Simulation Is All You Need

A Unifying algorithm for Network Revenue Management (NRM) and Dynamic Spatial Matching (DSM)

by Yash Kanoria (Columbia) on

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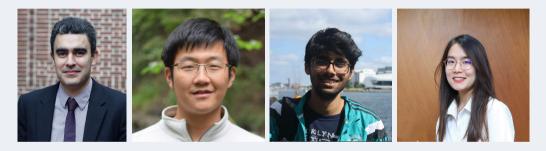
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» Collaborators



Omar Besbes Columbia DRO

Yilun Chen CUHK Shenzhen

Akshit Kumar Columbia DRO Wenxin Zhang Columbia DRO

| Motivation and Research Questions | | | | |
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» Network Revenue Management: Online Allocation with Resource Constraints



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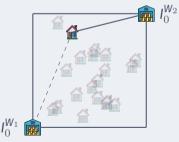


Budget Management in Ad Auctions



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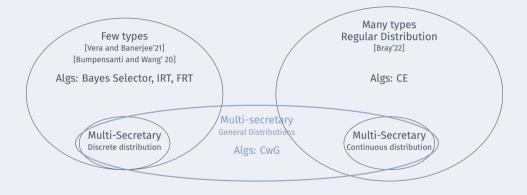






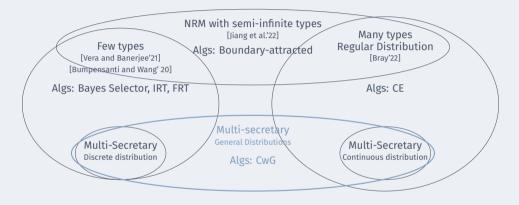






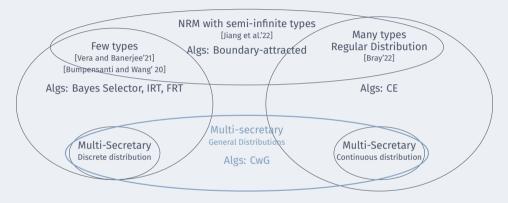
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» Landscape of NRM Problems

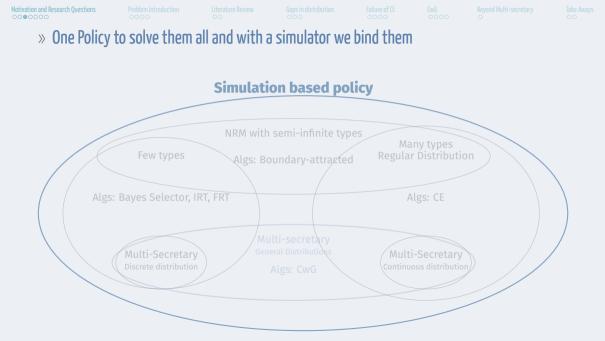


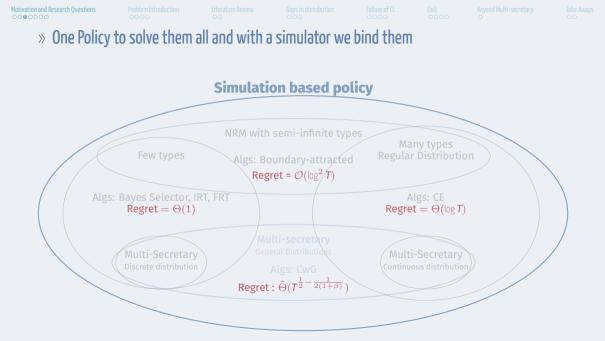
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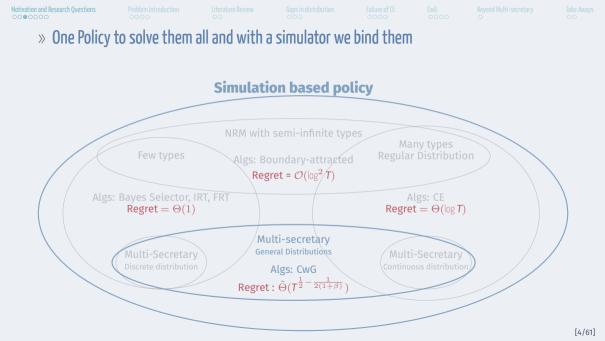
» Landscape of NRM Problems



Research Question: One policy to solve them all?







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Take Aways

» Dynamic Spatial Matching (DSM): Motivation

- * Ridehailing: spatial matching in two dimensions
- * Matching platforms
 - * Lodging e.g. Airbnb: supply and demand live in a multi-dimensional space (location, size, amenities, price, etc.)
 - * Labor e.g. Upwork: (expertise dimensions, price, duration, etc.)
- Network revenue management with a large number of demand types

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Take Aways

» DSM: Setting and Research Questions

- * Supply and demand which live in *d* dimensional space.
- * Cost of match distance between the matched pair.
- * *T* supply units are present beforehand.
- Demand arrives sequentially. Needs to be matched immediately with a supply unit.

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Take Aways

» DSM: Setting and Research Questions

- * Supply and demand which live in *d* dimensional space.
- * Cost of match distance between the matched pair.
- * *T* supply units are present beforehand.
- Demand arrives sequentially. Needs to be matched immediately with a supply unit.
- * How to match demand and supply to minimize spatial costs of matching under dynamic arrivals?
- * How large are the costs arising from spatial heterogeneity and uncertainty about the future in dynamic matching

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Take Away

» Summary of findings and talk outline

- $\ast~$ DSM with identical supply and demand distributions [K.]
 - Greedy matching suffices
 - $*\,$ Match distance \sim Nearest-neighbor-distance achievable, except one case
- * DSM with different supply and demand distributions [Chen, Akshit Kumar, K., Zhang]
 - * Greedy fails
 - * Simulate-Optimize-Assign-Repeat (SOAR) is near optimal
- Multisecretary problem with lumpy value distribution (a 1d DSM problem) [Besbes, Akshit Kumar, K.]
 - * The Certainty Equivalent policy and SOAR with one sample path fail
 - RAMS with multiple sample paths achieves optimal regret scaling
 - * RAMS works also for $d \geq 2$, and across NRM settings.

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Take Away:

» Talk outline

- DSM with identical supply and demand distributions
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 [O. Besbes, Akshit Kumar & K. '22]
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» Multi-secretary Problem

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Problem Statement

Given a sequence of *T* secretaries and a hiring budget *B*, a decision maker (DM) wants to hire the top *B* secretaries in terms of their ability.

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Take Away

» Multi-secretary Problem

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Details and Assumptions

* The secretaries arrive in an online fashion.

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Note: This is a 1d DSM problem, with an atomic "supply" distribution with B units at 1 and T - B units at 0.

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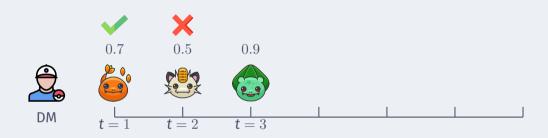


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DM t = 1 t = 2 t = 3 t = 4 t = 5



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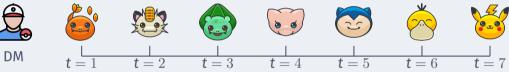


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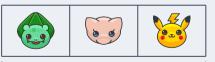




Online Policy

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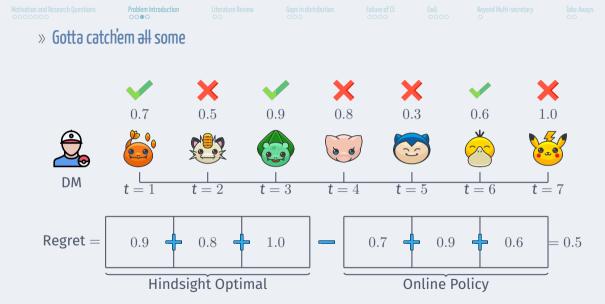
Hindsight Optimal

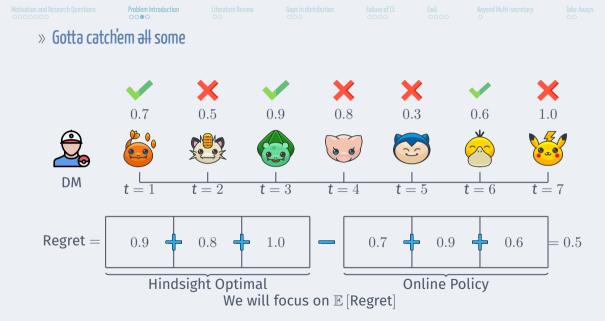


Online Policy

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| DM | t = 1 | t=2 | $\frac{1}{t=3}$ | $\frac{1}{t=4}$ | $\frac{1}{t=5}$ | t = 0 | 5 t = 7 |





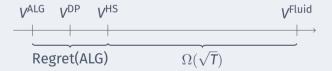




Multisecretary is a 1d DSM problem, with an atomic "supply" distribution with B units at 1 and T - B units at 0. $\Theta(\sqrt{T})$ optimal regret wrt fluid benchmark, which can be achieved by a trivial static policy.

Gap between fluid and hindsight benchmarks is already $\Omega(\sqrt{T})$.

As in the recent NRM literature, we adopt the tighter hindsight benchmark.

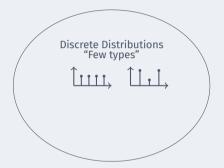


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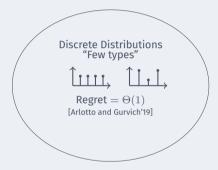
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Discrete Distributions "Few types"

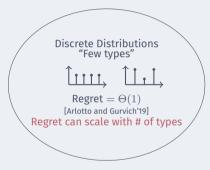
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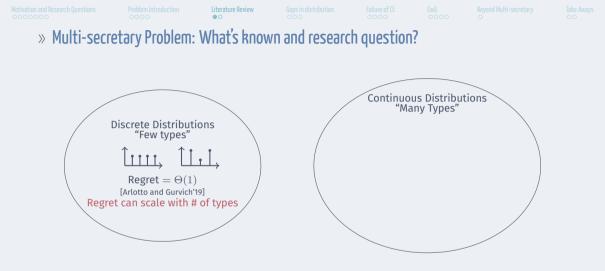


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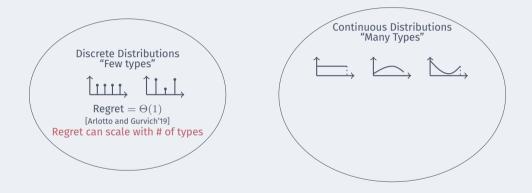


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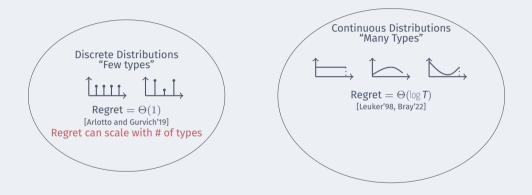




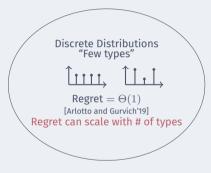


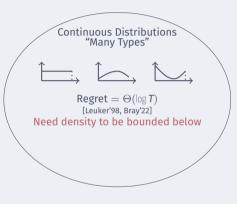


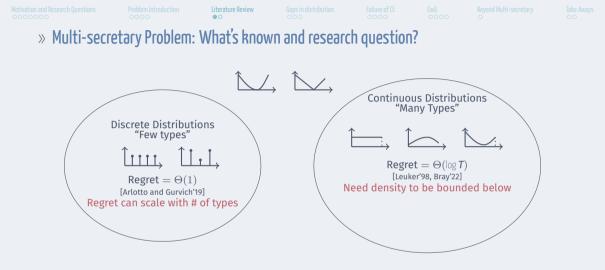


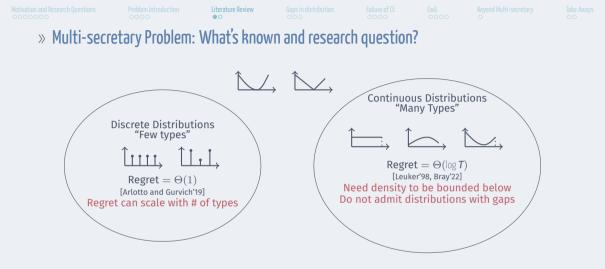


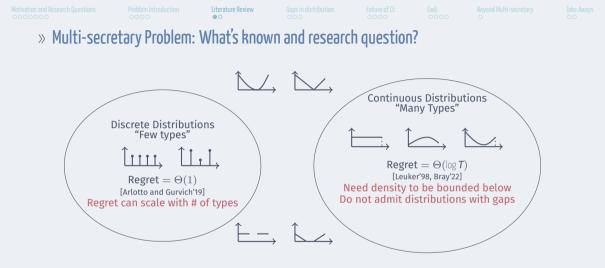


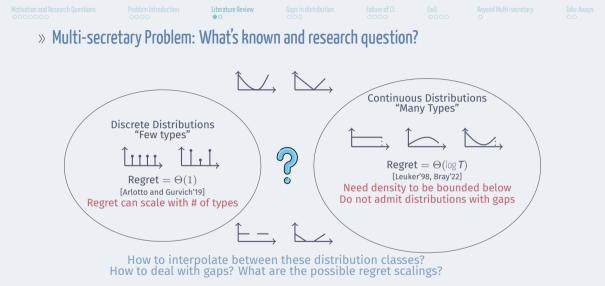












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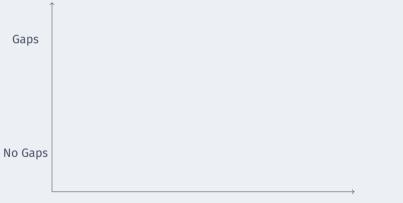
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» Punchline for the Multi-secretary Problem





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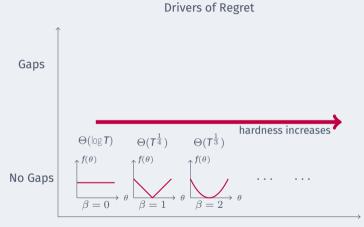
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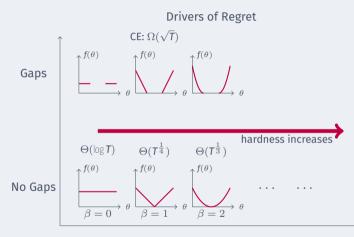
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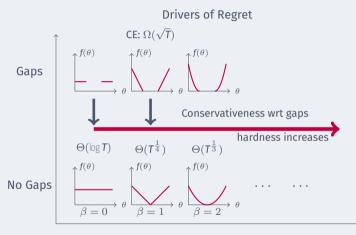
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Take Away:

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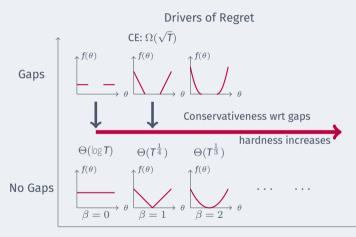
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Take Aways

» Punchline for the Multi-secretary Problem



- * Distribution shape is a **fundamental** driver of regret.
- Dealing with gaps is an algorithmic challenge.
- Novel Principle: Conservativeness wrt gaps (CwG)
- Simulation-based approach automatically pursues CwG

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Gaps in distribution

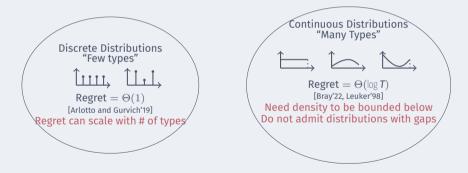
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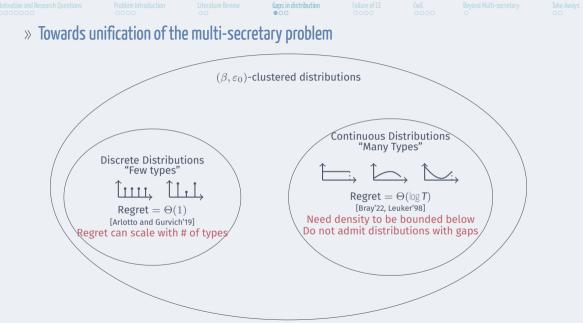
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Take Aways

» Towards unification of the multi-secretary problem





| | Gaps in distribution ○●○ | | |
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» Fundamental Limits

Universal Lower Bound

For every $\beta \in [0,\infty)$, there exists a distribution F_{eta} such that

$$\sup_{B \in [T]} \mathbb{E}_{F_{\beta}} \left[\mathsf{Regret}(\mathsf{DP}) \right] = \begin{cases} \Omega \left(\log T \right), & \beta = 0, \\ \Omega \left(T^{\frac{1}{2} - \frac{1}{2(1+\beta)}} \right), & \beta > 0. \end{cases}$$

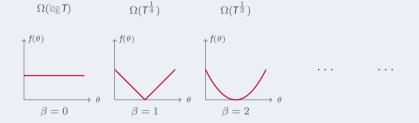
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Take Away

For $(m{eta}, \mathbf{1})$ -clustered distributions

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- Let B_t be the remaining budget at time t
- Compute the budget ratio

» Certainty Equivalent Control

- $br_t = rac{\text{Remaining Budget}}{\text{Remaining Time}} = rac{B_t}{T-t}$
- $\ast\,$ Define a quantile threshold $p_t^{ce}=1-br_t$
- * Define a ability threshold $\gamma_t^{ce} = \mathit{F}^{-1}(\mathit{p}_t^{ce})$
- * hire \iff $heta_t \geq \gamma_t^{ce}$

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» Certainty Equivalent Control

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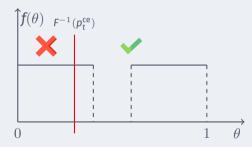
For Bi-modal Uniform Distribution

Let B_t be the remaining budget at time t

$$\mathsf{Budget}\ \mathsf{Ratio} = rac{\mathsf{Remaining}\ \mathsf{Budget}}{\mathsf{Remaining}\ \mathsf{Time}} = rac{B_t}{T-t}$$

CE Quantile Threshold
$$= 1 - \frac{B_t}{T-t} \triangleq p_t^{ce}$$

Decision: hire $\iff \theta_t \ge F^{-1}(p_t^{ce})$



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Regret Lower Bound

» Failure of Certainty Equivalent Control

Insufficiency of Certainty Equivalent Control

Assume that $F = \text{Unif}([0, \frac{1}{4}] \cup [\frac{3}{4}, 1])$, for B = T/2, we have

 $\mathbb{E}\left[\mathsf{Regret}(\mathsf{CE})\right] = \Omega\left(\sqrt{\mathsf{T}}\right)$

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Regret Lower Bound

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Insufficiency of Certainty Equivalent Control

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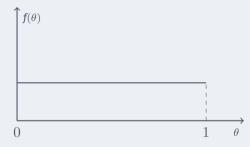
 $\mathbb{E}\left[\mathsf{Regret}(\mathsf{CE})\right] = \Omega\left(\sqrt{\mathsf{T}}\right)$

Remark

* Same scaling is achievable under a static threshold policy.

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» Why does CE fail?



Literature Review

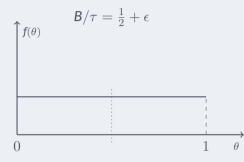
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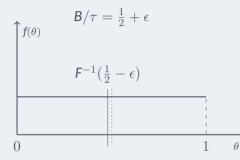
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Take Aways

» Why does CE fail?



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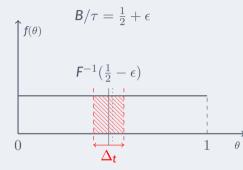
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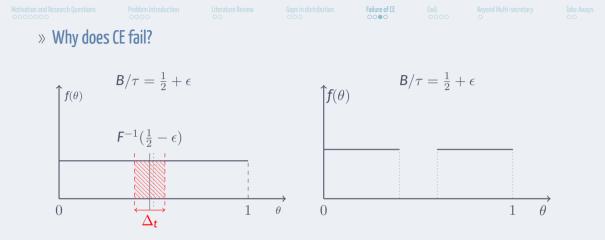
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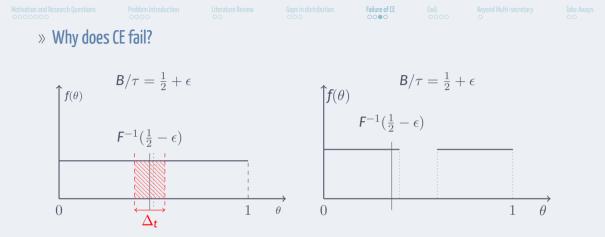
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Take Aways

» Why does CE fail?









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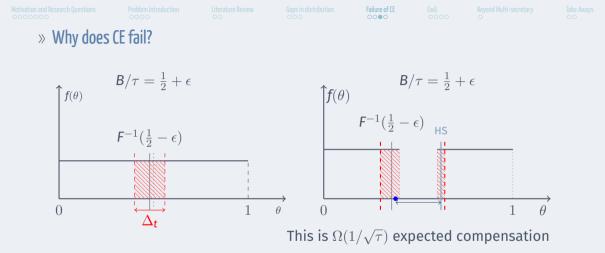


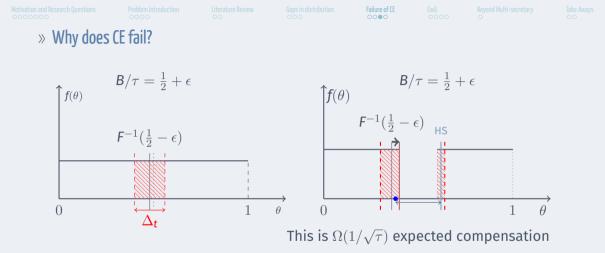
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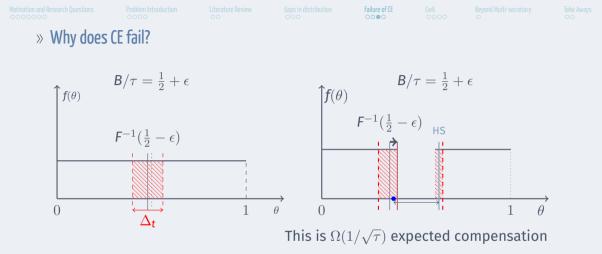
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 $F^{-1}(\frac{1}{2}-\epsilon)$

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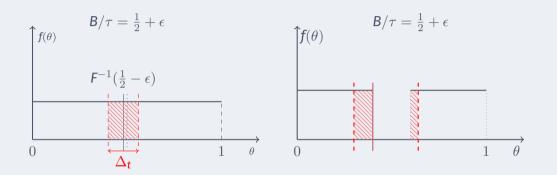




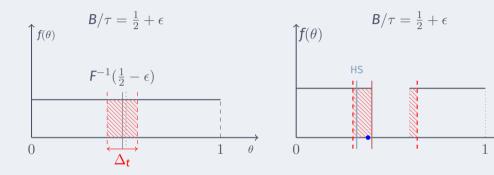


Conservativeness wrt gaps

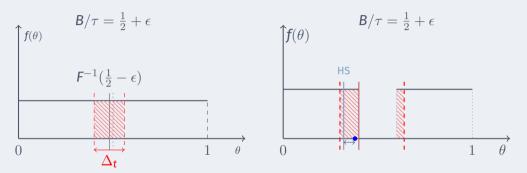












This is $\tilde{\mathcal{O}}(1/\tau)$ expected compensation

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Take Aways

» Good in theory but practically infeasible

- * What is the conservativeness parameter I should use?
- * How to find where these gaps are? What happens if gaps shift?
- * E.g., no chance of deploying for Amazon's fulfillment problem

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Take Aways

» Conservativeness with respect to gaps

Algorithmic Idea: Simulate into the future



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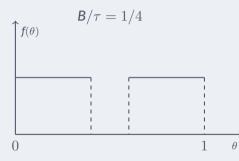
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» Conservativeness with respect to gaps



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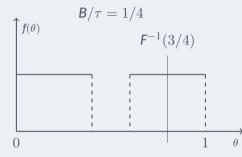
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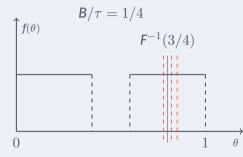
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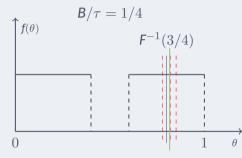
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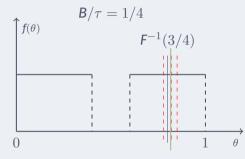
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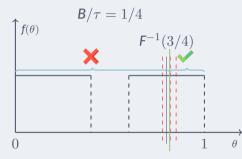
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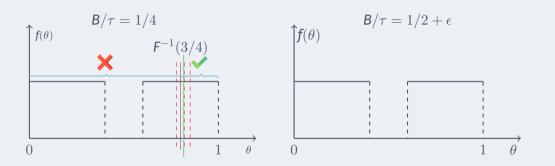


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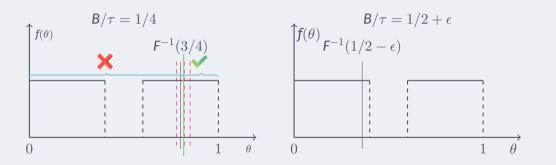
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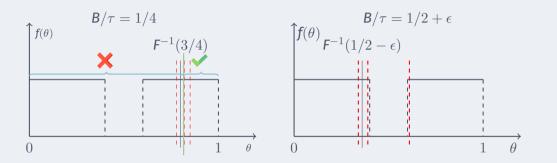
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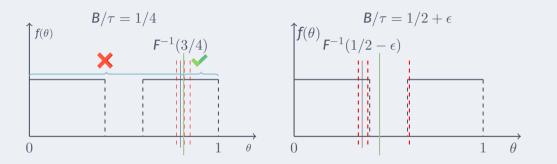
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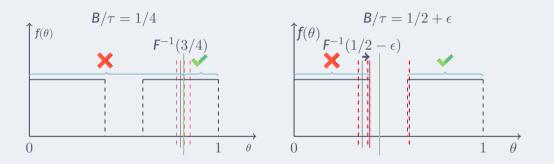
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Take Aways

» Conservativeness with respect to gaps

Algorithmic Idea: Simulate into the future



If far from a gap, use the CE threshold If close to gap, use the gap as threshold

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Take Away

Punchline

» Conservativeness with respect to gaps

Regret of RAMS Policy

If F is a $(\beta,\varepsilon_0)\text{-clustered}$ distribution, then

$$\mathbb{E}\left[\mathsf{Regret}(\mathsf{RAMS})\right] = \begin{cases} \mathcal{O}\left((\log T)^2\right), & \beta = 0, \\ \mathcal{O}\left(\mathsf{poly}(\log T)T^{\frac{1}{2} - \frac{1}{2(1+\beta)}}\right), & \beta > 0 \end{cases}$$

If *F* is a discrete distribution, $\mathbb{E}\left[\mathsf{Regret}\left(\mathsf{RAMS}\right)\right] = \mathcal{O}(1/\varepsilon_0)$

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Punchline

» Conservativeness with respect to gaps

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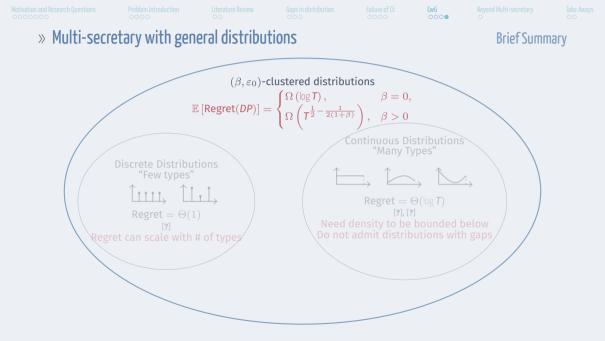
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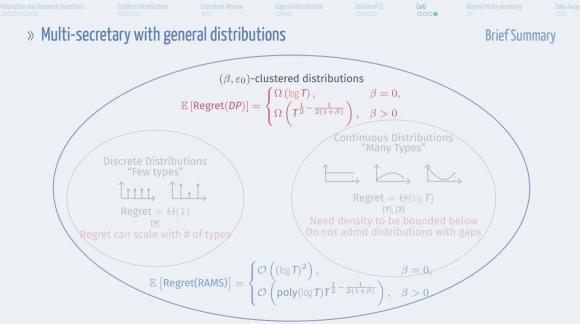
If F is a discrete distribution, $\mathbb{E}\left[\mathsf{Regret}\left(\mathsf{RAMS}\right)\right] = \mathcal{O}(1/\varepsilon_0)$

Remark

- * $F = \text{Unif}([0, \frac{1}{4}] \cup [\frac{3}{4}, 1])$, RAMS ($\mathcal{O}((\log T)^2)$) outperforms CE ($\Omega(\sqrt{T})$).
- $\ast\,$ Matches the universal lower bound upto polylog factors $\,\Rightarrow\,$ RAMS is near-optimal.









» One Policy to solve them all?

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- * The multi-secretary problem is special but RAMS is general: in each period, simulate several futures and choose the action which minimizes the expected "compensation" in hindsight. Compensation \equiv How to much we need to pay an agent who knows the future to take a particular action, for a given future.
- * Can be applied to NRM and stochastic online matching problems to recover almost all known guarantees in the literature.

n **Introduction**

Literature Review

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Take Away

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» One Policy to solve them all?

Proposition (RAMS is as good as any algorithm)

Given an NRM setting P, consider *any* algorithm A for P, such that with τ periods remaining, uniformly over the state, the expected compensation under A is bounded above by $\delta_{\tau}(A)$. Then RAMS achieves an expected compensation bounded uniformly by $\delta_{\tau} + 1/\tau^{1.1}$. As a result the regret of RAMS is bounded above by a constant plus the regret guarantee for algorithm A,

$$ext{Regret(RAMS)} \leq ext{Constant} + \sum_{ au=1}^{ au} \delta_{ au}(\mathsf{A}) \,.$$

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Take Aways

» What to take away from this talk?

Simple and practical simulation-based policy SOAR is broadly applicable:

- * RAMS (Repeatedly Act based on Multiple Sims) recovers the guarantees for almost all settings in the NRM literature (e.g., constant regret for finite types, $\log^2 T$ for semi-infinite types)
- * Establishes novel guarantees for dynamic spatial matching problems

Thank you!

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Take Aways

APPENDIX on (β, ε_0) -clustered distributions

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DSM Part II

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» $(eta,arepsilon_0)$ -clustered distribution



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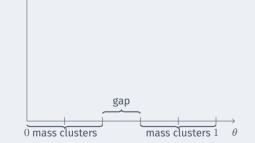
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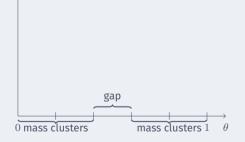
» $(eta,arepsilon_0)$ -clustered distribution





 $\text{Gap} \equiv \text{intervals}$ of positive length with zero mass

mass cluster \equiv interval with positive mass



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» $(oldsymbol{eta},arepsilon_0)$ -clustered distribution

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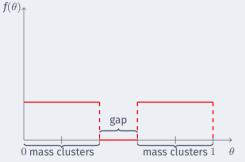
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 $\text{Gap} \equiv \text{intervals}$ of positive length with zero mass

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 $\beta = 0$ (mass accumulation around gaps)



» $(eta,arepsilon_0)$ -clustered distribution



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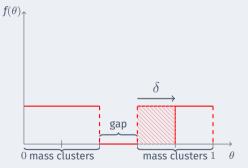
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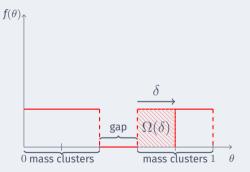


 $\text{Gap} \equiv \text{intervals}$ of positive length with zero mass

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 $\beta = 0$ (mass accumulation around gaps)

 $|F(m + \delta) - F(m)| \ge \delta$ on the same mass cluster







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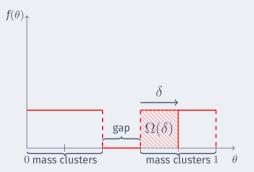
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» $(eta,arepsilon_0)$ -clustered distribution

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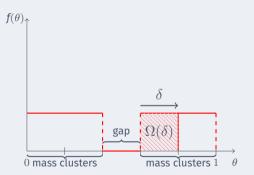
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 $\beta = 0$ (mass accumulation around gaps)

 $|F(m + \delta) - F(m)| \ge \delta$ on the same mass cluster

 $\mu(\text{mass clusters}) \geq \varepsilon_0$

For discrete distrbutions, $\beta = 0$, $\varepsilon_0 = \min_j p_j$

» $(oldsymbol{eta}, arepsilon_0)$ -clustered distribution

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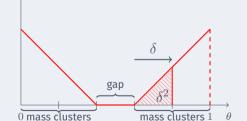
 $\mathsf{Gap} \equiv \mathsf{intervals}$ of positive length with zero mass

mass cluster \equiv interval with positive mass

 $\beta = 1$ (mass accumulation around gaps)

 $|\mathit{F}(\mathit{m}+\delta)-\mathit{F}(\mathit{m})|\geq \delta^2$ on the same mass cluster

 $\mu(\text{mass clusters}) \geq \varepsilon_0$







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APPENDIX on PART II

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home services

» Feature Based Dynamic Matching





- * Platform has a pool of *T* service providers, who live in *d* dimensional feature space.
 - * e.g., $Y_k = (price, rating)$



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 - * e.g., $Y_k = (price, rating)$
- * *T* customers arrive online and have i.i.d. preferences, i.e., weights over the features.
 - * e.g., $X_i = -($ sensitivity to price, sensitivity to rating)



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- * Match value is given as $\langle X_i, Y_k \rangle$



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 - * e.g., $X_i = -($ sensitivity to price, sensitivity to rating)
- * Match value is given as $\langle X_i, Y_k \rangle$
- * Both service provider and customer leave upon matching.
- $\ast~$ Supply and demand distributions are known and possibly different.

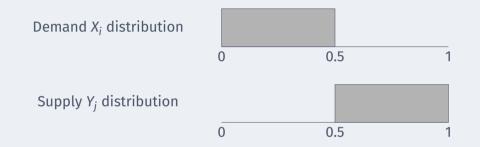


» Performance metric: regret with respect to fluid benchmark

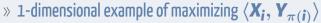
- * We aim to maximize the expected average match value $\frac{1}{T} \sum_{i=1}^{T} \langle X_i, Y_{\pi(i)} \rangle$
- * Fluid benchmark is the value of the optimal transport between the demand distribution and the supply distribution
- $\ast\,$ We aim to minimize the additive regret wrt the fluid benchmark. We want o(1) regret.
- * Problem is equivalent to minimizing $\frac{1}{T} \sum_{i=1}^{T} \|X_i Y_{\pi(i)}\|^2$

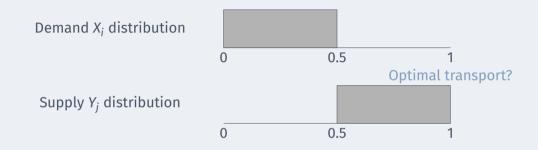


» 1-dimensional example of maximizing $\langle \pmb{X_i}, \pmb{Y_{\pi(i)}}
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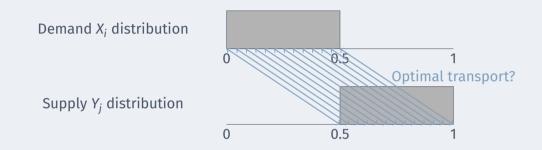




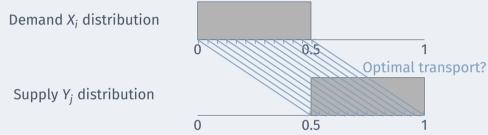


DSM Part II Problem Introduction Literature Review Gaps in distribution Failure of CE GwG Beyond Multi-secretary Take Aways

» 1-dimensional example of maximizing $\langle \pmb{X_i}, \pmb{Y_{\pi(i)}}
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- Optimal transport has value per match 0.208
- Greedy fails: produces a random matching, expected value per match is only 0.188



We introduce a simple forward-looking algorithm dubbed SOAR

Simulate

Optimize

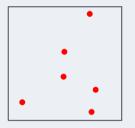
Assign

Repeat

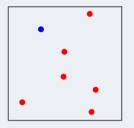
SOAR calculates each matching decision based on a simulation of the future, and hindsight optimization on that future.



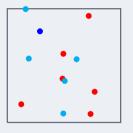






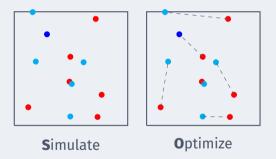




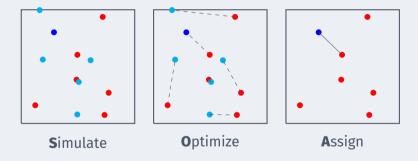


Simulate

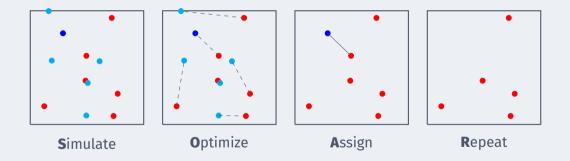


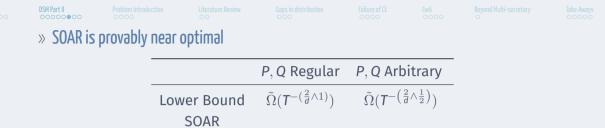












- $* \frac{1}{NND^2}$ is a lower bound on the regret.
- * For d = 1, the matching constraint leads to a tighter lower bound.
- * For irregular distributions, a simple example tells us $1/\sqrt{T}$ is a lower bound. $1/\sqrt{T} \gg 1/\text{NND}^2$ for $d \leq 3$.

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Lower Bound
$$\tilde{\Omega}(T^{-(\frac{2}{d}\wedge 1)}) = \tilde{\Omega}(T^{-(\frac{2}{d}\wedge \frac{1}{2})})$$

SOAR $\tilde{\mathcal{O}}(T^{-(\frac{2}{d}\wedge 1)}) = \tilde{\mathcal{O}}(T^{-(\frac{2}{d}\wedge \frac{1}{2})})$

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SOAR achieves the optimal regret scaling in all cases.

P, Q Regular P, Q Arbitrary

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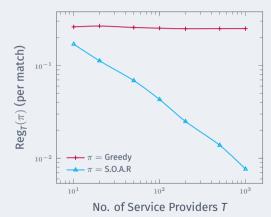
SOAR achieves the optimal regret scaling in all cases.

Proof idea: Expected regret incurred by SOAR's match when t periods remain is the same as the regret for offline matching of t pairs (which is larger than 1/t). Sum over t and divide by T. Result \sim regret for offline matching of T pairs.

» Numerical evaluation of SOAR's performance

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Figure: d = 1, demand ~ Unif(0, 1/2), supply ~ Unif(0, 1)



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- » Talk outline
 - DSM with identical supply and demand distributions
 - * Greedy matching suffices
 - *~ Match distance \sim Nearest-neighbor-distance achievable, except one case
 - DSM with different supply and demand distributions
 - * Greedy fails
 - * Simulate-Optimize-Assign-Repeat (SOAR) is near optimal
 - Multisecretary problem with lumpy value distribution (a 1d DSM problem)
 [O. Besbes, Akshit Kumar & K. '22]
 - * SOAR with one sample path fails
 - * RAMS with multiple sample paths achieves optimal regret scaling
 - * Works also for $d \geq 2$, and across NRM settings.



» Multi-secretary Problem

Problem Statement

Given a sequence of *T* secretaries and a hiring budget *B*, a decision maker (DM) wants to hire the top *B* secretaries in terms of their ability.

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Take Aways

» Multi-secretary Problem

Problem Statement

Given a sequence of *T* secretaries and a hiring budget *B*, a decision maker (DM) wants to hire the top *B* secretaries in terms of their ability.

Details and Assumptions

* The secretaries arrive in an online fashion.

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Note: This is a 1d DSM problem, with an atomic "supply" distribution with *B* units at 1 and T-B units at 0. $\Theta(\sqrt{T})$ optimal regret wrt fluid benchmark, which is trivial to achieve. We'll adopt a tighter benchmark to obtain algorithmic insights.

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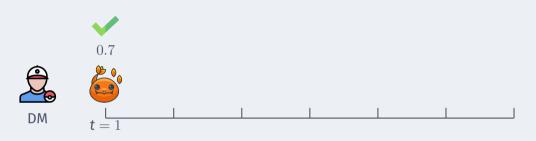
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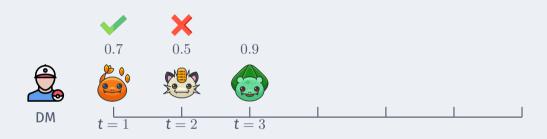
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» Gotta catch'em all some





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» Gotta catch'em all some





| 0 | DSM Part II | Problem Introduction ○●○○ | Literature Review | Gaps in distribution | Failure of CE | CwG 0000 | Beyond Multi-secretary ○ | Take Aways |
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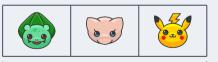




Online Policy

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Hindsight Optimal



Online Policy

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6

t=4

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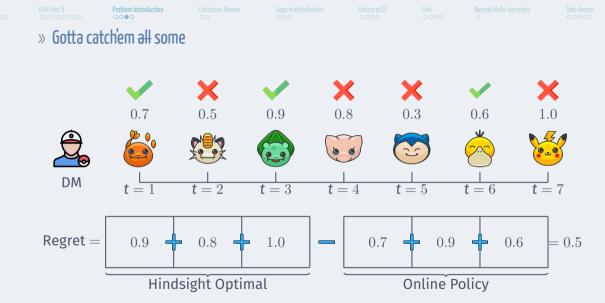
t=5

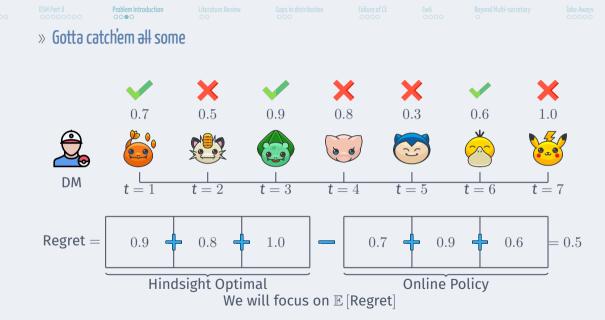
t=6





t=7







Multisecretary is a 1d DSM problem, with an atomic "supply" distribution with *B* units at 1 and T - B units at 0. $\Theta(\sqrt{T})$ optimal regret wrt fluid benchmark, which can be achieved by a trivial static policy.

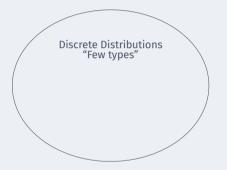
Gap between fluid and hindsight benchmarks is already $\Omega(\sqrt{T})$.

As in the recent NRM literature, we adopt the tighter hindsight benchmark.

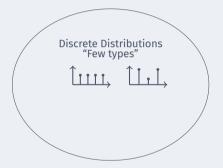


| | Literature Review | | | |
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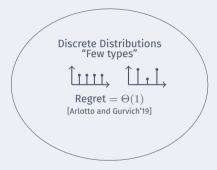




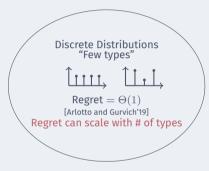




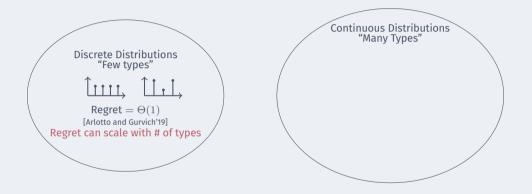




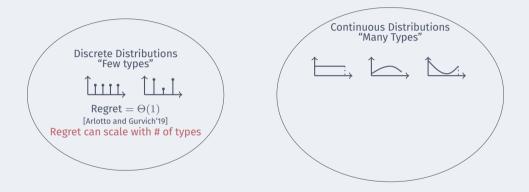




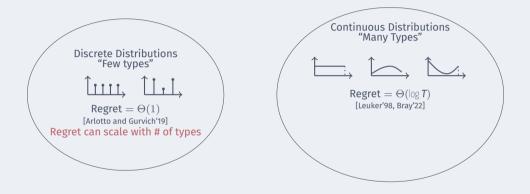




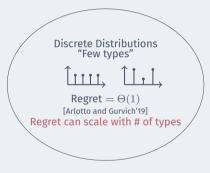


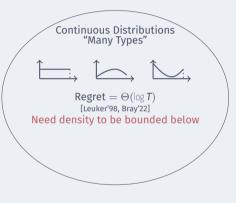


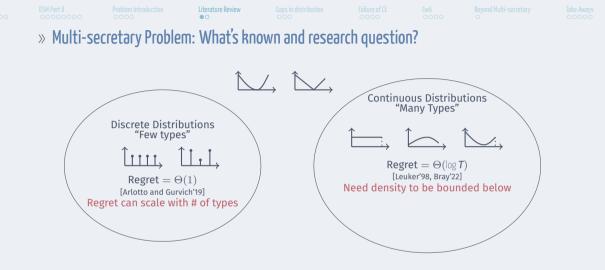


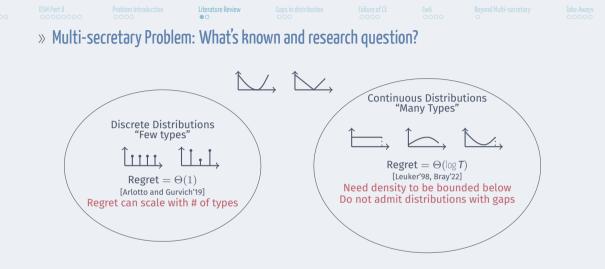


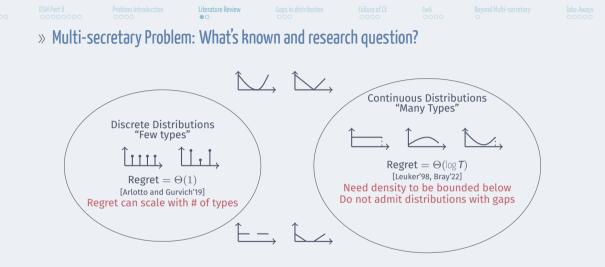
- DSM Part II Problem Introduction Literature Review Gaps in distribution Failure of CE CwG Beyond Multi-secretary
 - » Multi-secretary Problem: What's known and research question?

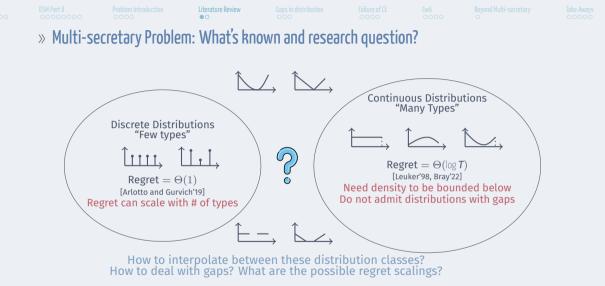












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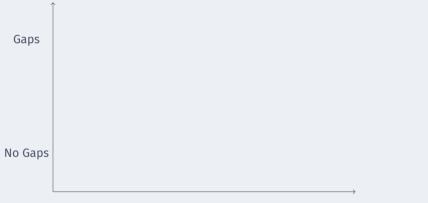
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Take Aways

» Punchline for the Multi-secretary Problem





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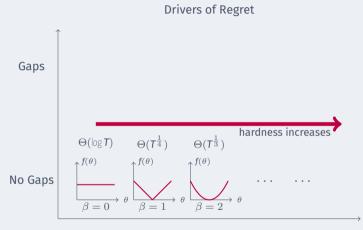
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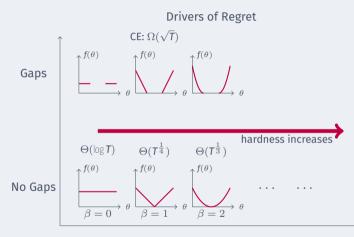
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» Punchline for the Multi-secretary Problem



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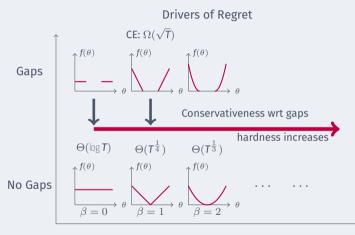
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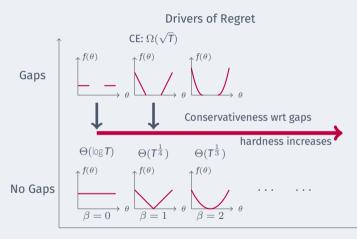
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Take Aways

» Punchline for the Multi-secretary Problem



0.

- * Distribution shape is a **fundamental** driver of regret.
- Dealing with gaps is an algorithmic challenge.
- Novel Principle: Conservativeness wrt gaps (CwG)
- multi-sim SOAR variant automatically pursues CwG

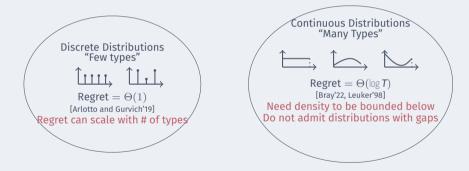
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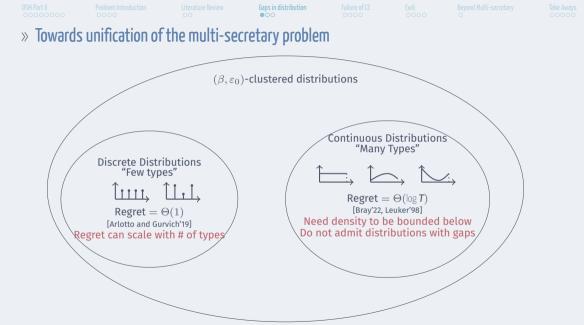
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Beyond Multi-se

Take Aways

» Towards unification of the multi-secretary problem





| | | Gaps in distribution ○●○ | | |
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» Fundamental Limits

Universal Lower Bound

For every $\beta \in [0,\infty)$, there exists a distribution F_{eta} such that

$$\sup_{B \in [T]} \mathbb{E}_{F_{\beta}} \left[\mathsf{Regret}(\mathsf{DP}) \right] = \begin{cases} \Omega \left(\log T \right), & \beta = 0, \\ \Omega \left(T^{\frac{1}{2} - \frac{1}{2(1+\beta)}} \right), & \beta > 0. \end{cases}$$

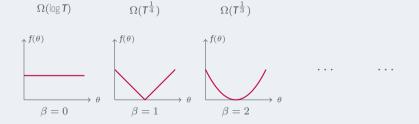
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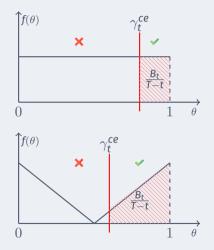
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Gaps in distribution

Beyond I

Take Aways

For $(m{eta}, \mathbf{1})$ -clustered distributions



- Let B_t be the remaining budget at time t
- * Compute the budget ratio

» Certainty Equivalent Control

- $br_t = rac{\text{Remaining Budget}}{\text{Remaining Time}} = rac{B_t}{T-t}$
- $\ast~$ Define a quantile threshold $p_t^{ce}=1-br_t$
- * Define a ability threshold $\gamma_t^{ce} = \mathit{F}^{-1}(\mathit{p}_t^{ce})$
- * hire $\iff \theta_t \geq \gamma_t^{ce}$

» Certainty Equivalent Control

For Bi-modal Uniform Distribution

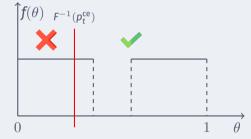
Let B_t be the remaining budget at time t

$$\mathsf{Budget}\ \mathsf{Ratio} = rac{\mathsf{Remaining}\ \mathsf{Budget}}{\mathsf{Remaining}\ \mathsf{Time}} = rac{B_t}{T-t}$$

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CE Quantile Threshold
$$= 1 - \frac{B_t}{T-t} \triangleq p_t^{ce}$$

Decision: hire $\iff \theta_t \ge F^{-1}(p_t^{ce})$



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Problem Introduct

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Take Aways

Regret Lower Bound

» Failure of Certainty Equivalent Control

Insufficiency of Certainty Equivalent Control

Assume that $F = \text{Unif}([0, \frac{1}{4}] \cup [\frac{3}{4}, 1])$, for B = T/2, we have

 $\mathbb{E}\left[\mathsf{Regret}(\mathsf{CE})\right] = \Omega\left(\sqrt{\mathsf{T}}\right)$

DSM Part II

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Review

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Take Aways

Regret Lower Bound

» Failure of Certainty Equivalent Control

Insufficiency of Certainty Equivalent Control

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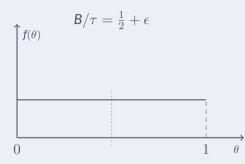
Remark

* Same scaling is achievable under a static threshold policy.

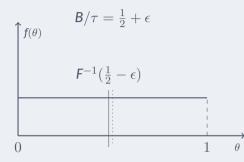
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| | | Failure of CE | | |



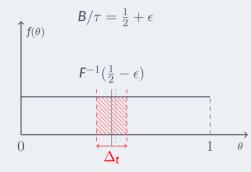
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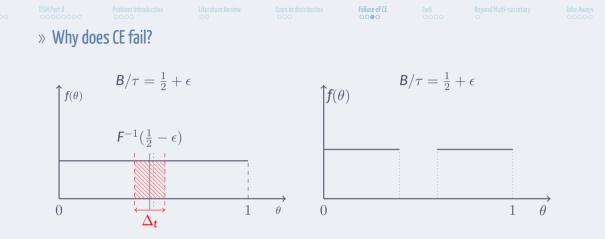


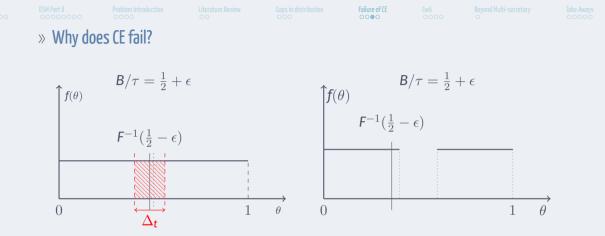
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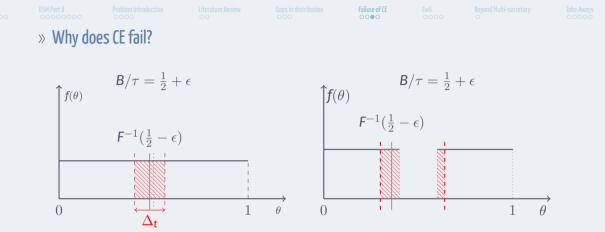


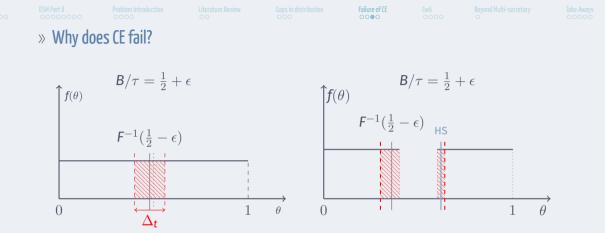
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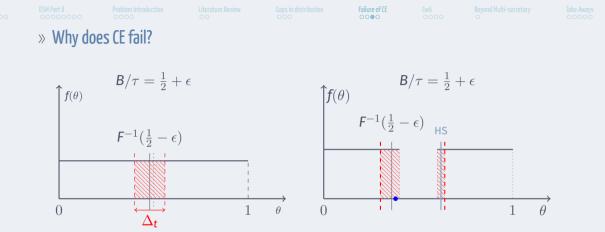


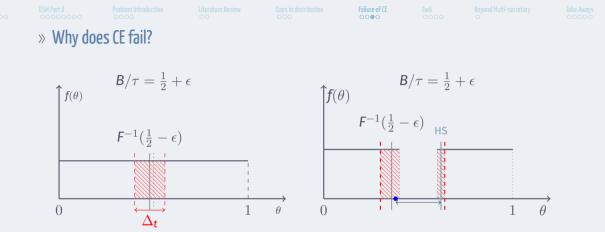


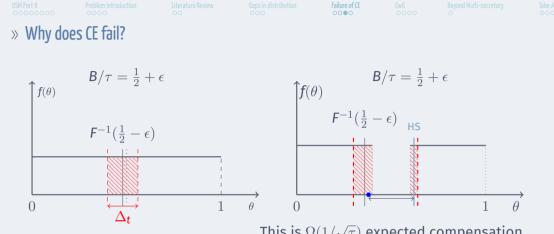




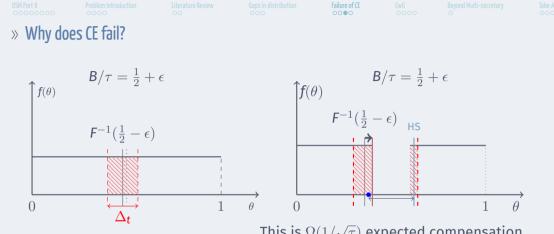




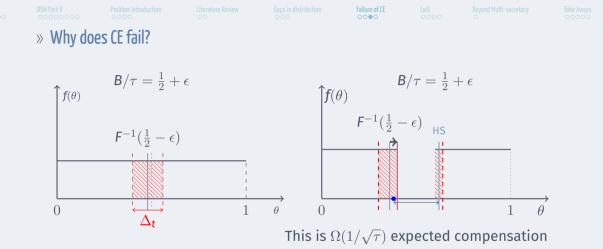




This is $\Omega(1/\sqrt{\tau})$ expected compensation

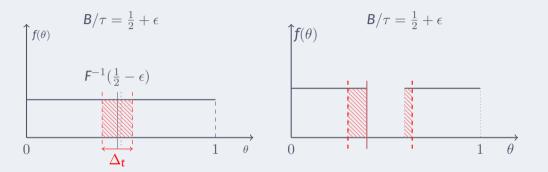


This is $\Omega(1/\sqrt{\tau})$ expected compensation

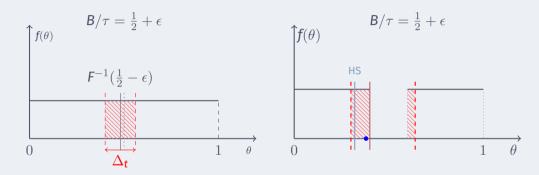


Conservativeness wrt gaps

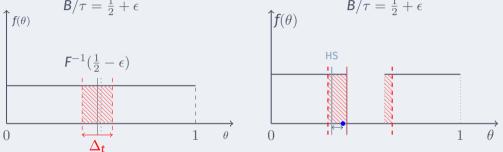












This is $\tilde{\mathcal{O}}(1/\tau)$ expected compensation

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Gaps in distribut

Failure of CE

CwG ●○○○ Beyond Multi-secretary

Take Aways

» Good in theory but practically infeasible

- * What is the conservativeness parameter I should use?
- * How to find where these gaps are? What happens if gaps shift?
- * E.g., no chance of deploying for Amazon's fulfillment problem

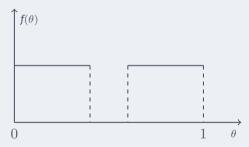


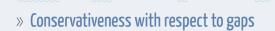
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Take Aways

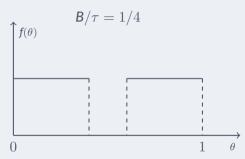
» Conservativeness with respect to gaps

Algorithmic Idea: Simulate into the future



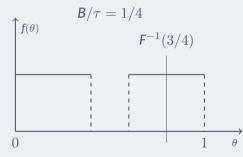


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Algorithmic Idea: Simulate into the future



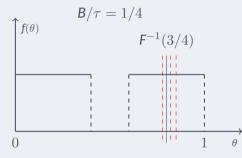


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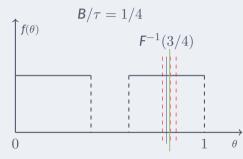
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Algorithmic Idea: Simulate into the future



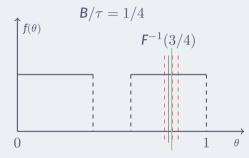


Algorithmic Idea: Simulate into the future



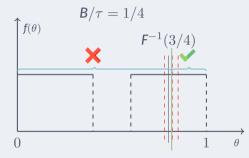


Algorithmic Idea: Simulate into the future

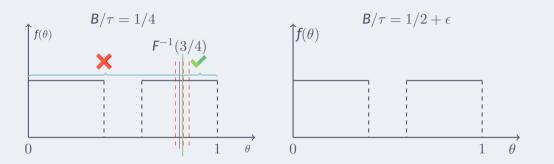




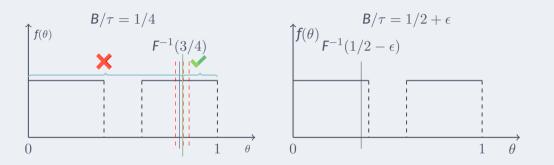
Algorithmic Idea: Simulate into the future



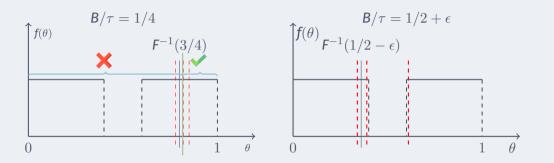




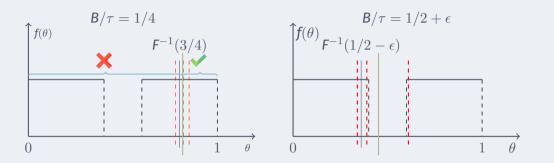




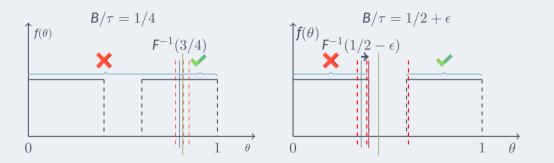












If far from a gap, use the CE threshold If close to gap, use the gap as threshold

Gaps in distr

Failure of CE

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Beyond Multi-secretar

Take Aways

Punchline

» Conservativeness with respect to gaps

Regret of RAMS Policy

If F is a $(\beta,\varepsilon_0)\text{-clustered}$ distribution, then

$$\mathbb{E}\left[\mathsf{Regret}(\mathsf{RAMS})\right] = \begin{cases} \mathcal{O}\left((\log T)^2\right), & \beta = 0, \\ \mathcal{O}\left(\mathsf{poly}(\log T)T^{\frac{1}{2} - \frac{1}{2(1+\beta)}}\right), & \beta > 0 \end{cases}$$

If F is a discrete distribution, $\mathbb{E}\left[\mathsf{Regret}\left(\mathsf{RAMS}\right)\right] = \mathcal{O}(1/\varepsilon_0)$

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Take Aways

Punchline

» Conservativeness with respect to gaps

Regret of RAMS Policy

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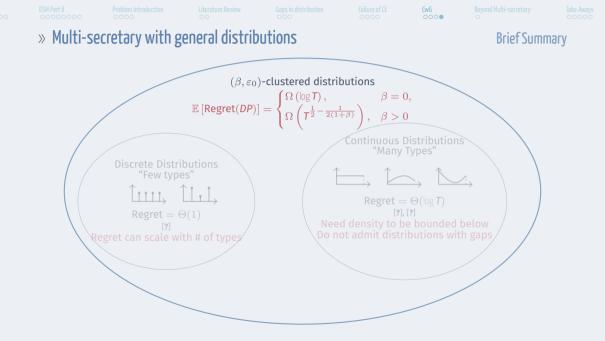
$$\mathbb{E}\left[\mathsf{Regret}(\mathsf{RAMS})\right] = \begin{cases} \mathcal{O}\left((\log T)^2\right), & \beta = 0, \\ \mathcal{O}\left(\mathsf{poly}(\log T)T^{\frac{1}{2} - \frac{1}{2(1+\beta)}}\right), & \beta > 0 \end{cases}$$

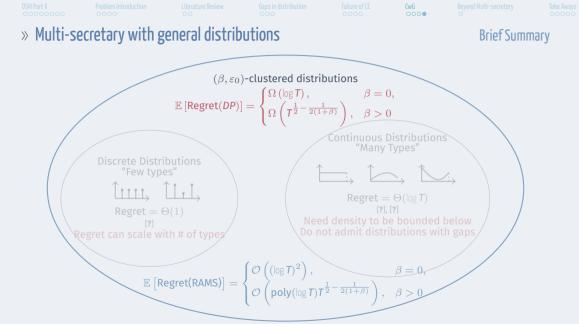
If F is a discrete distribution, $\mathbb{E}\left[\mathsf{Regret}\left(\mathsf{RAMS}\right)\right] = \mathcal{O}(1/\varepsilon_0)$

Remark

- * $F = \text{Unif}([0, \frac{1}{4}] \cup [\frac{3}{4}, 1])$, RAMS ($\mathcal{O}((\log T)^2)$) outperforms CE ($\Omega(\sqrt{T})$).
- $\ast\,$ Matches the universal lower bound upto polylog factors $\,\Rightarrow\,$ RAMS is near-optimal.









- * The multi-secretary problem is special but RAMS is general: in each period, simulate several futures and choose the action which minimizes the expected "compensation" in hindsight. Compensation ≡ How to much we need to pay an agent who knows the future to take a particular action, for a given future.
- * Can be applied to NRM and stochastic online matching problems to recover almost all known guarantees in the literature.

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Take Aways

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» One Policy to solve them all?

Proposition (RAMS is as good as any algorithm)

Given an NRM setting P, consider *any* algorithm A for P, such that with τ periods remaining, uniformly over the state, the expected compensation under A is bounded above by $\delta_{\tau}(A)$. Then RAMS achieves an expected compensation bounded uniformly by $\delta_{\tau} + 1/\tau^{1.1}$. As a result the regret of RAMS is bounded above by a constant plus the regret guarantee for algorithm A,

$$ext{Regret(RAMS)} \leq ext{Constant} + \sum_{ au=1}^{ au} \delta_{ au}(\mathsf{A}) \,.$$



» What to take away from this talk?

Simple and practical simulation-based policy SOAR is broadly applicable:

- * Recovers the guarantees for almost all settings in the NRM literature (e.g., constant regret for finite types, $\log^2 T$ for semi-infinite types)
- * Establishes novel guarantees for dynamic spatial matching problems

Thank you!

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Take Aways

APPENDIX on (β, ε_0) -clustered distributions

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» $(eta,arepsilon_0)$ -clustered distribution



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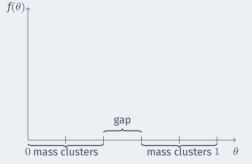
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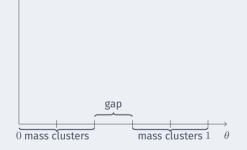
» $(eta,arepsilon_0)$ -clustered distribution





 $\text{Gap} \equiv \text{intervals}$ of positive length with zero mass

mass cluster \equiv interval with positive mass



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» $(oldsymbol{eta},arepsilon_0)$ -clustered distribution

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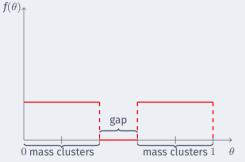
Take Aways



 $\text{Gap} \equiv \text{intervals}$ of positive length with zero mass

mass cluster \equiv interval with positive mass

 $\beta = 0$ (mass accumulation around gaps)



» $(eta,arepsilon_0)$ -clustered distribution



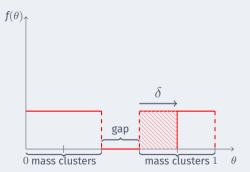
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 $\text{Gap} \equiv \text{intervals}$ of positive length with zero mass

mass cluster \equiv interval with positive mass

 $\beta = 0$ (mass accumulation around gaps)

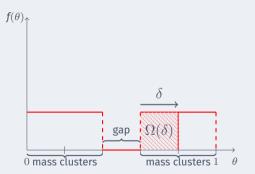
» $(eta,arepsilon_0)$ -clustered distribution

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CE

Take Aways





 $\text{Gap} \equiv \text{intervals}$ of positive length with zero mass

mass cluster \equiv interval with positive mass

 $\beta = 0$ (mass accumulation around gaps)

 $|F(m + \delta) - F(m)| \ge \delta$ on the same mass cluster





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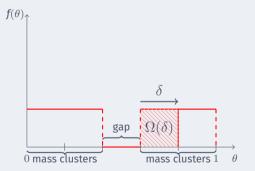
 $\text{Gap} \equiv \text{intervals}$ of positive length with zero mass

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 $\mu(\text{mass clusters}) \geq \varepsilon_0$



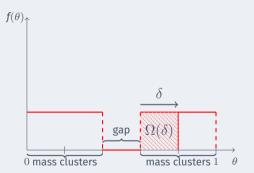
» $(eta,arepsilon_0)$ -clustered distribution

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 $\text{Gap} \equiv \text{intervals}$ of positive length with zero mass

mass cluster \equiv interval with positive mass

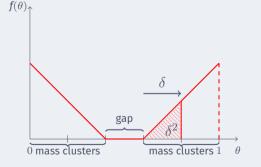
 $\beta = 0$ (mass accumulation around gaps)

 $|F(m + \delta) - F(m)| \ge \delta$ on the same mass cluster

 $\mu(\text{mass clusters}) \geq \varepsilon_0$

For discrete distrbutions, $\beta = 0$, $\varepsilon_0 = \min_j p_j$

» (β , ε_0)-clustered distribution



 $Gap \equiv intervals$ of positive length with zero mass

mass cluster \equiv interval with positive mass

 $\beta = 1$ (mass accumulation around gaps)

 $|F(m + \delta) - F(m)| > \delta^2$ on the same mass cluster

 μ (mass clusters) $\geq \varepsilon_0$

Examples