Conference on Calculus of Variations in Lille - 4th edition - June 24-28 2024

Report of Contributions

Contribution ID: **1** Type: **not specified**

The attractive log gas: propagation of chaos, stability and uniqueness questions

Monday, June 24, 2024 10:30 AM (1 hour)

We consider the dynamics of a system of particles with logarithmic attractive interaction, on the torus, at inverse temperature beta. We show phase transitions on the stability and uniqueness of the uniform distribution. Investigating the mean-field convergence of the system by the modulated free energy method, we deduce that uniform-in-time convergence is not always true. This is joint work with Antonin Chodron de Courcel and Matthew Rosenzweig.

Presenter: SERFATY, Sylvia (NYU Courant Institute)

Contribution ID: **2** Type: **not specified**

A free-discontinuity problem for smectic liquid crystals

Monday, June 24, 2024 11:30 AM (1 hour)

Smectic liquid crystals are a phase of matter in which the constituent molecules tend to align locally parallel to one another and to arrange themselves in layers. Experimental evidence shows that the configuration of the layers in smectic films may be rather complex, possibly with defects - that is, localised regions of sharp change in the orientation of the layers. Defects may occur at isolated points, along lines or surfaces. In this talk, we discuss a free-discontinuity variational problem for smectic A liquid crystals in two dimensions, set in the space SBV. We focus on a specific form of the energy functional, which penalises dislocations of the layers along the defects and is lower semicontinuous, so that minimisers exist. The talk is based on joint work with John M. Ball (Heriot-Watt University, Edinburgh and Hong Kong Institute of Advanced Studies) and Bianca Stroffolini (Università Federico II, Napoli).

Presenter: CANEVARI, Giacomo (Università degli studi di Verona)

Contribution ID: **3** Type: **not specified**

Asymptotic behavior of p-Harmonic mappings when p goes to 2

Monday, June 24, 2024 2:00 PM (30 minutes)

A mapping of nonzero topological degree from the boundary of a disk to the circle cannot be extended by a continuous mapping defined on the whole disk to the circle as homotopy theory asserts. However, this extension is possible if one allows the extended mapping to have discontinuous points, also called singularities. Geometrical and physical situations that we will describe motivate the problem of finding the 'best' extension possible in the sense of minimization of the p-Dirichlet energy when $p = 2$. However, this is only possible when $p < 2$ for reasons we will explain. We will describe the limiting behavior when p goes to 2 of the minimizers as well as the convergence of the singularities they carry. We will also take the opportunity to raise the question for the 3D variant as well as other manifolds than the circle.

Presenter: VAN VAERENBERGH, Benoît (Université catholique de Louvain)

Contribution ID: **4** Type: **not specified**

Variational Convergence of Liquid Crystal Energies to Line and Surface Energies

Monday, June 24, 2024 2:30 PM (1 hour)

Modelling a liquid crystal outside a colloidal particle with the Landau - de Gennes model when a uniform magnetic field is considered leads to a frustrated system. Line and point singularities are likely to appear. We derive, by means of variational convergence, a limiting model of a suitably rescaled energy that can be written in terms of geometric objects.
This is a joint work with D. Stantejsky and A. Chambolle.

Presenter: ALOUGES, François (ENS Paris-Saclay)

Contribution ID: **7** Type: **not specified**

On a variational model for epitaxial growth with dislocations

Tuesday, June 25, 2024 9:00 AM (1 hour)

In this talk, we consider a variational model which has been introduced in the literature to model the deposition of a thin crystalline film on a rigid substrate, allowing for the formation of dislocations. The energy functional takes into account the surface energy of the film's free surface, the elastic energy due to the crystallographic misfit between the film and the substrate, and the nucleation energy of dislocations. We discuss in particular the scaling law for the infimal energy. The upper bound constructions suggest that there are parameter regimes in which dislocations are energetically favorable. The lower bound builds on a new variant of a ball construction.

This talk is based on a recent joint work with Lukas Abel and Janusz Ginster (both HU Berlin).

Presenter: ZWICKNAGL, Barbara (Humboldt-Universität zu Berlin)

Contribution ID: **8** Type: **not specified**

Gradient flow solutions for porous medium equations with nonlocal Lévy-type pressure

Tuesday, June 25, 2024 10:00 AM (30 minutes)

We study a porous medium-type equation whose pressure is given by a nonlocal Lévy operator associated to a symmetric jump Lévy kernel. The class of nonlocal operators under consideration appears as a generalization of the classical fractional Laplace operator. For the class of Lévy-operators, we construct weak solutions using a variational minimizing movement scheme. The lack of interpolation techniques is ensued by technical challenges that render our setting more challenging than the one known for fractional operators. This is joint work with Guy Foghem and Markus Schmidtchen.

Presenter: PADILLA-GARZA, David (IST Austria)

Contribution ID: **9** Type: **not specified**

Nonoccurrence of Lavrentiev gap for a general minimization problem

Tuesday, June 25, 2024 11:00 AM (30 minutes)

We discuss the absence of Lavrentiev gap for minimization problems in the calculus of variations when the functional depends on the space variable, the function and the gradient. Namely, can we approximate a function u of finite energy by a sequence of Lipschitz functions whose energy converges to that of the original function? In general, it is not true, thus, we have to put some constraints on our functional. In our case, we require a natural condition balancing control on the variations of the space variable with growth with respect to those that depend on the gradient. The proof relies on this assumption combined with an approximation result on the graph of u.

Presenter: LLEDOS, Benjamin (Université catholique de Louvain)

Contribution ID: **10** Type: **not specified**

Closability of differential operators and structure of currents

Tuesday, June 25, 2024 11:30 AM (1 hour)

I will discuss recent results concerning the closability of certain directional derivative and Jacobian-type differential operators and their implications for the structure of flat chains and metric currents. Additionally, I will present a new, elementary proof of Ambrosio and Kirchhiem's flat chain conjecture, in the case of 1-dimensional currents. This conjecture asserts that metric currents in the Euclidean space correspond to Federer-Fleming flat chains. Our new proof sheds light on the obstructions that one needs to face towards a positive answer to the conjecture in full generality. This is based on joint works with G. Alberti, D. Bate, and A. Merlo.

Presenter: MARCHESE, Andrea (Università di Trento)

Contribution ID: **11** Type: **not specified**

Rigidity, Flexibility and Scaling for Models of Shape-Memory Alloys

Tuesday, June 25, 2024 2:00 PM (1 hour)

The modelling of shape-memory alloys displays a striking dichotomy between rigidity and flexibility. On the one hand, without any additional regularity solutions can be highly irregular and non-unique, they are very flexible. On the other hand, often, at higher regularity, which physically can be viewed as augmenting the model by an interfacial energy, the solutions become very rigid and obey strong kinematic constraints. In this talk I explore scaling as a mechanism distinguishing between these regimes and study scaling properties for selected model systems. If time permits, I relate these to nonlocal anisotropic isoperimetric problems. This is based on joint work with P. Cesana, J. Ginster, J. Taylor, A. Tribuzio, Ch. Zillinger and B. Zwicknagl.

Presenter: RÜLAND, Angkana (Universität Bonn)

Contribution ID: **12** Type: **not specified**

A Γ-convergence result for 2D type-I superconductors

Tuesday, June 25, 2024 3:00 PM (30 minutes)

In the Ginzburg-Landau nodel superconductors are characterized by a parameter κ called the Ginzburg-Landau parameter. If $\kappa < \frac{1}{\sqrt{2}}$ the superconductors are classified as type-I, if $\kappa > \frac{1}{\sqrt{2}}$ they are classified as type-II. While in type-II superconductors vortices appear, in type-I superconductors normal and superconducting regions are formed, separated by interfaces. In particular by the Meissner Effect, if ρ is the density of superconducting electrons and B is the magnetic field, it is observed that $\rho B \simeq 0$. Considering a 3D sample, it is experimentally observed that complex patterns appear at the surface. It is believed that these patterns are a manifestation of branching patterns inside the sample.

In a first work Conti Otto and Serfaty derive two regimes of parameters for the 3D type-I model, corresponding to uniform and non uniform branching patterns. Moreover, in a subsequent work Conti Otto Goldman and Serfaty prove a Γ-convergence result for the full 3D model in the case of uniform branching patterns.
In this talk I present a Γ-convergence result for the 2D type-I Ginzburg-Landau model in the crossover of the two regimes found in the former work. This is a first step in understanding how to extend the results of the latter work to the second regime.
With these hypothesis on the parameters the energy functional shares similarities with a Modica-Mortola type functional and in the limit Γ-converges to the area functional. To prove this result, it is necessary to carefully treat the global interaction between the phase of the complex order parameter u and the vector potential A, taking into account the gauge invariance satisfied by the functional.

This talk is based on an ongoing work with Michael Goldman and Alessandro Zilio.

Presenter: COSENZA, Alessandro (Université Paris Cité)

Contribution ID: **13** Type: **not specified**

BV curves of measures and the continuity equation

Tuesday, June 25, 2024 4:00 PM (1 hour)

Representation results for Lipschitz (or even absolutely continuous) curves $\mu : [0,T] \to \mathcal{P}_p(\mathbb{R}^d)$, $p > 1$, with values in
the Wasserstein space $(\mathcal{P}_p(\mathbb{R}^d), W_p)$ of Borel probability measures in $\mathbb{R}^d$ with finite p-moment provide a crucial tool to study evolutionary PDEs and geometric problems in a measure-theoretic setting.

They are strictly related to corresponding representation results for measure-valued solutions to the continuity equation, as a superposition of absolutely continuous curves solving a suitable differential equation.

In this talk we discuss the validity and the appropriate formulation of the above results in the case $p = 1$, for the space of probability measures with finite moment $\mathcal{P}_1(\mathbb{R}^d)$ endowed with the metric W_1. We will thus provide a suitable version of the superposition principle for curves of measures in $\mathcal{P}_1(\mathbb{R}^d)$ that are only of bounded variation with respect to the time variable.

Joint work with Stefano Almi (Napoli) and Giuseppe Savaré (Milano).

Presenter: ROSSI, Riccarda (Università degli Studi di Brescia)

Contribution ID: **14** Type: **not specified**

Self-adjoint problems in the optimization of non-linear pde models

Wednesday, June 26, 2024 9:00 AM (1 hour)

We consider optimization problems under partial differential equation constraints. It is assumed that the p.d.e. arises from the minimization of a convex non-linear (non-quadratic) energy. We prove that the optimization problem is self-adjoint when the objective function is the dual energy. In other words, the differential of the objective function with respect to the optimization variable does not involve any adjoint state. This result generalizes the well known fact that the so-called compliance is self-adjoint in the linear case (quadratic energy).

We show some applications for the shape and topology optimization of electrical machines in the 2-d magnetostatic context.

This is a joint work with Théodore Cherrière, Thomas Gauthey, Maya Hage Hassan, Xavier Mininger.

Presenter: ALLAIRE, Grégoire (École Polytechnique)

Contribution ID: **15** Type: **not specified**

Nonlocal-to-local analysis of energies in Micromagnetics

Wednesday, June 26, 2024 10:00 AM (30 minutes)

In this talk we first consider the nonlocal-to-local convergence of exchange energy functionals in Micromagnetics, extending the Bourgain-Brezis-Mironescu formula in order to encompass the scenario where also antisymmetric contributions are encoded.
In a first stage, the nonlocal approximation is given by a pointwise convergence result, obtaining as byproduct a rigorous justification of the so-called Dzyaloshinskii-Moriya interaction term. Then, also a Gamma-convergence argument is presented.
In the remaining time of the talk, in the modified setting where the nonlocal exchange interactions replace the classical local ones, we focus on the existence of minimizers for the micromagnetic energy functional.
Finally, by means of a nonlocal Poincaré-type inequality, we exhibit some conditions to obtain constant minimizing configurations in the spirit of the so-called Brown's Fundamental Theorem.

This is a joint work with E. Davoli, G. Di Fratta and L. Lombardini.

Presenter: GIORGIO, Rossella (TU Wien)

Contribution ID: **16** Type: **not specified**

Quantitative stochastic homogenization of variational models arising in fracture mechanics

Wednesday, June 26, 2024 11:00 AM (30 minutes)

I will present a recent quantitative result concerning the stochastic homogenization of the so-called Griffith type model arising in fracture mechanics : the energy $E_\varepsilon(u)$ for $u \in \mathrm{SBV}$ takes the form of
$\mathrm{E}_\varepsilon(u) = \int_{\Omega \setminus S_u} F\left(\frac{\cdot}{\varepsilon}, \nabla u\right) - f \cdot u + \int_{S_u} g\left(\frac{\cdot}{\varepsilon}\right) \, \mathrm{d}\mathcal{H}^{d-1}, where \mathrm{F} denotes the stored elastic energy, f the external forces, g the$
1 the scale of the microstructure. Since the work of Cagnetti, Dal Maso, Scardia and Zeppieri, the homogenized model has been identified qualitatively by taking the Γ-limit as $\varepsilon \downarrow 0$ in the equation above;
and in particular the two main constitutive properties of the system have been derived: the homogenized elastic energy and the homogenized fracture toughness, both given explicitly by means of cell-formulas. I will explain in this talk how we can derive quantitative estimates for the convergence of the cell-formula for the effective toughness. This is based on a joint work with Julian Fischer and Antonio Agresti.

Presenter: CLOZEAU, Nicolas (IST Austria)

Contribution ID: **17** Type: **not specified**

Constant sign and sign changing NLS ground states on metric graphs

Wednesday, June 26, 2024 11:30 AM (1 hour)

In this talk, we investigate existence and nonexistence of positive and nodal action ground states for the nonlinear Schrödinger equation on metric graphs.
For noncompact graphs with finitely many edges, we detect purely topological sharp conditions preventing the existence of ground states or of nodal ground states. We also investigate analogous conditions of metrical nature. The negative results are complemented by several sufficient conditions to ensure existence, either of topological or metrical nature, or a combination of the two.

This is based on joint work with Simone Dovetta (Politecnico di Torino (Italy)), Damien Galant (UPHF and UMons (Belgium)), Enrico Serra (Politecnico di Torino (Italy)), Christophe Troestler (UMons (Belgium)).

Presenter: DE COSTER, Colette (Université Polytechnique Hauts-de-France)

Contribution ID: **18** Type: **not specified**

Approximate mean curvature flows for general data, and their limits

Friday, June 28, 2024 11:00 AM (1 hour)

The classical mean curvature flow of regular surfaces may develop singularities in finite time and is not well defined beyond. Various extensions have been proposed which are meaningful for all positive times. However, they are generally not well defined or inconvenient for more general "surface-type" objects such as point clouds. In this talk, I will present a new notion of approximate mean curvature flow which is valid for very general objects. It is based on Brakke's construction of weak mean curvature flows and the adaptation due to Kim & Tonegawa. We first construct, for general varifolds and by iterated push-forwards, an approximate time-discrete mean curvature flow depending on both a given time step and an approximation parameter. This time-discrete flow converges, as the time step tends to 0, to a unique limit flow, that we call an approximate mean curvature flow. Our approach is fairly general: it provides an approximate notion of mean curvature flow for very general structures of any dimension and codimension, whether continuous surfaces in the classical sense or point clouds. I will discuss the properties of this approximate mean curvature flow, and the existence of a limit spacetime Brakke flow when the approximation parameter tends to 0. These results were obtained as part of Abdelmouksit Sagueni's PhD work cosupervised with Blanche Buet and Gian Paolo Leonardi.

Presenter: MASNOU, Simon (Université Claude Bernard Lyon 1)

Contribution ID: **21** Type: **not specified**

The extension of traces for Sobolev mappings between manifolds

Thursday, June 27, 2024 9:00 AM (1 hour)

Given compact Riemannian manifolds $\mathcal{M}$ and $\mathcal{N}$ and $p \in (1, \infty)$, the question of traces for Sobolev mappings consists in characterising the mappings from $\partial\mathcal{M}$ to $\mathcal{N}$ that can arises of maps in the first-order Sobolev space $\dot{W}^{1,p}(\mathcal{M}, \mathcal{N})$.
A direct application of Gagliardo's characterisation of traces for the linear spaces $\dot{W}^{1,p}(\mathcal{M}, \mathbb{R})$ shows that traces of maps in $\dot{W}^{1,p}(\mathcal{M}, \mathcal{N})$ should belong to the fractional Sobolev-Slobodeckij space $\dot{W}^{1-1/p,p}(\partial\mathcal{M}, \mathcal{N})$. There is however no reason for Gagliardo's linear extension to satisfy the nonlinear constraint imposed by $\mathcal{N}$ on the target.

In the case $p > \dim \mathcal{M}$, Sobolev mappings are continuous and thus traces of Sobolev maps are the mappings of $\dot{W}^{1-1/p,p}(\partial\mathcal{M}, \mathcal{N})$ that are also restrictions of continuous functions (F. Bethuel, F. Demengel, Extensions for Sobolev mappings between manifolds (1995)).
The critical case $p = \dim \mathcal{M}$ can be treated similarly thanks to their vanishing mean oscillation property (F. Bethuel, F. Demengel, Extensions for Sobolev mappings between manifolds (1995); H. Brezis, L. Nirenberg, Degree theory and BMO. I. Compact manifolds without boundaries (1995); R. Schoen, K. Uhlenbeck, A regularity theory for harmonic maps (1982)).

The case $1 < p < \dim \mathcal{M}$ is more delicate.
It was first proved that when the first homotopy $\pi_1(\mathcal{N}), \dots, \pi_{\lfloor p-1 \rfloor}(\mathcal{N})$ are trivial, then the trace operator from $\dot{W}^{1,p}(\mathcal{M}, \mathcal{N})$ to $\dot{W}^{1-1/p,p}(\partial\mathcal{M}, \mathcal{N})$ is surjective (R. Hardt, Lin F., Mappings minimizing the L^p norm of the gradient (1987)).
On the other hand, several conditions for the surjectivity have been known: topological obstructions require $\pi_{\lfloor p-1 \rfloor}(\mathcal{N})$ to be trivial whereas analytical obstructions arise unless the groups $\pi_1(\mathcal{N}), \dots, \pi_{\lfloor p-1 \rfloor}(\mathcal{N})$ are finite (F. Bethuel, A new obstruction to the extension problem for Sobolev maps between manifolds (2014)) and, when $p \geq 2$ is an integer, $\pi_{p-1}(\mathcal{N})$ is trivial (Trace theory for Sobolev mappings into a manifold (2021)).

In a recent work, I have completed the characterisation of the cases where the trace is surjective, proving that the known necessary conditions turn out to be sufficient (J. Van Schaftingen, The extension of traces for Sobolev mappings between manifolds).
I extend the traces thanks to a new construction which works on the domain rather than in the image. When $p \geq \dim \mathcal{M}$ the same construction also provides a Sobolev extension with linear estimates for maps that have a continuous extension, provided that there are no known analytical obstructions to such a control.

Presenter: VAN SCHAFTINGEN, Jean (Université catholique de Louvain)

Contribution ID: **22** Type: **not specified**

Convergence results for critical points of the one-dimensional Ambrosio-Tortorelli functional with an obstacle condition

Thursday, June 27, 2024 10:00 AM (30 minutes)

According to the variational approach to fracture introduced by Francfort and Marigo, the Mumford-Shah energy is commonly used to modelize brittle cracks in an elastic material. Ambrosio and Tortorelli proposed a variational phase field regularization of this functional which, through a Gamma-convergence result, leads to the convergence of global minimizers. This result gives however no information about the limiting behavior of general critical points.
This problem has been solved in dimension 1 by Francfort, Le and Serfaty, and partially extended to any dimension by Babadjian, Millot and Rodiac.

In this talk, we consider a 1-dimensional critical point of the Ambrosio-Tortorelli energy under an obstacle condition on the phase field variable. This problem can be interpreted a time discretization of a quasistatic evolution problem where one puts as the obstacle the
solution obtained in the previous time step. The obstacle condition now reads as an irreversibility condition (the crack can just increase in time). First, elliptic estimates for the obstacle problem yield the $C^{1,1}$ regularity of the critical points. The limits of such critical points turn out to be critical points of the Mumford-Shah energy that inherit the possible discontinuities generated by the obstacle sequence. Finally, the phase field term converges toward a Dirac mass, the mass of which can be computed using the so-called “equipartition of the energy” principle.

Presenter: RAKOVSKY, Martin (Université Paris-Saclay)

Contribution ID: **23** Type: **not specified**

Two examples of Steiner trees with a countable number of terminals and branching points

Thursday, June 27, 2024 11:00 AM (30 minutes)

The Steiner tree problem is a problem of connecting a given compact set by a shortest way. By a full Steiner tree we name a solution of the Steiner tree problem without vertices of degree 2.

I will talk about two solutions to the Steiner problem with given data differ by only one vertex. Every solution for the first data appears to be a full Steiner tree with an infinite number of branching points (id est points of degree 3) and every solution for the second data is a union of full Steiner trees on five vertices.

Presenter: TEPLITSKAYA, Yana (Université Paris-Saclay)

Contribution ID: **24** Type: **not specified**

Sharp stability for the Dirichlet spectrum near the ball

Thursday, June 27, 2024 11:30 AM (1 hour)

In this talk, we discuss the following question: knowing that the first Dirichlet-Laplacian eigenvalue of an open set is close to the one of the ball of same volume (which is the minimizer due to Kaber-Krahn's inequality), can we say that the other eigenvalues of this set are also close to the ones of the ball? More precisely we seek for quantitative estimates of the form $|\lambda_k(\Omega) - \lambda_k(B)| \leq C(\lambda_1(\Omega) - \lambda_1(B))^\alpha. We show that such an estimate is valid with \alpha = 1/2$ and that this is sharp in general, though it can be improved to $\alpha = 1$ if $\lambda_k(B)$ is simple. The proof of this last case requires the regularity analysis for minimizers of $\lambda_1 \pm \varepsilon\lambda_k$, which involves a vectorial free boundary problem.
We also provide an improved result for multiple eigenvalues, and we observe that our analysis leads to a reverse Kohler-Jobin inequality.

This is a joint work with Dorin Bucur, Mickaël Nahon and Raphaël Prunier.

Presenter: LAMBOLEY, Jimmy (Sorbonne Université)

Contribution ID: **25** Type: **not specified**

Fractional multiphase transitions & nonlocal minimal partitions: closed and open questions

Thursday, June 27, 2024 2:00 PM (1 hour)

I will present a convergence result for solutions of Allen-Cahn type systems with a multiple-well potential involving the usual fractional Laplacian in the regime of the so-called nonlocal minimal surfaces.

In the singular limit, solutions converge in a certain sense to stationary points of a nonlocal (or fractional) energy for partitions of the domain with (in general) non homogeneous surface tensions.

Then I will present partially regularity results and open questions concerning the limiting problem underlying the new features compared to classical minimal partition problems. This talk is based on joint works with Thomas Gabard.

Presenter: MILLOT, Vincent (Université Paris-Est Créteil)

Contribution ID: **26** Type: **not specified**

Biaxiality vs uniaxiality in Landau-de Gennes minimisers in 2D discs

Thursday, June 27, 2024 3:00 PM (30 minutes)

We consider the problem of minimising the (simplest) Landau-de Gennes (LdG) energy in two-dimensional discs, under axial symmetry, a physically relevant pointwise norm-constraint in the interior, and radial anchoring on the boundary. The goal is to study the uniaxial or biaxial character of minimisers. We show that the latter depends crucially on the value of a parameter $\lambda \geq 0$ appearing in front of the potential and penalising biaxiality. For λ large, minimisers are uniaxial. As λ decreases, biaxiality is less penalised and a threshold $\lambda_* > 0$ is met at which uniaxial and biaxial minimisers coexist. Below λ_*, all minimisers are biaxial. For all biaxial minimisers, complete biaxial escape occurs. The cornerstone of the argument consists in an energy gap between small and large maps in the associated minimisation problem for the Dirichlet integral (i.e., for $\lambda = 0$). Here, a map is called small if it does not escape the spherical cap containing the image of the boundary data, and large otherwise. The energy gap is made fully explicit by describing the set of optimal maps in both the small and the large case. A major difficulty in the analysis lies in dealing with a lack of compactness in minimising sequences.

This problem arose in a natural way in the framework of a broader investigation, carried out in a joint work with Vincent Millot and Adriano Pisante, of qualitative properties of LdG minimisers in 3D cylinders.

Presenter: DIPASQUALE, Federico Luigi (Scuola Superiore Meridionale)

Contribution ID: **27** Type: **not specified**

Blow-up limits for Griffith minimizers

Thursday, June 27, 2024 4:00 PM (1 hour)

The so called Griffith functional has been introduced to model the equilibrium state of a fracture in linearized elasticity. According to this model the equilibrium state of a fracture is defined as a minimizer of the functional
G(u,K) := $\int_{\Omega\setminus K} |e(u)|^2 \, \mathrm{d}x + \mathcal{H}^{N-1}(K)$,$among pairs(u,K)such that$K$is a subset of dimension$(N-1)$of \Omega \subset \mathbb{R}^N$ (the fracture), $u\colon \Omega \setminus K \to \mathbf{R}^N$ is a C^1 function (a displacement field) which satisfies a Dirichlet condition at the boundary $\partial\Omega$ and the matrix $e(u) := (Du + Du^T)/2$ is the symmetric part of the gradient of u.

The goal of this talk is to present a recent regularity result on Griffith minimizers and, in particular, a joint work with with Camille Labourie on blow-up limits and their (partial) classification in the plane.

For this purpose, we developed a new approach to the uniform concentration property of Dal Maso, Morel and Solimini in the vectorial case. The particular novelty of our proof is to avoid the use of the co-area formula, not available in the vectorial context.

Presenter: LEMENANT, Antoine (Université de Lorraine)

Contribution ID: **28** Type: **not specified**

Periodic partitions with minimal perimeter

Friday, June 28, 2024 9:00 AM (1 hour)

I will discuss existence and regularity of periodic tessellations of the Euclidean space, with possibly unequal cells, which minimize a general perimeter functional. In will present some examples in the planar case and some open problems.

Presenter: NOVAGA, Matteo (Università di Pisa)

Contribution ID: **29** Type: **not specified**

Concentration compactness phenomena for Willmore surfaces

Friday, June 28, 2024 10:00 AM (30 minutes)

The Willmore energy is fundamental in the study of curved surfaces and arise in various context, such as cell biology, optics, general relativity… Since the work of Mondino-Nguyen in 2018, the Willmore energy can also be understood as the only way to merge the study of minimal surfaces and conformal geometry. Despite its first appearance during the 1810s in the work of Germain and Poisson, the regularity of such surfaces has only been completely settled in the 2000s by Kuwert-Schätzle and Rivière. In order to understand the associated flow, min-max procedures or more generally Palais-Smale sequences, the next step is the study of compactness properties of Willmore surfaces. In this talk, I will present new results concerning the bubble tree convergence. In particular, we will discuss the full classification of bubbles.

Presenter: MARTINO, Dorian (Université Paris Cité)

Contribution ID: **31** Type: **not specified**

Shape Optimisation for nonlocal anisotropic energies

Wednesday, June 26, 2024 2:00 PM (1 hour)

We consider shape optimisation problems for sets of prescribed mass, where the driving energy functional is nonlocal and anisotropic. More precisely, we deal with the case of attractive/repulsive interactions in two and three dimensions, where the attraction is quadratic and the repulsion is given by an anisotropic variant of the Coulomb potential.

Under the sole assumption of strict positivity of the Fourier transform of the interaction potential, we show the existence of a threshold value for the mass above which the minimiser is an ellipsoid, and below which the minimiser does not exist. If, instead, the Fourier transform of the interaction potential is only nonnegative, we show the emergence of a dichotomy: either there exists a threshold value for the mass as in the case above, or the minimiser is an ellipsoid for any positive value of the mass.

This is joint work with Riccardo Cristoferi and Maria Giovanna Mora.

Presenter: SCARDIA, Lucia (Heriot-Watt University)

Contribution ID: **34** Type: **not specified**

Poster session

Monday, June 24, 2024 4:00 PM (2 hours)