Online Matching in Geometric Random Graphs

Thursday, September 26, 2024 10:30 AM (45 minutes)

We investigate online maximum cardinality matching, a central problem in ad allocation. In this problem, users are revealed sequentially, and each new user can be paired with any previously unmatched campaign that it is compatible with. Despite the limited theoretical guarantees, the greedy algorithm, which matches incoming users with any available campaign, exhibits outstanding performance in practice. Some theoretical support for this practical success was established in specific classes of graphs, where the connections between different vertices lack strong correlations - an assumption not always valid. To bridge this gap, we focus on the following model: both users and campaigns are represented as points uniformly distributed in the interval [0,1], and a user is eligible to be paired with a campaign if they are similar enough, i.e. the distance between their respective points is less than c/N, with c>0 a model parameter. As a benchmark, we determine the size of the optimal offline matching in these bipartite random geometric graphs. In the online setting and investigate the number of matches made by the online algorithm closest, which greedily pairs incoming points with their nearest available neighbors. We demonstrate that the algorithm's performance can be compared to its fluid limit, which is characterized as the solution to a specific partial differential equation (PDE). From this PDE solution, we can compute the competitive ratio of closest, and our computations reveal that it remains significantly better than its worst-case guarantee. This model turns out to be related to the online minimum cost matching problem, and we can extend the results to refine certain findings in that area of research. Specifically, we determine the exact asymptotic cost of closest in the ϵ -excess regime, providing a more accurate estimate than the previously known loose upper bound.

Presenter: SENTENAC, Flore (HEC Paris)

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