

Journées Jeunes EDPistes en France 2024

- Dates: 20-21-22 Mars 2024
- Localisation: Amphithéâtre Laurent Schwartz, bâtiment 1R3, Institut de Mathématiques de Toulouse
- Webpage: <https://indico.math.cnrs.fr/event/10254/>
- Comité scientifique: Karine Beauchard, David Dos Santos Ferreira, François Hamel, David Lannes, Evelyne Miot
- Comité local d'organisation: Jean-François Coulombel, Fanny Delebecque, Grégory Faye, Jérémy Heleine, Mihai Maris
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- Soutiens financiers: Réseau thématique AEDP, Institut de Mathématiques de Toulouse, Labex CIMI & ANR project Indyana

Programme scientifique

Mercredi 20 Mars

- 11h15-11h45: accueil
- 12h-14h : déjeuner @ l'Esplanade
- 14h-14h50 : Katharina Schratz
- 14h50-15h20 : Ricardo Grande Izquierdo
- 15h20-15h50 : Nicolas Camps
- 15h50-16h20 : Pause Café
- 16h20-16h50 : Mingmin Zhang
- 16h50-17h20 : Alain Blaustein
- 17h20-17h50 : Lucas Ertzbischoff

Jeudi 21 Mars

- 9h-9h50 : Jérémie Dardé
- 9h50-10h20 : Samuel Daudin
- 10h20-10h55 : pause café
- 10h55-11h25 : Imene Djebour
- 11h25-11h55 : Carlos Esteve-Yagüe
- 11h55-14h : buffet & session posters
- 14h-14h30 : Pei Su
- 14h30-15h : Juliette Dubois
- 15h-15h30 : Dimitri Cobb
- 15h30-16h00 : Pause Café
- 16h00-16h50 : Dragos Iftimie
- 16h50-17h20 : Quentin Chauleur
- 17h20-17h50 : Pierre Le Bris
- 19h : diner @ Les Caves de la Maréchale

Vendredi 22 Mars

- 9h-9h50 : Cécile Huneau
- 9h50-10h20 : Rodrigue Lelotte
- 10h20-10h55 : pause café
- 10h55-11h25 : Viviana Grasselli
- 11h25-11h55 : Marco Michetti
- 11h55-14h : déjeuner @ l'Esplanade

Titres et résumés des exposés

- Katharina Schratz (LJLL Sorbonne Université)

Titre: Resonances as a computational tool

Résumé: A large toolbox of numerical schemes for dispersive equations has been established, based on different discretization techniques such as discretizing the variation-of-constants formula (e.g., exponential integrators) or splitting the full equation into a series of simpler subproblems (e.g., splitting methods). In many situations these classical schemes allow a precise and efficient approximation. This, however, drastically changes whenever non-smooth phenomena enter the scene such as for problems at low regularity and high oscillations. Classical schemes fail to capture the oscillatory nature of the solution, and this may lead to severe instabilities and loss of convergence. In this talk I present a new class of resonance based schemes. The key idea in the construction of the new schemes is to tackle and deeply embed the underlying nonlinear structure of resonances into the numerical discretization. As in the continuous case, these terms are central to structure preservation and offer the new schemes strong geometric properties at low regularity.

- Ricardo Grande Izquierdo (SISSA)

Titre: Étude statistique de la formation des vagues extrêmes

Résumé: Nous nous intéresserons à la formation de vagues extrêmes, en haute mer, en adoptant un point de vue probabiliste. Nous identifierons en premier lieu le premier terme du développement asymptotique de la probabilité d'occurrence d'une telle vague lorsque la hauteur de la vague tend vers l'infini. Si une vague extrême survient, quelle est la donnée initiale la plus probable qui l'a produite ? Nous répondrons à cette question dans le régime faiblement non linéaire en donnant une caractérisation probabiliste de l'ensemble de vagues extrêmes aussi bien dans le cadre de l'équation NLS que dans le cadre des équations Water-Waves.

- Nicolas Camps (Nantes Université)

Titre: Nonlinear Schrödinger equations on compact surfaces

Résumé: This talk is devoted to the general study of the long-time dynamics of solutions to nonlinear Schrödinger equations (NLS) on compact surfaces. In this context, weak dispersion and nonlinear resonances can cause energy cascades from low to high frequency scales of oscillations. Meanwhile, one can use the Galerkin approximation and extend methods from the study of finite-dimensional Hamiltonian systems to show stability in certain regimes.

We present a dynamical approach based on Birkhoff normal forms to prove long-time stability on Diophantine tori, as well as a statistical approach in which we prove the invariance of the Gibbs measure for the cubic NLS on the sphere.

The results are based on joint work with Joackim Bernier, and ongoing joint works with Gigliola Staffilani, and Nicolas Burq, Chenmin Sun, Nikolay Tzvetkov.

- Mingmin Zhang (Institut de Mathématiques de Toulouse)

Titre: The logarithmic Bramson correction for Fisher-KPP equations on the lattice \mathbb{Z}

Résumé: In the talk, I will present the logarithmic Bramson correction for Fisher-KPP equations on the lattice \mathbb{Z} , that is the level sets of solutions with step-like initial conditions are located at position $c_*t - (3/(2\lambda_*)) \ln t + O(1)$ as $t \rightarrow +\infty$ for some explicit positive constants c_* and λ_* . This extends a well-known result of Bramson in the continuous setting to the discrete case using only PDE arguments.

- Alain Blaustein (Pennsylvania State University)

Titre: Asymptotic analysis and simulation of collisional Vlasov-Poisson models

Résumé: This presentation focuses on collisional Vlasov-Poisson systems. These kinetic models are of primary interest, as they encode the multiple scales that arise in a plasma, ranging from fluid-like behavior when collisions dominate to wave interactions in weakly collisional regimes. We present quantitative results that capture the scales of both the continuous model and its discretized formulation. Then, we carry out various simulations which highlight our theoretical results (phase transition between regimes, plasma instabilities).

- Lucas Ertzbischoff (Imperial College London)

Titre: On the hydrostatic limit of the Euler-Boussinesq equations

Résumé: I will talk about the hydrostatic approximation of the 2d Euler-Boussinesq system, describing the evolution of an inviscid stratified fluid where the vertical length scale is much smaller than the horizontal one. Even though of importance in oceanography, the justification of the hydrostatic limit in this context has remained an open problem. I will discuss some recent results showing that some instability mechanisms may prevent this limit to hold.

This is joint work with R. Bianchini (CNR Rome) and M. Coti Zelati (Imperial College London).

- Jérémi Dardé (Institut de Mathématiques de Toulouse)

Titre: Coût d'observabilité en temps petit de l'équation de la chaleur 1D

Résumé: L'estimation du coût d'observabilité en temps petit de l'équation de la chaleur 1D (et, par dualité, celle du coût de contrôle en temps petit de la même équation), est une longue histoire qui commence dans les années 80, et n'est toujours pas terminée.

Dans cet exposé, j’expliquerai comment, dans un travail avec Sylvain Ervedoza (2019), nous avons amélioré l’estimation par au-dessus de ce coût. Ce sera l’occasion de parler d’inégalité de Carleman, de principe d’incertitude, de principe de Phragmén-Lindelöf et de transformées conformes de Schwarz–Christoffel.

- Samuel Daudin (Université Nice Côte d’Azur)

Titre: The convergence problem in mean-field control theory and related PDEs over the space of probability measures

Résumé: The goal of this talk is to discuss recent progress in the convergence problem in mean-field control theory.

We are interested in control problems involving a large number of (controlled) interacting particles subject to independent noises of Brownian type. When the number of particles tends to infinity, the problem simplifies into a control problem of mean-field type, set on the space of probability measures over the euclidean space. I will present some recent progress in the quantitative analysis of this convergence. More precisely I will discuss an approach based on a suitable mollification of the value function of the limiting problem. By dynamic programming, this value function solves in a weak viscosity sense a semi-linear Hamilton-Jacobi equation over the set of probability measures. We regularize it via sup-convolution in a well-chosen functional Hilbert space in order to produce approximations that are almost classical (sub)-solutions to the dynamic programming equation. Projecting these approximations in finite dimension, we can compare them with the value functions of the particle systems and obtain sharp rates of convergence.

This is based on a joint works with François Delarue, Joe Jackson and Ben Seeger.

- Imene Djebour (Cergy Paris Université)

Titre: Existence of strong solutions for a compressible fluid-solid interaction system with Navier slip boundary conditions

Résumé: We consider a fluid-structure interaction system coupling a viscous fluid governed by the compressible Navier-Stokes equations and a rigid body immersed in the fluid and modeled by the Newton’s law. In this work, we consider the Navier slip boundary conditions. Our aim is to show the local in time existence and uniqueness of the strong solution to the corresponding problem. The main step of this work is that we use Lagrangian change of variables in order to handle the transport equation and to reduce the problem in the initial domain. Therefore, it brings some extra nonlinear terms in the boundary conditions. The strategy is based on the study of the linearized system with nonhomogeneous boundary conditions and on the Banach fixed point theorem.

- Carlos Esteve-Yagüe (University of Cambridge)

Titre: A finite-difference based variational approach for solving Hamilton-Jacobi equations in high-dimensional domains

Résumé: It is well-known that the value function associated to a given optimal control problem or differential game can be characterised as the viscosity solution of an associated Hamilton-Jacobi equation. Numerical methods based on finite-differences are guaranteed to approximate the viscosity solution, provided the numerical scheme has the correct monotonicity. However, these grid-based methods suffer from the curse of dimensionality when the dimension of the domain is high. In this talk, I will present a variational approach to approximate the viscosity solution, consisting in the minimisation of a functional involving a Lax-Friedrichs discretisation of the Hamiltonian. I will show that, by choosing an appropriate numerical scheme, one can ensure uniqueness of a critical point for the functional. This in turn implies that the gradient flow associated to this functional converges to the unique global minimiser, which additionally can be proven to be close to the viscosity solution. In practice, the solution to the minimisation problem can be approximated by means of a Neural Network, trained through stochastic gradient descent, which simulates the gradient flow associated to the functional.

- Pei Su (Laboratoire de Mathématiques d’Orsay)

Titre: Regularity issue for the system describing elastic structure interacting with Navier-Stokes equations

Résumé: We are interested in the interaction of a viscous incompressible fluid with an elastic structure, where the structure is located on a part of the fluid boundary. It reacts to the surface forces induced by the fluid and deforms the reference domain Ω to Ω_η . The fluid equations are coupled with the structure via the kinematic condition and the action-reaction principle on the interface.

We first study the 2D visco-elastic shell interacts with 3D Navier-Stokes equations. Especially in a general reference geometry (the shell deforms along the normal direction of the flexible boundary), we prove a counterpart of the classical Ladyzhenskaya-Prodi-Serrin condition yielding conditional regularity and uniqueness of a solution. This requires additionally the deformation of the shell is Lipschitz continuous.

Then we consider a 1D perfectly elastic plate, deforming vertically in flat case, interacts with 2D Navier-Stokes equations, which thereby gives a hyperbolic evolution. We show the new regularity result for this parabolic-hyperbolic coupled system. It turns out that the "parabolic effect" of the fluid suffices to regularize the solution to the coupled fluid-structure system which is previously known for the Navier-Stokes equations in fixed domains.

This is based on joint work with D. Breit (Clausthal), P. Mensah (Clausthal) and S. Schwarzacher (Uppsala).

- Juliette Dubois (RWTH Aachen)

Titre: Two formulations for acoustic and surface waves in a free-surface, vertically stratified ocean

Résumé: I will present two formulations for a linear model describing the propagation of acoustic and surface gravity waves in a free-surface, stratified ocean. The first formulation, already studied in previous works, is obtained by the linearization of the compressible Euler equations, written in Lagrangian coordinates. The second formulation uses a new variable which can be understood as a generalized potential, allowing for a natural decomposition into rotational and irrotational velocity components. I will show that both formulations are well-posed and present some simulations.

- Dimitri Cobb (Universität Bonn)

Titre: Solutions de Yudovich non-bornées pour les équations d'Euler 2D

Résumé: Dans cet exposé, nous étudierons les solutions non-bornées des équations d'Euler incompressibles en deux dimensions d'espace. Ces solutions trouvent leur intérêt dans le fait que les espaces habituels de solutions (p. ex. basés sur une condition d'énergie finie comme L^2) ne respectent pas certaines des symétries du problème : l'invariance de Galilée et l'invariance d'échelle. Par ailleurs, les solutions d'énergie infinie ont une importance certaine dans plusieurs problèmes concrets, typiquement la géophysique.

Après avoir présenté le problème et un aperçu de la littérature sur le sujet, nous donnerons notre résultat : l'existence et l'unicité de solutions de Yudovich sous la condition d'une croissance en racine carrée du champ des vitesses. La démonstration, que nous esquisserons, est basée sur une décomposition intégrale de la pression, ainsi que des bilans d'énergie locaux conduisant à des estimations des solutions dans des espaces de type Morrey locaux.

Ce travail a été réalisé en collaboration avec Herbert Koch (Universität Bonn).

- Dragos Iftimie (Institut Camille Jordan)

Titre: On collapses of single-signed point-vortices with the boundary

Résumé: We consider the point-vortex system in a domain and assume the masses to be single-signed. We investigate the possibility of collapse with the boundary. We prove that such collapses are not possible in the case of the disk and of the half-plane. For general domains, we give a necessary condition for a collapse with the boundary to occur. This is joint work with M. Donati and L. Godard-Cadillac.

- Quentin Chauleur (Université de Lille)

Titre: Nonlinear waves on lattices and continuum limit

Résumé: In this talk we will be interested in the dynamics of dispersive PDEs on infinite lattices. In particular, we will highlight how the dispersive properties of the solutions, which are weaker than the one on the continuous setting, can be used in order to study the continuum limit of such systems as the step size of the grid tends to zero. We will also provide some perspectives on the subject.

- Pierre Le Bris (IHES)

Titre: Uniform in time propagation of chaos for the 2D vortex model

Résumé: We are interested in a system of particles in singular mean-field interaction and wish to prove that, as the number of particles goes to infinity, two given particles within that system become "more and more" independent, a phenomenon known as propagation of chaos. The interaction we will focus on comes from the Biot-Savart kernel, for which the nonlinear limit of the particle system satisfies the vorticity equation, arising from the 2D incompressible Navier-Stokes system.

We build upon a recent work of P.-E. Jabin and Z. Wang to obtain a uniform in time convergence. The approach consists in computing the time evolution of the relative entropy of the joint law of the particle system with respect to the nonlinear limit. We prove time-uniform bounds on the limit, as well as a logarithmic Sobolev inequality. From the latter, the Fisher information appearing in the entropy dissipation yields a control on the relative entropy itself, inducing the time uniformity.

This is joint work with A. Guillin and P. Monmarché.

- Cécile Huneau (CNRS et Ecole Polytechnique)

Titre: Burnett's conjecture in General Relativity

Résumé: In this work, I will present a work in collaboration with Jonathan Luk where we prove that weak limits of solutions to Einstein vacuum equations, in some setting, converge to solutions to Einstein equations coupled to a massless Vlasov field. The proof uses the microlocal defect measures of Tartar and Gérard, and compensated compactness.

- Rodrigue Lelotte (ENPC & Inria)

Titre: Strictly-Correlated Electrons from the viewpoint of optimal transport and their dissociation at infinity

Résumé: The Strictly-Correlated Electrons (SCE) is a formalism of Density-Functional Theory (DFT) used to approximate ground-state energies of strongly-correlated quantum systems. From a mathematical viewpoint, it is obtained as the semi-classical limit of the Levy-Lieb functional, which is one of the central objects to DFT. I will present this problem, which arises as a multimarginal optimal transport of a special kind, and present a recent result of mine on the dissociation at infinity of such systems, reminiscent of the ionisation conjecture in quantum physics.

- Viviana Grasselli (Institut Elie Cartan de Lorraine - Metz)

Titre: Zero resonant states for the Schrödinger operator

Résumé: The Schrödinger operator on the whole space \mathbb{R}^d gives rise to a dispersive equation, meaning that the mass of the solution spreads towards infinity, and these dispersive properties are tightly linked to its spectrum. Resonances can be seen as a generalisation of eigenvalues: they are complex numbers for which the eigenvalue equation admits a non L^2 solution. Their dynamical interpretation is that the imaginary part of a resonance determines the speed of dispersion of a resonant state. In this talk we will analyze resonances in zero, which are known to be an obstacle to dispersion. For a rather general class of potentials we will see when zero is a resonance or an eigenvalue and some properties of the associated state.

- Marco Michetti (University of Paris Saclay)

Titre: The Robin Harmonic Measure on non smooth domains

Résumé: We analyze the boundary behavior of solutions to elliptic PDE with prescribed Robin data in rough domains. In particular, we construct a “Robin elliptic measure” and demonstrate the surprising fact that this measure is (quantitatively) mutually absolutely continuous with respect to surface measure on a wide class of domains that includes the complement of certain fractals.

Titres et résumés des posters

- [Théo Belin](#)

Titre: Quantitative estimates for non-autonomous operators in L^p maximal regularity

Résumé: In this work I will present a method to estimate the continuity constant of the linear Cauchy problem associated to a class of nonautonomous L^p maximally regular operator, relatively continuous in time. Giving explicit estimates of this constant is useful for the global existence of strong solutions in a large class of nonlinear equations. The method relies on a perturbation method used in the context of relatively continuous operators which usually yields nonquantitative estimates. However, under mild integrability conditions on the pointwise autonomous regularity constant it is possible to give an upper bound of the constant, which is explicit in terms of three quantities associated to the pointwise maximal regularity and time regularity of the operator. The estimates are used to find new growth conditions of nonlinearities that allow the global existence of strong solutions of abstract quasilinear equations through the use of the Schauder's fixed point theorem.

- [Sana Ben Hafsia](#)

Titre: Existence results for problems involving non local operator with weight and a critical nonlinearity

Résumé: Recently, a great attention has been focused on the study of fractional and non-local operators of elliptic type, both for the pure mathematical research and in view of concrete real-world applications.

We consider the non local minimizing problem on $\mathbb{H}_0^s(\Omega) \subset L^{q_s}(\Omega)$, with $q_s := \frac{2n}{n-2s}$, $s \in]0, 1[$ and $n \geq 3$:

$$\inf_{\substack{u \in \mathbb{H}_0^s(\Omega) \\ \|u\|_{L^{q_s}(\Omega)}=1}} \int_{\mathbb{R}^n} p(x) \left(\int_{\mathbb{R}^n} \frac{|u(x) - u(y)|^2}{|x - y|^{n+2s}} dy \right) dx - \lambda \int_{\Omega} |u(x)|^2 dx, \quad (0.1)$$

where Ω is a bounded domain in \mathbb{R}^n , $p : \mathbb{R}^n \rightarrow \mathbb{R}$ is a given positive weight presenting a global minimum $p_0 > 0$ at $a \in \Omega$ and λ is a real constant. The objective of this work is to show that minimizers do exist for some k, s, λ and n .

We are interested also to prove non-existence of solutions of a minimizing problem involving fractional Laplacian with weight and we study non ground state solutions thanks to the Mountain Pass Theorem.

- [Jordan Berthoumieu](#)

Titre: Minimizing travelling waves for nonlinear Schrödinger equations

Résumé: The poster deals with special solutions to the one-dimensional nonlinear Schrödinger equation $iu_t + u_x x + f(|u|^2)u = 0$ that are the travelling waves solutions of the form $u(t, x) =$

$\psi(x - ct)$. These special solutions were shown to be orbitally stable for the Gross-Pitaevskii equation by a variational argument. This argument relies on understanding a minimization problem under constraints, especially the compactness of the pseudo-minimizing sequences for this problem. We adapt this argument in order to prove a similar result of orbital stability in a more general framework.

- Rayan Fahs

Titre: On the semi-classical analysis of Schrödinger operators with linear electric potentials in a bounded domain

Résumé: The aim of this poster is to establish the asymptotic expansion of the eigenvalues of the Stark Hamiltonian, with a strong uniform electric field and Dirichlet boundary conditions on a smooth bounded domain of \mathbb{R}^N , $N \geq 1$. In the strong electric field limit, we derive, under certain local convexity conditions, a full asymptotic expansion of the low-lying eigenvalues. To establish our main result, we perform the construction of quasi-modes. The "optimality" of our constructions is then established thanks to a reduction to model operators and localization estimates. Additionally, we apply our techniques to find a full asymptotic expansion of the low-lying eigenvalues of a more general operator $-h^2\Delta + V(\mathbf{x})$, where h is a small parameter and V is a smooth real potential satisfying some additional conditions.

- Marina Ferreira

Titre: Anomalous self-similar behaviour in coagulation equations with source

Résumé: The Smoluchowski coagulation equation was proposed in 1916 as a model for the size distribution of particles undergoing binary coalescence. These are nonlinear nonlocal integro-differential equations which exhibit rich behaviour depending on the coagulation rate kernel considered and many analytical questions remain open, such as the precise longtime behaviour for general coagulation kernels. This poster focuses on the longtime behaviour of solutions which have linearly increasing mass. Our formal analysis indicates that the qualitative behaviour depends on the kernel parameters. More precisely, we argue that the long-time behaviour is still self-similar, but the scaling depends on the kernel parameters and it differs from the usual one which is commonly found in these equations.

- Vincent Laheurte

Titre: Contrôlabilité en hautes fréquences des systèmes hyperboliques linéaires du premier ordre

Résumé: On s'intéresse au problème classique de la contrôlabilité de systèmes d'évolution linéaires du premier ordre, via une approche inspirée de la méthode des multiplicateurs de Komornik. On regarde en particulier le coût associé à des conditions initiales localisées ou, dans le cas dispersif, semi-classiquement localisées.

- Dorian Martino

Titre: About Willmore surfaces : a foot into the analysis of conformal geometry

Résumé: The Willmore energy has been introduced in the 19th century as a measure of the bending of a two dimensional membrane. From an analytical view-point, the Euler-Lagrange equation is a nonlinear critical elliptic equation of order 4. Thanks to the works of Kuwert-Schätzle (2001) and Rivière (2008), Willmore surfaces are smooth. However, Marque proved in 2019 that the space of Willmore surfaces is not compact. On this poster, I will describe some ideas and new results about the study of singularities arising in the study of limits and Palais-Smale sequences of Willmore surfaces.

- [Paul Pace](#)

Titre: Méthodes Sparse-PIC d'ordre élevées

Résumé: D'abord je présenterai brièvement les spécificités des méthodes dites Particle In Cell (PIC). Puis en particulier du formalisme parcimonieux (Sparse) dans ce contexte. Ensuite j'illustrerai l'intérêt pour ces méthodes, d'intégrer des noyaux d'interpolations et de projections d'ordre élevé à travers des simulations d'exemples classiques issus de la physique des plasmas.

- [Hugo Parada](#)

Titre: Approximate controllability of coupled fractional parabolic equations

Résumé: This work investigates the approximate controllability of coupled nonlocal fractional heat equations within one-dimensional domains, employing internal or exterior controls, focusing on scenarios where $s \in (1/2, 1)$. Our results are based on unique continuation properties for the fractional Laplacian and a spectral analysis of the coupling operator.

- [Adrien Tendani Soler](#)

Titre: Sur l'espace atteignable de quelques systèmes paraboliques

Résumé: La problématique générale des espaces atteignables peut être résumée de la manière suivante: *Pour un système contrôlé donné. étant donné un état initial u_i et un temps $T > 0$, décrire l'espace $R(u_i, T)$ des états finaux u_f que l'on peut atteindre à partir de u_i au temps T .* Déterminer l'espace atteignable $R(u_i, T)$ des systèmes contrôlés est l'un des principaux problèmes de la théorie du contrôle. Donner une caractérisation précise des états qui peuvent être atteints en un certain temps $T > 0$ est une question encore largement ouverte: même pour l'équation de la chaleur à coefficients constants en une dimension et contrôlée depuis la frontière, la caractérisation complète de l'espace atteignable, en termes d'espaces de Bergman, n'a été obtenue que très récemment. La description de l'espace atteignable pour l'équation de la chaleur a reçu une grande attention ces dernières années. Dans mon poster je présenterai mes résultats sur l'espace atteignable pour le système de Stokes où de nouvelles difficultés apparaissent liées à la non-localité de la pression et à la propagation de la condition de divergence nulle.

Diner de conférence – 19h Jeudi 21 Mars

- Restaurant: Les Caves de la Maréchale
- Localisation: 3 Rue Jules Chalande, 31000 Toulouse
- Webpage: <https://www.lescavesdelamarechale.com>

Déjeuners des Mercredi 20 et Vendredi 22 Mars

- Restaurant: Brasserie L'Esplanade
- Localisation: sur le campus, proche de la station de métro Université Paul Sabatier
- Webpage: <https://www.crous-toulouse.fr/restaurant/brasserie-lesplanade/>