Journées Jeunes EDPistes en France 2024

Rapport sur les contributions

https://indico.math.cnrs.fr/e/10254

ID de Contribution: **1** Type: **Non spécifié**

Resonances as a computational tool

mercredi 20 mars 2024 14:00 (50 minutes)

A large toolbox of numerical schemes for dispersive equations has been established, based on different discretization techniques such as discretizing the variation-of-constants formula (e.g., exponential integrators) or splitting the full equation into a series of simpler subproblems (e.g., splitting methods). In many situations these classical schemes allow a precise and efficient approximation. This, however, drastically changes whenever non-smooth phenomena enter the scene such as for problems at low regularity and high oscillations. Classical schemes fail to capture the oscillatory nature of the solution, and this may lead to severe instabilities and loss of convergence. In this talk I present a new class of resonance based schemes. The key idea in the construction of the new schemes is to tackle and deeply embed the underlying nonlinear structure of resonances into the numerical discretization. As in the continuous case, these terms are central to structure preservation and offer the new schemes strong geometric properties at low regularity.

Orateur: SCHRATZ, Katharina

ID de Contribution: **2** Type: **Non spécifié**

Étude statistique de la formation des vagues extrêmes

mercredi 20 mars 2024 14:50 (30 minutes)

Nous nous intéresserons à la formation de vagues extrêmes, en haute mer, en adoptant un point de vue probabiliste. Nous identifierons en premier lieu le premier terme du développement asymptotique de la probabilité d'occurrence d'une telle vague lorsque la hauteur de la vague tend vers l'infini. Si une vague extrême survient, quelle est la donnée initiale la plus probable qui l'a produite ? Nous répondrons a cette question dans le régime faiblement non linéaire en donnant une caractérisation probabiliste de l'ensemble de vagues extrêmes aussi bien dans le cadre de l'équation NLS que dans le cadre des équations Water-Waves.

Orateur: GRANDE IZQUIERDO, Ricardo

ID de Contribution: **3** Type: **Non spécifié**

Nonlinear Schrödinger equations on compact surfaces

mercredi 20 mars 2024 15:20 (30 minutes)

This talk is devoted to the general study of the long-time dynamics of solutions to nonlinear Schrödinger equations (NLS) on compact surfaces. In this context, weak dispersion and nonlinear resonances can cause energy cascades from low to high frequency scales of oscillations. Meanwhile, one can use the Galerkin approximation and extend methods from the study of finite-dimensional Hamiltonian systems to show stability in certain regimes.

We present a dynamical approach based on Birkhoff normal forms to prove long-time stability on Diophantine tori, as well as a statistical approach in which we prove the invariance of the Gibbs measure for the cubic NLS on the sphere.

The results are based on joint work with Joackim Bernier, and ongoing joint works with Gigliola Staffilani, and Nicolas Burq, Chenmin Sun, Nikolay Tzvetkov.

Orateur: CAMPS, Nicolas

ID de Contribution: **4** Type: **Non spécifié**

The logarithmic Bramson correction for Fisher-KPP equations on the lattice Z

mercredi 20 mars 2024 16:20 (30 minutes)

In the talk, I will present the logarithmic Bramson correction for Fisher-KPP equations on the lattice Z, that is the level sets of solutions with step-like initial conditions are located at position $c_*t - (3/(2\lambda_*))\ln t + O(1)$ as $t \longrightarrow +\infty$ for some explicit positive constants c_* and λ_*. This extends a well-known result of Bramson in the continuous setting to the discrete case using only PDE arguments.

Orateur: ZHANG, Mingmin

ID de Contribution: **5** Type: **Non spécifié**

Asymptotic analysis and simulation of collisional Vlasov-Poisson models

mercredi 20 mars 2024 16:50 (30 minutes)

This presentation focuses on collisional Vlasov-Poisson systems. These kinetic models are of primary interest, as they encode the multiple scales that arise in a plasma, ranging from fluid-like behavior when collisions dominate to wave interactions in weakly collisional regimes. We present quantitative results that capture the scales of both the continuous model and its discretized formulation. Then, we carry out various simulations which highlight our theoretical results (phase transition between regimes, plasma instabilities).

Orateur: BLAUSTEIN, Alain

ID de Contribution: **6** Type: **Non spécifié**

On the hydrostatic limit of the Euler-Boussinesq equations

mercredi 20 mars 2024 17:20 (30 minutes)

I will talk about the hydrostatic approximation of the 2d Euler-Boussinesq system, describing the evolution of an inviscid stratified fluid where the vertical length scale is much smaller than the horizontal one. Even though of importance in oceanography, the justification of the hydrostatic limit in this context has remained an open problem. I will discuss some recent results showing that some instability mechanisms may prevent this limit to hold.

This is joint work with R. Bianchini (CNR Rome) and M. Coti Zelati (Imperial College London).

Orateur: ERTZBISCHOFF, Lucas

ID de Contribution: **7** Type: **Non spécifié**

Coût d'observabilité en temps petit de l'équation de la chaleur 1D

jeudi 21 mars 2024 09:00 (50 minutes)

L'estimation du coût d'observabilité en temps petit de l'équation de la chaleur 1D (et, par dualité, celle du coût de contrôle en temps petit de la même équation), est une longue histoire qui commence dans les années 80, et n'est toujours pas terminée.

Dans cet exposé, j'expliquerai comment, dans un travail avec Sylvain Ervedoza (2019), nous avons amélioré l'estimation par au-dessus de ce coût. Ce sera l'occasion de parler d'inégalité de Carleman, de principe d'incertitude, de principe de Phragmén-Lindelöf et de transformées conformes de Schwarz–Christoffel.

Orateur: DARDE, Jeremi

ID de Contribution: **8** Type: **Non spécifié**

The convergence problem in mean-field control theory and related PDEs over the space of probability measures

jeudi 21 mars 2024 09:50 (30 minutes)

The goal of this talk is to discuss recent progress in the convergence problem in mean-field control theory.

We are interested in control problems involving a large number of (controlled) interacting particles subject to independent noises of Brownian type. When the number of particles tends to infinity, the problem simplifies into a control problem of mean-field type, set on the space of probability measures over the euclidean space. I will present some recent progress in the quantitative analysis of this convergence. More precisely I will discuss an approach based on a suitable mollification of the value function of the limiting problem. By dynamic programming, this value function solves in a weak viscosity sense a semi-linear Hamilton-Jacobi equation over the set of probability measures. We regularize it via sup-convolution in a well-chosen functional Hilbert space in order to produce approximations that are almost classical (sub)-solutions to the dynamic programming equation. Projecting these approximations in finite dimension, we can compare them with the value functions of the particle systems and obtain sharp rates of convergence.

This is based on a joint works with François Delarue, Joe Jackson and Ben Seeger.

Orateur: DAUDIN, Samuel (Université Côte d'Azur)

ID de Contribution: **9** Type: **Non spécifié**

Existence of strong solutions for a compressible fluid-solid interaction system with Navier slip boundary conditions

jeudi 21 mars 2024 10:55 (30 minutes)

We consider a fluid-structure interaction system coupling a viscous fluid governed by the compressible Navier-Stokes equations and a rigid body immersed in the fluid and modeled by the Newton's law. In this work, we consider the Navier slip boundary conditions. Our aim is to show the local in time existence and uniqueness of the strong solution to the corresponding problem. The main step of this work is that we use Lagrangian change of variables in order to handle the transport equation and to reduce the problem in the initial domain. Therefore, it brings some extra nonlinear terms in the boundary conditions. The strategy is based on the study of the linearized system with nonhomogeneous boundary conditions and on the Banach fixed point theorem.

Orateur: DJEBOUR, Imene

ID de Contribution: **10** Type: **Non spécifié**

A finite-difference based variational approach for solving Hamilton-Jacobi equations in high-dimensional domains

jeudi 21 mars 2024 11:25 (30 minutes)

It is well-known that the value function associated to a given optimal control problem or differential game can be characterised as the viscosity solution of an associated Hamilton-Jacobi equation. Numerical methods based on finite-differences are guaranteed to approximate the viscosity solution, provided the numerical scheme has the correct monotonicity. However, these grid-based methods suffer from the curse of dimensionality when the dimension of the domain is high. In this talk, I will present a variational approach to approximate the viscosity solution, consisting in the minimisation of a functional involving a Lax-Friedrichs discretisation of the Hamiltonian. I will show that, by choosing an appropriate numerical scheme, one can ensure uniqueness of a critical point for the functional. This in turn implies that the gradient flow associated to this functional converges to the unique global minimiser, which additionally can be proven to be close to the viscosity solution. In practice, the solution to the minimisation problem can be approximated by means of a Neural Network, trained through stochastic gradient descent, which simulates the gradient flow associated to the functional.

Orateur: ESTEVE-YAGÜE, Carlos

ID de Contribution: **11** Type: **Non spécifié**

Regularity issue for the system describing elastic structure interacting with Navier-Stokes equations

jeudi 21 mars 2024 14:00 (30 minutes)

We are interested in the interaction of a viscous incompressible fluid with an elastic structure, where the structure is located on a part of the fluid boundary. It reacts to the surface forces induced by the fluid and deforms the reference domain Ω to Ω_η. The fluid equations are coupled with the structure via the kinematic condition and the action-reaction principle on the interface.

We first study the 2D visco-elastic shell interacts with 3D Navier-Stokes equations. Especially in a general reference geometry (the shell deforms along the normal direction of the flexible boundary), we prove a counterpart of the classical Ladyzhenskaya-Prodi-Serrin condition yielding conditional regularity and uniqueness of a solution. This requires additionally the deformation of the shell is Lipschitz continuous.

Then we consider a 1D perfectly elastic plate, deforming vertically in flat case, interacts with 2D Navier-Stokes equations, which thereby gives a hyperbolic evolution. We show the new regularity result for this parabolic-hyperbolic coupled system. It turns out that the "parabolic effect"of the fluid suffices to regularize the solution to the coupled fluid-structure system which is previously known for the Navier-Stokes equations in fixed domains.

This is based on joint work with D. Breit (Clausthal), P. Mensah (Clausthal) and S. Schwarzacher (Uppsala).

Orateur: SU, Pei

ID de Contribution: **12** Type: **Non spécifié**

Two formulations for acoustic and surface waves in a free-surface, vertically stratified ocean

jeudi 21 mars 2024 14:30 (30 minutes)

I will present two formulations for a linear model describing the propagation of acoustic and surface gravity waves in a free-surface, stratified ocean. The first formulation, already studied in previous works, is obtained by the linearization of the compressible Euler equations, written in Lagrangian coordinates. The second formulation uses a new variable which can be understood as a generalized potential, allowing for a natural decomposition into rotational and irrotational velocity components. I will show that both formulations are well-posed and present some simulations.

Orateur: DUBOIS, Juliette

ID de Contribution: **13** Type: **Non spécifié**

Solutions de Yudovich non-bornées pour les équations d'Euler 2D

jeudi 21 mars 2024 15:00 (30 minutes)

Dans cet exposé, nous étudierons les solutions non-bornées des équations d'Euler incompressibles en deux dimensions d'espace. Ces solutions trouvent leur interêt dans le fait que les espaces habituels de solutions (p. ex. basés sur une condition d'énergie finie comme L^2) ne respectent pas certaines des symétries du problème : l'invariance de Galilée et l'invariance d'échelle. Par ailleurs, les solutions d'énergie infinie ont une importance certaine dans plusieurs problèmes concrets, typiquement la géophysique.

Après avoir présenté le problème et un aperçu de la littérature sur le sujet, nous donnerons notre résultat : l'existence et l'unicité de solutions de Yudovich sous la condition d'une croissance en racine carrée du champ des vitesses. La démonstration, que nous esquisserons, est basée sur une décomposition intégrale de la pression, ainsi que des bilans d'énergie locaux conduisant à des estimations des solutions dans des espaces de type Morrey locaux.

Ce travail a été réalisé en collaboration avec Herbert Koch (Universität Bonn).

Orateur: COBB, Dimitri

ID de Contribution: **14** Type: **Non spécifié**

On collapses of single-signed point-vortices with the boundary

jeudi 21 mars 2024 16:00 (50 minutes)

We consider the point-vortex system in a domain and assume the masses to be single-signed. We investigate the possibility of collapse with the boundary. We prove that such collapses are not possible in the case of the disk and of the half-plane. For general domains, we give a necessary condition for a collapse with the boundary to occur. This is joint work with M. Donati and L. Godard-Cadillac.

Orateur: IFTIMIE, Dragos (Institut Camille Jordan - Université Lyon 1)

ID de Contribution: **15** Type: **Non spécifié**

Nonlinear waves on lattices and continuum limit

jeudi 21 mars 2024 16:50 (30 minutes)

In this talk we will be interested in the dynamics of dispersive PDEs on infinite lattices. In particular, we will highlight how the dispersive properties of the solutions, which are weaker than the one on the continuous setting, can be used in order to study the continuum limit of such systems as the step size of the grid tends to zero. We will also provide some perspectives on the subject.

Orateur: CHAULEUR, Quentin

ID de Contribution: **16** Type: **Non spécifié**

Uniform in time propagation of chaos for the 2D vortex model

jeudi 21 mars 2024 17:20 (30 minutes)

We are interested in a system of particles in singular mean-field interaction and wish to prove that, as the number of particles goes to infinity, two given particles within that system become « more and more » independent, a phenomenon known as propagation of chaos. The interaction we will focus on comes from the Biot-Savart kernel, for which the nonlinear limit of the particle system satisfies the vorticity equation, arising from the 2D incompressible Navier-Stokes system.

We build upon a recent work of P.-E. Jabin and Z. Wang to obtain a uniform in time convergence. The approach consists in computing the time evolution of the relative entropy of the joint law of the particle system with respect to the nonlinear limit. We prove time-uniform bounds on the limit, as well as a logarithmic Sobolev inequality. From the latter, the Fisher information appearing in the entropy dissipation yields a control on the relative entropy itself, inducing the time uniformity.

This is joint work with A. Guillin and P. Monmarché.

Orateur: LE BRIS, Pierre

ID de Contribution: **17** Type: **Non spécifié**

Burnett's conjecture in General Relativity

vendredi 22 mars 2024 09:00 (50 minutes)

In this work, I will present a work in collaboration with Jonathan Luk where we prove that weak limits of solutions to Einstein vacuum equations, in some setting, converge to solutions to Einstein equations coupled to a massless Vlasov field. The proof uses the microlocal deffect measures of Tartar and Gérard, and compensated compactness.

Orateur: HUNEAU, Cécile (CNRS et Ecole Polytechnique)

ID de Contribution: **18** Type: **Non spécifié**

Strictly-Correlated Electrons from the viewpoint of optimal transport and their dissociation at infinity

vendredi 22 mars 2024 09:50 (30 minutes)

The Strictly-Correlated Electrons (SCE) is a formalism of Density-Functional Theory (DFT) used to approximate ground-state energies of strongly-correlated quantum systems. From a mathematical viewpoint, it is obtained as the semi-classical limit of the Levy-Lieb functional, which is one of the central objects to DFT. I will present this problem, which arises as a multimarginal optimal transport of a special kind, and present a recent result of mine on the dissociation at infinity of such systems, reminiscent of the ionisation conjecture in quantum physics.

Orateur: LELOTTE, Rodrigue

ID de Contribution: **19** Type: **Non spécifié**

Zero resonant states for the Schrödinger operator

vendredi 22 mars 2024 10:55 (30 minutes)

The Schrödinger operator on the whole space R^d gives rise to a dispersive equation, meaning that the mass of the solution spreads towards infinity, and these dispersive properties are tightly linked to its spectrum. Resonances can be seen as a generalisation of eigenvalues: they are complex numbers for which the eigenvalue equation admits a non L^2 solution. Their dynamical interpretation is that the imaginary part of a resonance determines the speed of dispersion of a resonant state. In this talk we will analyze resonances in zero, which are known to be an obstacle to dispersion. For a rather general class of potentials we will see when zero is a resonance or an eigenvalue and some properties of the associated state.

Orateur: GRASSELLI, Viviana

ID de Contribution: **20** Type: **Non spécifié**

The Robin Harmonic Measure on non smooth domains

vendredi 22 mars 2024 11:25 (30 minutes)

We analyze the boundary behavior of solutions to elliptic PDE with prescribed Robin data in rough domains. In particular, we construct a "Robin elliptic measure" and demonstrate the suprising fact that this measure is (quantitatively) mutually absolutely continuous with respect to surface measure on a wide class of domains that includes the complement of certain fractals.

Orateur: MICHETTI, Marco