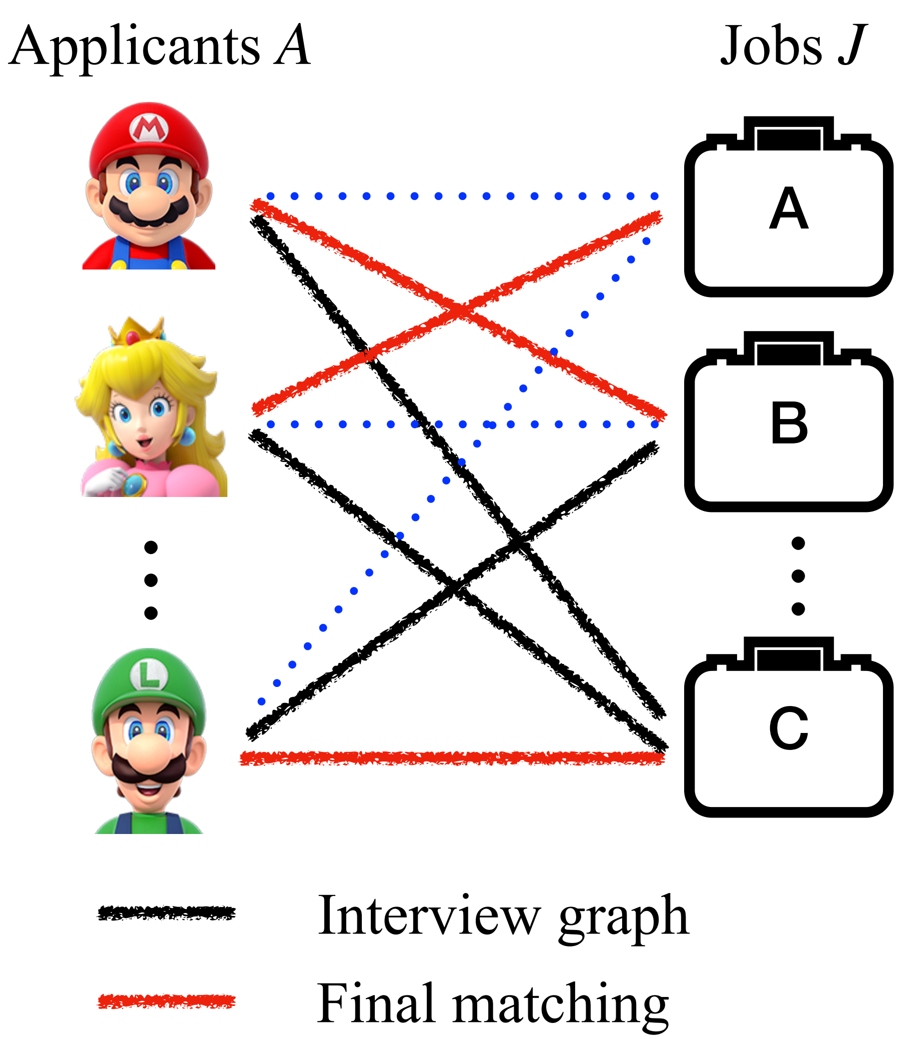

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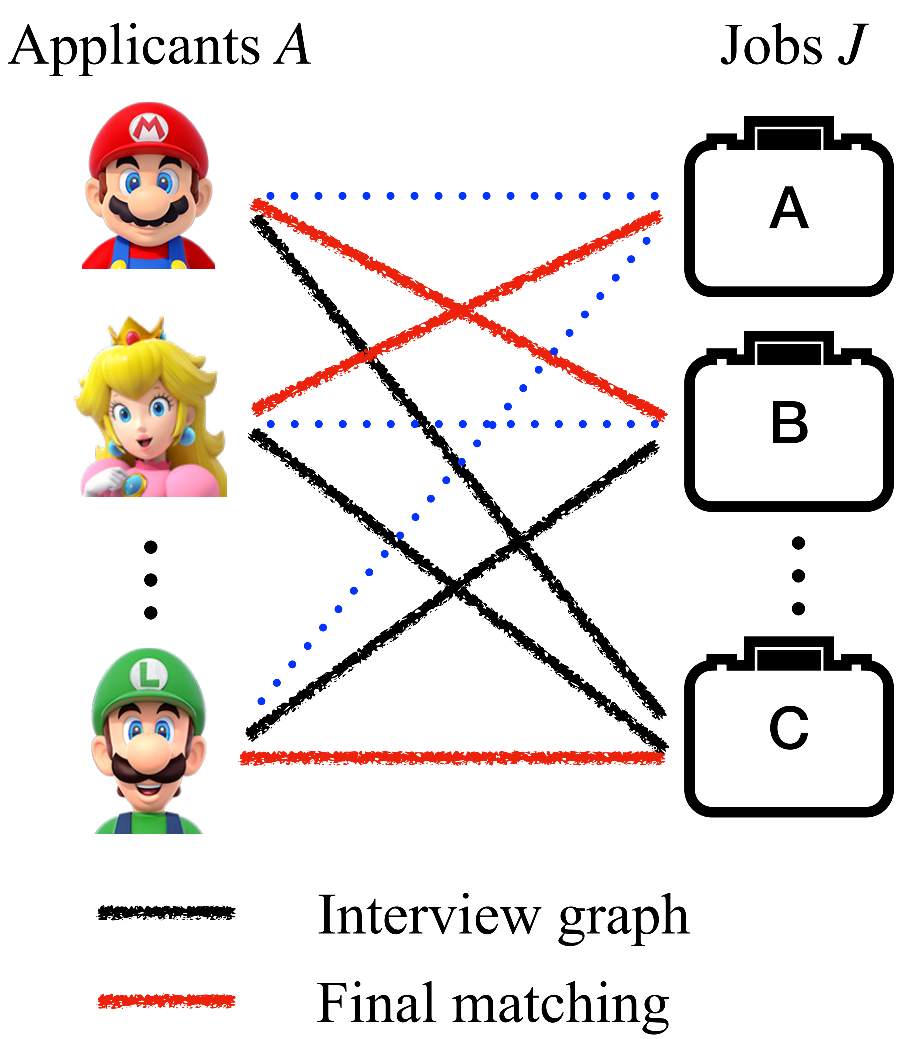

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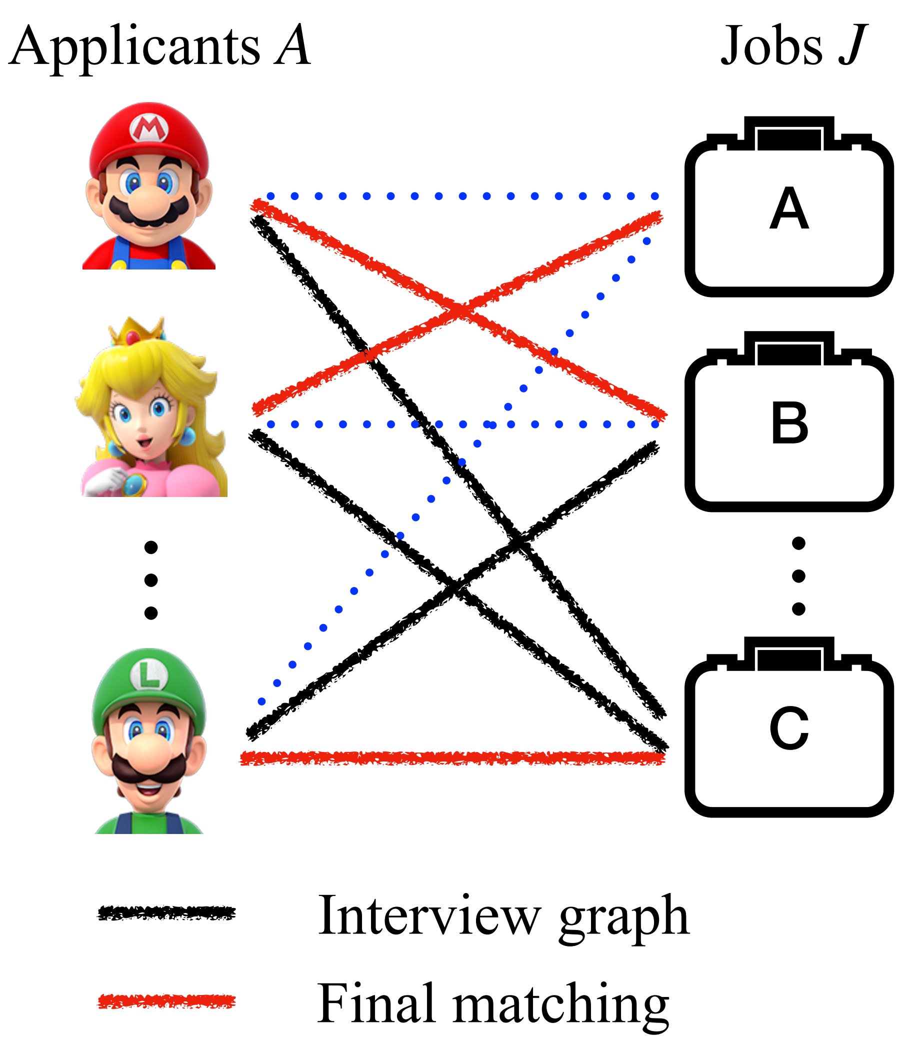

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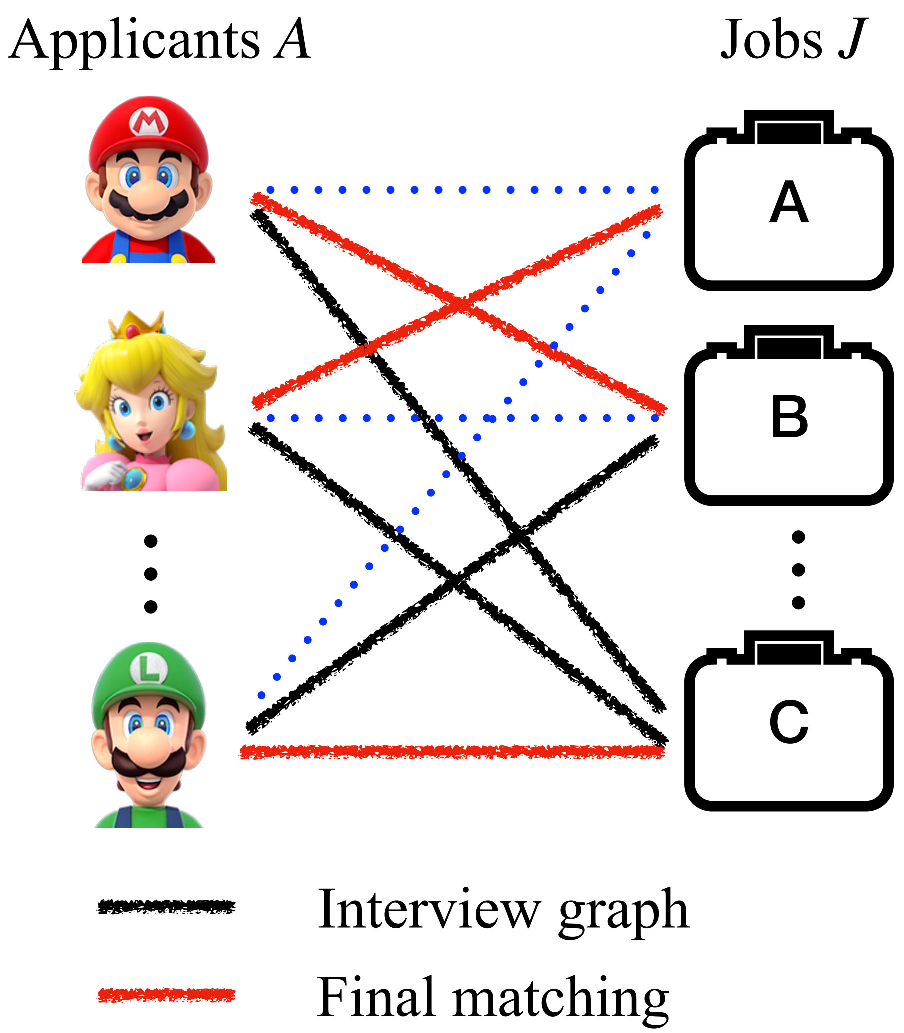

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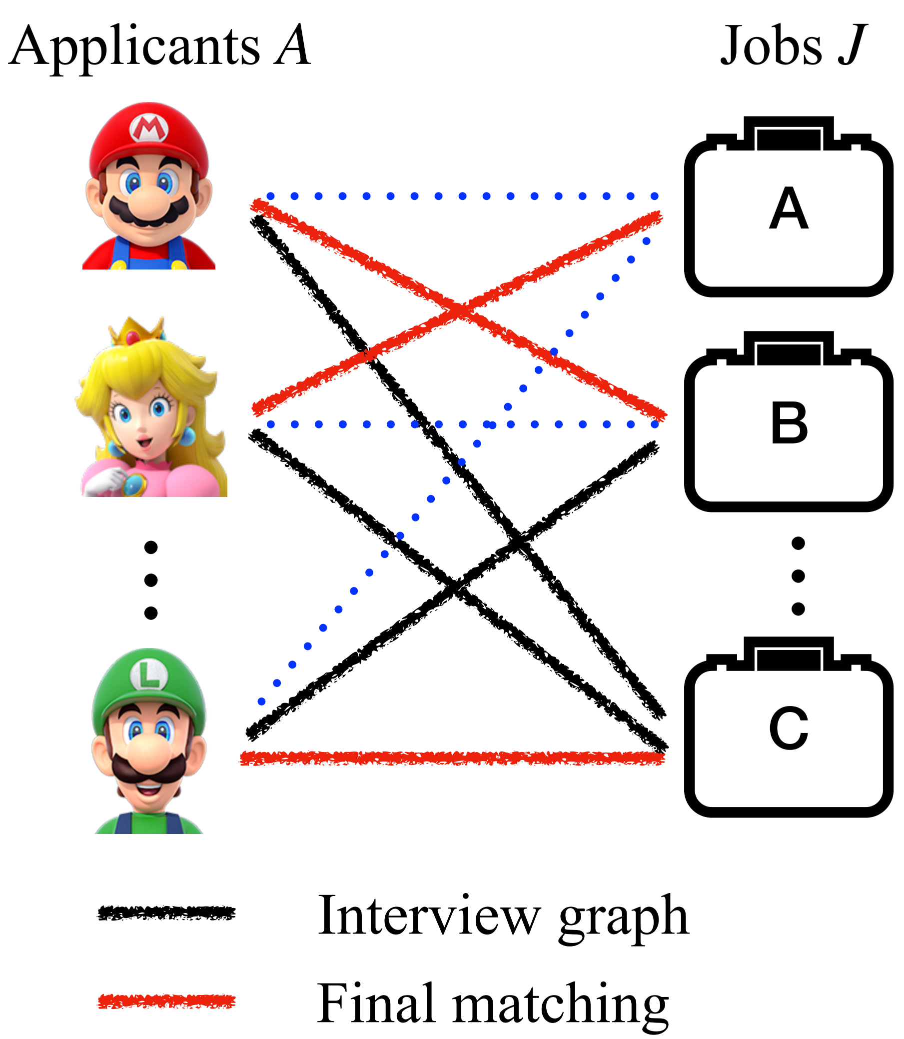

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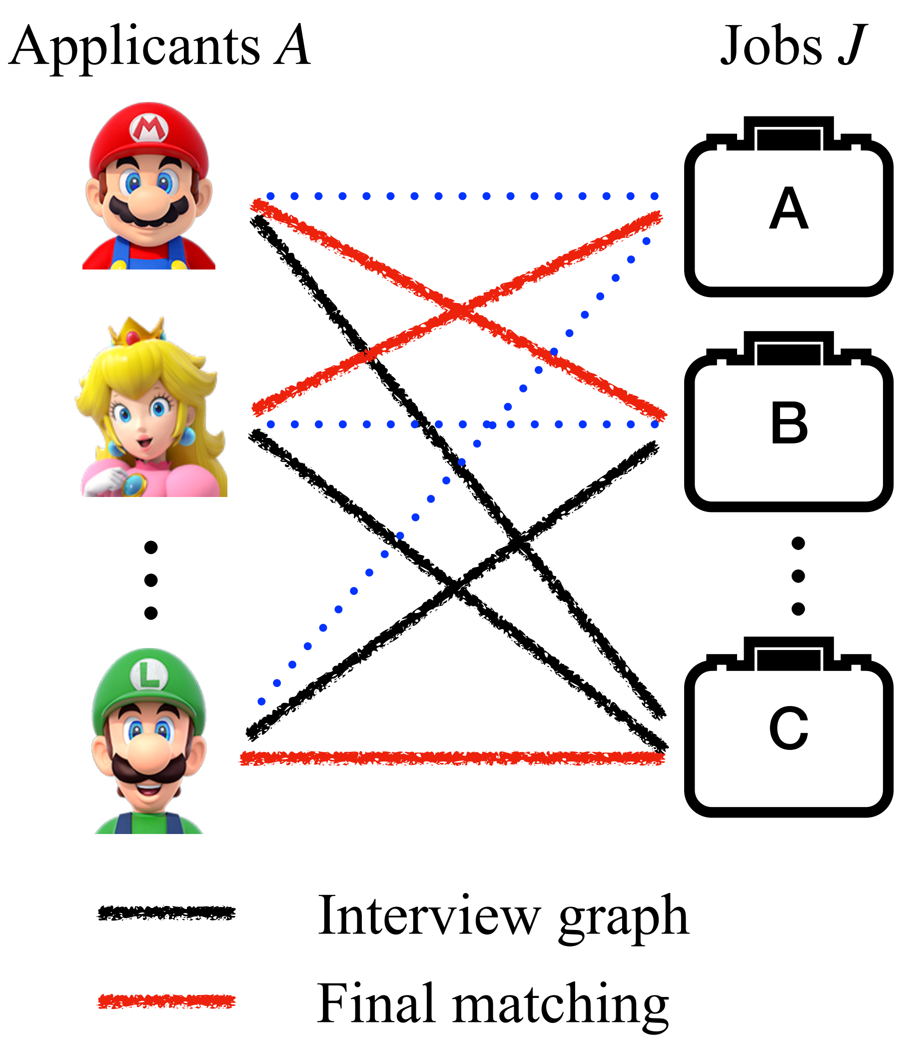

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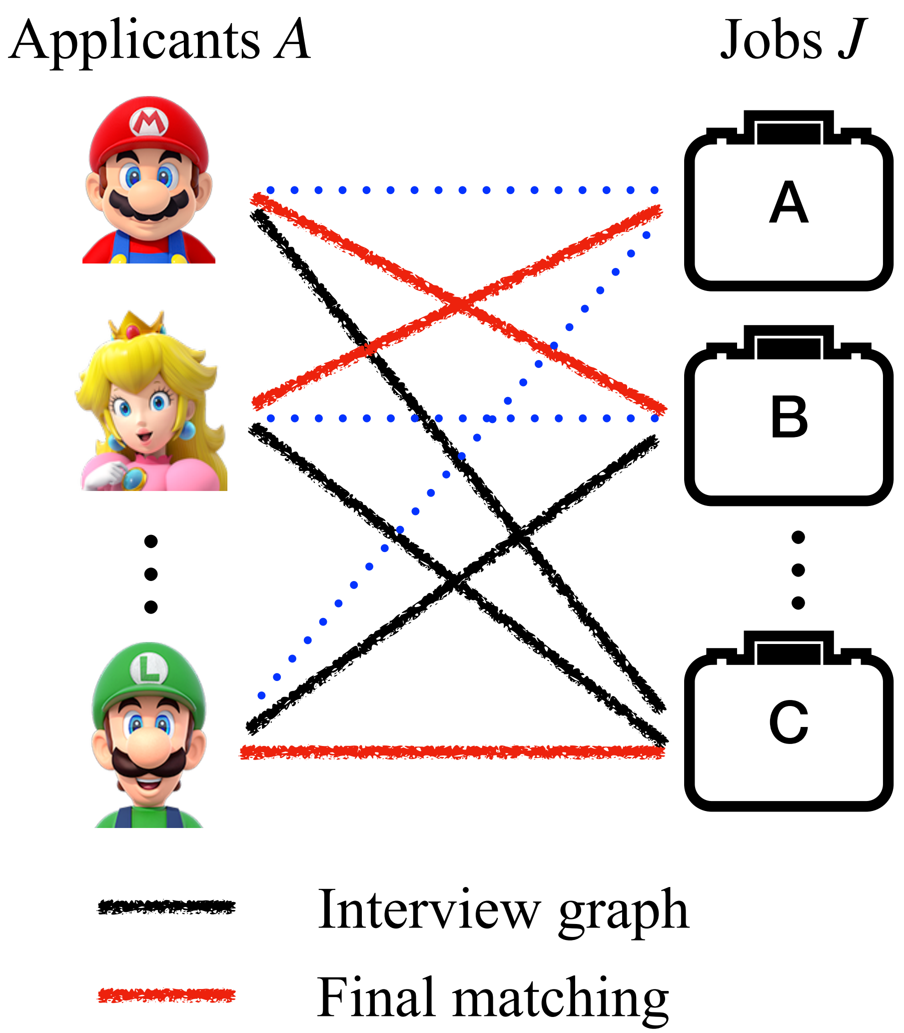

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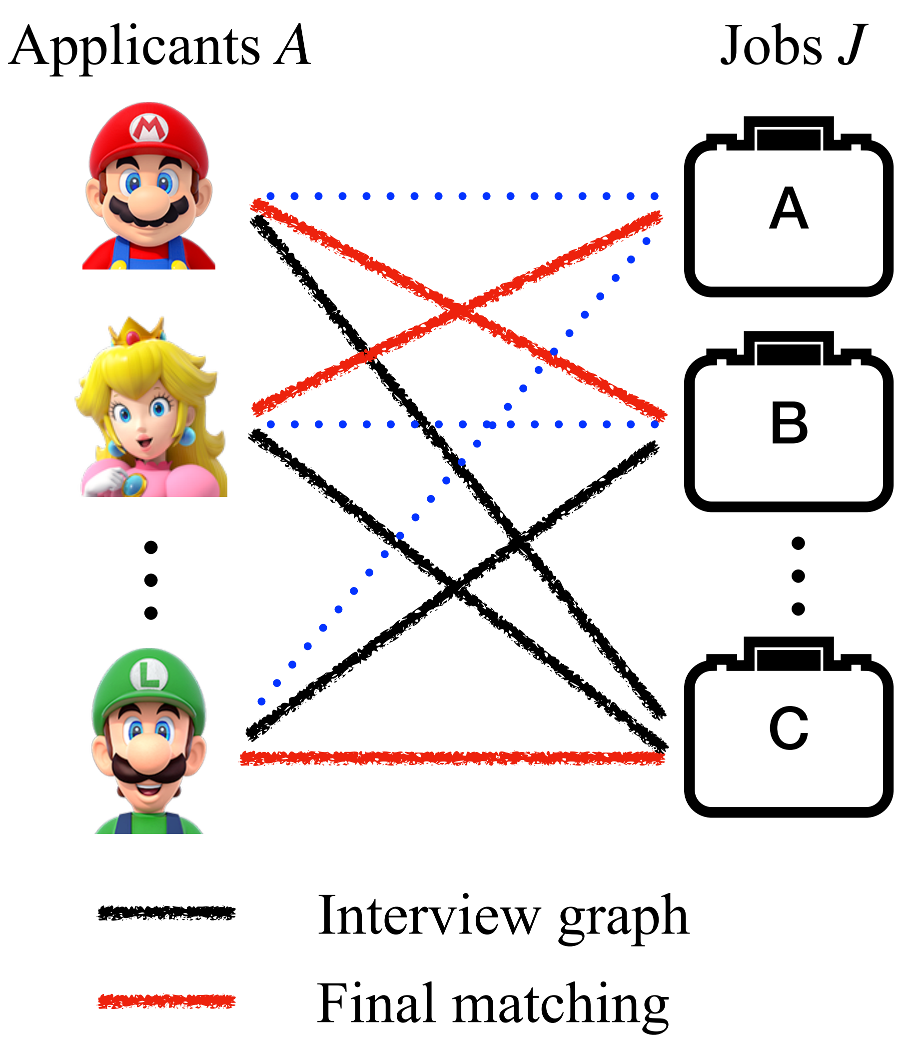

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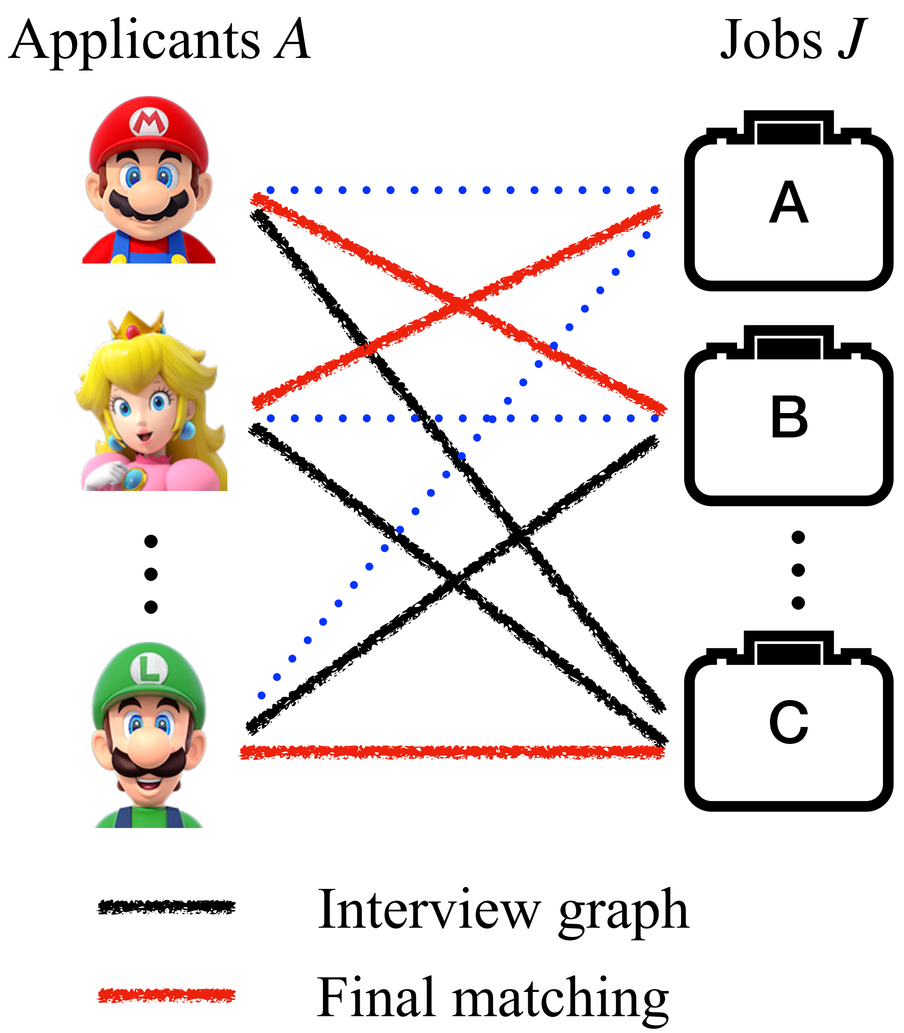

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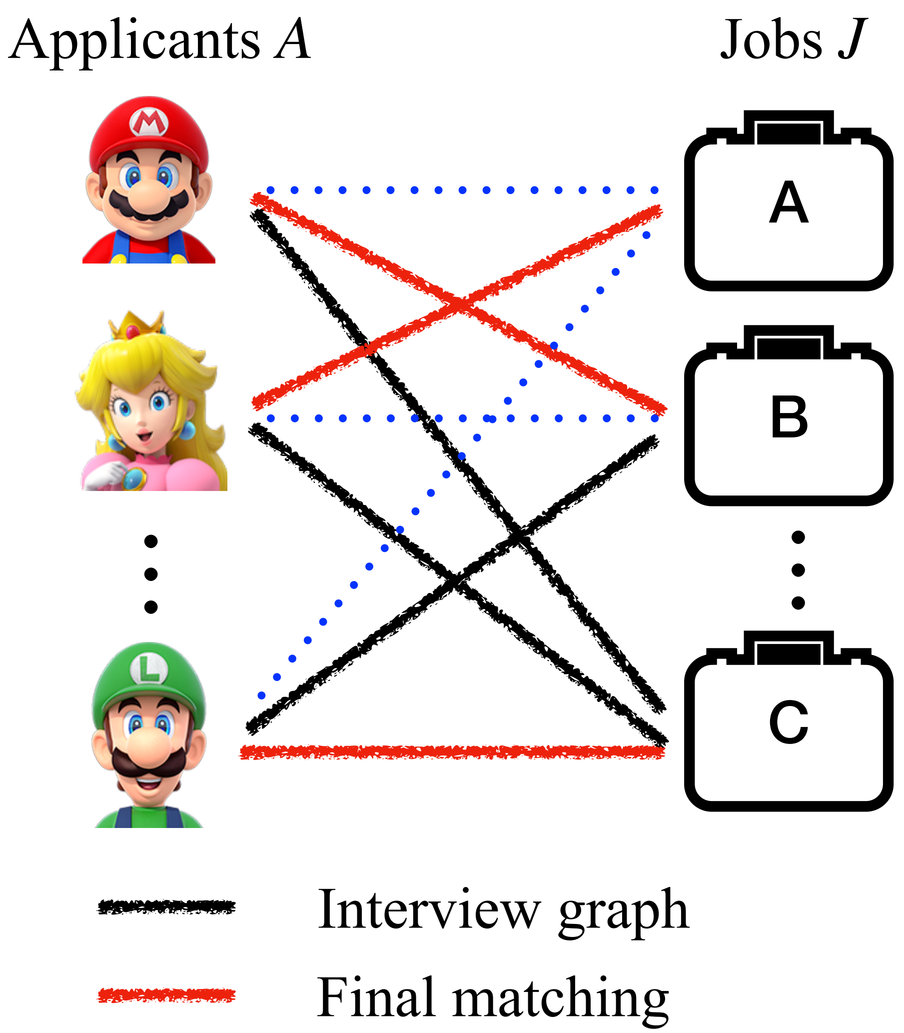

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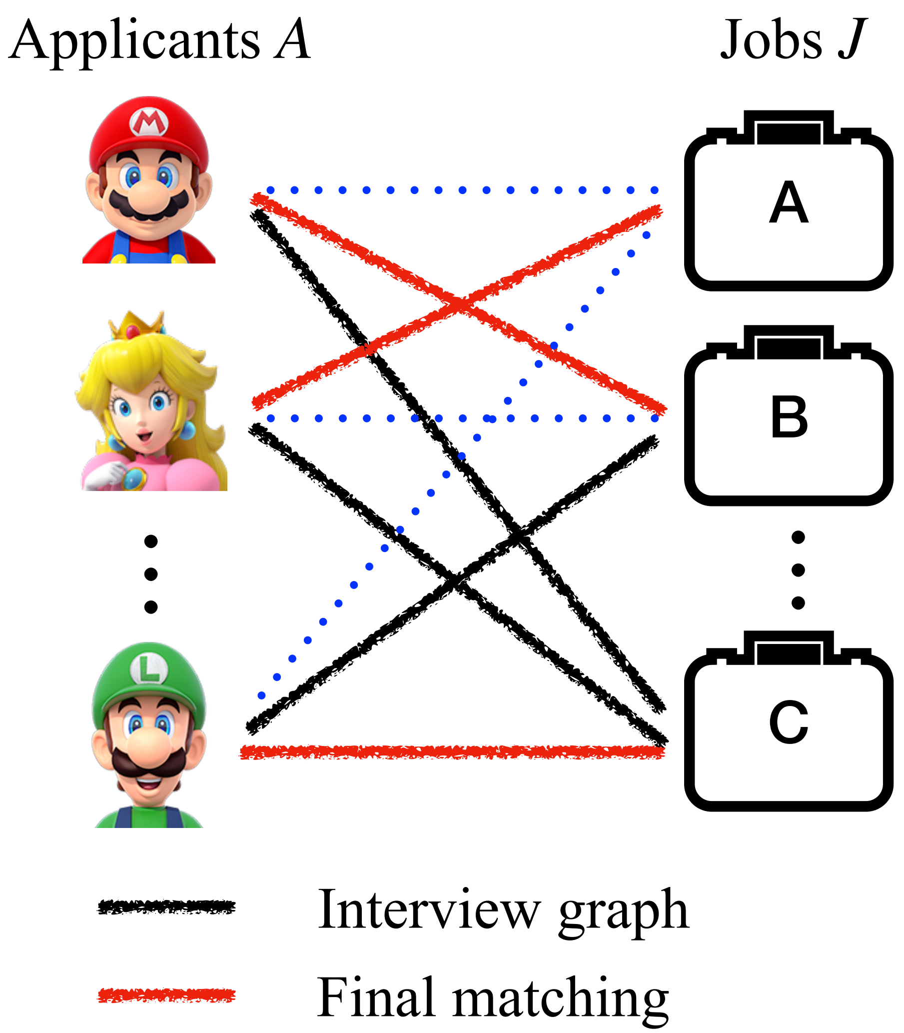

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Focus: From signaling to interviews$\Longrightarrow$ interim stability + reduce congestions

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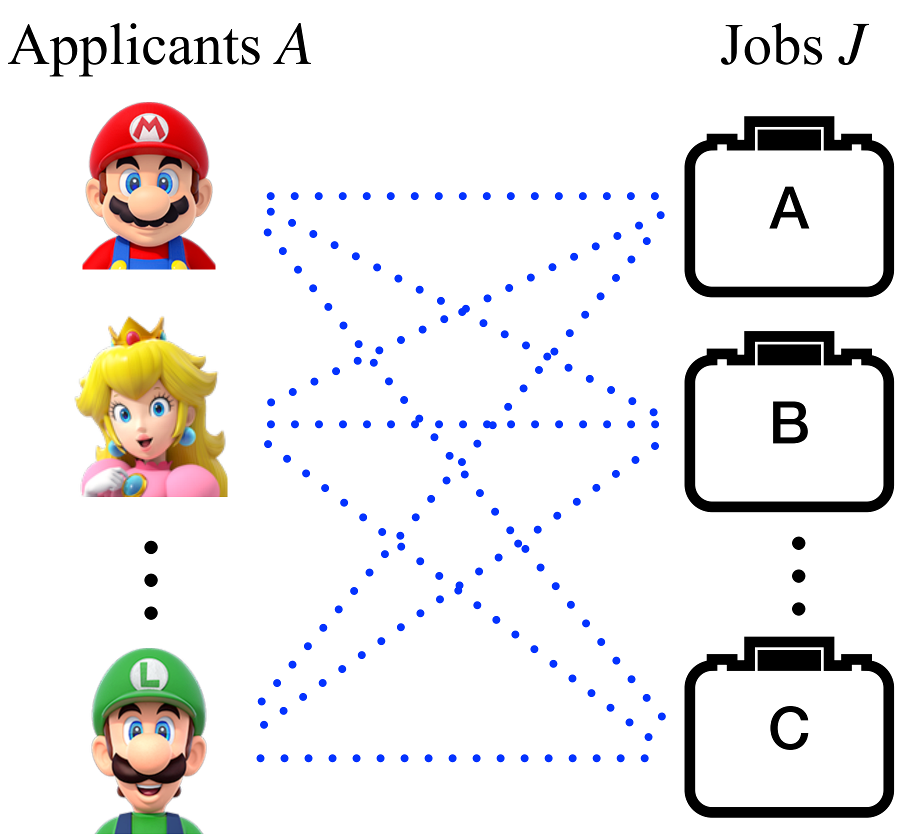

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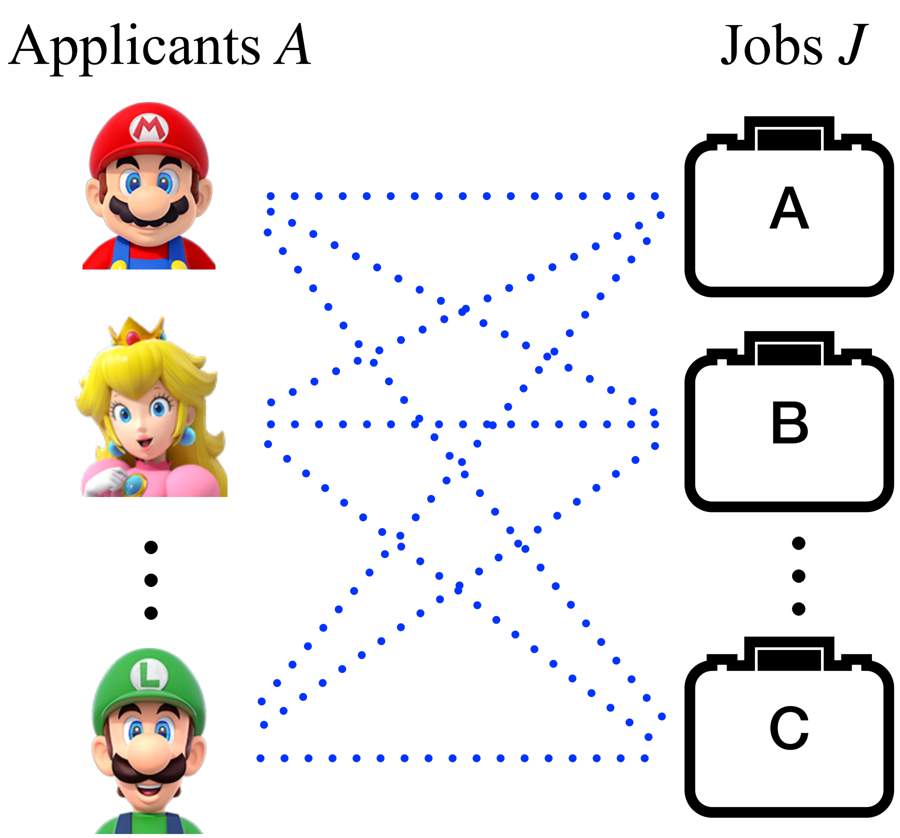

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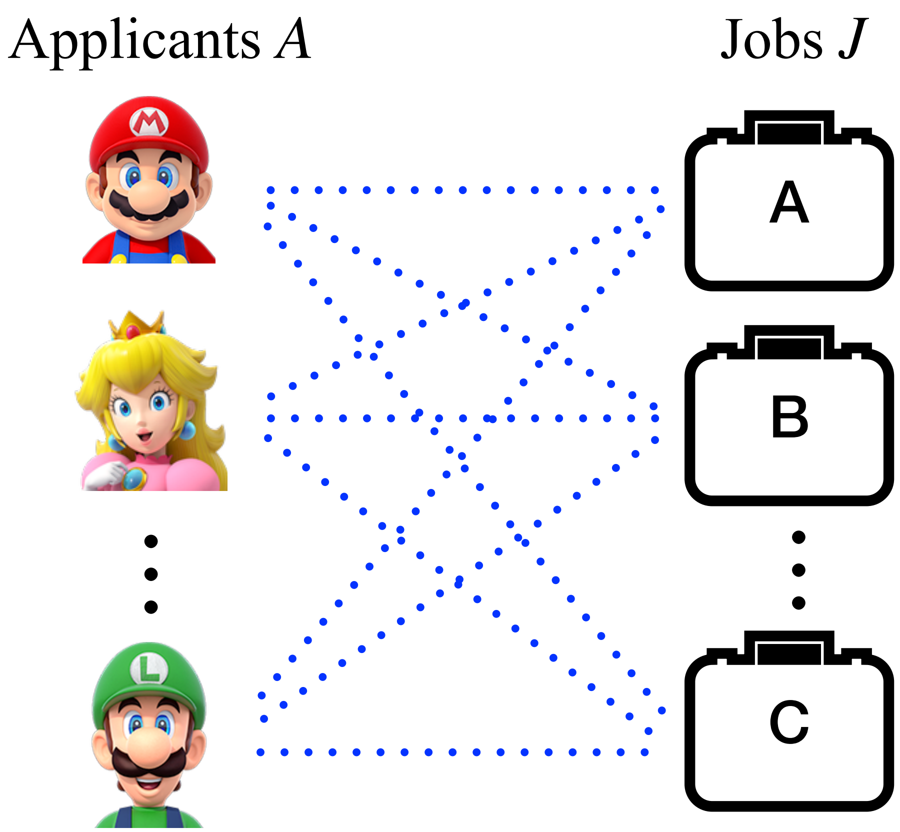

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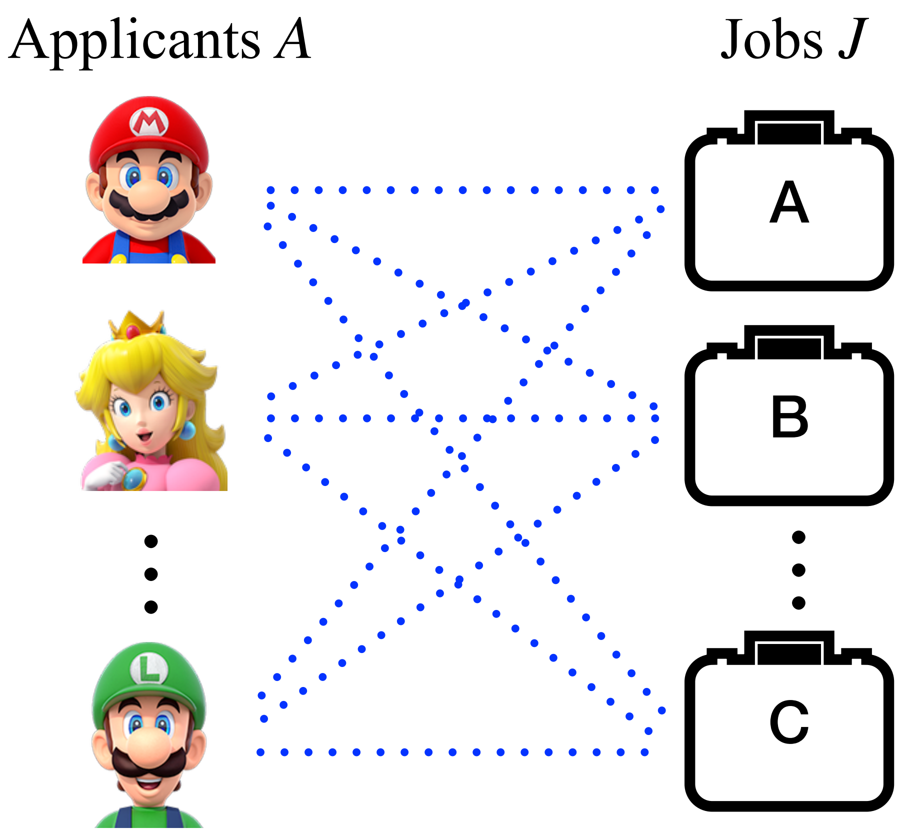

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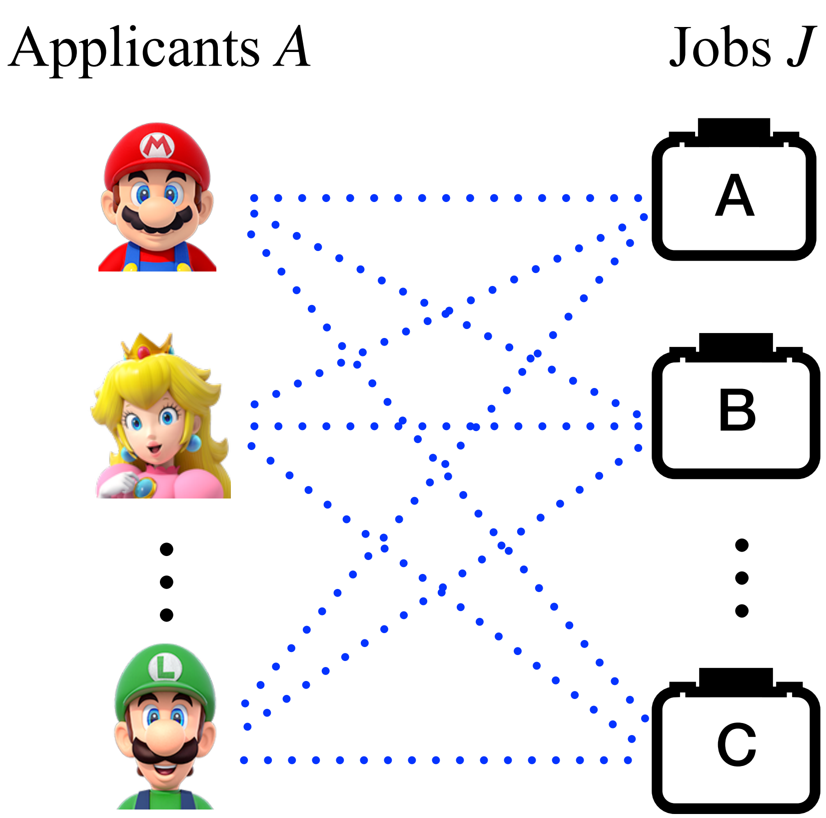
Every agent's pre-interview utilities are i.i.d. generated;

Every agent's post-interview utilities are also i.i.d. generated marginally.

From signaling to interviews

One-side signaling: Each agent on the “chosen” side signals its top d preferred candidates based on the **pre-interview** utilities



applicant-signaling with $d = 2$

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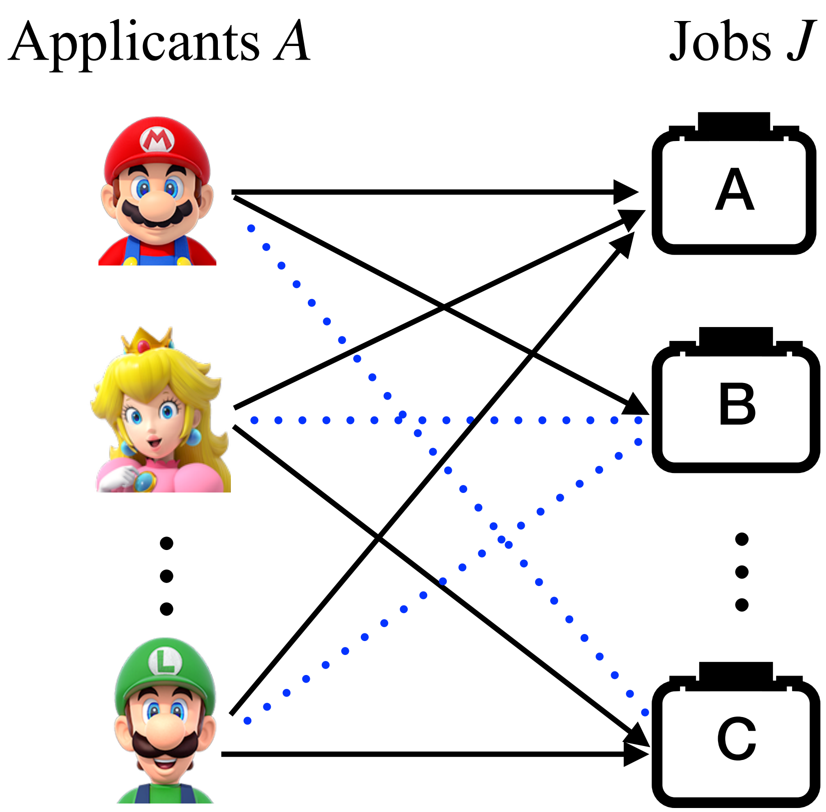

applicant-signaling with $d = 2$

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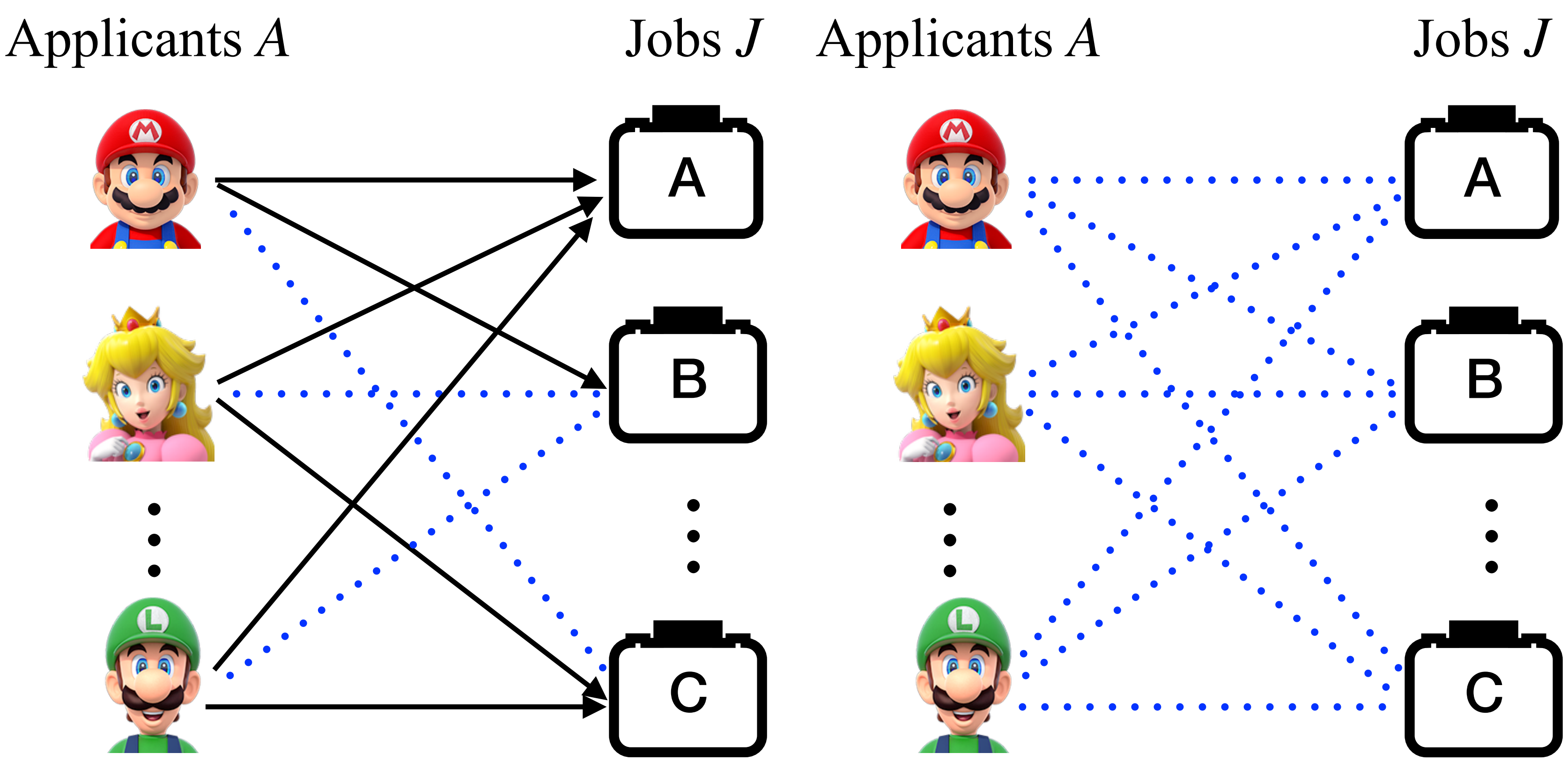

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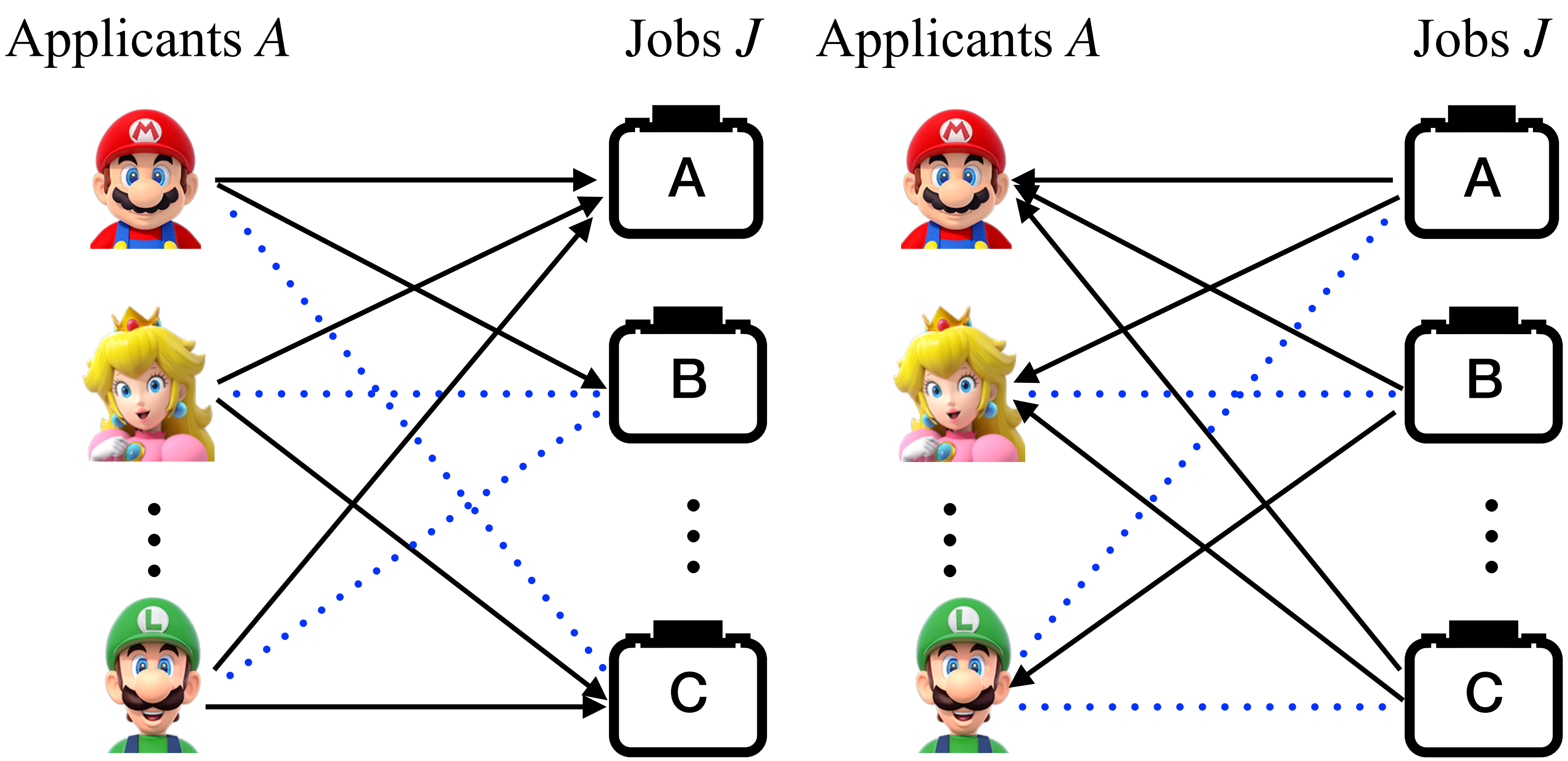

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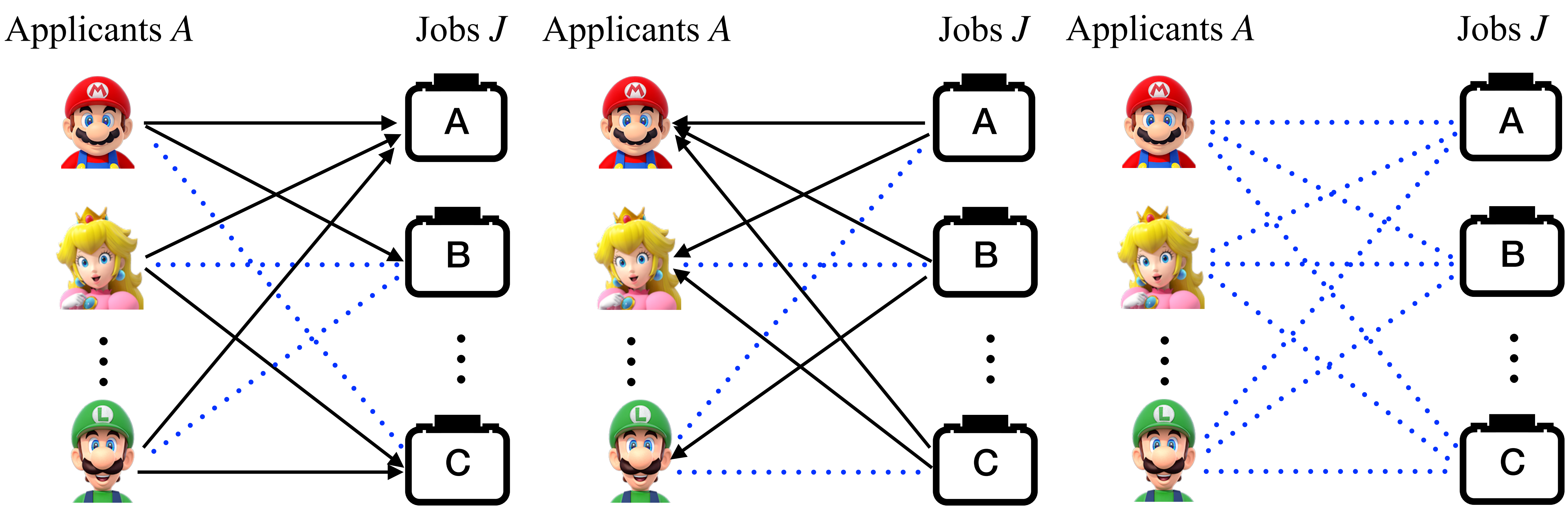

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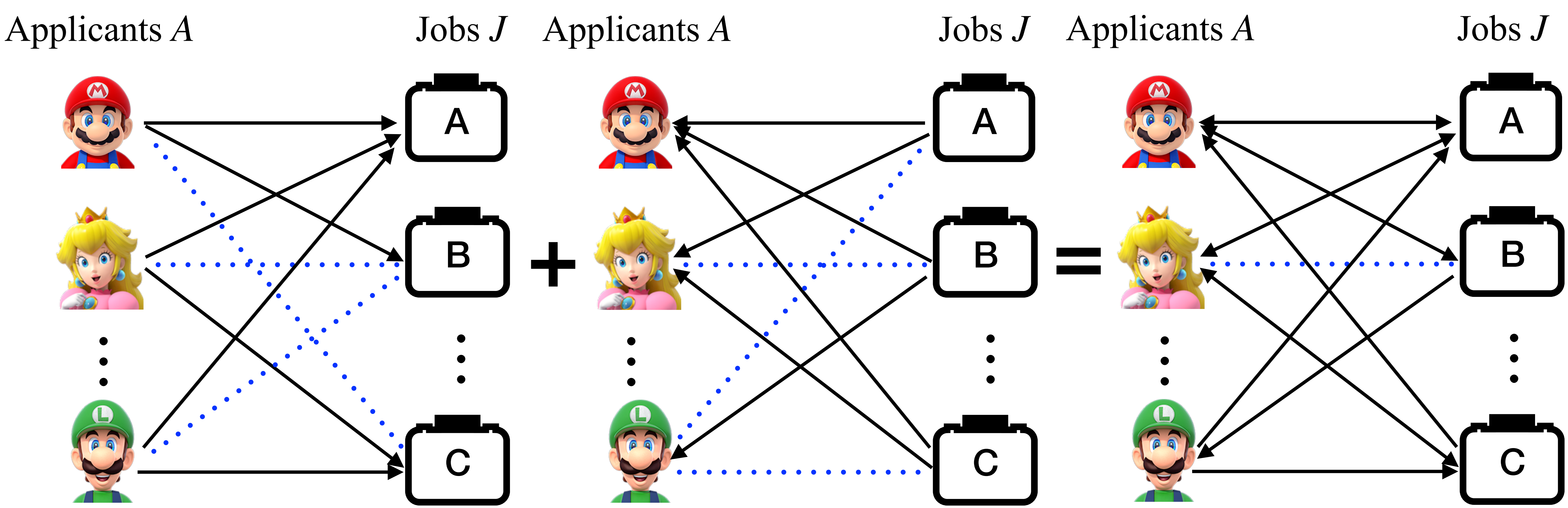

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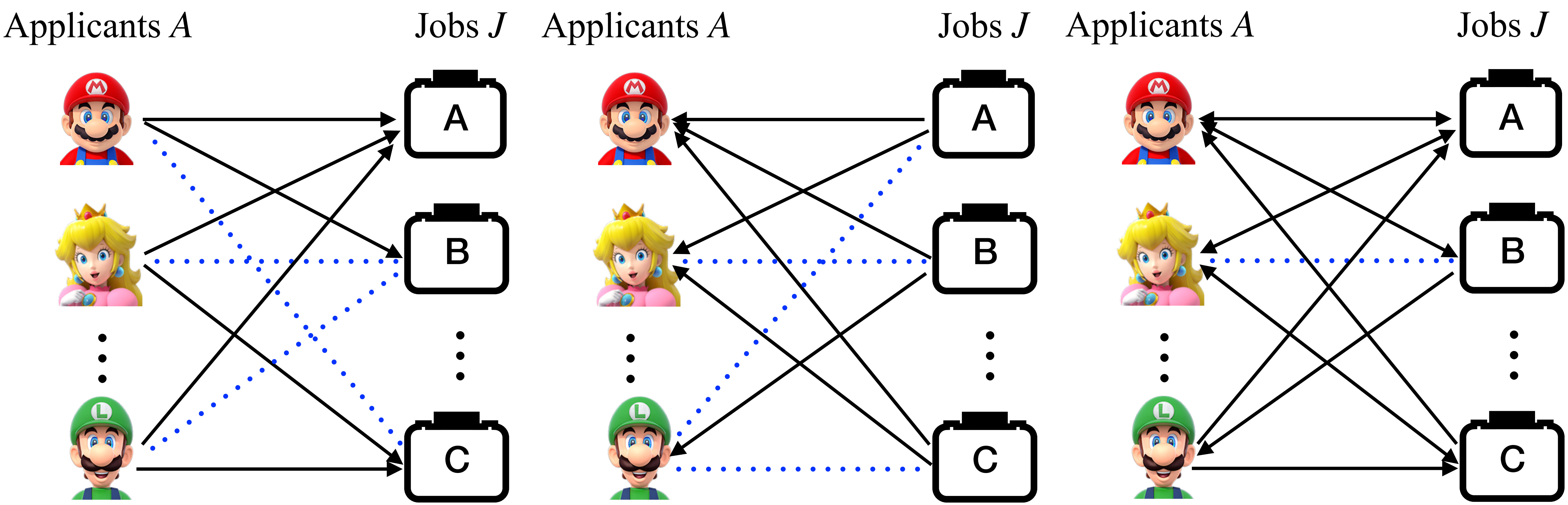

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Interview graph is formed by **including all pairs** where **at least one party has signaled to the other.**

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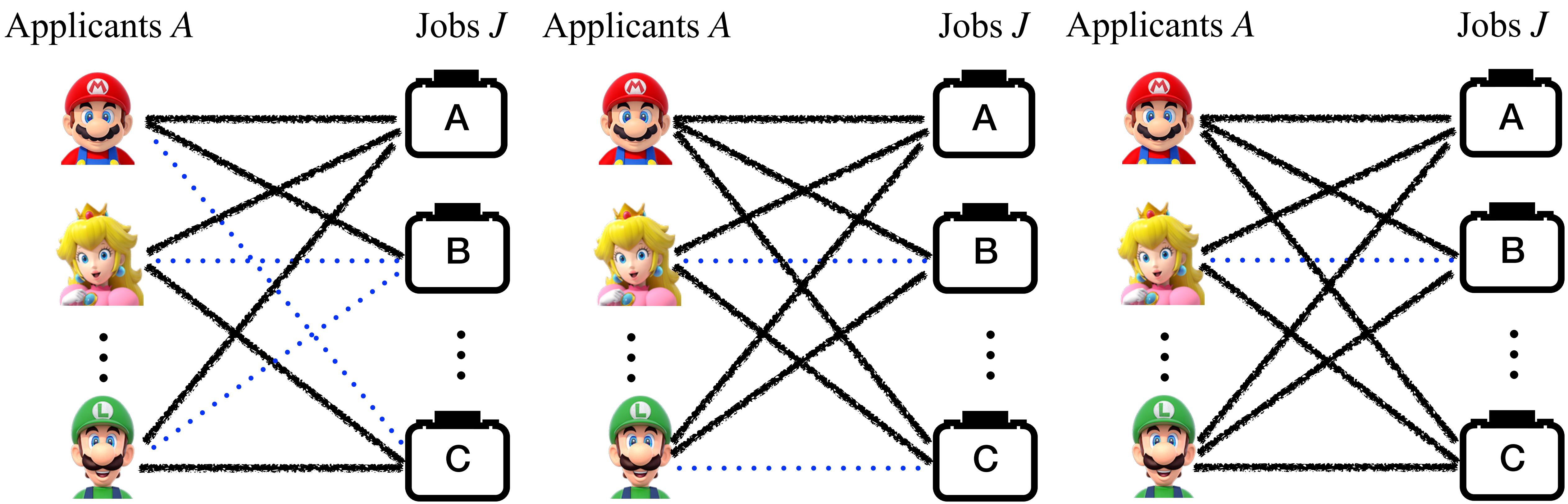

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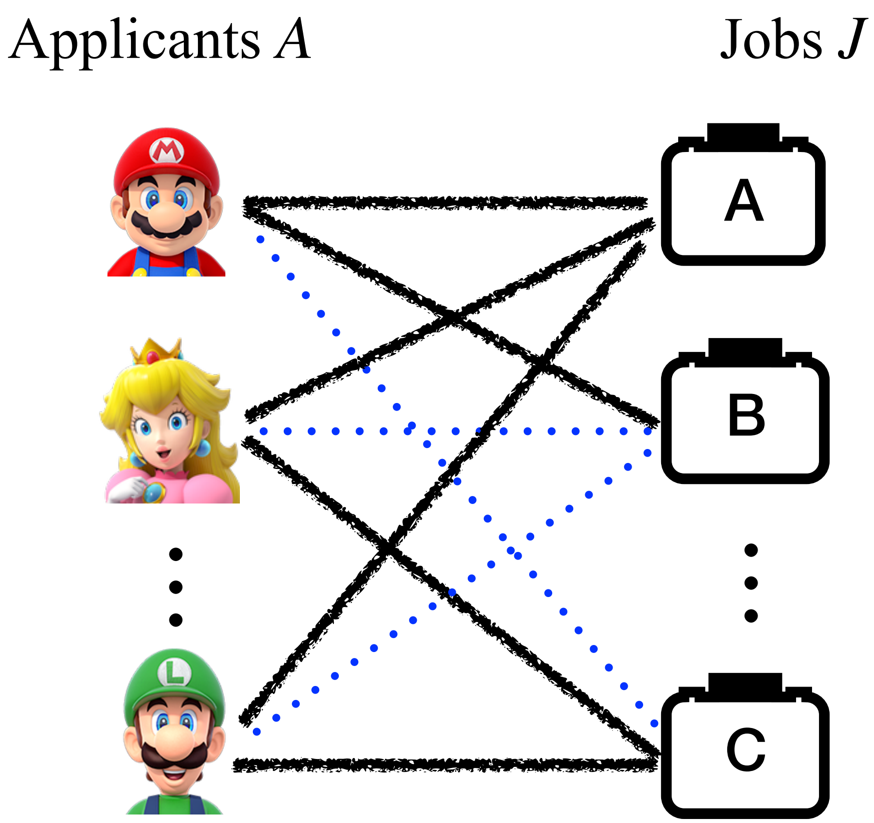
Main results on single-tiered market



$H \triangleq$ interview graph constructed by **applicant-signaling** with d

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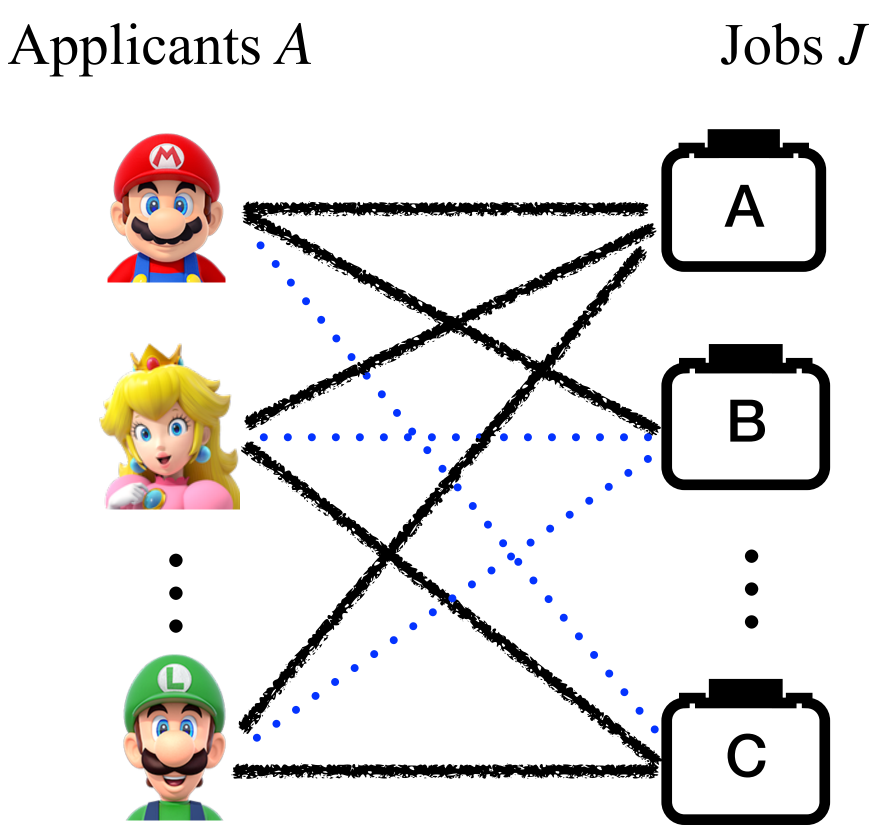

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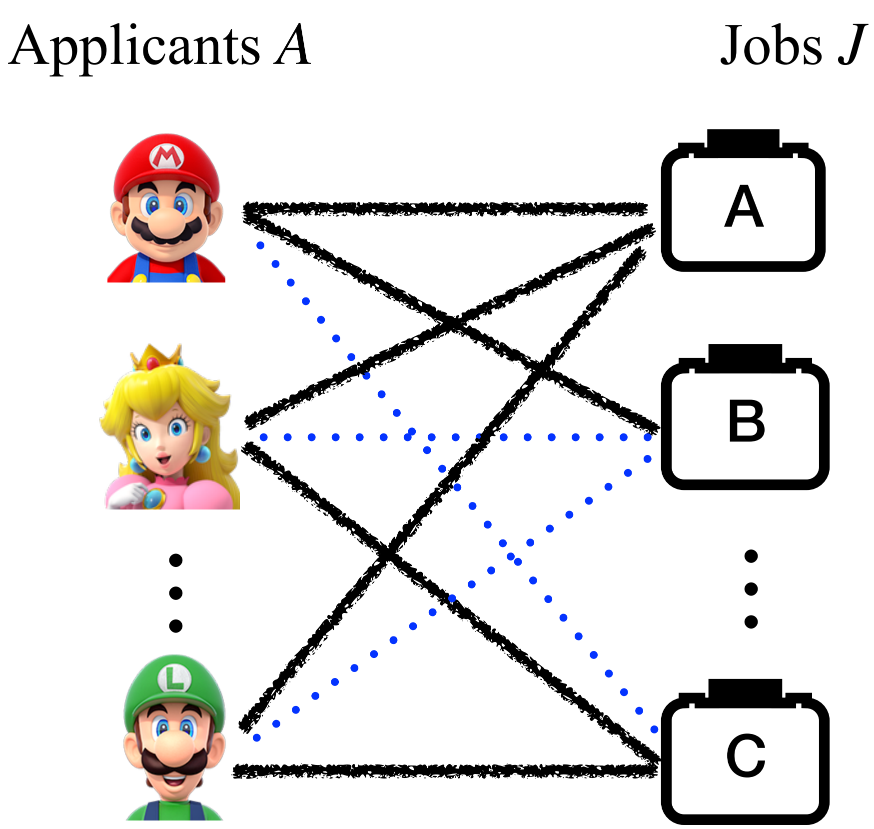

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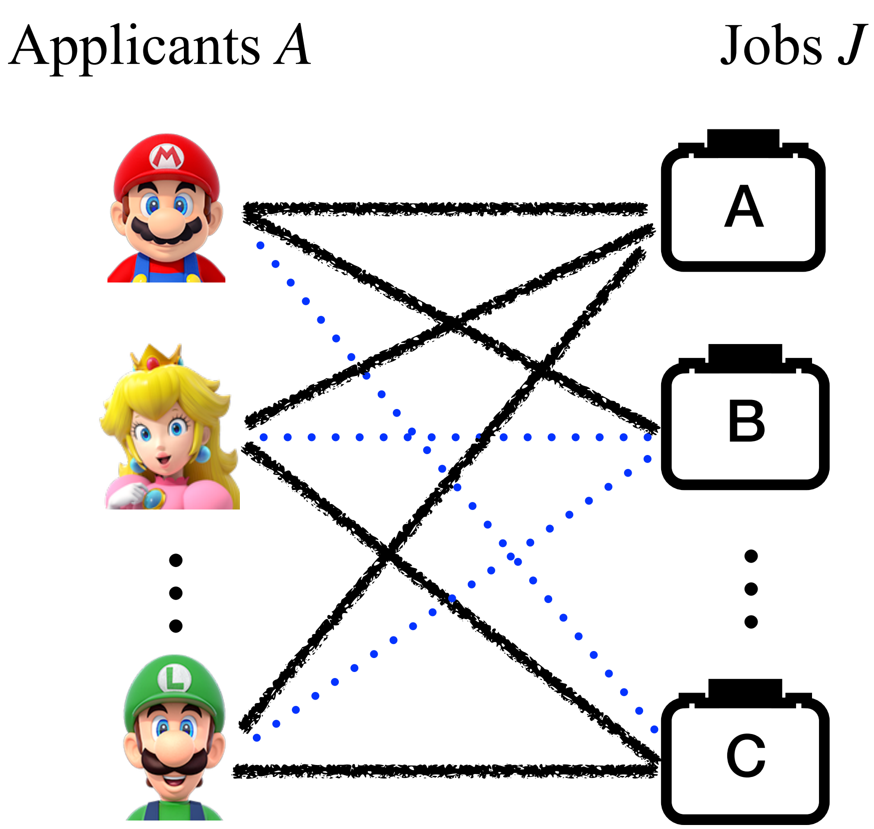

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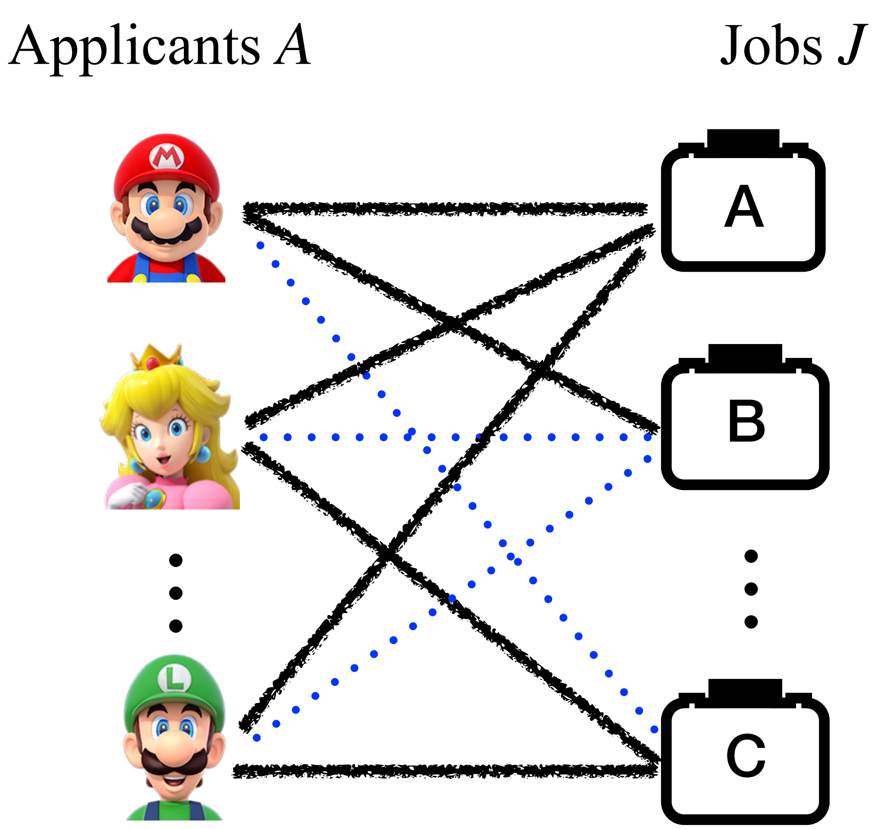

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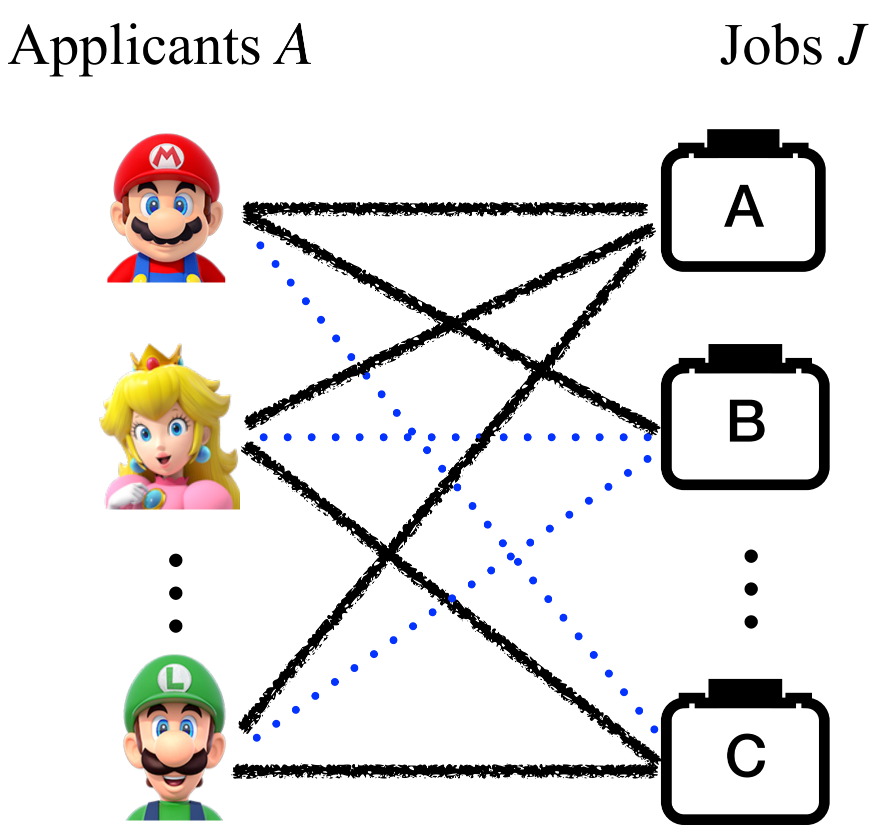
For market with **sparse** signaling:

- **Weakly** imbalanced market, **either short-side or long-side signaling**
- **Strongly** imbalanced market, **only short-side signaling**

$\Longrightarrow$ almost interim stability

Main results on single-tiered market



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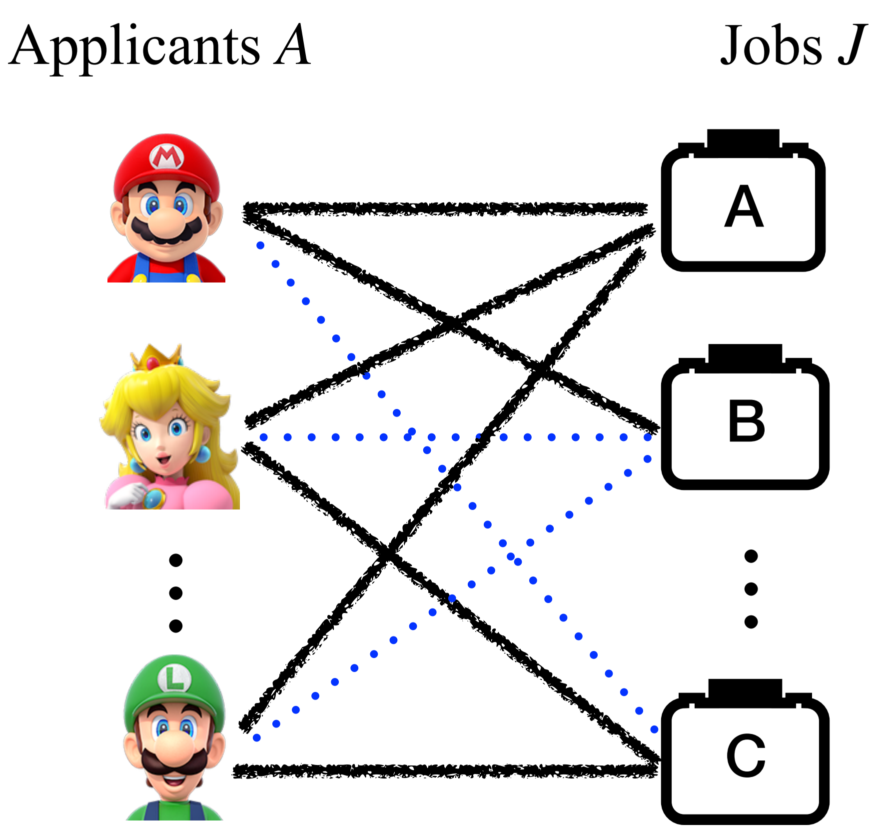

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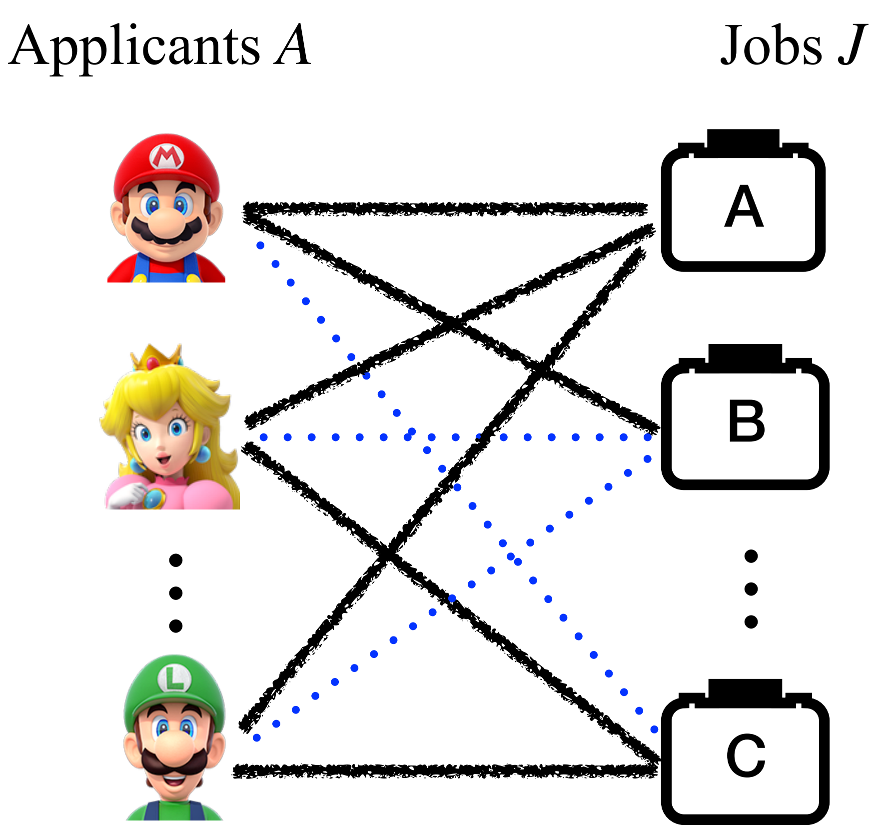

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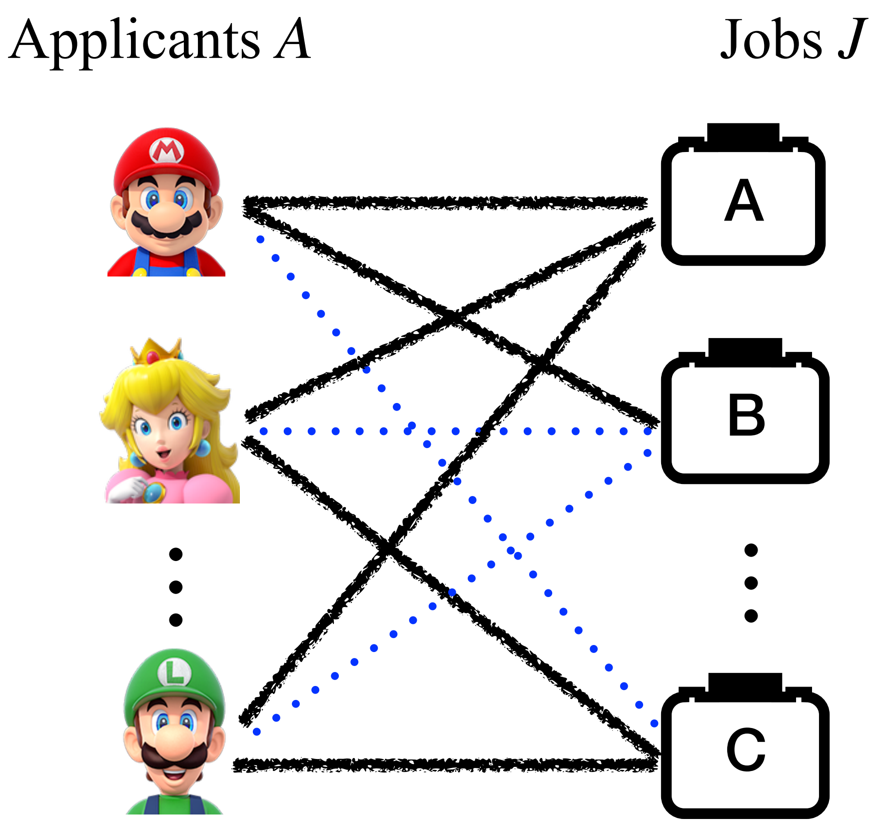

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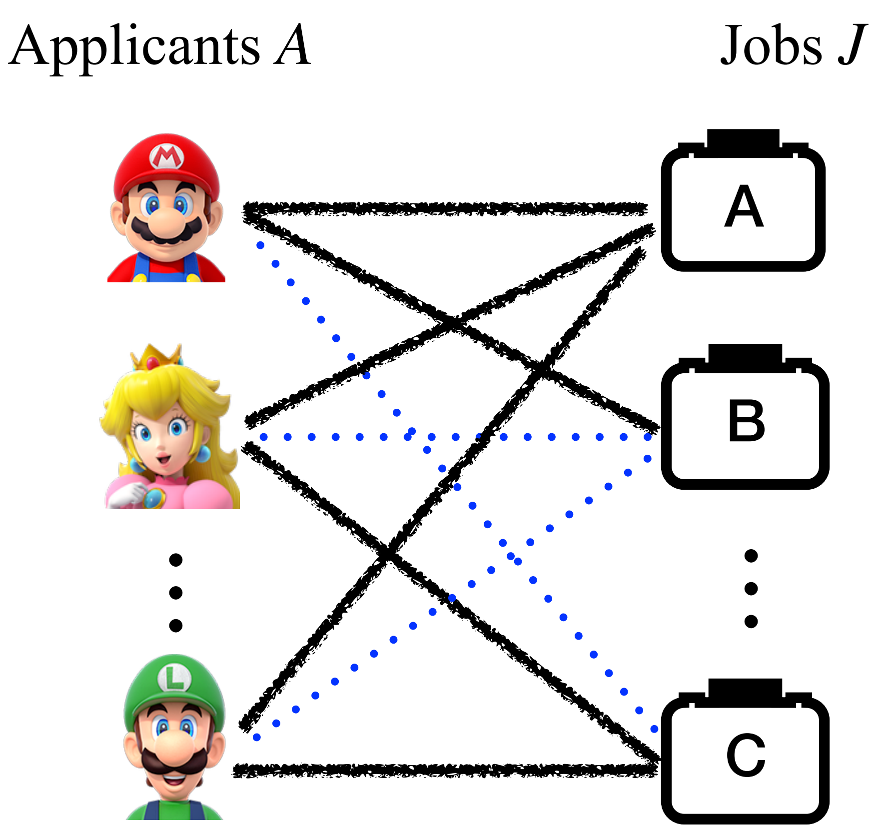

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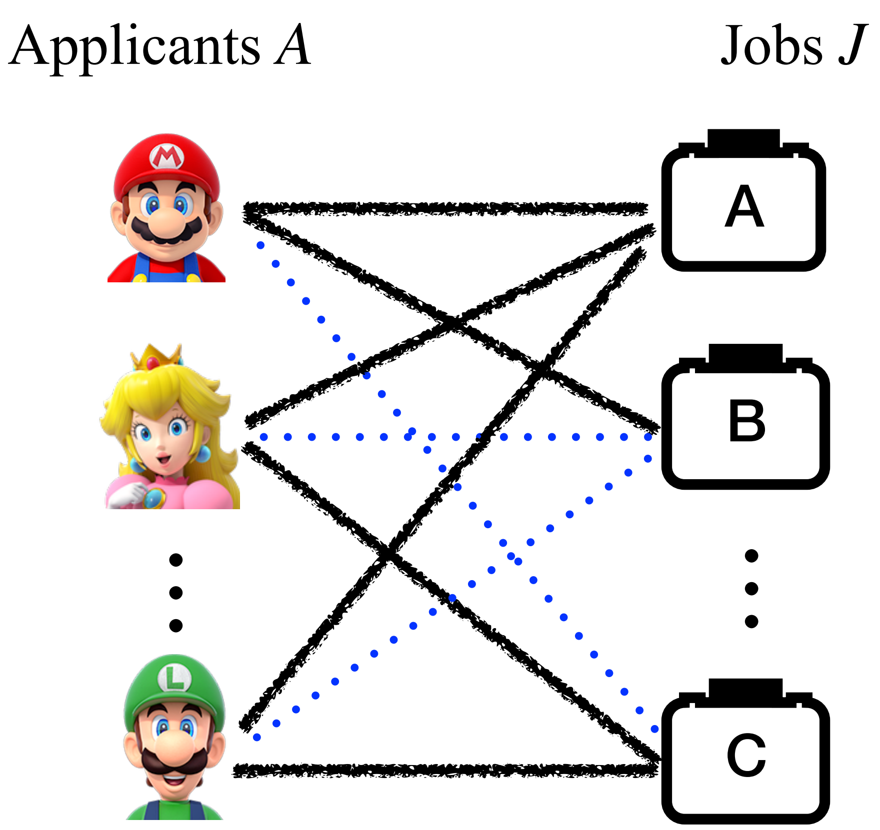

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Moreover, if the market is strongly imbalanced, $d \geq \Omega(\log n)$ also works.

Proof sketch for one-side signaling

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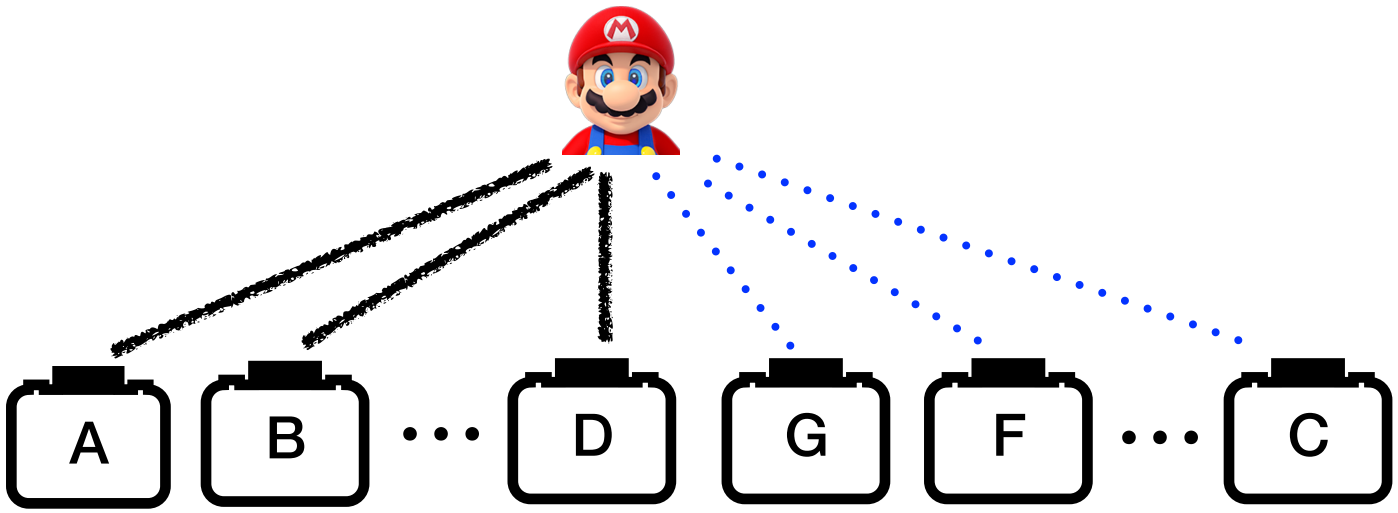

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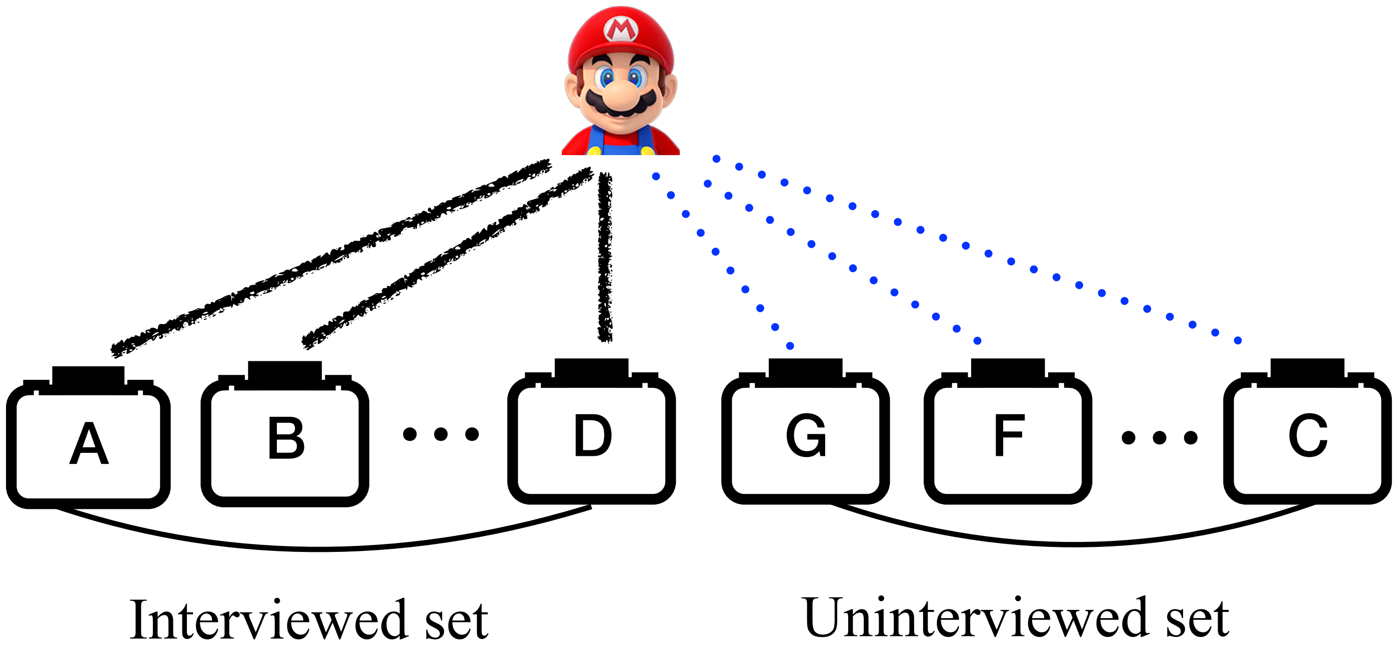

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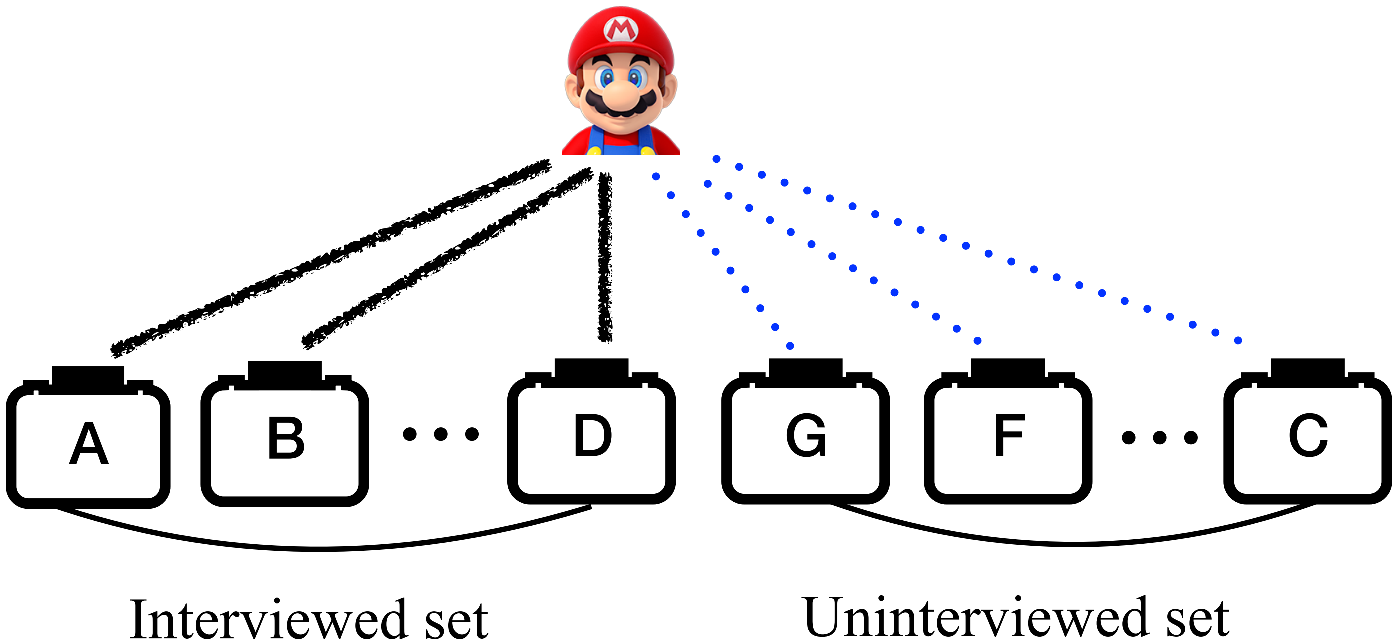

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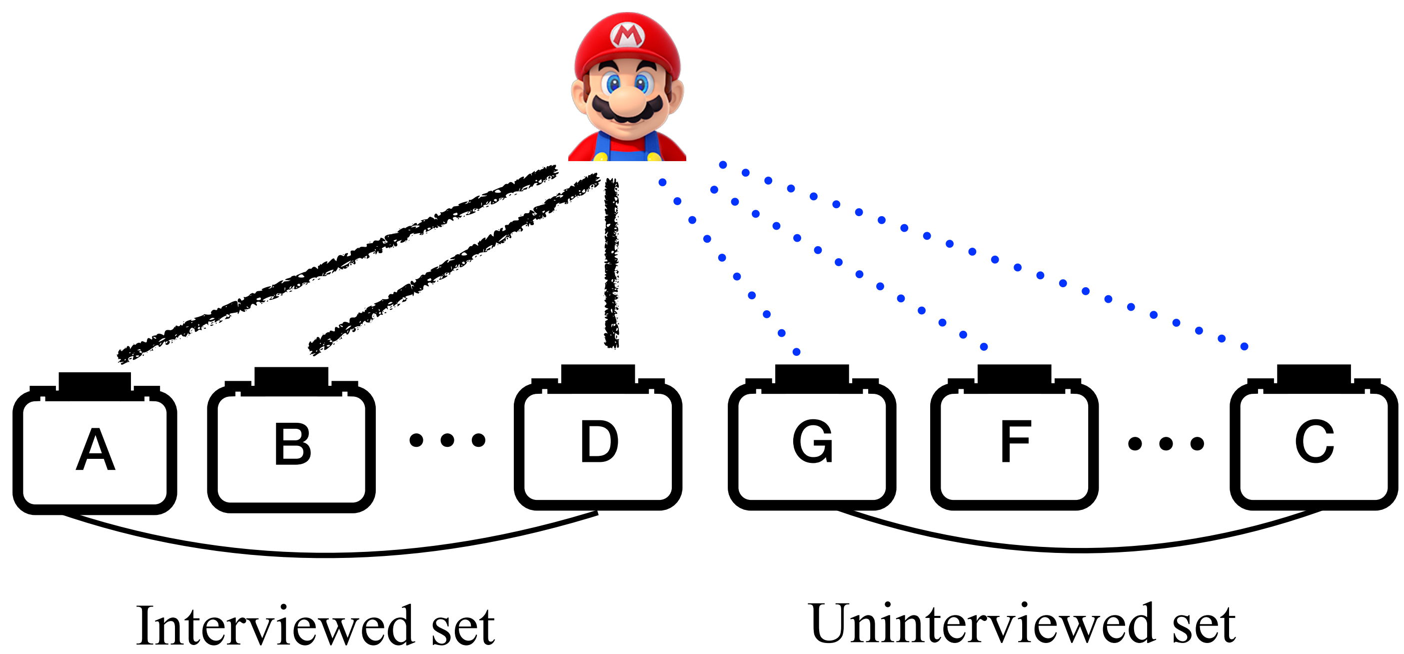

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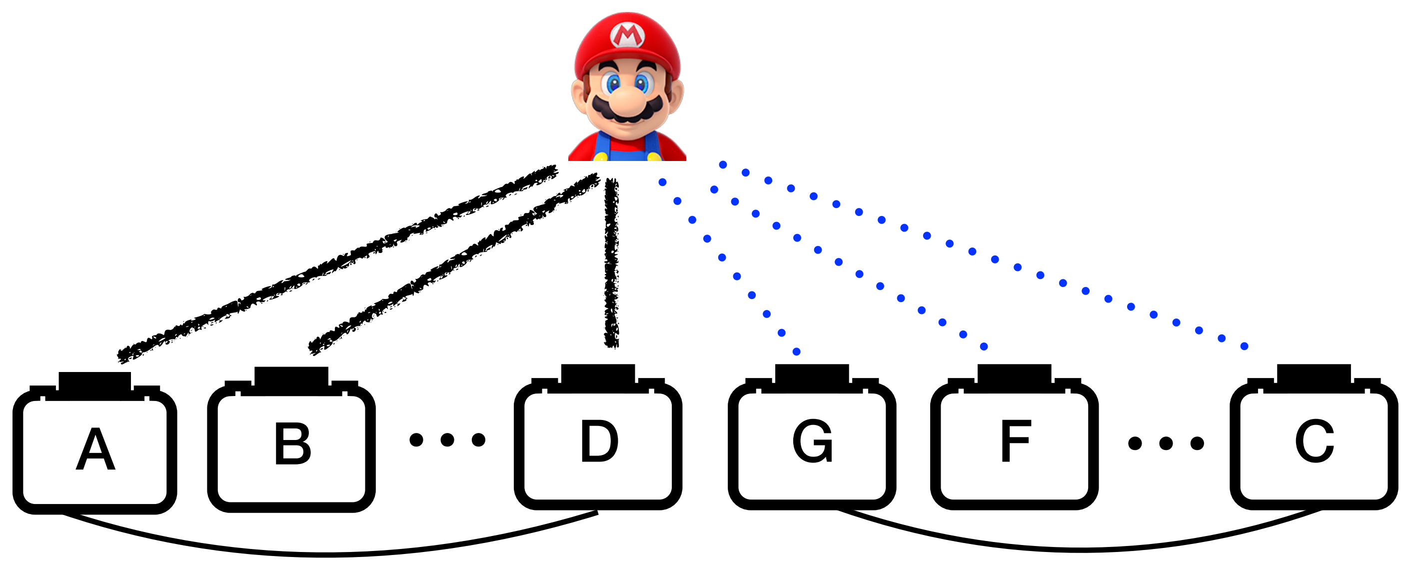

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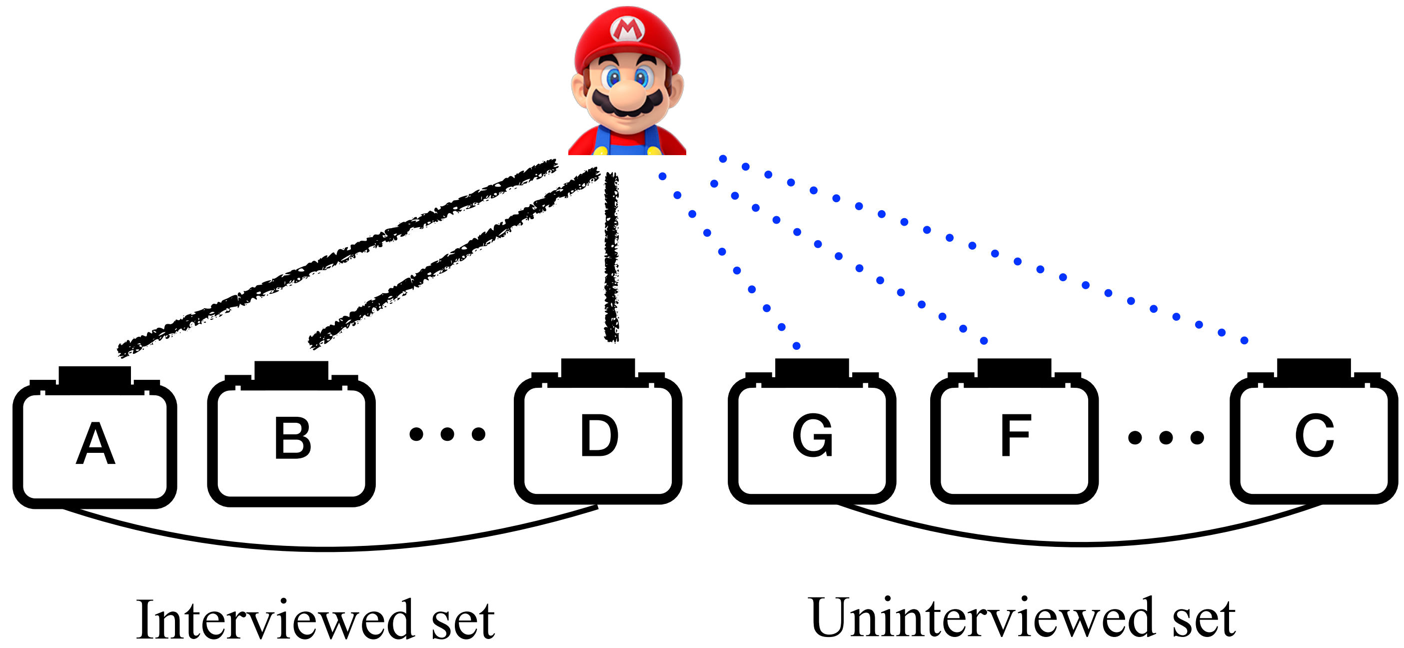

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To prove interim stability:

- for every applicant a, determine if there exists a job j with $A_{a,j} \geq 0$ that is available to a.

Leveraging over local information

For every applicant a, determine if exists a job j with $A_{a,j} \geq 0$ that is available to a.

Step 1: **truncation** on **local neighborhood** of a:

Step 2: find stable matching on local neighborhood:

Step 3: **message-passing** on tree

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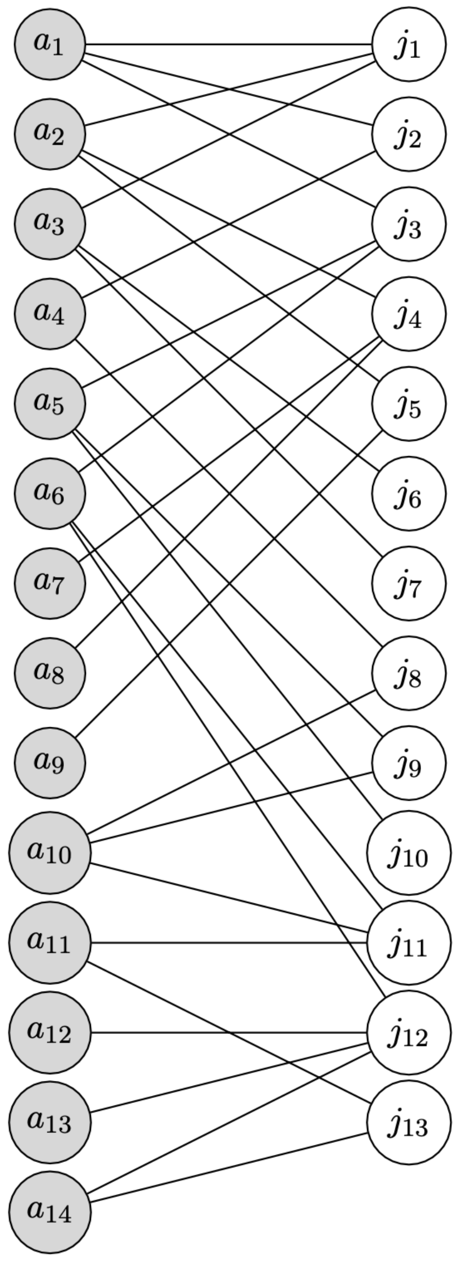

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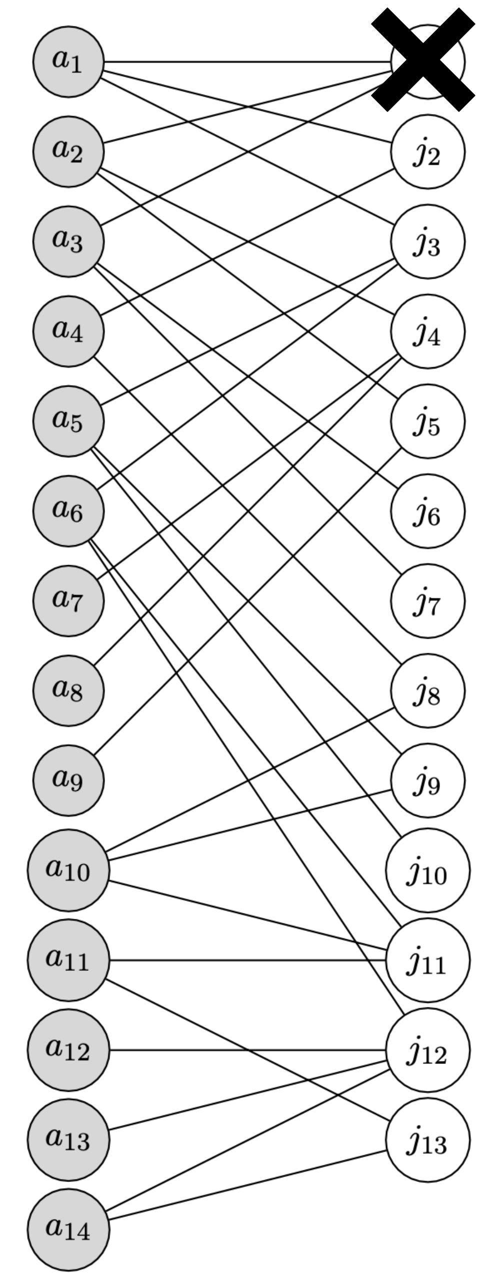

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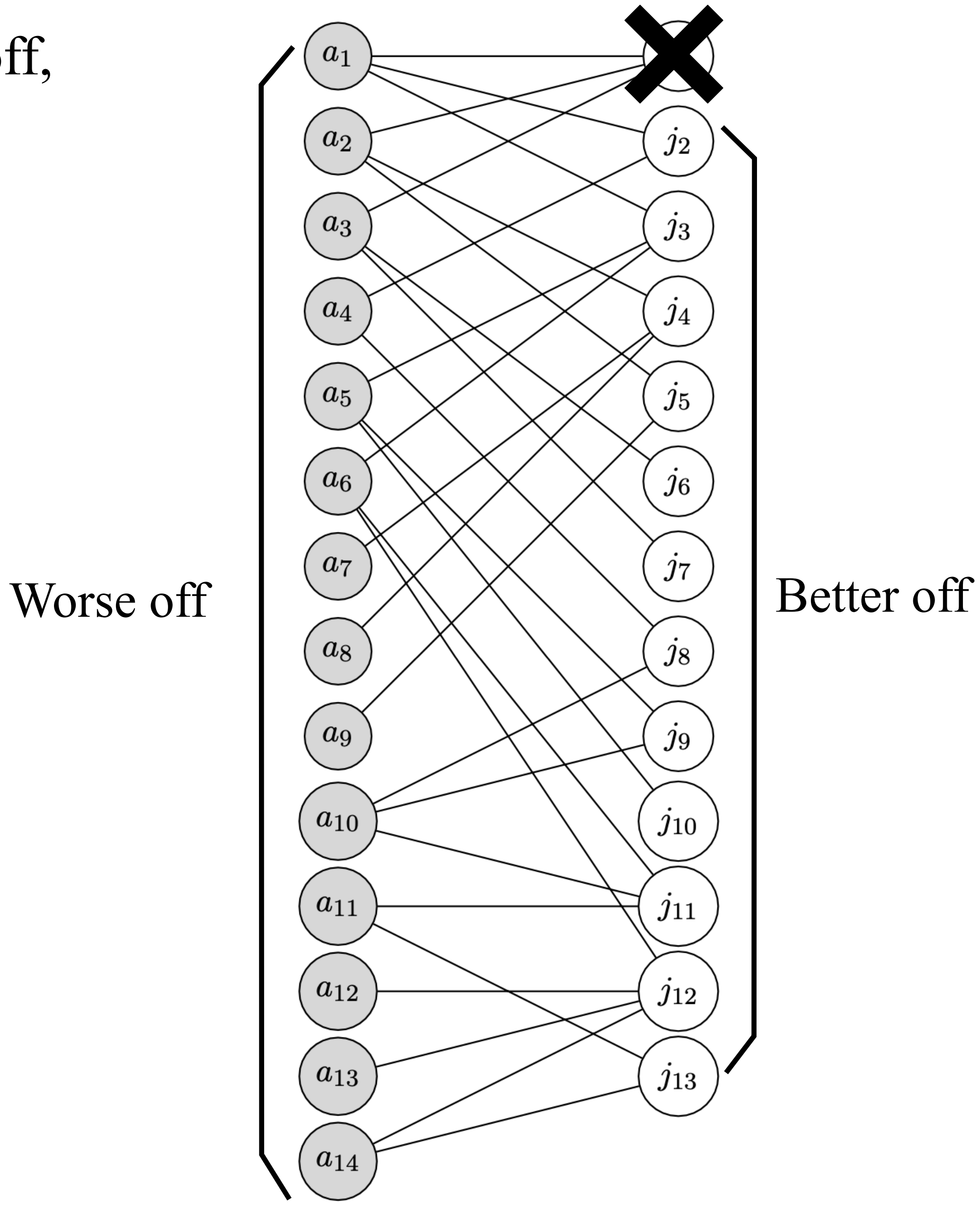

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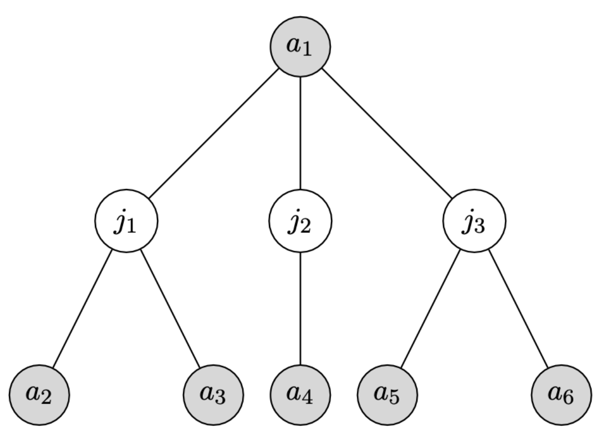
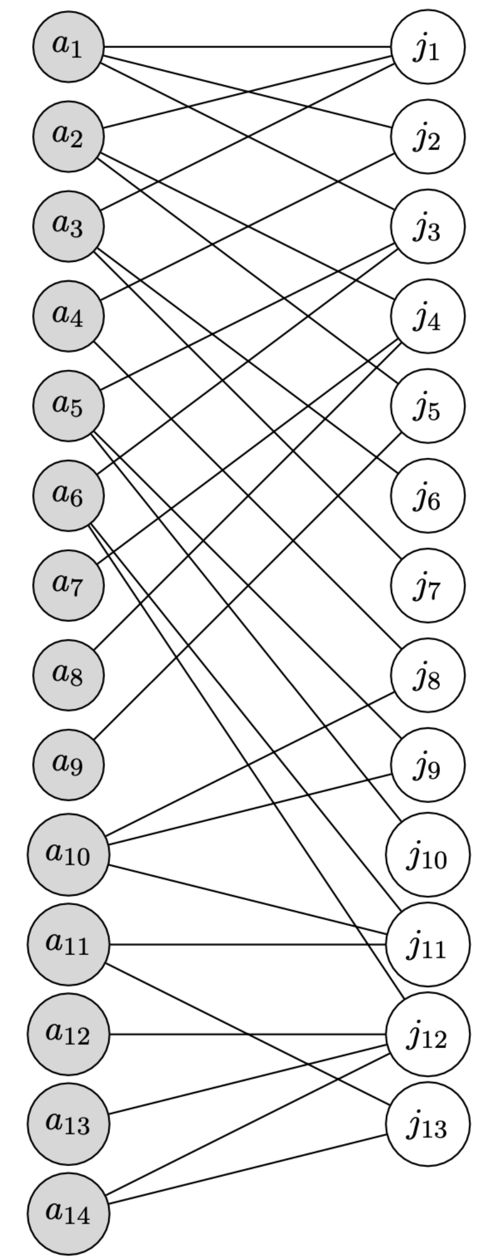


$H_2(a_1)$: 2-hop neighborhood of a_1



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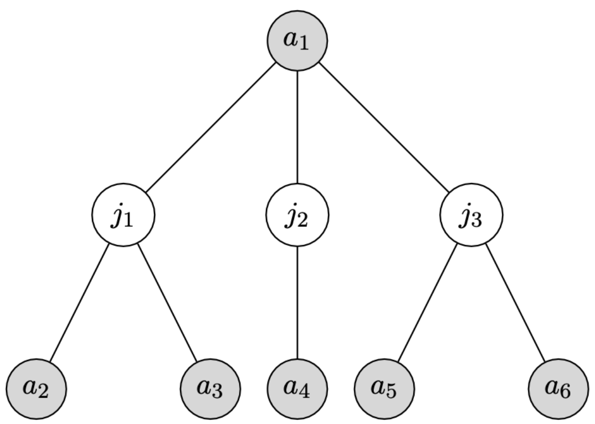

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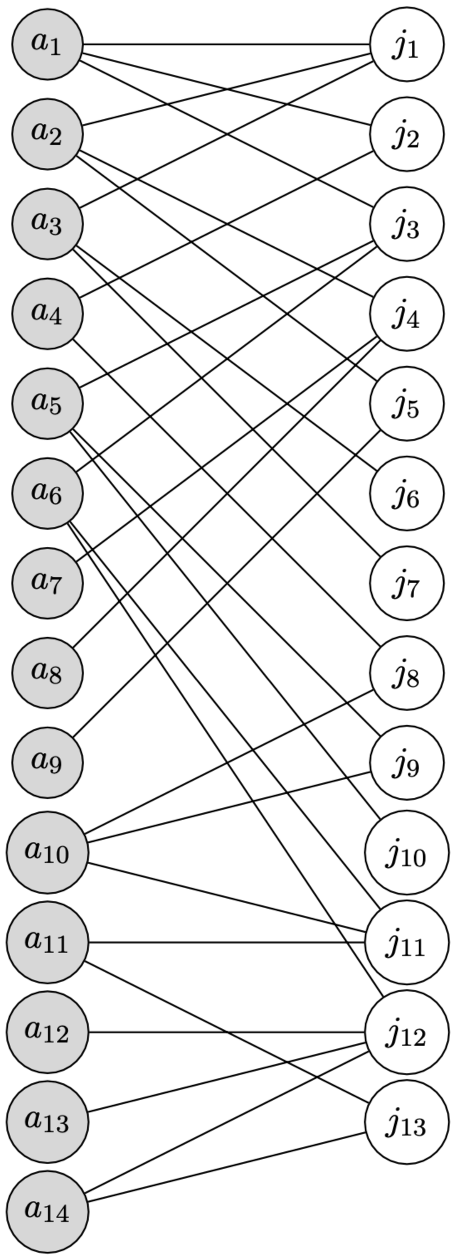
Remove agents on depth 3



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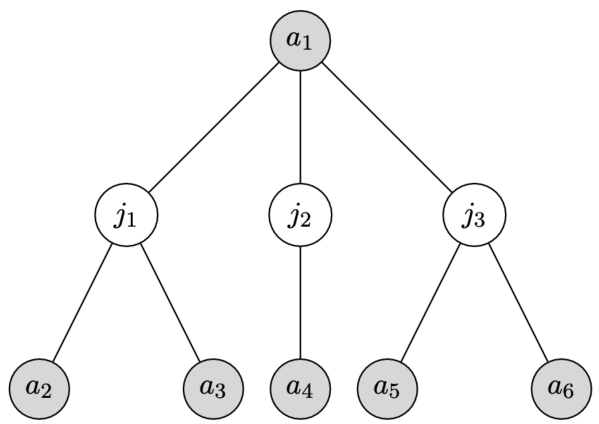

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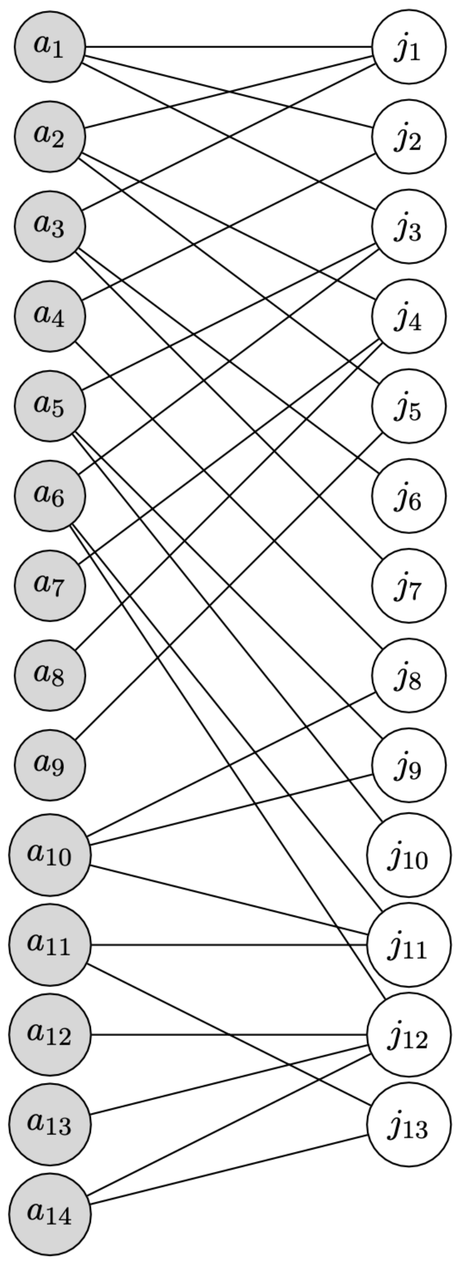

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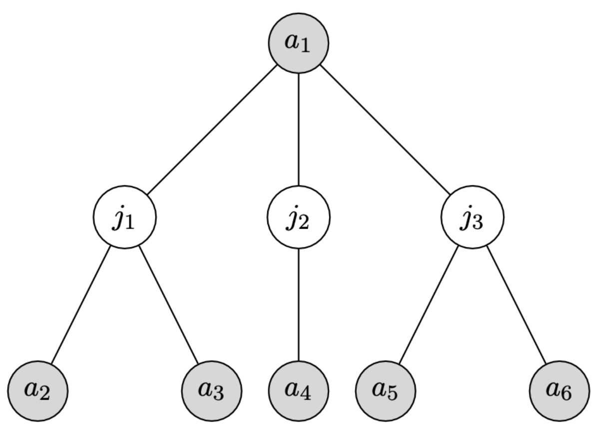

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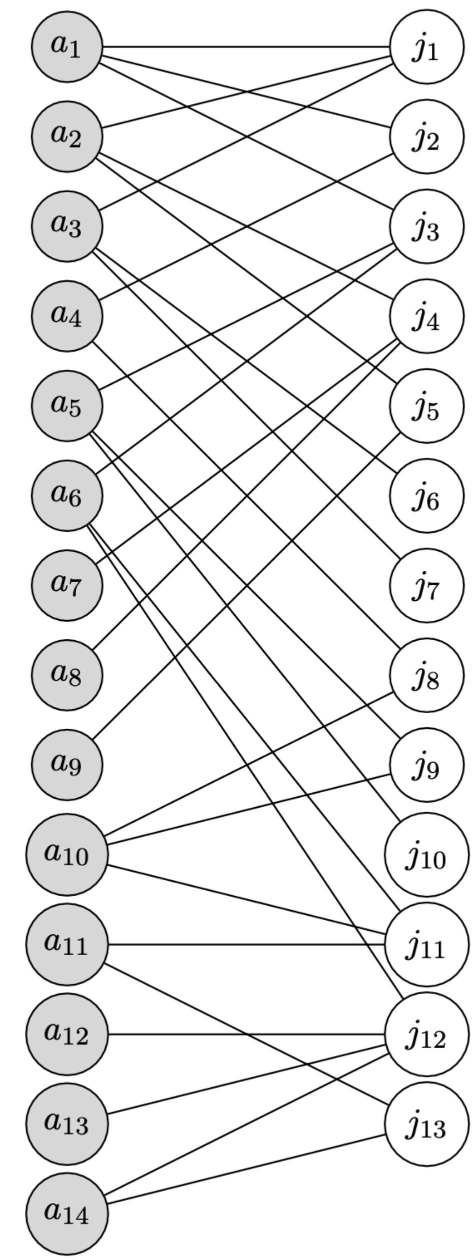

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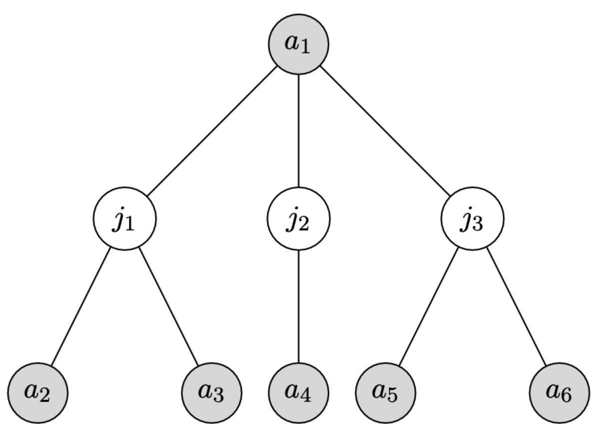

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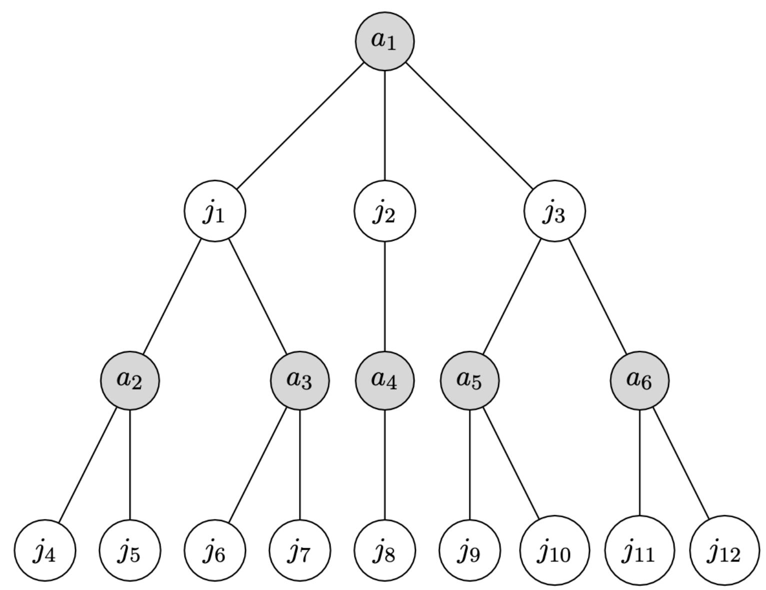

$H_3(a_1)$: 3-hop neighborhood of a_1

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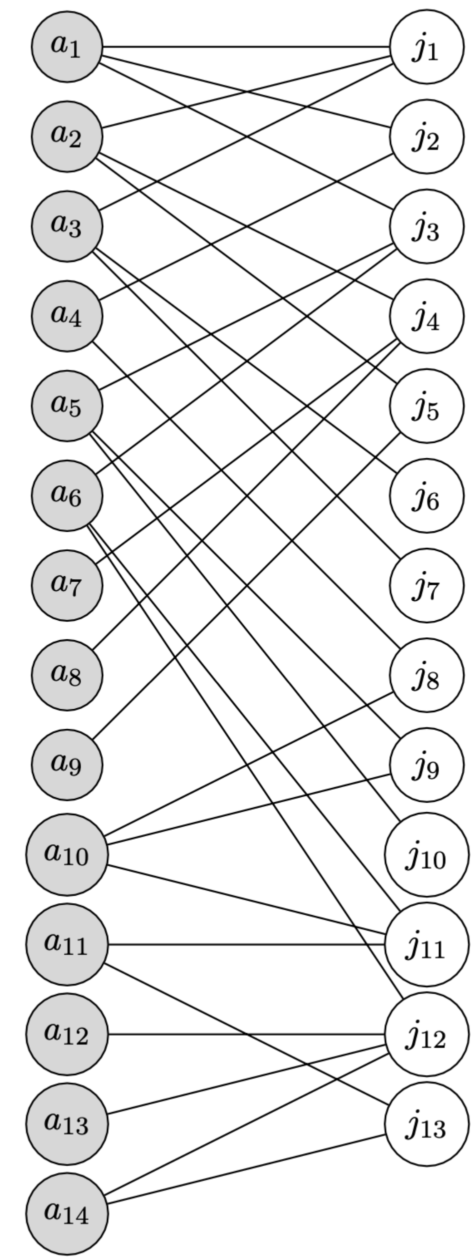

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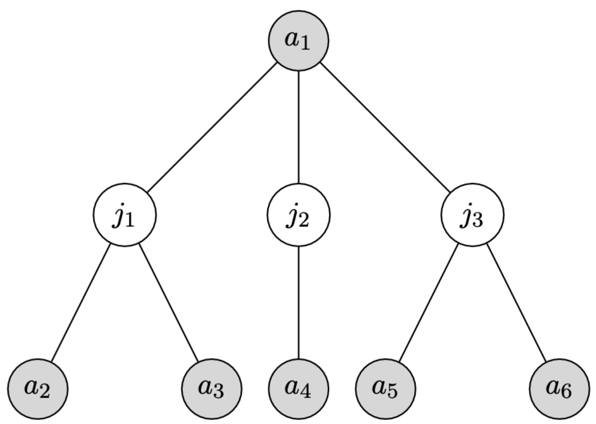

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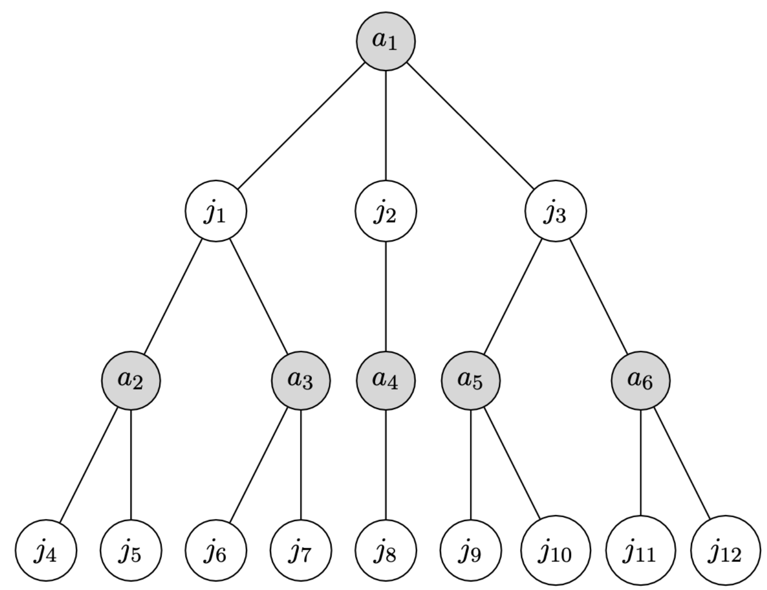

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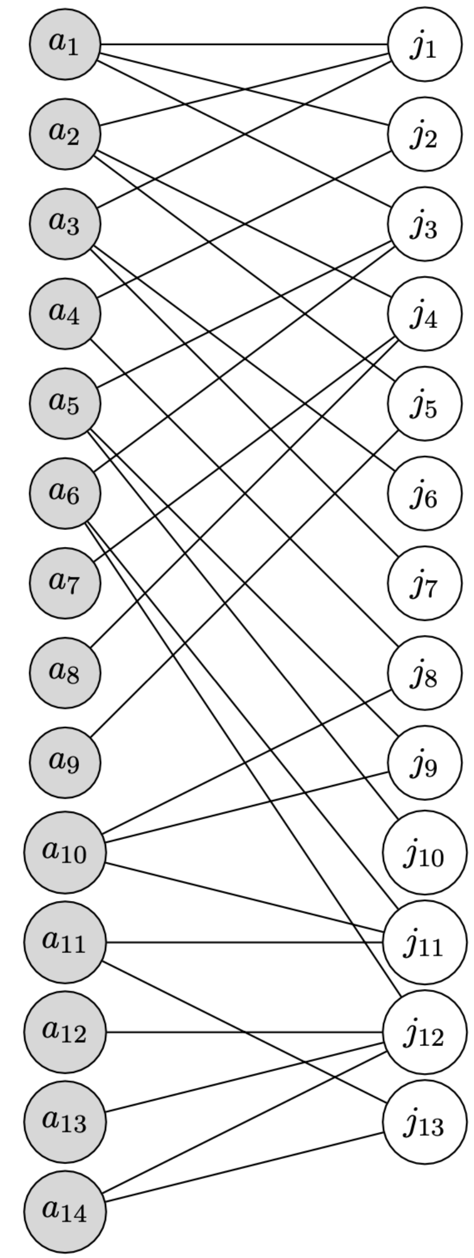

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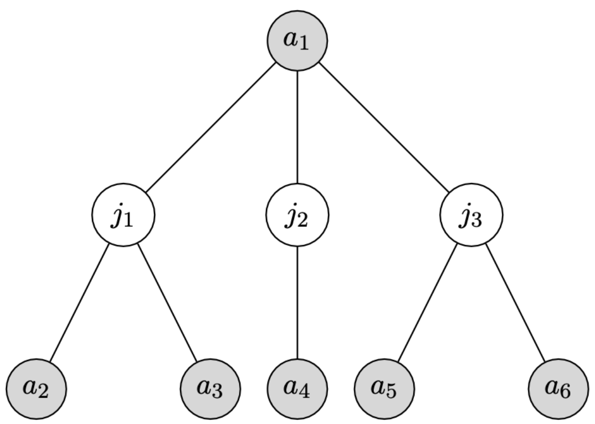

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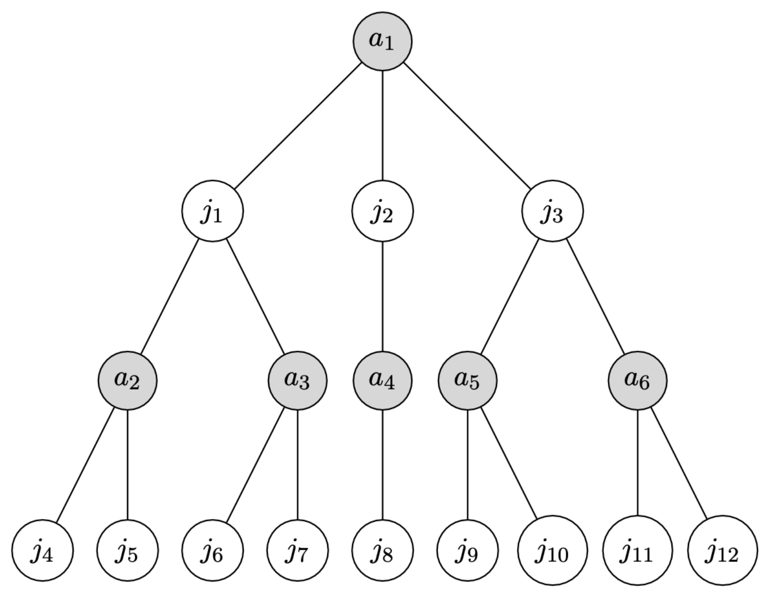

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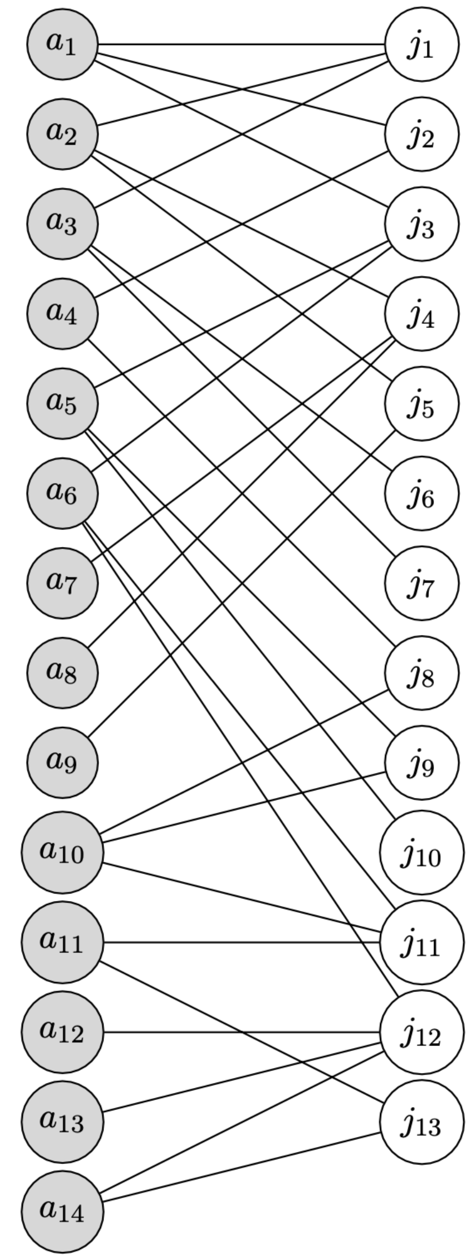

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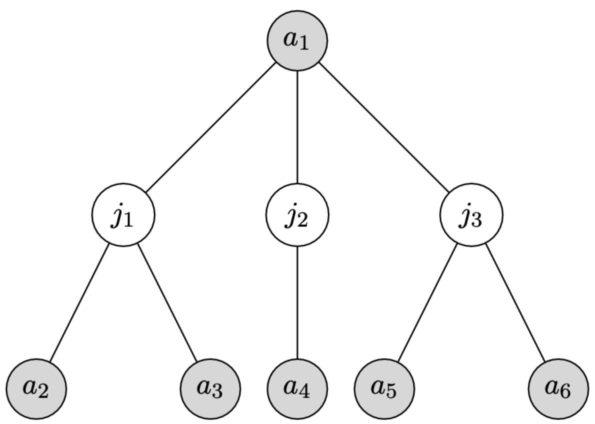

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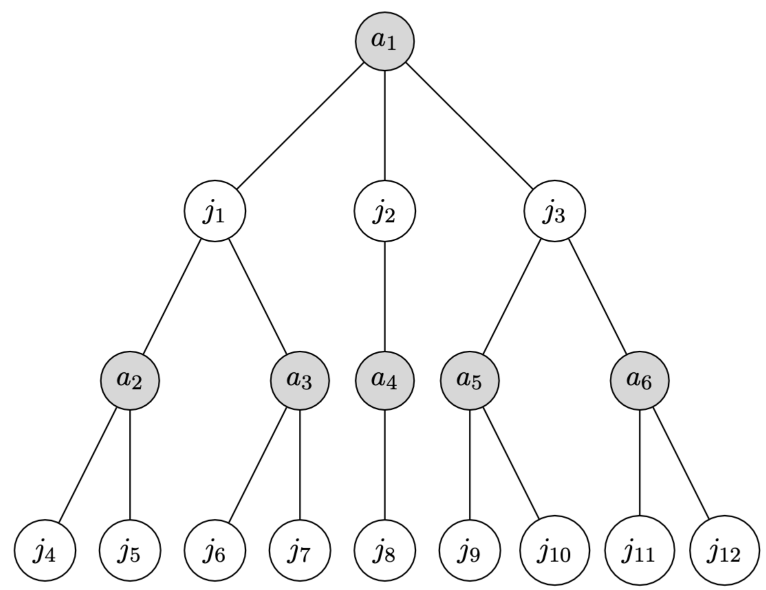

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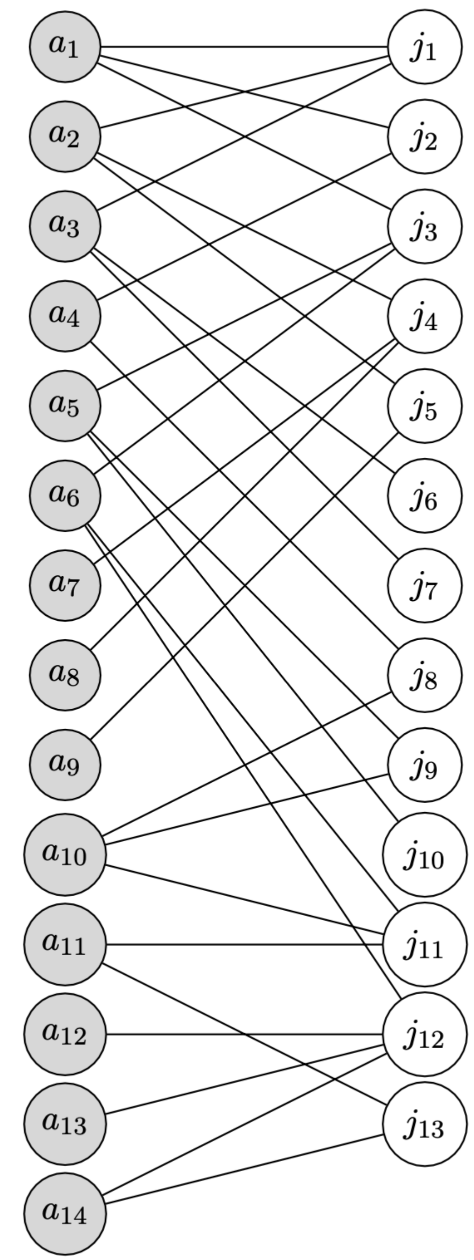

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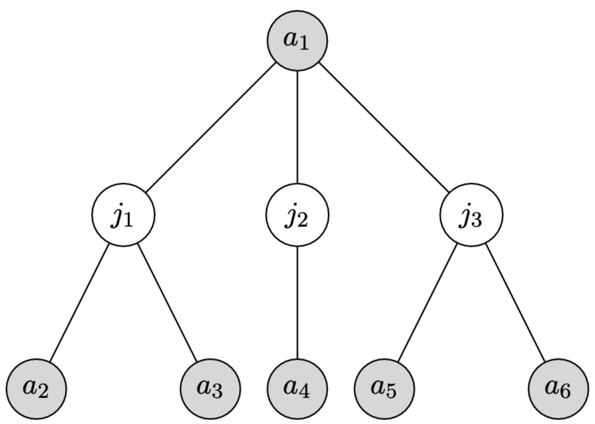

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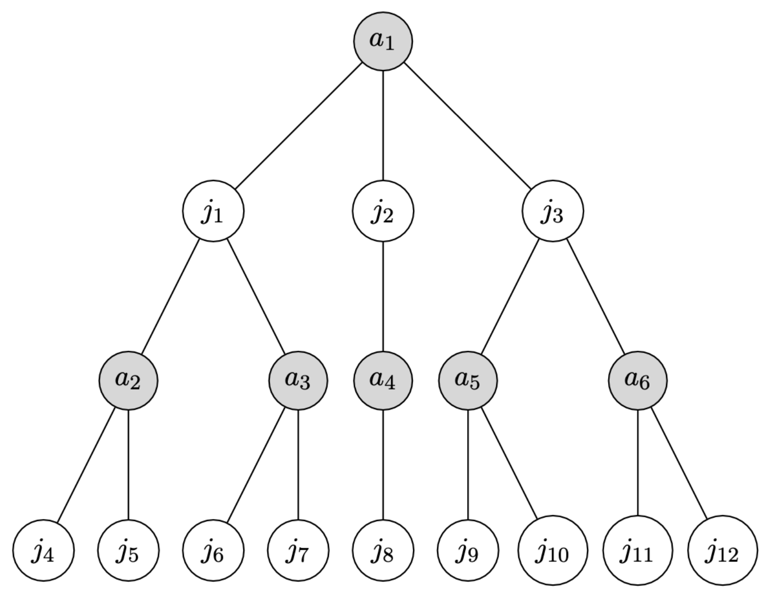

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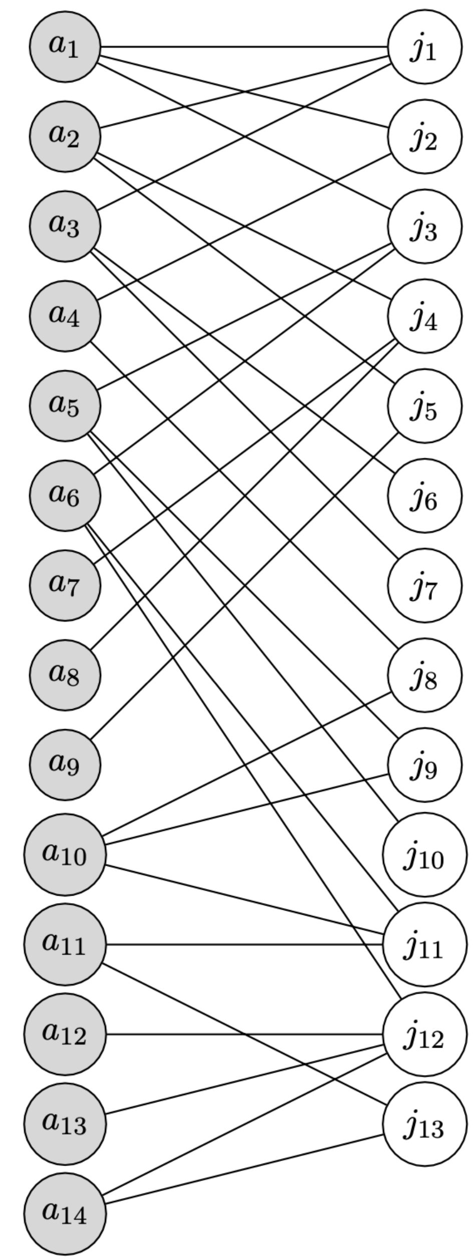

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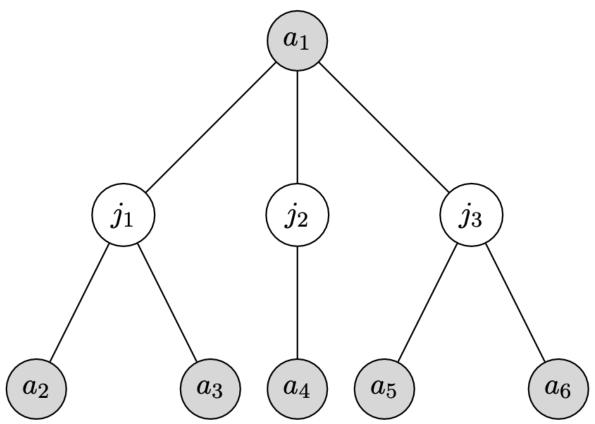

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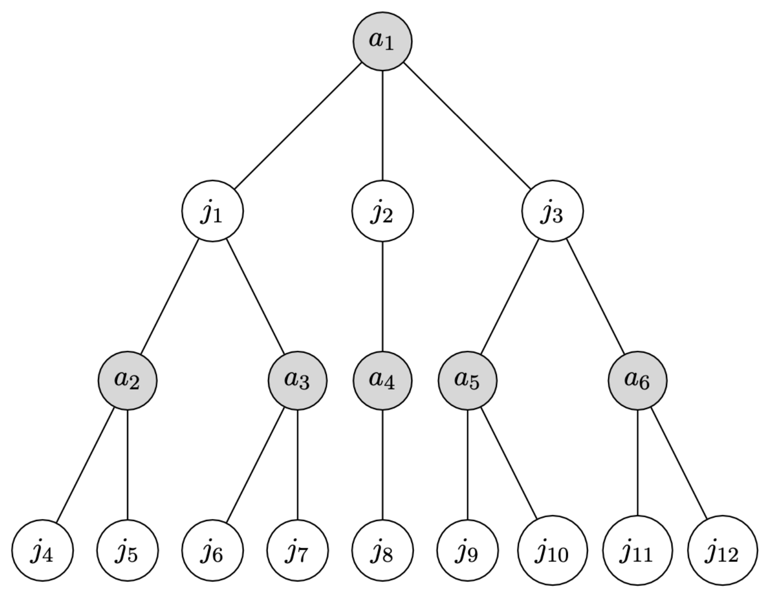

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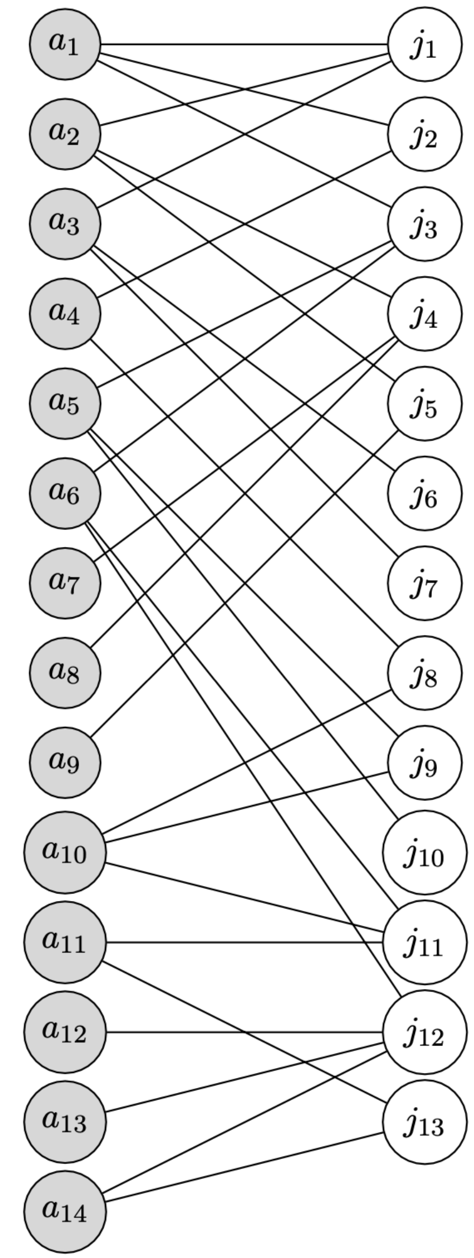

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$\Longrightarrow$ Focus on local neighborhood

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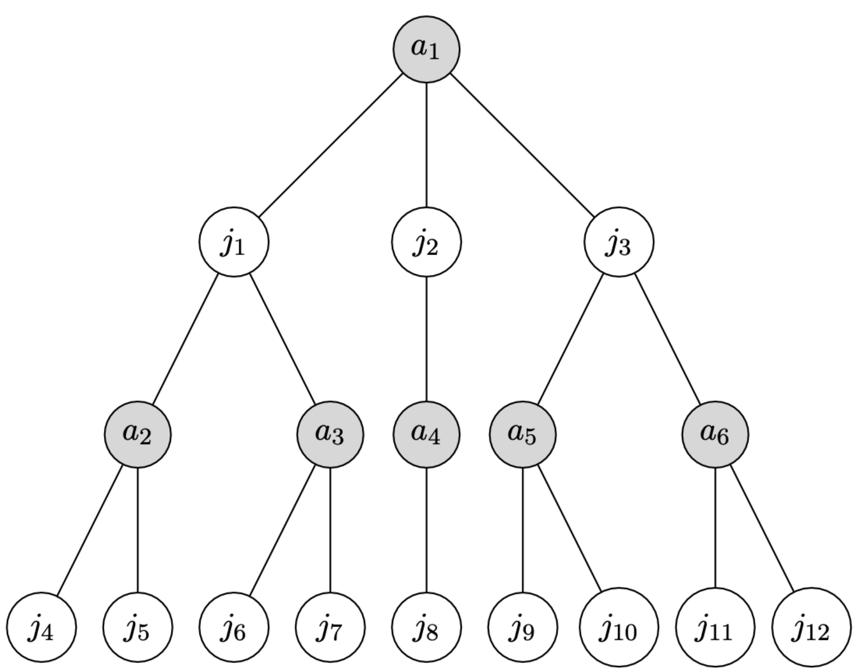

Hierarchical proposal-passing algorithm on tree

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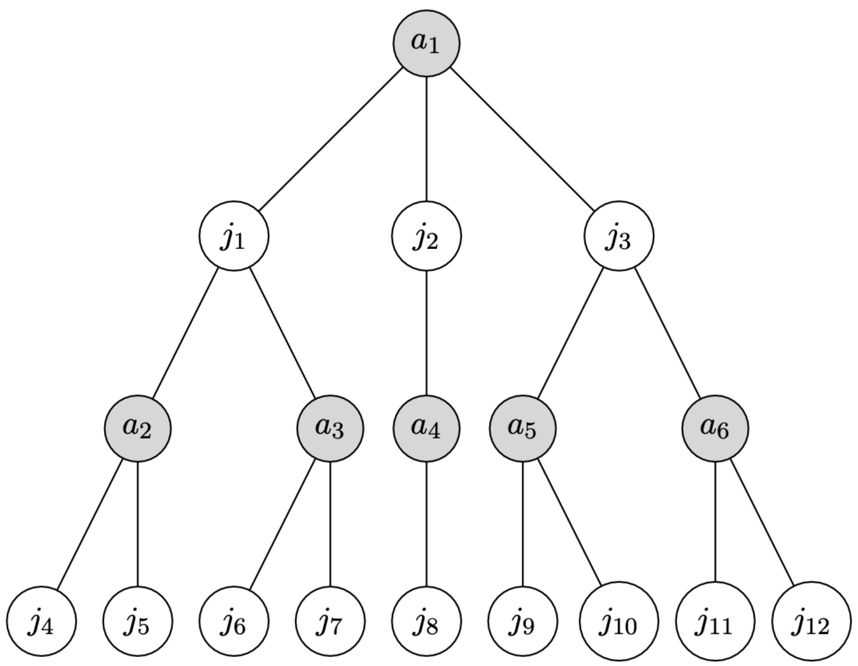

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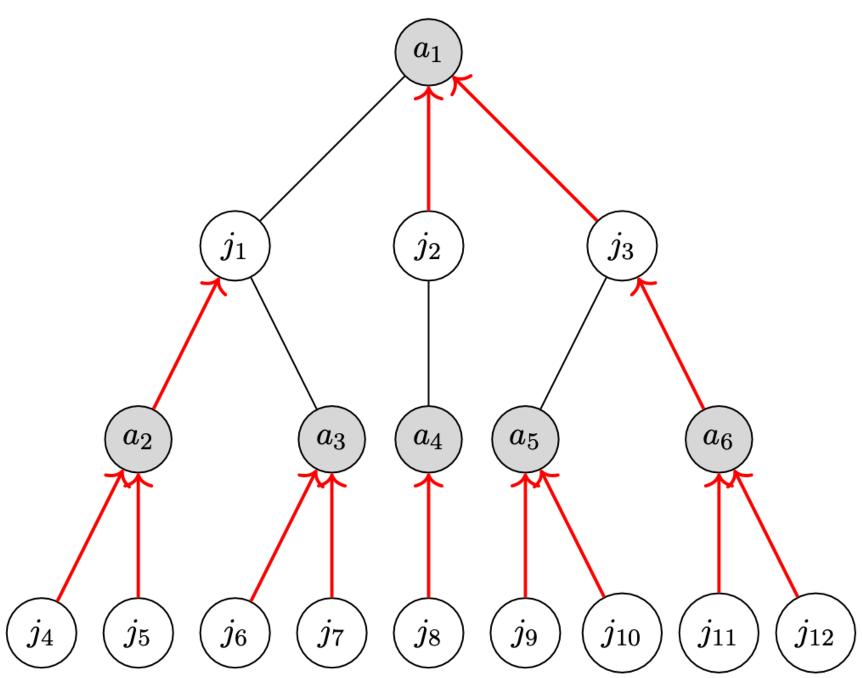

(a) The proposing phase.

Hierarchical proposal-passing algorithm on tree

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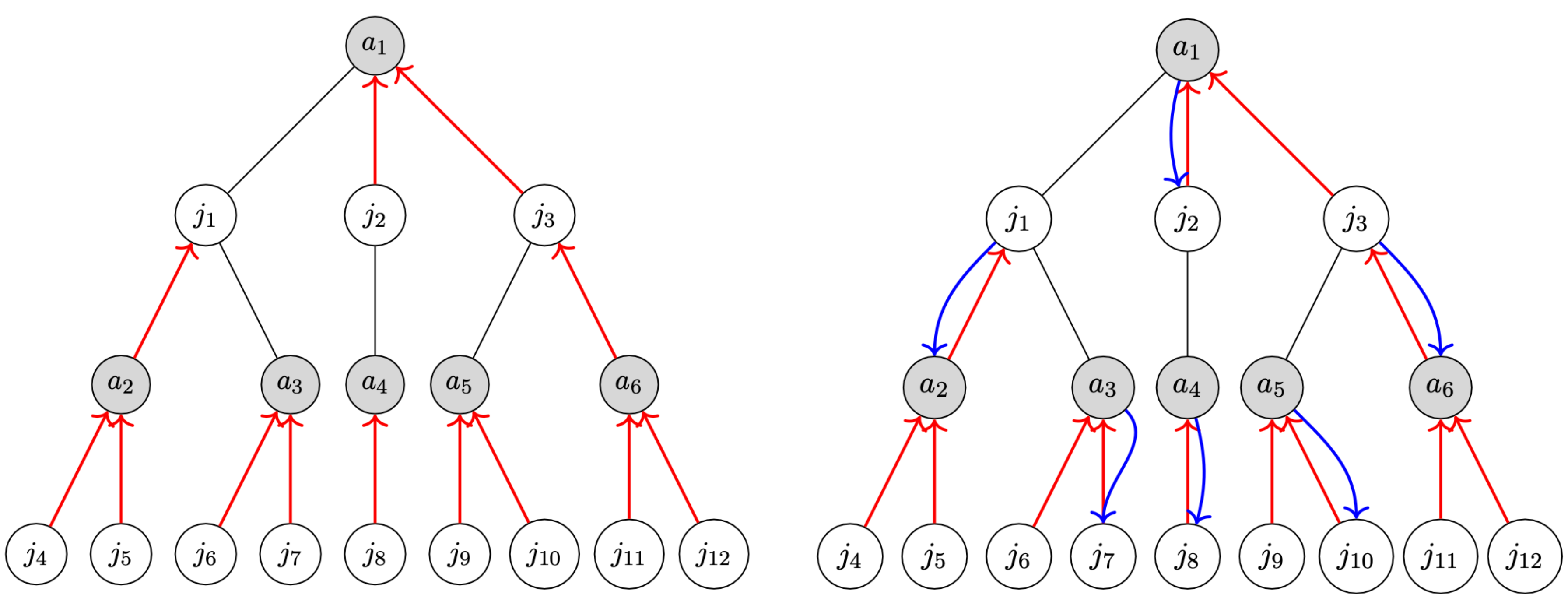

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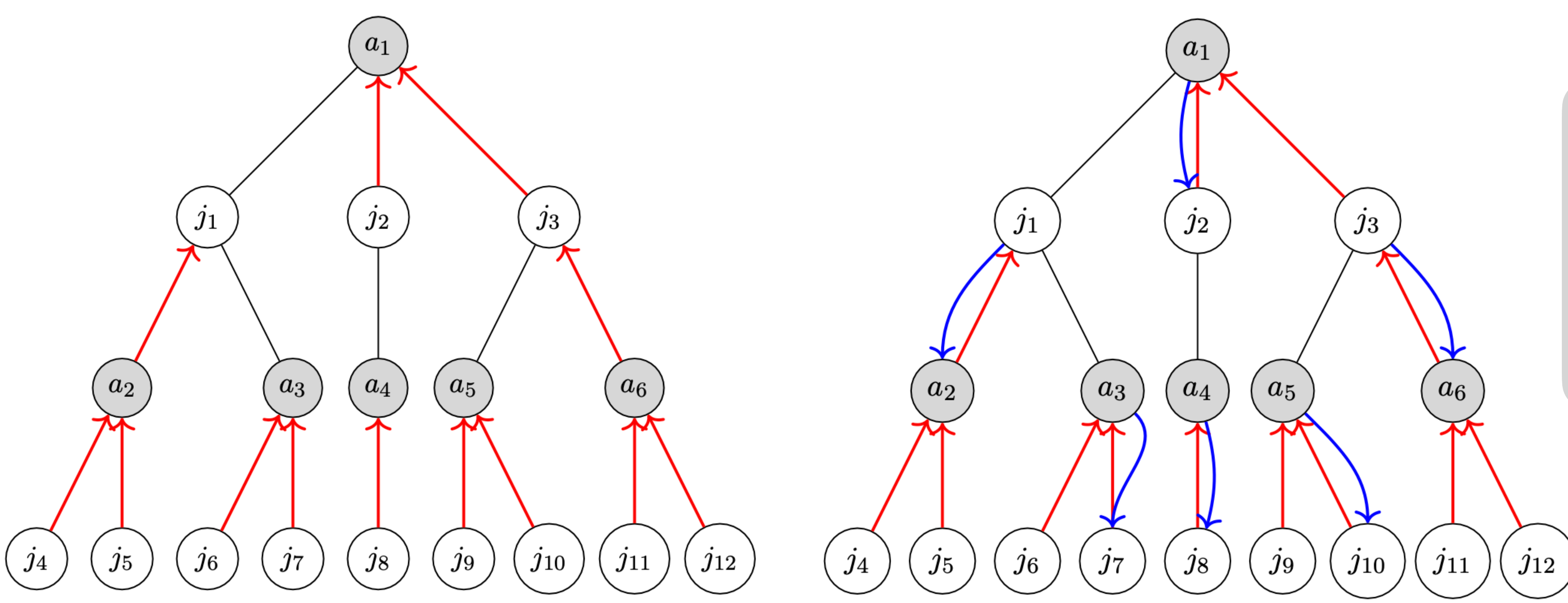

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Claim: If a node **proposes** to its parent, the proposing node must be **available** to its parent on the tree.

Hierarchical proposal-passing algorithm on tree

Step 3: message-passing on tree

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  - Suppose each node proposes to j with probability p
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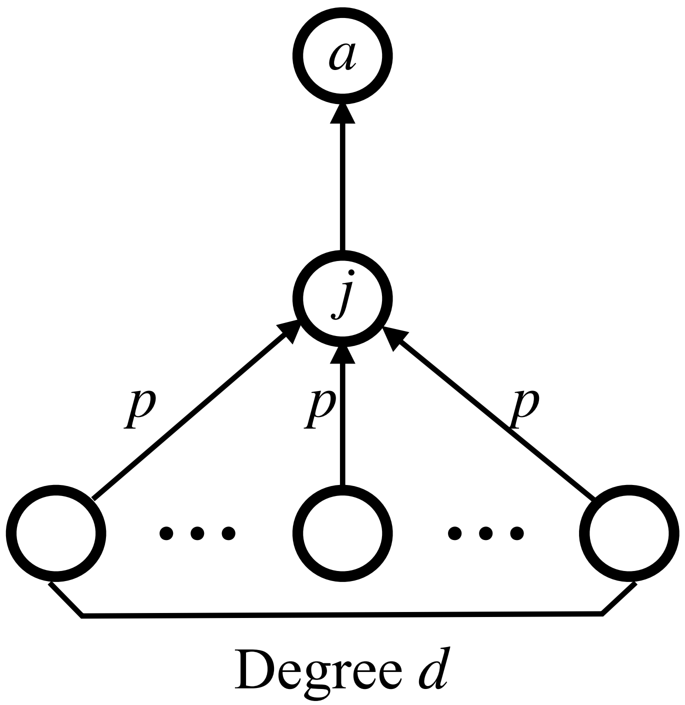

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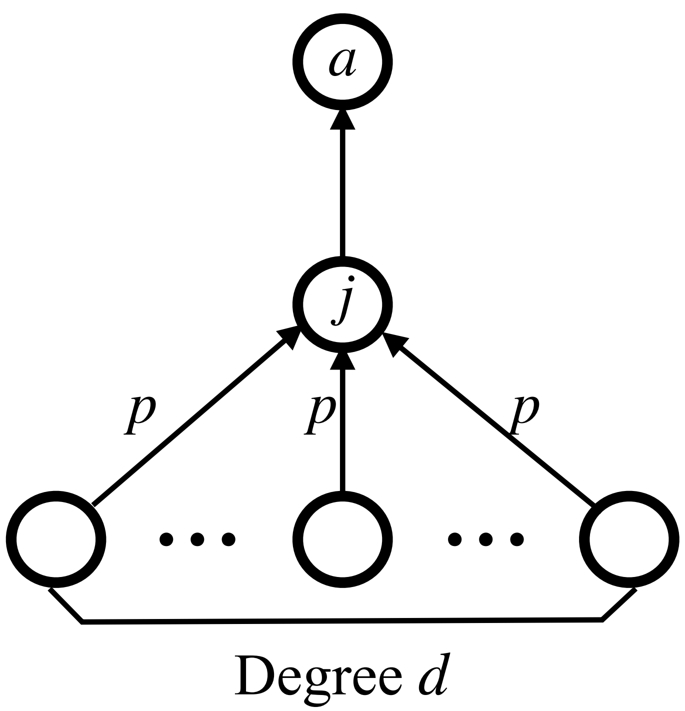

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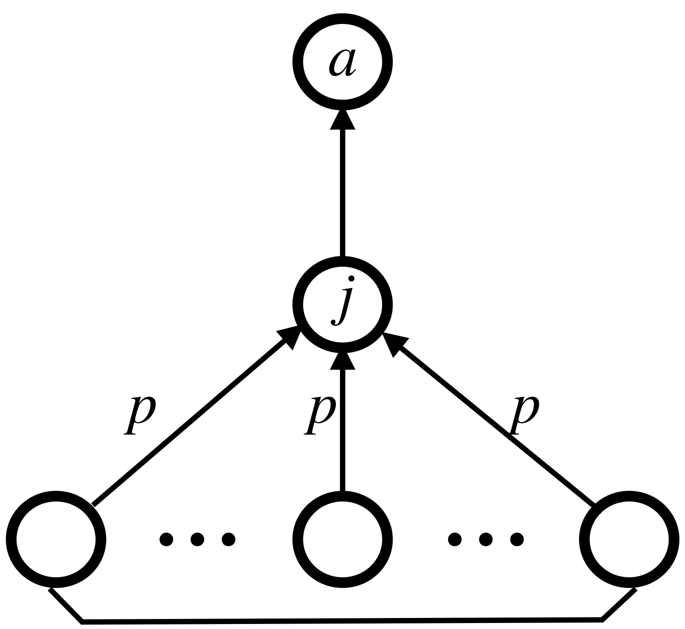

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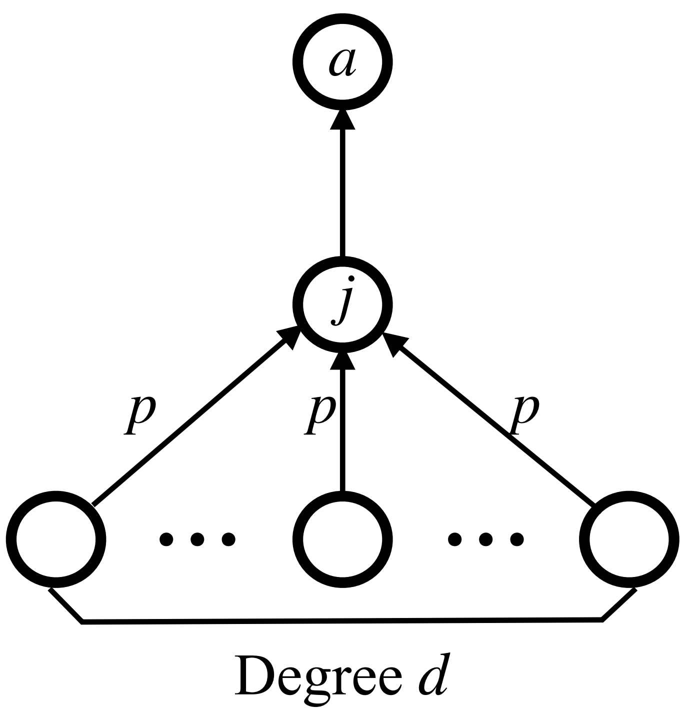

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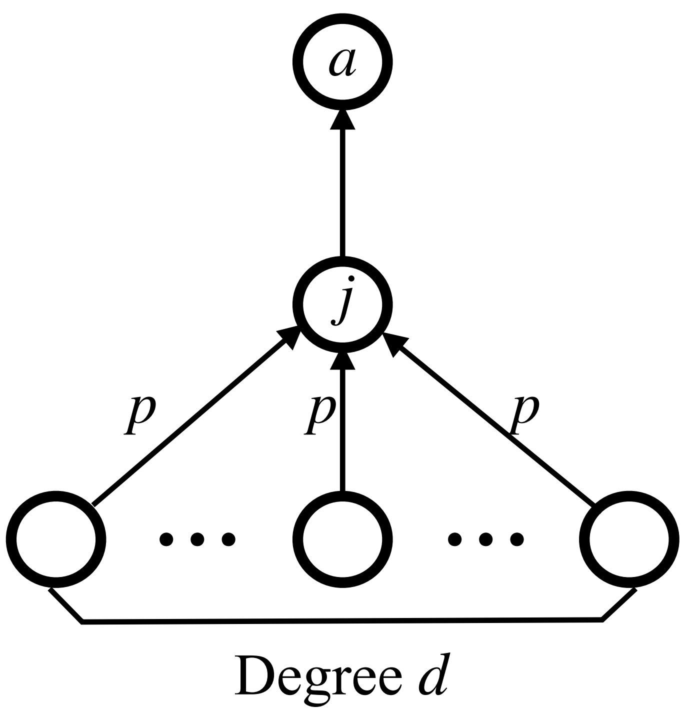

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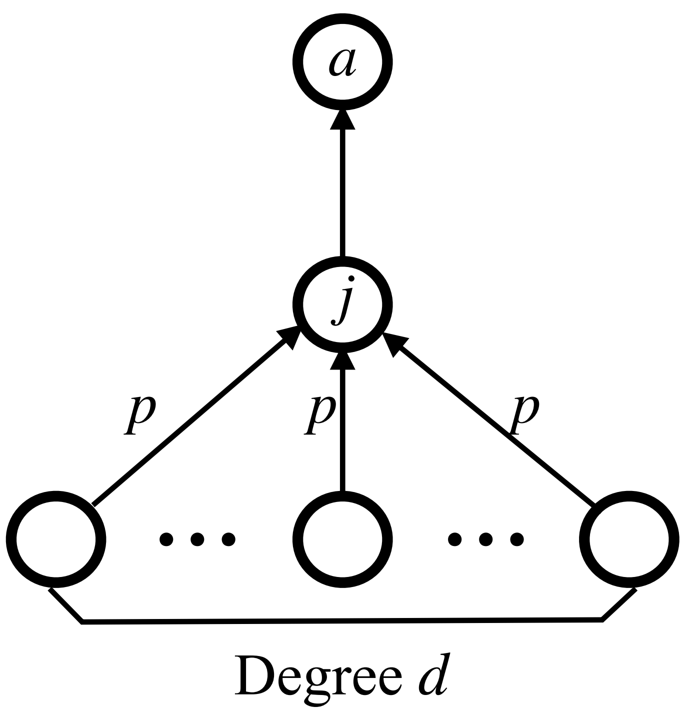

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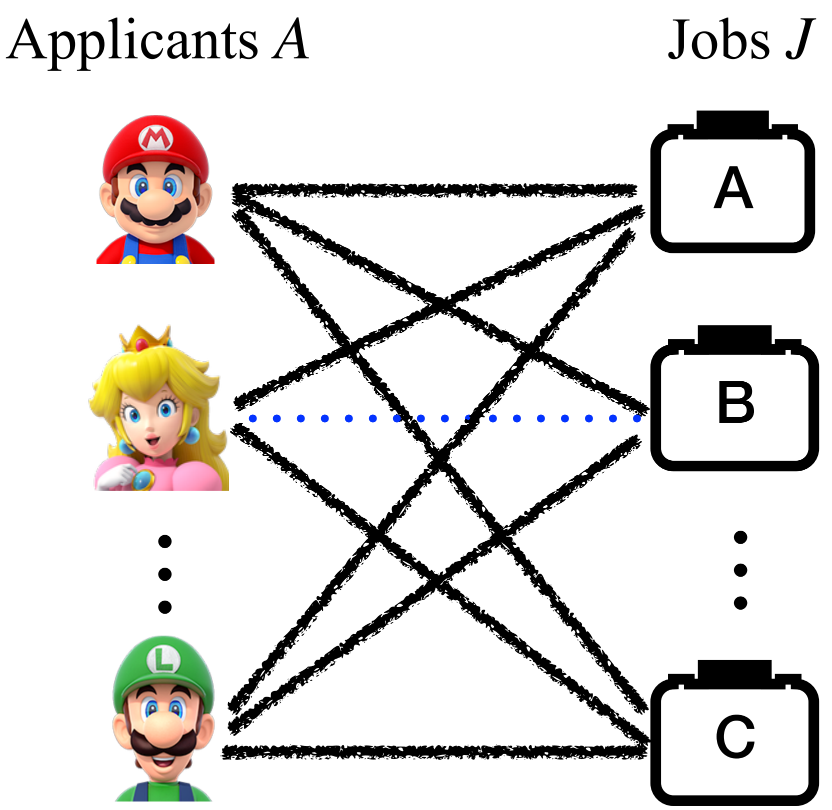
Main results on single-tiered market



$H \triangleq$ interview graph constructed by **both-side-signaling** with d

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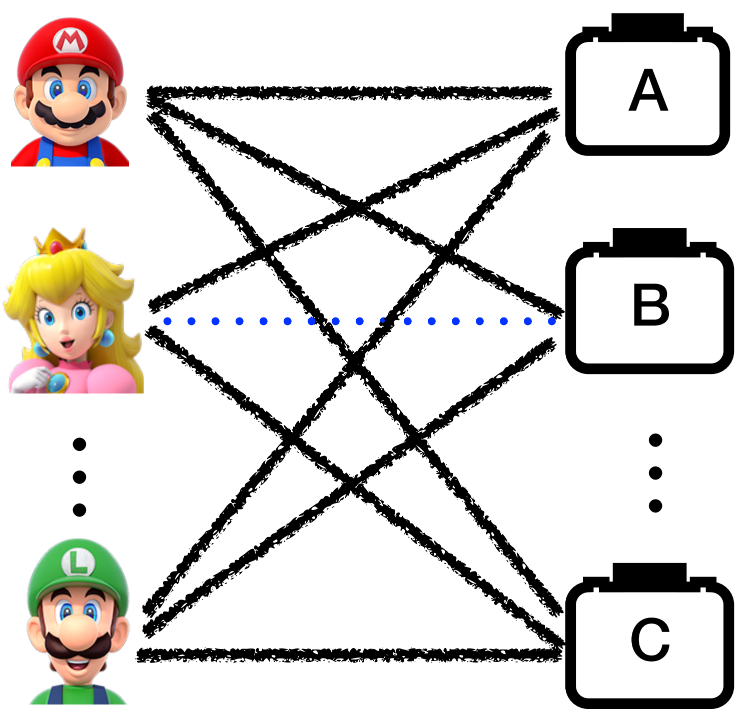
Applicants A Jobs J



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Consider balanced market: $|A| = |J|$

Theorem: sparse signaling regime: $\omega(1) \leq d \leq o(\log n)$

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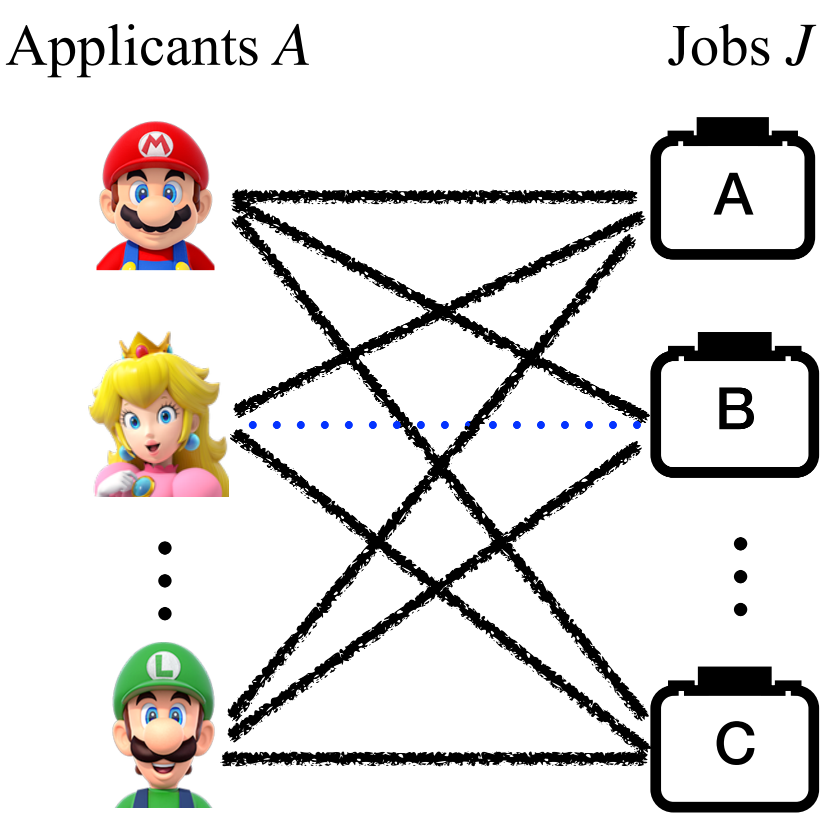

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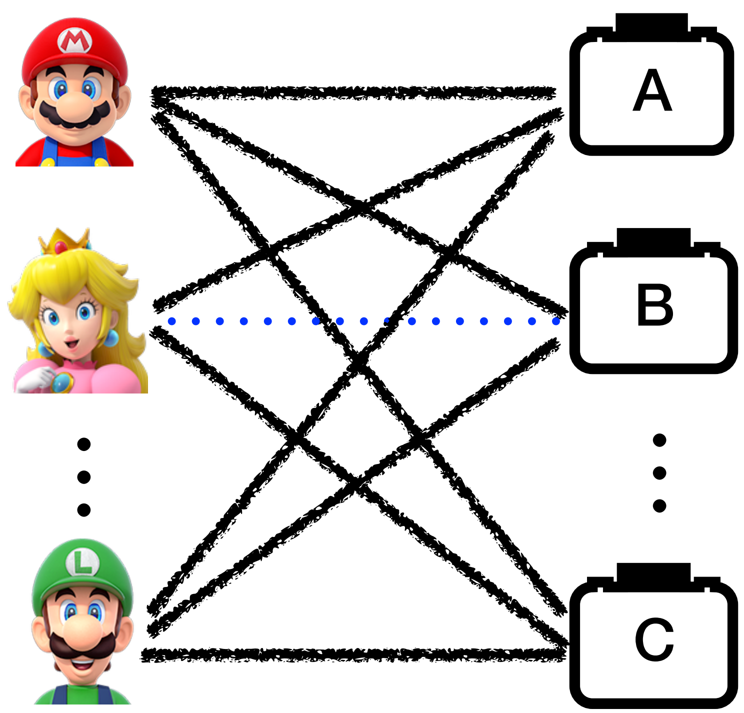

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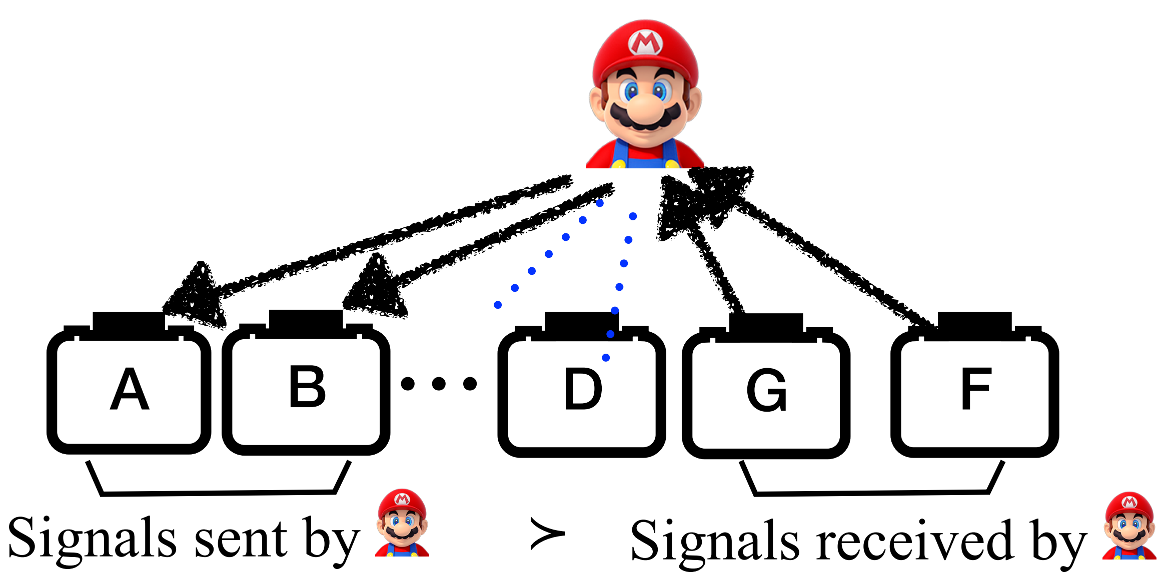

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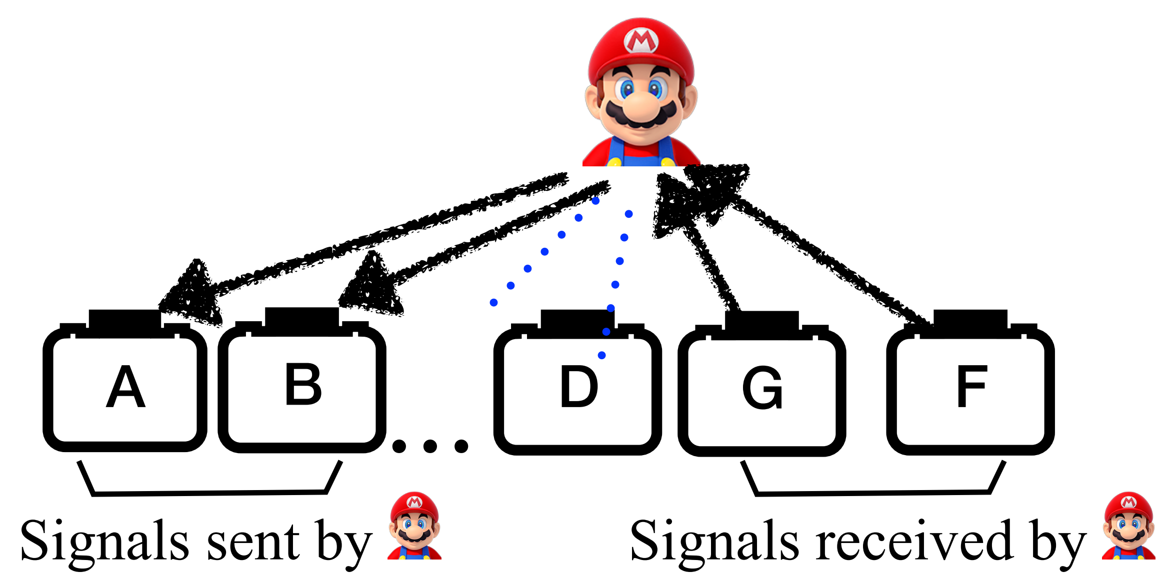
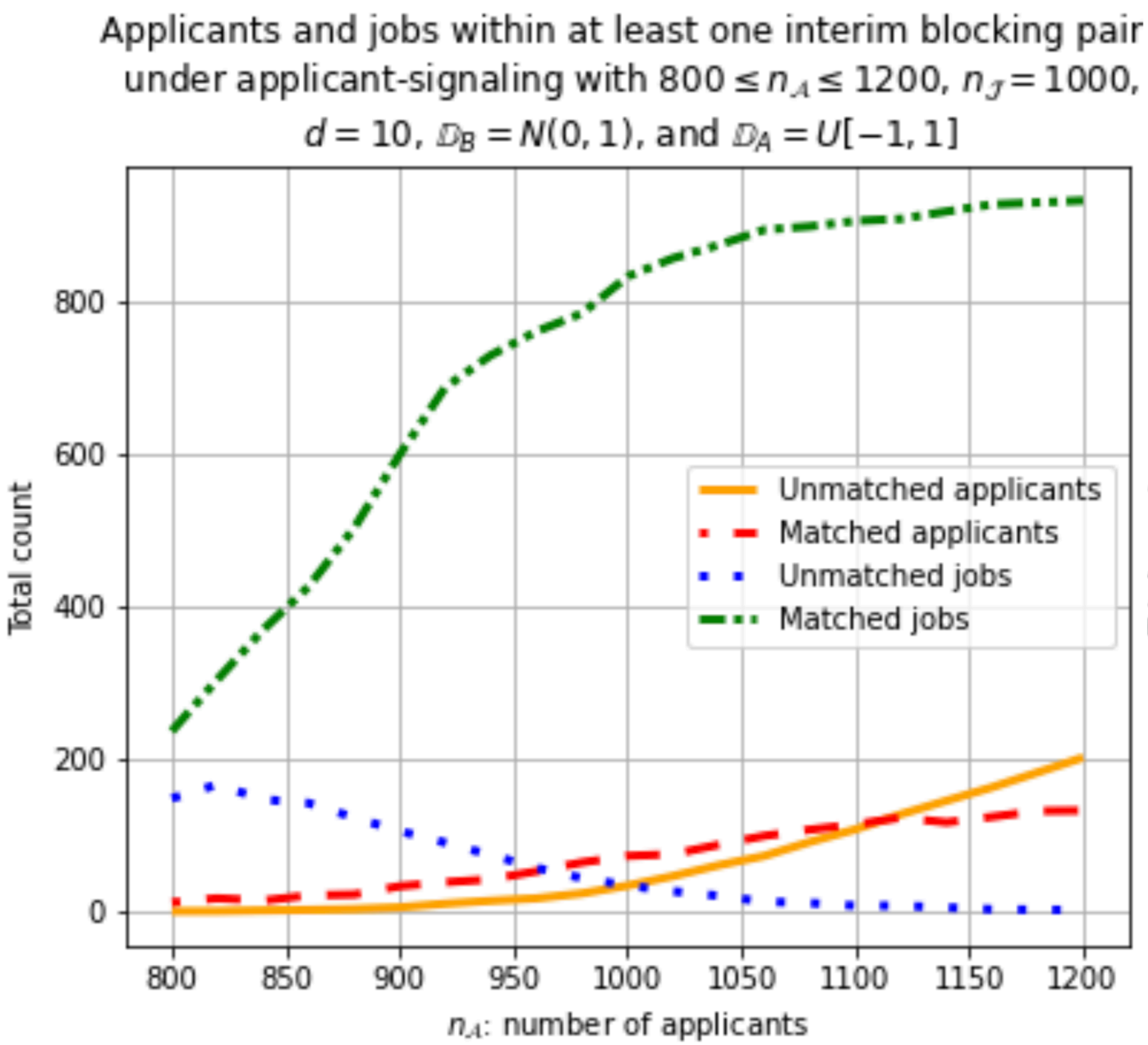
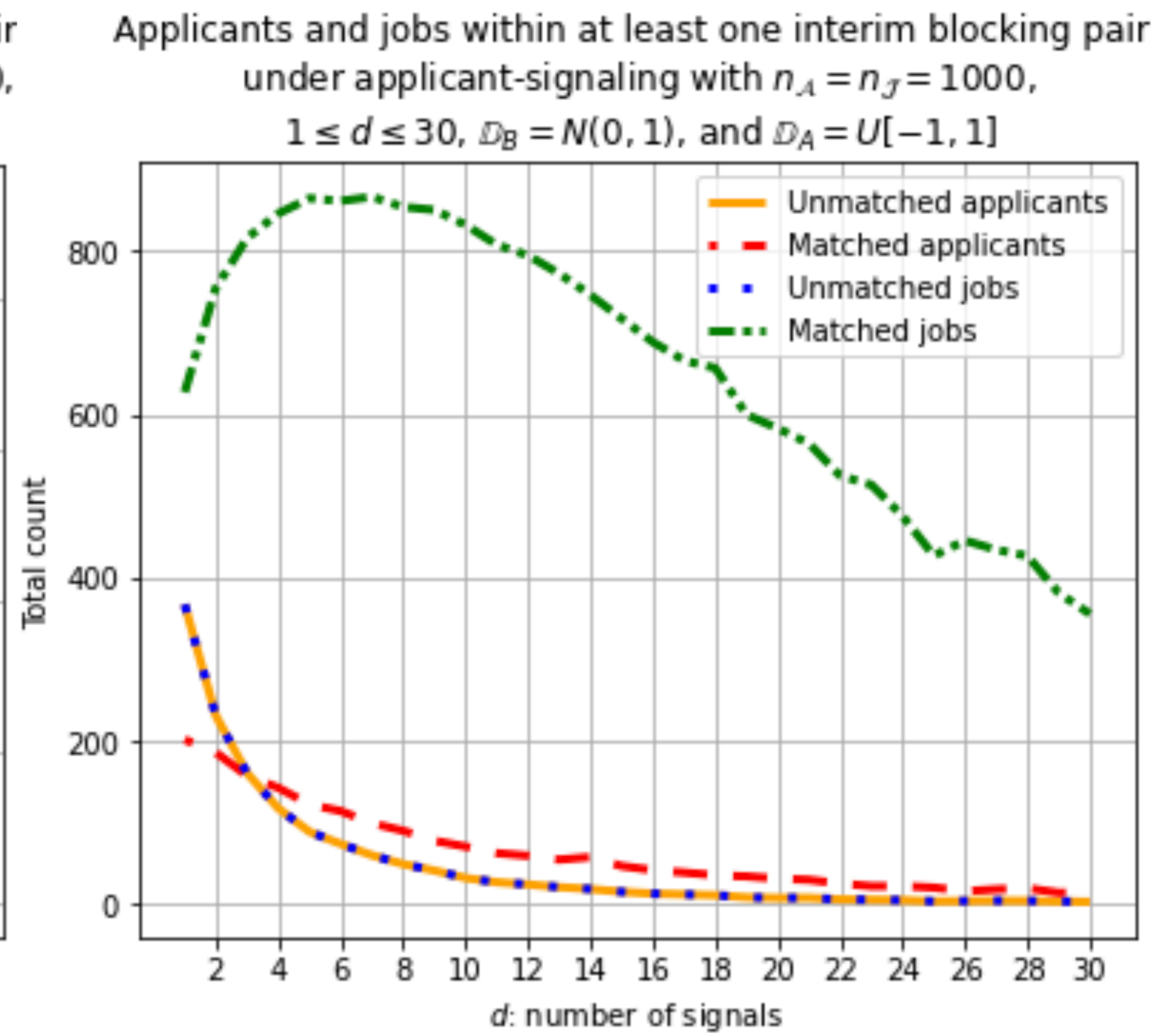


no pre-interview shocks (preference i.i.d.)

Simulation results





Applicant-signaling

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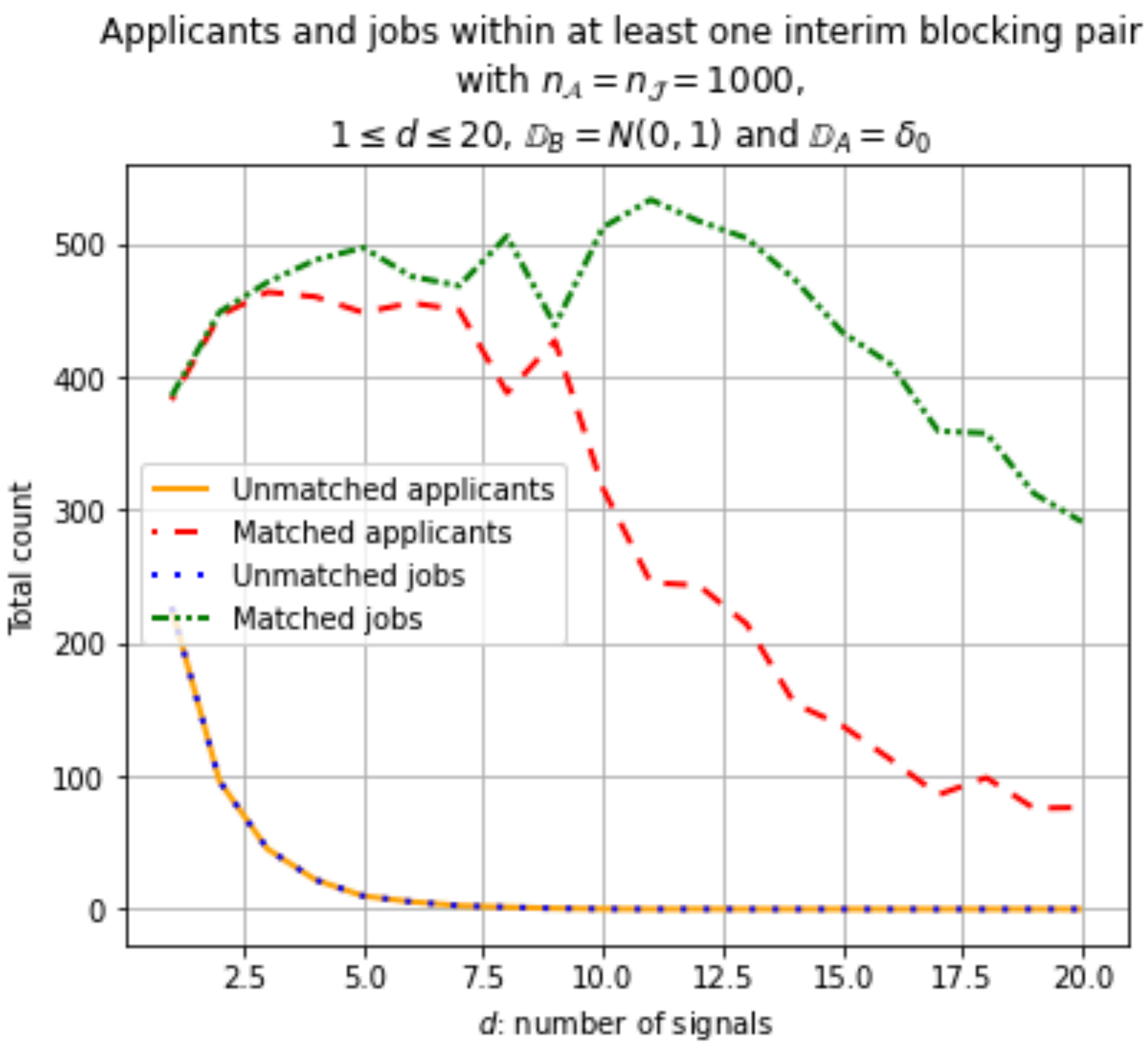

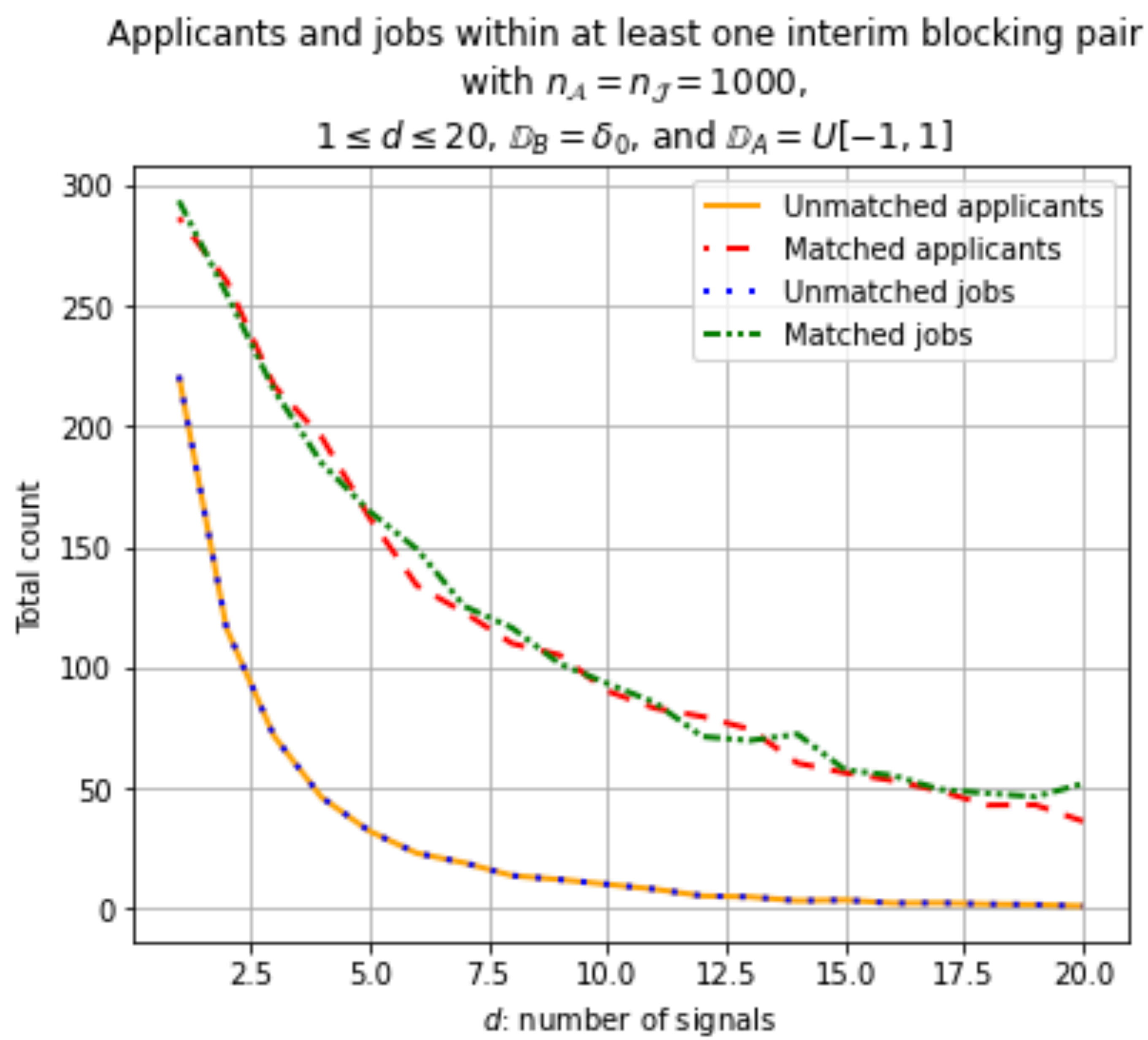

Both-side-signaling

Conclusion and open problems

- Conclusion
  - Study single-tiered and multi-tiered market
    - How signaling mechanism, market structure and number of signals impact on the achievement of almost interim stability and perfect interim stability.
  - Methodology:
    - Develop a **message-passing algorithm** that efficiently **determines interim stability** and match outcomes by leveraging their **local neighborhood** structure
- Open problems:
  - Vertical heterogeneity;
  - Sequential signaling;
  - Application of message-passing algorithm to real-world datasets;
  - Many-to-many matchings to construct interview graph;
  - Preferences generated from mallow distribution.
- Draft is available upon request (hysophie@upenn.edu)

Proof sketch for both-side signaling (sparse)

- Suppose there are no post-interview shocks
  - Given $d = o(\log n)$, the signals sent out by the agent is almost **disjoint** with the signals she receives.
  - Even after interview, agent **strictly prefer** the candidates she signals to $\succ$ candidates signals to her.
  - Suppose we run applicants proposing DA.
    - Phase 1: applicants first propose to the jobs that they signal to.

      Since $d = o(\log n)$, there will be **a non-negligible fraction of** agents remain unmatched.
    - Phase 2: unmatched applicant start to **propose to jobs that send signal to her**

      Jobs prefer the new proposals, since those are the signals sent out by them

      $\Longrightarrow$ triggers a long rejection chain

Incentive to deviate {
$\Longrightarrow$ constant fraction of applicants are matched with jobs that signal to them
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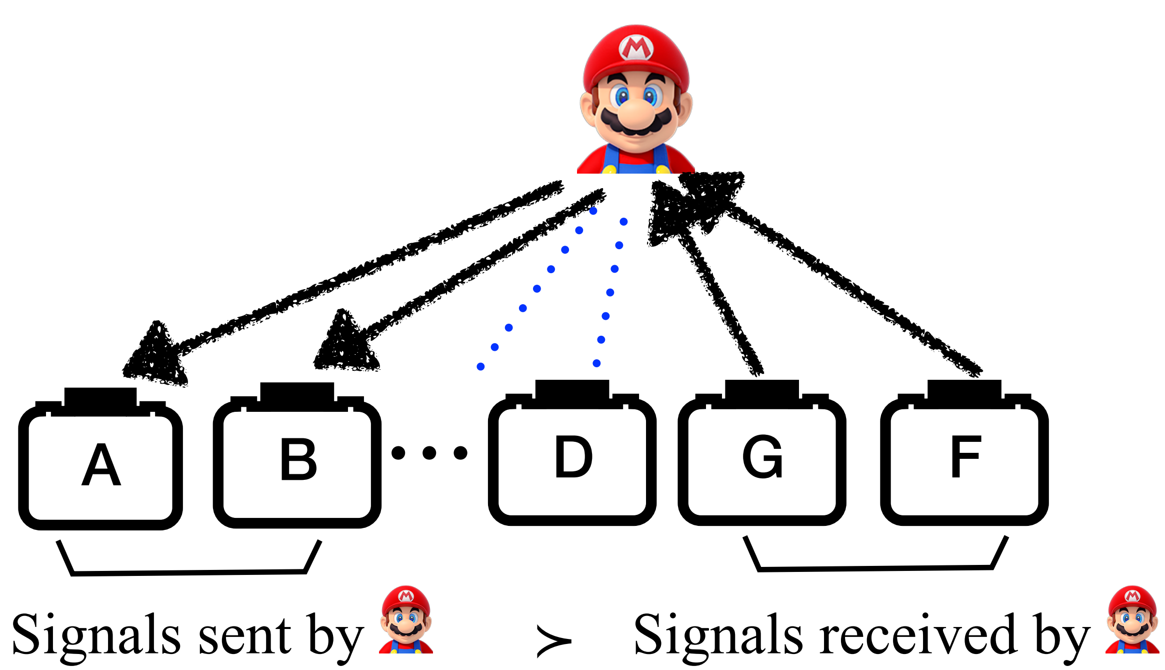

Proof sketch for sparse signaling regime

- Suppose there are no pre-interview shocks:
  - After interview, preferences over its neighbors are i.i.d. generated;
  - Similar as the one-side signaling.